

**Law of One Price in International  
Commodity Markets:  
A Fractional Cointegration Analysis**

Samarendu Mohanty, Darnell B. Smith, E. Wesley F. Peterson,  
and William H. Meyers

*Working Paper 96-WP 155*  
February 1996

# **Law of One Price in International Commodity Markets: A Fractional Cointegration Analysis**

Samarendu Mohanty, Darnell B. Smith, E. Wesley F. Peterson,  
and William H. Meyers

*Working Paper 96 - WP 155*  
February 1996

**Center for Agricultural and Rural Development  
Iowa State University  
Ames, IA 50011-1070**

*Samarendu Mohanty is a FAPRI post-doctoral research associate; Darnell B. Smith is managing director of FAPRI; E. Wesley F. Peterson is an associate professor of agricultural economics, University of Nebraska-Lincoln; and William H. Meyers is associate director of CARD and co-director of FAPRI.*

For more information about the content of this paper, please contact S. Mohanty, Iowa State University, 475 Heady Hall, Ames, Iowa 50011; phone (515) 294-6296; e-mail [smohant@card.iastate.edu](mailto:smohant@card.iastate.edu).

## **ABSTRACT**

This paper examines the Law of One Price (LOP) in the International Commodity markets using a fractional cointegration analysis. Out of nine pairs of price series examined, fractional cointegration supports the existence of the LOP in eight cases as compared to three cases as in standard cointegration process.

## **LAW OF ONE PRICE IN THE INTERNATIONAL COMMODITY MARKETS: A FRACTIONAL COINTEGRATION ANALYSIS**

The Law of One Price (LOP) is an important component of most, if not all, international trade models because this assumption allows researchers to use a single representative price for all regions. The LOP is also a condition for purchasing power parities to hold (Ardeni 1989). In addition, commodity price arbitrage is a major building block of standard models of exchange rate determination. On the other hand, deviation from LOP can explain the short-run volatility of exchange rates and “overshooting effects” (Ardeni 1989).

In the last decade, a number of studies have been conducted to test for the LOP in international commodity markets. Unfortunately, no consensus has been reached regarding this hypothesis. Most of these studies have failed to support the hypothesis of the LOP. Officer (1986) conducted 40 tests and found support for the LOP in only four cases. Among the remaining 36 tests, he rejected the LOP in 28, with eight having mixed results. Later, Ardeni (1989) tested for the LOP using a bivariate cointegration testing framework and in most cases his results failed to support the LOP as a long-run process. Recently, Goodwin (1992) used multivariate cointegration tests to examine the LOP in the international wheat market. He concluded that the LOP holds good only if transportation costs are taken into account.

Among the different methods used in previous studies, a particular time series modeling technique, cointegration, has become a common approach (see for example Ardeni 1989; Goodwin 1992). Cointegration analysis is appropriate for testing the LOP because most of the price variables analyzed are nonstationary. In the presence of nonstationarity, estimation of relationships between or among economic variables using standard estimation procedures is not appropriate (Ardeni 1989). On the other hand, the theory of cointegration allows for the testing of long-run relationships between or among economic variables in the presence of nonstationarity.

Mixed findings from cointegration suggest some problems associated with testing the LOP. A test of the long-run LOP entails proper modeling of the low-frequency dynamics of economic variables and their equilibrium relationships while allowing for significant deviations from equilibrium in the short run (Cheung and Lai 1993). Empirical results depend crucially on the power of the statistical

technique employed to separate low-frequency from high-frequency dynamics. Thus, a method that can distinguish between high and low frequencies and detect long-run relationships in the noisy data seems to be desirable (Cheung and Lai 1993).

This study examines the LOP using a fractional cointegration approach introduced by Granger (1986). This approach combines the concept of cointegration and fractional differencing introduced by Engle and Granger (1987), Granger and Joyeux (1980), and Hosking (1981). Even though the concept was introduced in the mid-1980s, its empirical significance was not appreciated until recently.

