Effects of Site-Specific Management on the Application of Agricultural Inputs

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Abstract

Site-specific management of inputs in agricultural production is receiving increasing attention because of new technologies and concerns about excessive input use. This paper provides a microeconomic analysis of its implications. It shows that profit decreases with an increase in the variability of input requirements, but that the input and production effects can be quite complicated. The effects of moving from uncertainty about input requirements to variable requirements are also identified. An empirical study of nitrogen fertilization suggests that site-specific management may reduce input use substantially, but the production and profitability impacts may not be large.

Key words: negative externality, site-specific management, uncertainty, variability.

Effects of Site-Specific Management on the Application of Agricultural Inputs

Nitrogen and water share many common attributes. Both are essential for crop growth, and both can be made available to the crop from either soil endowment or applied sources. Farmers have control over applied water and nitrogen levels, but soil endowments are variable and may be unknown. The use of both nutrients is of considerable interest to society because nitrogen losses can contaminate water supplies and irrigation water often has a higher marginal value off the farm than on it. A major concern among agronomists, environmentalists, and the water industry, is the "excess" application of nitrogen by producers (Nielsen and Lee; Office of Technology Assessment). Where the applied nutrient is distributed at a single rate across a field, it has been shown that this "excess" could be explained by the interaction between the crop production technology and the distribution of soil-stored nutrient (Babcock; Feinerman, Letey, and Vaux).

New technologies now make it technically feasible to map soil nutrient status in small areas of a field, and to use this information together with the guidance of global positioning satellites to apply the appropriate input level at a given position (Brunoehler). This technology permits the producer to move from a uniform, or single, application rate technology (SAR) to a site-specific, or variable, application rate technology (VAR). Similar advances have occurred in animal production. The availability of VAR gives rise to the question of its implications for profit, output, and input decisions. Profit implications are of interest to producers and to the providers of the agricultural machinery, computer hardware and software, and geographic information systems (GIS) required to operationalize VAR. The output effect is of interest to the food industry, while the effect on input use is of concern to nutrient suppliers, non-agricultural water users and water quality/non-point pollution controllers.

Variability vs. Uncertainty

There is a clear differentiation between the concepts of variability and uncertainty. Variability is where the soil-stored nutrient status is known, but not constant, across a field. Uncertainty is where the status is not known. There are two distinct ways in which the soil-stored nutrient can be variable. Spatial variability is where, at a particular point in time, the soil status is not uniform across the fields' two dimensions. Temporal variability is where the soil status changes over time, but at a particular point in time it is constant across the field.

Temporal variability is isomorphic to the spatial variability case when the decision-maker is risk-neutral. Each year the producer identifies the soil nutrient status and distributes nutrient accordingly. If one ignores time discounts, an integration over profit, or production, or input use with respect to the *a priori* soil nutrient probability distribution makes the results under temporal variability identical to the spatial variability case where the integration is with respect to the mass distribution of soil nutrient across all field locations. Therefore, results pertaining to the spatial variability case carry over to the temporal case with one qualification and one interpretative exception. As qualification, assume that the distribution of soil stored nutrient is independently and identically distributed from year to year. This is unlikely when nutrient carryover occurs, and in rotations where different crops have different nutrient demands and leave different soil residue profiles. In interpretation, instead of actual impacts of distributional changes in soil nutrient status, we must talk of *a priori* expected impacts. This paper shall focus on spatial rather than temporal impacts. Theoretical work on the economics of variability is limited, but studies have been conducted by Katz, by Chavas and Larson, and by Chavas, Kristjanson, and Matlon, while Niven as well as Fiez, Miller, and Pan, among others, have done empirical work.

As with variability, there are two distinct ways in which soil nutrient status can be uncertain. Spatial uncertainty is where, at a given time point, the status at any plot location is unknown. The producer will apply a uniform rate of nutrient across the field. Here, if the cumulative density function (cdf) of soil-stored nutrient is known with certainty, then producer risk preferences do not matter because integration over the spatial dimensions maps, one for one, each application rate onto a deterministic per plot production and profit. While production and profit at a point may be stochastic, the law of large numbers ensures that whole field production and profit are deterministic functions of the uniform nutrient application rate. For water, uniform application under spatial uncertainty has been studied by Feinerman, Letey, and Vaux, who found that differences in the shape of the production function can influence optimum water application in non-intuitive ways.

Temporal uncertainty is where at any point in time the fields' nutrient status is invariant to location, but total soil nutrient availability is unknown. There is a large literature devoted to the study of anologues to the temporal case. Here, because of temporal non-substitutability of utility, producer risk preferences matter. Some of the most recent and general work in this area has been by Ormiston and Schlee (1992; 1993). Work of relevance to this study has been conducted by Chiao and Gillingham who studied the impact of non-uniform distribution methods on optimum application rates, Babcock and Blackmer who looked at the value of resolving soil nitrogen uncertainty, and Babcock, Carriquery, and Stern who use a Bayesian approach to estimate the value of incomplete resolution of soil nitrogen uncertainty.

When a producer adopts a VAR technology, the information environment he faces changes from spatial uncertainty to spatial variability. Correspondingly, the problem faced changes from choosing a SAR to choosing a function that maps known soil nutrient status onto application rate. This change from uncertainty to variability is the focus of this study. To understand the economic effects of the change we must understand economic decision-making under spatial uncertainty and under spatial variability. As the literature review above suggests, the economic consequences of uncertainty have received more attention than the consequences of variability. In the next section of this paper we develop the theory of production under variability when soil-stored and applied nutrients are additive. The following section considers the more general case. Then the variability results are connected with the production under uncertainty results. And finally we present an empirical study of variable rate nitrogen fertilizer application. In this way insights will be developed on the effect on input use, production, and profit of moving from a SAR to a VAR technology.