In general, both cointegration and fractional cointegration analyses enable us to test for long-run relationships between the economic variables or the mean reverting behavior of equilibrium errors with few restrictions on short-run dynamics, but they differ in the way the hypotheses are being tested. Cointegration tests for unit roots, the order of integration of equilibrium errors assumed to be either 0 or 1 with the presumption that the order of integration is an integer. On the other hand, fractional cointegration analysis allows equilibrium errors to follow a fractionally cointegrated process, such that the order of integration is a fraction between 0 and 1 (Granger 1986). This is of particular relevance because fractionally cointegrated equilibrium errors can be shown to be mean reverting though they exhibit significant persistence in the short-run. In other words, fractionally cointegrated economic variables may have a long-run relationship although it might take a longer time for the fractionally cointegrated variables to return to equilibrium. Thus, by avoiding the discrete hypothesis of unit roots/ no unit roots in the equilibrium, this method permits analysis of a wider range of mean reversion behavior than standard cointegration analysis. This gain in flexibility in testing subtle mean reverting dynamics is shown to be vital in the proper evaluation of the LOP.

### **The LOP and Fractional Cointegration Tests**

The LOP suggests that prices for a single homogenous commodity expressed in a common currency are the same both at home and abroad. This implies a long-run relationship between the prices and is typically written as

$$P_t = \alpha + \beta P_t^* + \epsilon_t, \quad (1)$$

where  $P_t$  and  $P_t^*$  are the domestic and foreign prices for a specific commodity, expressed in a common currency,  $\alpha$  is some constant, and  $\epsilon_t$  is the error term capturing deviation from the LOP. All variables are in levels and expressed in logarithmic forms. Typically, the LOP is supported if  $\beta$  is not

significantly different from one. As pointed out by Goodwin, Grennes, and Wohlgemant (1990), prices may vary in a nonsynchronous manner within a band created by transportation costs and in that case, any value of  $\beta$  could be consistent with the LOP. But the presence of nonstationarity in variables makes the hypothesis tests regarding the value of  $\alpha$  and  $\beta$  estimated from the conventional model unreliable (Stock 1987). Thus, researchers like Ardeni (1989) and Goodwin (1992) have used cointegration tests to overcome the unreliability of testing the hypothesis using conventional regression models. If  $P_t$  and  $P_t^*$  are found to be cointegrated (i.e.,  $\epsilon_t$  is a mean-reverting process) then any shock to the system will die out, which means the LOP holds between the two prices.

An equilibrium relationship between  $P_t$  and  $P_t^*$ , represented by a vector  $X_t$ , can be characterized in a linear combination  $Z_t = \alpha X_t$ . If  $P_t$  and  $P_t^*$  are integrated of order  $I(d)$ , then their linear combination in general also will be integrated of the same order. If a vector  $\alpha$  exists such that  $Z_t$  is  $I(d - b)$  with  $b > 0$ , however,  $P_t$  and  $P_t^*$  are said to be integrated of order  $(d, b)$ . But if  $Z_t = 0$ , then there is a long-run equilibrium.

The typical case considered in empirical work is one in which  $b = d = 1$ , meaning that the components of  $X_t$  are  $I(1)$  and the equilibrium error  $Z_t$  is  $I(0)$ . Thus, the mean reversion behavior of the equilibrium error is the primary interest in testing for long-run equilibrium relationships among economic variables. But the procedure developed by Engle and Granger (1987), which has been widely used for testing cointegration, involves regressing  $P_t$  on  $P_t^*$  (or  $P_t^*$  on  $P_t$ ) and then testing to determine if the residual is integrated of order zero  $I(0)$  or not, using a unit root test. This procedure tests whether the residual is  $I(0)$  or  $I(1)$ . If the residual is found to be  $I(0)$ , then the null hypothesis of no cointegration is rejected.

Since cointegration requires the equilibrium error to be mean reverting, limiting the hypothesis to  $I(1)$  and  $I(0)$  is restrictive because the equilibrium error could be mean reverting without being exactly  $I(0)$ . A fractionally integrated error term will also display mean-reverting behavior (Granger and Joyeux 1980; and Hosking 1981).