Variability and an Additive Technology

Throughout this analysis x_S is the random soil endowment of the nutrient at a point, x_A is the level of nutrient applied by the decision-maker at a point, w is the nonstochastic, per unit nutrient price, and P is the nonstochastic output price. Land is assumed to be uniform in agronomic properties, except for the availability of the soil nutrient. Therefore, at all points on the land surface the same production function applies. In this section we consider only the increasing, concave, point-specific production function[1]

(1) $$Q = F(x_S + x_A).$$

Later, we consider nonhomogeneous nutrients with a more general production function. We first consider a change in the distribution function of x_S. The initial cdf of x_S is denoted by $G_0(x_S)$, and the new cdf is denoted by $G_1(x_S)$. These distributions are supported in the interval [a, b], where a must be non-negative and b is assumed to be finite, but can be infinite. Where differentials are taken, they may be represented by any of the notations , $\frac{\partial F}{\partial x}$, or F_x where appropriate. If a variable, z, is the i th argument in the function, then F_i may also be used to denote the differentiation.

Returns over fertilizer costs for a unit area of land are

(2) $$\pi = P F(x_S + x_A) - w x_A,$$

with profit maximizing first-order condition

(3) $$P F_{x_A} - w = 0.$$

By concavity, the second order condition is assured. Denote the optimum level of applied input by x^*.

As previously discussed, temporal and spatial variability are isomorphic when decision-makers are risk-neutral. We will apply the model to spatial variability but inferences can be drawn about the temporal case *mutatis mutandis*. The producer is assumed to know the availability of soil nutrient at each point on the land surface. This knowledge may be completely represented by the nutrient availability mapping $x_S(y, z)$, where y and z are spatial coordinates. At each spatial point, equation (3) is solved to give $x_A(y, z)$. A first degree stochastically dominating shift (FSD) in x_S from the initial to the new distribution, cannot increase the total application of x_A over all the field. In fact, if we assume that all of the initial and new mass distributions of x_S are contained in the interior of $[0, x^*]$, then by differentiating (3) partially with respect to x_A and x_S, treating x_A as an implicit function of x_S, we get

(4) $$\frac{\partial x_A}{\partial x_S} = -1.$$

Thus, there is a one for one reduction in the application rate as x_S increases. In this case a FSD in x_S does not change total nutrient available at a point, so total production does not change. Production will be constant across all points of the field. Profit will change due to savings in the cost of nutrients. Profit increases by

(5) $$\Delta\pi = w\int_a^b [G_0(x_S) - G_1(x_S)]\, dx_S,$$

$$\int_a^b [G_0(x_S) - G_1(x_S)]\, dx_S$$

where Δ is the change operator, and can be shown, through an integration by parts, to be the change in mean value of x_S at a point. If some, but not all, of the new x_S is contained in $[0, x^*]$, then some of the extra soil nutrient will not be compensated for by reductions in application rates, output will increase, and profit will increase by a magnitude less than (5). Production will

(6) $$\int_a^t [G_0(x_S) - G_1(x_S)]\, dx_S \leq 0 \quad \text{for all } t \in [a, b],$$

not be spatially uniform, being larger at points where the new x_S is contained in the semi-open interval $(x^*, b]$ than at points where the new x_S is contained in $[0, x^*]$.

Next, consider a general mean preserving spread (mps) in the Rothschild and Stiglitz sense. The mps requires that the following two conditions hold

If the mps occurs completely in the interval $[0, x^*]$, then the aggregate applied nutrient level, output level, and profit level remain unaltered because of the complete substitution effect shown

(7) $$\int_a^b [G_0(x_S) - G_1(x_S)]\, dx_S = 0.$$

by (4) above. If some of the mps occurs in the semi-open interval $(x^*, b]$, then the effect on aggregate applied nutrient level, output level, and profit level is ambiguous. The following can be said, however. If the mps induces a decrease in mean x_A, then total nutrient availability falls, and the concavity of the production function will ensure that total output falls under a mps in x_S. However, if x_A increases so that total nutrient availability increases then the two effects will counteract to render ambiguity.

Because a second degree stochastically dominating shift (SSD) in x_S can be decomposed into a FSD and a mean preserving contraction (mpc) (Makowski; Hadar and Seo), we can infer the effects of an SSD from the above analysis. If all of an SSD shift occurs in $[0, x^*]$, then applied input use falls, profit rises according to (5) above, and output does not change. If some of the SSD occurs in $(x^*, b]$, then the effects are ambiguous. If total nutrient availability rises, then the mpc effect and production function concavity will ensure that total output rises. In this case profit will also rise.

It should be noted that, except for the costs of collecting spatial information and of varying application rates, if the original and new soil nutrient distributions are strictly positive only in the interval $[0, x^*]$, then the only statistics relevant in determining profit and aggregate applied input requirements are the original and shifted total soil nutrient levels. Further, in this case the production level is independent of the soil nutrient distribution and shifts in distribution. Because of complete substitutability between x_S and x_A, this result holds true regardless of the number of factors in the production function. Elementary comparative statics on the first-order conditions will verify this.[2]

Variability and a Nonadditive Technology

To this point we have considered only the situation where the applied nutrient and the soil-stored nutrient are additive. This additivity assumption is not appropriate if the two nutrient sources are not chemically identical, or if one source is more readily available to the plant.[3] We will next analyze the application decision when the two sources are not additive. This we do by considering the following general, concave production function for a point on the landscape,

$$(8) \qquad Q = F(x_S, x_A),$$

which gives the profit-maximizing first-order condition at a point,

$$(9) \qquad PF_2 - w = 0.$$

It is assumed that x_S is never so large as to make it uneconomic to apply nutrient at any point. When x_S is known to the producer, a soil nutrient contingent choice of x_A, $x_A(x_S)$, can be made to satisfy (9) for each value of x_S. We now propose

PROPOSITION 1. A mps in the variability of x_S decreases total field profit.

Proof. Solving (9), we get the input choice function $x_A(x_S)$. Using (9), the change in x_A with respect to x_S that holds (9) constant is

$$\text{(10)} \quad \frac{\partial x_A}{\partial x_S} = -\frac{F_{12}}{F_{22}},$$

Substituting $x_A(x_S)$ into the profit function results in

$$\text{(11)} \qquad \pi = P\,F[x_S, x_A(x_S)] - w\,x_A(x_S).$$

Differentiating, and using the envelope theorem, gives $d\pi / dx_S = P F_1$. Differentiating again gives

$$\frac{d^2\pi}{dx_S^2} = P\left(F_{12}\frac{\partial x_A}{\partial x_S} + F_{11}\right) = P\left(\frac{F_{11}F_{22} - F_{12}^2}{F_{22}}\right).$$

Concavity of the production function ensures that the second derivative of π with respect to x_S is negative. Therefore, by the Rothschild and Stiglitz generalization of Jensen's inequality, a mps in x_S will decrease profit.

An implication is that, given a fixed total amount of soil-stored nutrient, the greater the locational variability the lower the value of land. We can also state

PROPOSITION 2. A mean preserving spread in the variability of x_S increases (decreases) total field production if $F_1 - F_2F_{12}/F_{22}$ is increasing (decreasing) in x_S.

Proof. By Rothschild and Stiglitz, a mps increases (decreases) the expected value of a convex (concave) function. We will identify the conditions under which production is convex (concave) in x_S. Differentiating the production function with respect to x_S we get $F_1 - F_2F_{12}/F_{22}$. The result follows from signing a further differentiation, and using the Rothschild and Stiglitz result.

The resulting second derivative is

$$\frac{d^2F}{dx_S^2} = \left(\frac{F_{11}F_{22} - F_{12}^2}{F_{22}}\right) + \frac{F_2}{F_{22}^2}\left(2F_{12}F_{122} - F_{22}F_{112} - \frac{F_{12}^2F_{222}}{F_{22}}\right). \tag{12}$$

Due to the complexity of the expression, no global comparative static results should generally be expected. A simpler result is

PROPOSITION 3. An increase in the variability of x_S will increase (decrease) the use of total field x_A if F_{12}/F_{22} is decreasing (increasing) in x_S.

Proof. We will identify the conditions under which x_A is convex (concave) in x_S. Use the first-order condition to get $x_A(x_S)$. From the first-order condition, the gradient is $dx_A/dx_S = -F_{12}/F_{21}$. The result follows from differentiating this expression, and applying the Rothschild and Stiglitz result on concave functions.

The resulting second derivative is

$$\frac{d^2x_A}{dx_S^2} = \frac{1}{F_{22}^2}\left(2F_{12}F_{122} - F_{22}F_{112} - \frac{F_{12}^2F_{222}}{F_{22}}\right), \tag{13}$$

as has been reported previously by Katz. This expression is also rather difficult to sign. A relationship between Propositions 2 and 3 is provided by the following corollary,

COROLLARY 1. An increase in the variability of x_S that leads to a decrease in the use of total field x_A always leads to a decrease in total field output.

Proof. From Proposition 2, an increase in the variability of x_S decreases output if

$$\frac{d\left(F_1 + F_2\dfrac{\partial x_A}{\partial x_S}\right)}{dx_S} = \left(\frac{F_{11}F_{22} - F_{12}^2}{F_{22}}\right) - F_2\frac{d\left(F_{12}/F_{22}\right)}{dx_S} < 0.$$

The first right hand expression is negative due to the concavity of the production function. The positivity of the full derivative in the second right hand expression is a necessary and sufficient condition for x_A to decrease with a mps in x_S.

The result is due to the direct effect of a mps on yield and the reduction in total field x_A both acting in the same direction. Having developed the economic implications of variability, in the next section we will connect these results to the literature on production under uncertainty. This connection will allow us to study the transition from a SAR technology to a VAR technology.

Moving from Uncertainty to Variability

From Propositions 2 and 3, it is clear that unambiguously signing the effects of spatial variability on output and application rates may not always be possible for the general production function. In this section we will show that, perhaps somewhat counterintuitively, when one moves from uncertainty to variability with the same distribution of the variable by acquiring information about the distribution of x_S, the effect on production and input use may be easier to sign. Let x_A^* be the optimal SAR, let $x_A^*(\bar{x}_S)$ be the optimal SAR when x_S has mean $\bar{x}_S$ and zero variance, and let σ_S^2 be the variance of the soil nitrogen distribution. The results obtained below are based on second-order approximations which have low errors for tight distributions (low variances) of x_S.