A fractionally integrated process can be represented as

$$C(L)(1-L)^d Z_t = D(L) V_t, \quad (2)$$

where  $C(L) = 1 - C_1L - \dots - C_pL^p$ ,  $D(L) = 1 + D_1L + \dots + D_qL^q$ .  $v_t$  is iid  $(0, \sigma^2)$ . Equation (2) is referred to as the *autoregressive fractionally integrated moving average* (ARFIMA) model and is similar to the standard *autoregressive moving average* (ARIMA) model where  $d$  is restricted to

integers. In the ARFIMA model,  $d$  can take any real value. According to Hosking (1981), for the  $d$  value between 0 and 0.5 ( $0 < d < 0.5$ ), the autocorrelation of  $Z_t$  shows a hyperbolic decay at a rate proportional to  $k^{2d-1}$  as compared with faster geometric decay in a standard ARMA process, where  $d = 1$ . The distinction between  $d = 1$  and  $d < 1$  is also crucial in the mean reversion property of  $Z_t$  and the cointegration property of  $P_t$  and  $P_t^*$ . For  $d < 1$ , the effect of any shock will die out slowly, whereas for  $d = 1$ , it will remain forever (Cheung and Lai 1993).

### Testing for Fractional Cointegration

As with standard cointegration tests, equation (1) is estimated and its residuals examined to determine if  $\epsilon_t$  is  $I(d)$ , where  $d < 1$ . Unlike standard cointegration tests, where the distinct hypotheses of  $I(1)$  and  $I(0)$  are tested using the unit root test, fractional cointegration requires direct estimation of the integration parameter  $d$ . According to Diebold and Rudebusch (1991) and Sowell (1990a), standard unit root tests such as the Dickey-Fuller test may have lower power against fractional alternatives. Thus, in this paper, a test based on spectral regression, developed by Geweke-Porter-Hudak (GPH), is used to test for fractional cointegration.

Fractional integration behavior of a series  $Z_t$  can be seen from its spectral density  $f_z(w)$ , which behaves like  $w^{-2d}$  as  $w \rightarrow 0$ . For  $d > 0$ ,  $f_z(w)$  is unbounded at frequency  $w = 0$  rather than bounded as for a stationary ARMA series (Cheung and Lai 1993). Geweke-Porter-Hudak use this relationship to develop a procedure to estimate fractional integration behavior. An integrated series, as in (2), where the error term is a stationary linear process with spectral density function  $f_u(w)$ , which is finite, is bounded away from zero and continuous on the interval  $[-\pi, +\pi]$ . The spectral density function of  $Z_t$ , where  $t = 1, 2, \dots, T$ , is

$$f_z(w) = (\sigma^2/2\pi) 4 \sin^2(w)^{-d} f_u(w) . \quad (3)$$

Taking the logarithm of both sides of (3),

$$\text{Log}[f(w)] = \text{Log}[\sigma^2 f_u(0)/2\pi] - d \text{Log}[4 \sin^2(w/2)] + \text{Log}[f_u(w)/f_u(0)] . \quad (4)$$

Adding  $I(w_j)$  on both sides of (4) and evaluating at harmonic frequencies  $w_j = 2\pi j/T$  ( $J = 0, 1, 2, \dots, T-1$ ), (4) yields

$$\text{Log}[I(w_j)] = \text{Log}[\sigma^2 f_u(0)/2\pi] - d \text{Log}[4 \sin^2(w_j/2)] + \text{Log}[f_u(w_j)/f_u(0)] + \text{Log}[I(w_j)/f(w_j)] . \quad (5)$$

where  $I(w_j)$  is the periodogram of the series  $Z$  at frequency  $w_j$  and is defined as

$$I(w) = \frac{1}{2\pi T} \left[ \left( \sum_{t=1}^T e^{itw} (Z_t - \bar{Z}) \right)^2 \right] . \quad (6)$$

For low-frequency ordinates  $w_j$  at near zero, say  $j \leq n \leq T$ , the term  $\text{Log}[f_u(w_j) / f_u(0)]$  in (5) becomes negligible compared with the other terms. In that case, it may be estimated using this simple linear regression equation:

$$\text{Log}[I(w_j)] = c - d \text{Log}[4 \sin^2(w_j/2)] + \eta_t , \quad (7)$$

where  $c$  and  $\eta_t$  are equal to  $\text{Log}(\sigma^2 f_u(0)/2\pi)$  and  $\text{Log}[I(w_j) / f(w_j)]$ ,  $j = 1, 2, \dots, n$  ( $n = g(T) < T$ ). The theoretical variance of  $\eta_t$  is known to be equal to  $\Pi^2/6$ , which is often imposed in estimation to raise efficiency (Cheung and Lai 1993). The notation  $n = g(T)$  is an increasing function of  $T$ . It has been shown by Geweke, Porter, and Hudak (1983) that least squares regression of (7) can provide a consistent estimate of the parameter  $d$ .