PROPOSITION 4. For tight distributions of x_S, the change in expected profits, $\Delta E[\pi]$, from acquiring *ex-ante* site-specific information is approximately

$$-\sigma_S^2\frac{P}{2}\frac{F_{12}^2}{F_{22}} - \frac{P}{2}[x_A^* - x_A^*(\bar{x}_S)]^2F_{22}.$$

Proof. We take the difference of second-order approximations of profit when x_S is variable but known and expected profits when x_S is uncertain. First, take a second-order Taylor series expansion of profit around the mean, $\bar{x}_S$:

$$(14) \qquad _S)] \approx \pi[\bar{x}_S, x_A(\bar{x}_S)] + (x_S - \bar{x}_S)\pi'[\bar{x}_S, x_A(\bar{x}_S)] + \frac{1}{2}(x_S - \bar{x}_S)^2\pi''[\bar{x}$$

where the prime indicates a complete first derivative with respect to x_S (including the effect through x_A), and the double prime indicates a complete second derivative with respect to x_S. Noting that $\pi''[\bar{x}_S, x_A(\bar{x}_S)] = P\,(F_{11}F_{22} - F_{12}^2)/F_{22}$, and taking expectations of both sides of (14) results in

$$(15) \qquad E\{\pi[x_S, x_A(x_S)]\} \approx \pi[\bar{x}_S, x_A(\bar{x}_S)] + \frac{1}{2} P\,\sigma_S^2 \left[\frac{F_{11}F_{22} - F_{12}^2}{F_{22}}\right].$$

Now we need an expression for expected profits under uncertainty. Profit at any location is a function of two variables: x_S and x_A^*. Take a Taylor-series expansion of profit at any spatial location around the point $\bar{x}_S$ and the input choice $x_A^*(\bar{x}_S)$, where $x_A^*(\bar{x}_S)$ denotes the optimal input level when there is no spatial variability:

$$(16) \qquad) \approx \pi[\bar{x}_S, x_A^*(\bar{x}_S)] + (x_S - \bar{x}_S)\,\pi_1 + [x_A^* - x_A^*(\bar{x}_S)]\,\pi_2 + \frac{1}{2}(x_S -$$
$$+ (x_S - \bar{x}_S)[x_A^* - x_A^*(\bar{x}_S)]\,\pi_{12} + \frac{1}{2}[x_A^* - x_A^*(\bar{x}_S)]^2 \pi_{22}.$$

Taking expectations and substituting the appropriate expressions for the derivatives gives

$$(17) \qquad E[\pi(x_S, x_A^*)] \approx \pi[\bar{x}_S, x_A^*(\bar{x}_S)] + \frac{P}{2}\{\sigma_S^2 F_{11} + [x_A^* - x_A^*(\bar{x}_S)]^2 F_{22}\}.$$

Subtracting (17) from (15), and noting that the choice $x_A(\bar{x}_S)$ under variability equals the choice $x_A^*(\bar{x}_S)$ under uncertainty, results in

$$(18) \qquad = E\{\pi[x_S, x_A(x_S)]\} - E[\pi(x_S, x_A^*)] \approx -\sigma_S^2 \frac{P}{2} \frac{F_{12}^2}{F_{22}} - \frac{P}{2}[x_A^* - x_A^*($$

which completes the proof.

COROLLARY 2. An increase in the variability of x_S increases $\Delta E[\pi]$.

Proof. Differentiating $\Delta E[\pi]$ with respect to σ_S^2 results in

$$(19) \qquad \frac{d\Delta E[\pi]}{d\sigma_S^2} \approx -\frac{P}{2}\frac{F_{12}^2}{F_{22}} - P[x_A^* - x_A^*(\bar{x}_S)]F_{22}\frac{\partial x_A^*}{\partial \sigma_S^2}.$$

When $dx_A^*/d\sigma_S^2 > 0$; then $x_A^* > x_A(\bar{x}_S)$, and $d\Delta E[\pi]/d\sigma_S^2 > 0$. When $\partial x_A^*/\partial\sigma_S^2 < 0$; then $x_A^* < x_A(\bar{x}_S)$, and $d\Delta E[\pi]/d\sigma_S^2 > 0$.

Thus, as one would expect, the value of moving to known variability increases as spatial variability increases. This last proof raises the question as to how increases in spatial variability affect the change in mean input use as one moves from uncertainty to known variability.

PROPOSITION 5. For tight distributions of x_S, an increase in spatial variability changes the difference in expected input use, $\Delta E[x_A]$, by approximately

$$\frac{1}{2F_{22}^2}\left[2F_{12}F_{122} - F_{22}F_{112} - \frac{F_{22}^2 F_{112}}{F_{22} + [x_A^* - x_A^*(\bar{x}_S)]F_{222}} - \frac{F_{12}^2 F_{222}}{F_{22}}\right].$$

Proof. Again we will take the difference in second-order approximations. A second-order approximation around $\bar{x}_S$ obtains

$$(20) \qquad x_A(x_S) \approx x_A(\bar{x}_S) + (x_S - \bar{x}_S)\frac{dx_A}{dx_S} + \frac{(x_S - \bar{x}_S)^2}{2}\frac{d^2x_A}{dx_S^2}.$$

Taking expectations of both sides, differentiating with respect to σ_S^2, and substituting in for the expression d^2x_A/dx_S^2 results in

$$(21) \qquad \frac{dE[x_A(x_S)]}{d\sigma_S^2} \approx \frac{1}{2F_{22}^2}\left[2F_{12}F_{122} - F_{22}F_{112} - \frac{F_{12}^2 F_{222}}{F_{22}}\right].$$

Now we need to approximate the difference between the marginal product of applied inputs and the price ratio at a given location in the field,

$$(22) \qquad {}_{S}, x_A^*) \; - \; \frac{w}{P} \;\approx\; F_2[\bar{x}_S, x_A^*(\bar{x}_S)] \; - \; \frac{w}{P} \; + \; (x_S - \bar{x}_S) F_{12} \; + \; [x_A^* - x_A^*(\bar{x}_S)$$

$$\frac{1}{2}(x_S - \bar{x}_S)^2 F_{112} \; + \; (x_S - \bar{x}_S)[x_A^* - x_A^*(\bar{x}_S)] F_{122} \; + \; \frac{1}{2}[x_A^* - x_A^*(\bar{x}_S)]^2 F$$

Taking the expectation of both sides and using the first-order condition results in

$$(23) \qquad E[F_2(x_S, x_A^*)] \; - \; \frac{w}{P} \; = \; 0$$

$${}_{S}, x_A^*(\bar{x}_S)] \; - \; \frac{w}{P} \; + \; [x_A^* - x_A^*(\bar{x}_S)] F_{22} \; + \; \frac{1}{2}\sigma_S^2 F_{112} \; + \; \frac{1}{2}[x_A^* - x_A^*(\bar{x}_S$$

Differentiating both sides of (23) with respect to σ_S^2 and solving for $dx_A^* / d\sigma_S^2$ results in

$$(24) \qquad \frac{dx_A^*}{d\sigma_S^2} \;\approx\; -\frac{1}{2} \frac{F_{112}}{F_{22} + [x_A^* - x_A^*(\bar{x}_S)] F_{222}}.$$

Take the difference between (21) and (24) to obtain

$$(25) \qquad \frac{A]}{\;} \;\approx\; \frac{1}{2F_{22}^2}\left[2F_{12}F_{122} - F_{22}F_{112} + \frac{F_{22}^2 F_{112}}{F_{22} + [x_A^* - x_A^*(\bar{x}_S)] F_{222}} - \frac{F_1}{\;}\right.$$

which completes the proof.

Note that when $x_A^* = x_A^*(\bar{x}_S)$, that is, our departure point is where there is no uncertainty, then (25) becomes

$$(26) \qquad \frac{d\Delta E[x_A]}{d\sigma_S^2} \;\approx\; \frac{1}{2F_{22}^2}\left[2F_{12}F_{122} \; - \; \frac{F_{12}^2 F_{222}}{F_{22}}\right].$$

The conventional wisdom is that (26) is often negative, particularly when w is small relative to the average value of x_A. That is, adoption of site-specific farming practices should decrease input use. F_{112} is the Rothschild and Stiglitz concavity condition applied to the first-order condition. This is removed from the variability effect to purge it of the impact of uncertainty, and leave only technical substitution impacts associated with certain variability. If $F_{12} < 0$, as is likely in our case, then the local effect of moving from uncertainty to variability is to increase average input use if $F_{122} < 0$ and $F_{222} > 0$, while the effect is to decrease average input use if $F_{122} > 0$ and $F_{222} < 0$. The effect is indeterminate if these two third derivatives have the same sign.

Having approximated the input effects, it should be possible to identify the production effects of site-specific information.

PROPOSITION 6. For tight distributions of x_S, an increase in spatial variability changes the difference in expected yield, $\Delta E[F(x_S, x_A^*)]$, from acquisition of information by approximately

$$-\frac{F_{12}^2}{2F_{22}} + \frac{F_2}{2F_{22}^2}\left[2F_{12}F_{122} - F_{22}F_{112} - \frac{F_{12}^2 F_{222}}{F_{22}}\right] - \{F_2 + [x_A^* - x_A^*(\bar{x}_S)]F_{22}\}\frac{\partial x_A^*}{\partial \sigma_S^2}.$$

Proof. Using equation (12), the expectation of a second-order expansion of yield under variability is

(27) $$; x_A(x_S)] \approx F[\bar{x}_S, x_A(\bar{x}_S)] + \frac{\sigma_S^2}{2}\left[F_{11} + 2F_{12}\frac{dx_A}{dx_S} + F_{22}\left(\frac{dx_A}{dx_S}\right)^2 + F_2\right.$$

And the expectation of a second-order expansion of yield under uncertainty is

(28) $$; x_A^*)] \approx F[\bar{x}_S, x_A^*(\bar{x}_S)] + [x_A^* - x_A^*(\bar{x}_S)]F_2 + \frac{\sigma_S^2}{2}F_{11} + \frac{[x_A^* - x_A^*(\bar{x}_S)}{2}$$

Taking the difference between (27) and (28), and differentiating with respect to σ_S^2 gives

$$\frac{d\Delta E[F[x_S, x_A^*]]}{d\sigma_S^2} \approx -\frac{F_{12}^2}{2F_{22}} + \frac{F_2}{2F_{22}^2}\left(2F_{12}F_{122} - F_{22}F_{112} - \frac{F_{12}^2 F_{222}}{F_{22}}\right) - \{F_2 + [x_A^* - x_A^*(\bar{x}_S)]F_{22}\}\frac{dx_A^*}{d\sigma_S^2}, \tag{29}$$

which completes the proof.

When $x_A^* = x_A^*(\bar{x}_S)$, (24) and (29) give

$$\frac{d\Delta E[F(x_S, x_A^*)]}{d\sigma_S^2} \approx -\frac{F_{12}^2}{2F_{22}} + \frac{F_2}{2F_{22}^2}\left(2F_{12}F_{122} - \frac{F_{12}^2 F_{222}}{F_{22}}\right) \tag{30}$$

Under this condition, we can use propositions 4, 5, and 6 to write

$$P\frac{d\Delta E[F(x_S, x_A^*)]}{d\sigma_S^2} = \frac{d\Delta E[\pi]}{d\sigma_S^2} + w\frac{d\Delta E[x_A]}{d\sigma_S^2}. \tag{31}$$

That is, the change in expected profit due to the shift from uncertainty to variability added to the change in expected cost due to the shift equals the change in expected revenue due to the shift. A logical inference from (31) is

COROLLARY 3. A shift from uncertainty to variability in x_S that leads to an increase in mean use of x_A always leads to an increase in mean output.

Proof. The proof follows from equation (31) and the fact that expected profit must increase with the shift.