The choice of the number of low-frequency ordinates,  $n$ , used in the GPH regression necessarily involve judgment by the researcher (Cheung and Lai 1993). A value of  $n$  that is too large will contaminate the estimate of  $d$  due to medium- or high-frequency ordinates, whereas a value that is too small will result in an imprecise estimate due to limited degrees of freedom (Cheung and Lai 1993). Cheung and Lai conducted a Monte Carlo experiment to obtain the size of  $n$  for their sample of 76 and used range values of  $\mu$  for the sample size function,  $n = T^\mu$ . Use of this range value provides information on the sensitivity of the results to the choice of  $n$ . Based on the simulation results, they found better performance for  $\mu = 0.55, 0.575$ , and  $0.6$ . In another article, Cheung (1993) used  $\mu = 0.5$ , which is commonly used to test for fractional integration and also reports results for  $\mu = 0.45$  and  $0.55$  to check the sensitivity of the results.

In addition to determining the optimum size for  $n$ , Cheung and Liu also measured the power of the GPH test compared with conventional unit root test. Using the simulation results, they showed that the GPH test performs at least as well as the augmented Dickey-Fuller (ADF) test against the usual



autoregressive alternatives but against the fractional alternative, the GPH test performs significantly better than the ADF test. They also confirmed results from other studies that show that the power of either the GPH or the ADF test rises as the sample size increases. But for a sample size of 200 or fewer, the GPH test has a potential power advantage over the ADF test.

### Data, Estimation, and Results

The quarterly price series used to test the LOP through fractional cointegration includes a small group of commodities (wheat, wool, sugar, tea, tin, and zinc) and four countries (Australia, Canada, United Kingdom, and United States). These commodities and countries are similar to the ones used by Ardeni (1989) with the exception that most of the series used in this study are updated and in few cases, nonavailability of data forced us to abandon some price series. All the data are collected from various issues of *International Financial Statistics* (International Monetary Fund). A brief description of each price series, along with its sample range, is provided in Appendix A. All prices are expressed in U.S. dollars.

Before testing for fractional cointegration, it is necessary to check the order of integration of each price series. To verify the order of integration, the ADF test was performed.

The ADF test is based on the regression

$$\Delta Z_t = \alpha_0 + \beta Z_{t-1} + \sum_{i=1}^m \delta_i \Delta Z_{t-i} + \varepsilon_t, \quad (8)$$

where  $\Delta$  is the first difference operator and  $\varepsilon_t$  is the stationary error term. The number of lags to include in the equation was determined using the Akaike information criterion and was found to be four for all price series. The importance of including a constant without a time trend was addressed by Dickey, Bell, and Miller (1986) and Miller and Russek (1990). Based on their suggestion, ADF equations were estimated with an intercept and no time trend. All the price series are expressed in logarithmic form. The null hypothesis of nonstationarity or unit roots is tested using a t-test on the  $\beta$  coefficient. The null hypothesis is rejected if  $\beta$  is significantly negative. In addition, the GPH test was also conducted to measure the order of integration of individual price series using (7). The sample size for the GPH regression was determined using  $n = T^\mu$ . In this study, we chose  $\mu = 0.5, 0.55,$  and  $0.575$ , considering our sample size and the findings of other studies. In estimating (7), the error

variance was restricted to its theoretical value of  $\pi^2/6$ . Using the GPH test, the unit root hypothesis can be tested by determining whether or not the GPH estimate of  $d$  is significantly different from 1.

Both ADF and GPH test results for each price series are presented in Table 1. The ADF test statistic indicates that, for all the series examined, the null hypothesis of nonstationarity or unit roots cannot be rejected, even at the 10 percent significance level. Since lag order determination using statistical tests alone has been criticized, the ADF test was performed using different lag orders. These alternative representations did not alter the ADF results. The GPH test statistics, reported in Table 1, fail to reject the  $I(1)$  hypothesis and confirm the findings of ADF tests. In general, both tests support the hypothesis that the price series have unit roots.