This result arises because site-specific information improves the efficiency of nitrogen use. If nitrogen is used more efficiently and if more of it is used, then mean production must increase. Having developed the theory, in the next section we will apply it to a nitrogen use problem in the Palouse area of eastern Washington.

Data and Dynamic Considerations

The Palouse region of eastern Washington state has a distinctive topography, consisting of fertile rolling hills. Hill slope averages about 13%, and Mulla et al. have concluded that soil fertility varies considerably within a field. This suggests a need for good information on fertility. The principal rotation in the eastern Palouse consists of winter wheat followed by spring barley and then a legume. The legume is either dried pea or spring lentil, and wheat is the most profitable crop. As wheat follows the legume and legumes endow the soil with high but variable amounts of nitrogen, the region may be particularly well suited for variable rate technology.

Fiez, Miller, and Pan conducted a series of nitrogen experiments on white winter wheat at two silt loam soil type locations in eastern Whitman county on the border with Idaho. The variety chosen was Madsen, commmon in the area. At both locations (Pullman and Farmington) the experiments were carried out over the successive years 1990 and 1991. The previous crop was lentils at Farmington and peas at Pullman. This previous cropping pattern is true in both years because the fields at each location were not the same in the two years. In 1990 five different applications rates (0, 50, 75, 100, and 125 lb/acre) of aqua ammonia were applied at planting. In 1991 a sixth application rate (25 lb/acre) was added. Also considered were each of the four basic landscape positions (south backslope, shoulder, north backslope, and footslope). The rainfall was average for the region in both years (20.4 inches from September to September in 1989-1990, and 20.8 inches in the following year), and irrigation was not used.

For each block of replications (five application rates in 1989-1990, and six application rates in the following year), preplant inorganic residual soil nitrogen was measured in a 60 inch soil profile from the surface. Nitrogen mineralization was also imputed from readings of soil organic matter. These two sources of nitrogen were summed to give a measure of the preplant soil nitrogen status. A total of 340 plot yields were recorded, twelve less than the number of planted plots. These twelve 1991 Farmington shoulder slope plots were eliminated because winterkill severely reduced the plant stand.

Farm level white wheat price was assumed to be \$3.50 bu/acre, and the price of applied nitrogen was assumed to be \$0.31/lb (Painter, Hinman, and Burns). However, not all nitrogen applied in a year is actually taken up in that year. Depending on weather conditions and the nature of the agriculture, a non-negligible fraction may be carried over as soil nitrogen to the following crop year. In a study of sorghum production in Australia's Northern territory, Kennedy et al. set the range of carryover at between 20% and 40%. For Iowa, Fuller estimated a carryover for continous corn of about 32%. This carryover should be accommodated in any static optimization model. We shall adopt the approach of Kennedy, outlined below.

Let carryover be a constant proportion of the sum of applied and soil nitrogen,

$$x_S^{t+1} = (x_S^t + x_A^t)\alpha, \tag{32}$$

where α is the fractional carryover and the superscript denotes the annual time period. Then the producer faces the dynamic programming problem

$$\max_{x_A^t,\, x_A^{t+1},\, \ldots} \sum_{i=0}^{\infty} \rho^i [P\,F(x_S^{t+i} + x_A^{t+i}) - w\,x_A^{t+i}] \tag{33}$$

subject to the carryover equation, nonnegative nitrogen applications, and an initial soil nitrogen endowment. Here, ρ is the annual discount factor. Assuming a concave production function, Kennedy developed an optimal steady state decision rule. That rule is
to apply at time t the nitrogen level which solves

$$\rho P \frac{\partial F(x_S^t + x_A^t)}{\partial x_A} = w - \rho\alpha w \tag{34}$$

Given constant prices and a concave production function, carryover equation (32) ensures that soil and applied nitrogen levels will converge to time invariant steady state levels. In equation

(34), note that discounted marginal value of product is set to a fraction of input price rather than the full input price. This effective input price, $(1 - \rho\alpha)w$, is decreasing in the discount rate and in the carryover fraction.

Estimation and Results

To capture the production and input effects developed in the theory section above, it is necessary to estimate a production function that is flexible to the third order. We chose the general cubic production function,

$$\text{(35)} \qquad) = b_0 + b_S x_S + b_A x_A + b_{SS} x_S^2 + b_{SA} x_S x_A + b_{AA} x_A^2 + b_{SSS} x_S^3 + b_{S} \\ + b_{SAA} x_S x_A^2 + b_{AAA} x_A^3 + c_1 A + c_2 B + c_3 F + c_4 R + c_5 R ,$$

where A is a dummy variable equal to one when the observation is at the Farmington site. B, F, R are dummy variables equal to one when the observation is a south backslope, a footslope, and a north backslope, respectively. R is a dummy variable set equal to one in the 1990-1991 crop year. Yield is measured in bu/acre, and nitrogen in lb/acre. Regression results are presented in table 1. The coefficients of the powers of x_S are all strongly significant. So also are the location and year coefficients, though the site coefficient is not. The terms with applied nitrogen in them are not individually significant. An F test was run to test for the collective significance of these terms. The result is presented in table 2, and it was found that they were collectively significant at the 1% level. The b_{SS} and b_{AA} coefficients are negative, and so are consistent with concavity. The b_{SA} coefficient is negative, which is consistent with the two sources of nitrogen being substitutes. The perfect substitutability specification

$$\text{(36)} \qquad F(x_S, x_A) = d_0 + d_1(x_S + x_A) + d_2(x_S + x_A)^2 + d_3(x_S + x_A)^3 \\ + c_1 A + c_2 B + c_3 F + c_4 R + c_5 R$$

was tested for (see table 2), and rejected at the 1% level.

The coefficient b_{SSA} is positive, suggesting that a more uncertain distribution of soil nitrogen would increase demand for applied nitrogen. This is consistent with Babcock's (1992) suggestion that nitrogen is applied as insurance against the possibility of low soil nitrogen. The model was also tested for the hypothesis that the coefficients of own power terms are equal; i.e., imposing the restrictions that $b_S = b_A$, $b_{SS} = b_{AA}$, and $b_{SSS} = b_{AAA}$. This is a less stringent version of the perfect substitution hypothesis. The test result is presented in table 2, and the restrictions were also rejected at the 1% level.

Because the comparative static conditions arrived at in the theory sections are not simple, it was not possible to construct confidence intervals around point estimations to test statistically for production and input use impacts of increases in variability and of moving from uncertainty to variability. Point estimates are provided in table 3. These estimates are evaluated at the mean level of soil nitrogen over all plots, 198.6 lb/acre, and the expected profit maximizing level of applied nitrogen when the white wheat price is \$3.50/bu and the effective nitrogen price is \$0.22/lb, This profit maximizing level of nitrogen is 63 lb/acre. Applying equation (34), if the discount rate, ρ, equals 0.93, and the actual nitrogen price is \$0.31/lb, then an effective nitrogen price of \$0.22/lb is consistent with a carryover fraction of 0.312, a reasonable proportion.[4]

From the second order conditions, we conclude that the production function is concave. To preserve space, the three conditions are not presented in table 3. When x_A is endogenized due to a variable but certain distribution of x_S, then the production function is concave with second derivative equal to -0.00822 at the point of evaluation. Therefore, a mps in the variability of x_S is expected to decrease mean production, in accordance with Proposition 2. With a variable but certain distribution of x_S, then $x_A(x_S)$ is concave with second derivative equal to -0.02719 at the point of evaluation, indicating that a mps in the variability of x_S should decrease mean input use, in accordance with Proposition 3.

As the last three results in the table are the evaluations of derivatives with respect to σ_S^2, to fully interpret their implications it is necessary to have knowledge about the magnitudes of σ_S^2. The means and variances of x_S for the two sites and four topographical locations are presented in table 4. Standard deviations are about 25 lb/acre, except for two sites where some very high outliers caused high standard deviations. For the purpose of interpreting table 3, we will assume a standard deviation of 35 lb/acre. In this case mean profit increases by a meagre $0.00015(35)^2 =$ \$0.184/acre, hardly enough to sustain a management intensive innovation. The effect on mean input use is more substantial, however, at $- 0.01891(35)^2 = - 23$ lb/acre. But this reduction in input use does not seem to affect mean output much because it appears to fall by only about $-0.00121(35)^2 = - 1.48$ bu/acre. It would appear that, while the input use impacts may be significant, the profit implications might not be. As the above results are just approximations and the model is in place to provide more exact results, next we will provide simulations to confirm our inferences.

First, we will evaluate optimal decisions for the expected profit maximizer without access to site-specific soil nitrogen readings. We will then optimize when site-specific information is available, and compare the two sets of results. We assume that our expected profit maximizer knows the spatial distribution of soil nitrogen for each of the four slopes and two fields considered in the experiments. We estimate optimal nitrogen application rates for each of the slopes and fields. From Painter, Hinman, and Burns, variable costs other than nitrogen were assumed to be \$94.23/acre, and fixed costs were assumed to be \$121.26/acre.

Table 5 presents the results for the expected profit maximizer without access to site-specific information. At each site the first-order condition was solved for the value of x_A that resulted in the highest average profit at that site. At this rate, average yield and profit were calculated. As shown in table 4, Farmington shoulder and Pullman north backslope locations have sample standard deviations of soil nitrogen that are much greater than the other locations. These large values presented us with difficulties because the regularity conditions of the production function were violated at some soil nitrogen levels. For this reason, the standard deviations at these plots were reduced by 25 lb/acre when solving for both SAR and VAR nitrogen application rates. They were still almost twice as high as standard deviations for other plots.

The mean application rate under SAR technology, giving each slope X location combination equal weight, is 58 lb/acre. Typical rates in this region are 90-100 lb/acre, so our average rate is somewhat low. Expected yield under the SAR technology varies from 99.4 bu/acre on the Pullman shoulder sites down to 82.2 bu/acre on the Pullman north backslope sites, where soil nitrogen was high enough to make it unprofitable to apply any fertilizer (see table 4). Mean profit for the SAR technology ranges from $60/acre to $120/acre.

To calculate optimal application rates under VAR, the first two moments of the soil nitrogen distribution at each of the eight slope X location combinations was used to given the moments of a random normal variate. Then the first-order condition (9) is solved for each of 3,000 random draws from this distribution. Because the production function is cubic, there are two nitrogen application rate solutions for each draw, but only one is relevant. Profit and output were calculated for the optimal nitrogen application for each draw, and then profit, output, and input use were averaged over all draws. The results are presented in parentheses in table 5.

The average nitrogen application rate under VAR is less than under SAR for all locations except for the Pullman north backslope, where the optimal rate is zero. The average reduction on the other seven sites is 17 lb/acre, which represents a 25.5% reduction. Overall, under the VAR technology average nitrogen application is 44.9 lb/acre, a 22.6% reduction. The largest decrease is on the shoulder slopes at Farmington where soil nitrogen variance is high. At this Farmington site, much of the nitrogen is applied under SAR as assurance against being caught short of nitrogen. Under VAR, this assurance is no longer necessary.

Average yield under VAR is 87.1 bu/acre, down 0.7 bu/acre from the SAR scenario. Average profit is $79.91/acre, up only $0.37/acre from the SAR scenario. There would appear to be little economic incentive for producers in this area to adopt this technology for nitrogen application.