Having confirmed that the price series are integrated of order one, we conducted cointegration tests using both ADF and GPH tests. The ADF test for cointegration between two series involves regressing one series on the other and testing the residuals from these equations for unit roots using the ADF test. ADF test statistics for the residual of each pair of price series are presented in Table 2. In most cases, the null hypothesis of no cointegration was not rejected. Out of nine pairs of price series, the hypothesis of no cointegration was not rejected in six, cases even at the 10 percent significance level. Only in the cases of U.S. and Australian wheat prices ( $PW_{US}$  and  $PW_{AU}$ ); U.S. and Canadian wheat prices ( $PW_{US}$  and  $PW_{CA}$ ); and U.S. and U.K. tea prices ( $PT_{US}$  and  $PT_{UK}$ ), the hypothesis of no cointegration was rejected either at the 5 or 10 percent significance level. For each pair of price series, reverse cointegration regression was also performed.<sup>1</sup> The results did not differ appreciably. In addition, cointegration was also tested for each pair of price series using Johansen's maximum likelihood procedure.<sup>2</sup> This yielded results similar to those found with the ADF tests.

In the next step, cointegration was tested using the GPH test for the same pairs of price series. This involves estimating (7) for the residuals obtained from each pair of series. The  $\mu$  values are

---

<sup>1</sup>For example, for  $PW_{US}$  and  $PW_{AU}$ , the ADF test statistic is based on the residual obtained from regressing  $PW_{US}$  on  $PW_{AU}$ . Reverse cointegration test include regressing  $PW_{AU}$  on  $PW_{US}$  and testing its residual for unit roots. This procedure was performed for each pair of series.

<sup>2</sup>Johansen's maximum likelihood procedure is based on the following error correction representation.

$$\Delta Z_t = \sum_{j=1}^k \alpha_j \Delta Z_{t-j} + \theta(r)Z_{t-1} + \epsilon_t$$

where  $Z_t$  is the  $2 \times 1$  vector of  $I(1)$  processes. The rank of  $\theta(r)$  equals the number of cointegrating vectors, which is tested by maximum eigenvalue and trace statistics. The critical values for these statistics were obtained from Johansen and Juselius (1992). This equation was estimated with each pair of price series.

similar to the ones used for testing the order of integration of individual price series. As before, the error variance was also restricted to its theoretical value ( $\pi^2/6$ ). The estimated  $d$  values, along with  $F$  statistics for null hypotheses of  $d = 1$  and  $d = 0$ , are also presented in Table III. In most cases, the results vary slightly across the different values of  $\mu$ . This suggests that the results are not sensitive to the choice of  $\mu$ . As seen in Table 3, the null hypothesis of  $d = 1$  was rejected in all but one case, implying the presence of cointegration and possibly fractional cointegration between each of these eight pairs of prices. The only case where the null hypothesis of  $d = 1$  was not rejected includes U.S. and UK sugar prices ( $PS_{UK}$  and  $PS_{AU}$ ). Of the eight cases, where the null hypothesis of  $d = 1$  is rejected in five cases, and the null hypothesis of  $d = 0$  was also rejected. This means that estimates of  $d$  lie between 0 and 1 for those five cases, suggesting the possibility of fractional cointegration. The GPH test results thus provide wider and more significant support for the LOP than the ADF test results do.

The GPH test results are particularly interesting when compared with those based on the ADF test for cointegration in individual cases. As shown in Table 2, the ADF tests support cointegration among three pairs of price series, U.S. and Australian wheat prices ( $PW_{US}$  and  $PW_{AU}$ ); U.S. and Canadian wheat prices ( $PW_{US}$  and  $PW_{CA}$ ); and U.S. and UK tea prices ( $PT_{US}$  and  $PT_{UK}$ ). For these three cases, the residuals are integrated of order 0, or  $d = 0$ . Comparing the GPH test results for these price series, it can be seen (Table 3) that the null hypothesis of  $d = 0$  cannot be rejected for these three pairs of price series. This suggests that both ADF and GPH tests produce similar results if the estimated value of  $d$  is 0. For the remaining six pairs of price series, the ADF test fails to find cointegration between each pair of price series, whereas the GPH test finds evidence of fractional cointegration in all but one pair of prices series,  $PS_{UK}$  and  $PS_{AU}$ . Thus, although full cointegration is rejected by both sets of tests for these price series, there is evidence of fractional cointegration, which supports the existence of long-run tendencies for the LOP to hold.