Summary and Conclusions

This paper has demonstrated the complexity of the impacts of site-specific information on decision-making. While information has positive value, its effects on production and input use are not clear. In practice, it seems likely that moving from input use under uncertainty to input use under variability will decrease mean input use because uncertainty may be causing privately excessive levels of input use as insurance against the possibility of low yields. Empirical results provide evidence for these conclusions.

However, our results suggest a low value for site-specific information, so there may be little incentive to adopt the technology. As reported by Brunoehler, Lowenberg-DeBoer estimates the sampling and mapping costs of site-specific management at about $7.25/acre. Lowenberg-DeBoer goes on to suggest that it would be difficult to cover this cost through reducing average input levels. In general, the value of site-specific management may vary from site to site, crop to crop, and input to input. For example, the variable application of expensive patented pesticides may be quite profitable. Because we have controlled for year, our results do not suggest that soil testing is irrelevant. As Babcock and Blackmer have demonstrated, there may be considerable demand for the resolution of temporal uncertainty.

The results suggest interesting policy implications for governments. Suppose that uncertainty is causing privately excessive applications of inputs that cause negative externalities, and the government owns global positioning infrastructure. If this infrastructure is not congested, then its services are public goods. If the fees for positioning services make site-specific management unprofitable, then to reduce the magnitudes of the negative externalities, the government might consider refraining from charging fees.

Footnotes

1. If x_S is considered not as a soil nutrient endowment but as an index of technical productivity instead, then variability in x_S can be considered as heterogeneity in technology. For nitrogen, the effect of the introduction of GIS on nitrogen application when there are heterogeneous soil conditions in a field has been studied by Niven (1994). The index of technical productivity interpretation can be applied to all production functions considered in this paper.

2. The production function $Q = F(x_S + x_1, x_2)$, where x_1 and x_2 are chosen inputs, is of little interest when x_S is variable but known. If x_S does not exceed x^*, then x_1 supplements x_S to give x^* always, and the comparative statics matrix has a zero determinant. That is, x_2 is unaffected by the distribution of x_S.

3. This is particularly true when we generalize and consider, for example, substitutability between homegrown fodder and purchased, quality-controlled, concentrate animal feed.

4. Because our formula for computing the carryover fraction, equation (34), is predicated on the assumption of perfect substitutability, an assumption that was rejected, the calculated carryover fractions can only be considered to be rough approximations.

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Table 1. Econometric Estimates of the Cubic Production Function Parameters

Parameter	Estimate	t-statistic
b_0	-164.27	- 6.78
b_S	2.85283	9.83
b_A	0.312845	1.43
b_{SS}	- 0.00979577	- 8.64
b_{SA}	- 0.00161579	- 1.15
b_{AA}	- 0.00105106	- 0.47
b_{SSS}	0.00001058	7.44
b_{SSA}	0.00000207	0.85
b_{SAA}	0.00000485	0.88
b_{AAA}	0.000000235	- 0.02
c_1	- 0.81479	- 0.82
c_2	- 8.32977	- 5.76
c_3	- 9.90051	- 7.20
c_4	- 13.2722	- 8.64
c_5	- 2.44571	- 2.05

Table 2. Tests for Nested Specifications of the Cubic Production Function

Test	F-value	Degrees of Freedom and Significance
Collective significance of terms with x_A in them	5.819	6, 325 1%
Perfect substitutability between x_S and x_A	25.81	6, 325 1%
Coefficients of own powers are set equal	28.01	3, 325 1%
Cubic terms are set equal to zero	14.10	4, 325 1%

Table 3. Point Estimates of Comparative Static Effects

Comparative Static	Relevant Proposition	Estimate	Conclusion
Concavity of $F(x_S, x_A)$ in its arguments	Proposition 1	**	Concave
Curvature of $F[x_S, x_A(x_S)]$ in x_S	Proposition 2	- 0.00822	mps decreases mean production
Curvature of $x_A(x_S)$ in x_S	Proposition 3	- 0.02719	mps decreases mean input use
Approximate value of information about x_S	Proposition 4	0.00015	low increase in profit
Input effect of information on x_S	Proposition 5	- 0.01891	significant decrease in mean input use
Approximate production effect of information on x_S	Proposition 6	- 0.00121	low decrease in production

Table 4. Moments of the Distribution of Soil Nitrogen

Site	Topographical Location	Mean lb/acre	Standard deviation lb/acre
Farmington	South backslope	176.1	33.53
	Footslope	200.8	24.36
	North backslope	186.1	22.66
	Shoulder	216.3	86.25
Pullman	South backslope	168.8	19.46
	Footslope	187.3	25.67
	North backslope	253.7	81.06
	Shoulder	193.6	19.15

Table 5. Profit, Yield, and Nitrogen Application by Site, Location, and Technology (Variable Rate Technology Results are in Parentheses)

Nitrogen @ $0.22/lb				
Farmington	South backslope	59.54 (59.64)	83.00 (82.14)	71.9 (57.8)
	Footslope	85.43 (85.56)	89.23 (88.32)	53.5 (39.6)
	North backslope	61.76 (62.50)	83.14 (82.66)	64.4 (53.4)
	Shoulder	98.76 (99.22)	94.65 (91.70)	79.3 (30.3)
Pullman	South backslope	62.47 (63.48)	83.85 (83.84)	71.8 (67.1)
	Footslope	75.79 (75.83)	87.21 (86.36)	64.6 (50.9)
	North backslope	72.46 (72.90)	82.24 (83.19)	0 (13.1)
	Shoulder	120.03 (120.11)	99.42 (98.70)	58.6 (46.8)