## Conclusion

This paper examines the long-run LOP for international commodity prices using a generalized notion of cointegration called *fractional cointegration*. The analysis of fractional cointegration allows the equilibrium error to be a fractionally integrated process and avoids the strict  $I(1)$  and  $I(0)$  distinction assumed in previous studies. Evidence supporting the LOP requires the equilibrium error to be mean reverting. Since fractionally integrated equilibrium errors identify a wide range of mean reversion behavior, it is important to consider this possibility when evaluating the existence of the long-run LOP.

In this study, fractional cointegration analysis is applied to nine pairs of price series. The empirical results shows that these series are fractionally cointegrated even when the hypothesis of cointegration has been rejected. Of the nine cases, fractional cointegration supports the existence of the LOP in eight cases, compared with three cases in the standard cointegration process. These results suggest the notion that there is a long-run tendency for the LOP to hold for these commodity prices.

Another interesting aspect of these results is revealed in comparing them with Ardeni's LOP results from standard cointegration techniques for similar series. Ardeni (1989) found cointegration in the same three cases for which we found  $d$  values to be zero, suggesting that both standard cointegration technique and fractional cointegration provide similar results if the series are fully cointegrated (i.e., equilibrium errors are integrated of order zero). But differences arise in which standard cointegration techniques reject the hypothesis of cointegration. Based on the GPH test, it can be concluded that these series are fractionally cointegrated (although full cointegration,  $d=0$ , is also rejected in these cases). Overall, our results do not contradict Ardeni's findings, but rather, adds to them by identifying those cases that are not fully cointegrated but still have long-run relationships.

Table 1. Unit Roots Results Using ADF and GPH Tests

Price	ADF Test Statistics	GPH Test		
		$\mu=0.50$	$\mu=0.55$	$\mu=0.575$
PW <sub>AU</sub>	-2.021	0.972	0.416	0.379
PW <sub>US</sub>	-2.177	1.173	1.125	1.441
PW <sub>CA</sub>	-1.787	0.179	1.003	1.309
PWO <sub>AU</sub>	-2.412	2.228	1.329	2.023
PWO <sub>UK</sub>	-2.485	2.023	1.855	2.549
PS <sub>AU</sub>	-2.288	1.767	1.315	1.743
PS <sub>UK</sub>	-2.136	1.472	0.742	1.344
PT <sub>US</sub>	-1.174	1.218	2.11	1.908
PT <sub>UK</sub>	-2.203	0.611	2.329	1.397
PZ <sub>US</sub>	-1.569	4.67**	2.761	1.733
PZ <sub>CA</sub>	-0.071	0.597	0.127	0.488
PZ <sub>UK</sub>	-1.686	4.074**	2.932	2.062

\*\* indicates significance at 10 percent level.

ADF tests critical values for 100 observations are -3.17 and -2.91 at 5 and 10 percent significance level and for 50 observations, the corresponding significance levels are -3.58 and -2.93. (Engle and Granger, and Fuller). The GPH test statistics are F- statistics from the spectral regression. For the GPH test, the null hypothesis of  $d=1$  is tested against the alternative  $d \neq 1$ .

Table 2. Cointegration results using ADF tests

Price	ADF Test Statistics
$PW_{US}$ and $PW_{AU}$	-4.697*
$PW_{US}$ and $PW_{CA}$	-3.3**
$PW_{CA}$ and $PW_{AU}$	-2.019
$PWO_{UK}$ and $PWO_{AU}$	-1.917
$PS_{UK}$ and $PS_{AU}$	-1.505
$PT_{US}$ and $PT_{UK}$	-4.078*
$PZ_{US}$ and $PZ_{CA}$	-2.386
$PZ_{US}$ and $PZ_{UK}$	-2.484
$PZ_{CA}$ and $PZ_{UK}$	-2.393

\* indicates significance at the 5 percent level and \*\* at the 10 percent level.

ADF tests critical values for 100 observations are -3.17 and -2.91 at 5 and 10 percent significance level and for 50 observations, the corresponding significance levels are -3.58 and -2.93 (Engle and Granger, and Fuller).

Table 3. Cointegration results using GPH tests

Prices	$\mu=0.50$			$\mu=0.55$			$\mu=0.575$		
	d	H <sub>0</sub> : d=1	H <sub>0</sub> : d=0	d	H <sub>0</sub> : d=1	H <sub>0</sub> : d=0	d	H <sub>0</sub> : d=1	H <sub>0</sub> : d=0
PW <sub>US</sub> and PW <sub>AU</sub>	0.363	3.962**	0.857	0.394	3.624**	1.024	0.269	4.24**	0.482
PW <sub>US</sub> and PW <sub>CA</sub>	0.19	3.952**	0.218	0.313	4.334**	0.899	0.158	11.737*	0.414
PW <sub>CA</sub> and PW <sub>AU</sub>	0.488	4.659**	7.248*	0.522	4.107**	8.246*	0.626	3.796**	10.612*
PWO <sub>UK</sub> and PWO <sub>AU</sub>	0.391	4.21**	3.654**	0.367	4.01**	3.561**	0.563	3.54**	4.985*
PS <sub>UK</sub> and PS <sub>AU</sub>	0.749	3.023	26.997*	0.883	0.886	50.471*	0.988	0.008	63.976*
PT <sub>US</sub> and PT <sub>UK</sub>	0.429	6.136*	3.48	0.208	9.306*	0.643	0.332	9.25*	2.284
PZ <sub>US</sub> and PZ <sub>CA</sub>	0.668	3.968**	15.953*	0.528	7.395*	9.235*	0.445	4.357**	2.794
PZ <sub>US</sub> and PZ <sub>UK</sub>	0.572	5.72*	9.486*	0.623	5.577*	15.172*	0.635	8.285*	25.167*
PZ <sub>CA</sub> and PZ <sub>UK</sub>	0.61	5.659*	13.86*	0.552	11.845*	18.05*	0.431	18.886*	10.828*

\* indicates significance at the 5 percent level and \*\* at the 10 percent level.

The GPH test statistics are F- statistics from the spectral regression. The null hypothesis of d=1 and d=0 are tested against the alternative d≠1 and d≠0.

## APPENDIX A.

## Description of Price Series Used for the Analysis

Name	Description	Sample Range
<u>Wheat</u>		
PW <sub>AU</sub>	Australian wheat export price, unit value (US\$/bushel)	1966:1-1993:4
PW <sub>US</sub>	U.S. wheat export price, No.1 HRW, Gulf (US\$/bushel)	1966:1-1993:4
PW <sub>CA</sub>	Canadian wheat export price, HRS, unit value (US\$/bushel)	1966:1-1990:2
<u>Wool</u>		
PWO <sub>AU</sub>	Australian wool export price (cents/kg)	1975:1-1993:4
PWO <sub>UK</sub>	U.K. wool import price Australia-New Zealand 50's: UK dominion(cents/kg)	1975:1-1993:4
<u>Sugar</u>		
PS <sub>AU</sub>	Australian sugar import price (cents/kg), unit value	1975:1-1993:4
PS <sub>UK</sub>	Sugar London daily spot price (cents/lb)	1975:1-1993:4
<u>Tin</u>		
PTN <sub>US</sub>	U.S. tin mid-month spot price, all origins, New York (U.S. cents/lb)	1966:1-1985:4
PTN <sub>UK</sub>	Tin price(all origin, London): London metal exchange spot CIF, European port, (U.S. cents/lb)	1966:1-1985:4
<u>Tea</u>		
PT <sub>US</sub>	Tea mid month price import price,cents/lb	1966:1-1985:4
PT <sub>UK</sub>	Tea, London auction price, UK, warehouse CIF, cents/lb	1966:1-1985:4
<u>Zinc</u>		
PZ <sub>US</sub>	Zinc FOB price, New York, U.S. cents/lb	1966:1-1993:4
PZ <sub>CA</sub>	Canadian zinc import Price, unit value, U.S. cents/lb	1966:1-1989:2
PZ <sub>UK</sub>	Zinc spot Price, London metal exchange, CIF, 98% pure,cents/lb	1966:1-1989:2



## REFERENCES

- Ardeni, P. G. "Does Law of One Price Really Hold for Commodity Prices." *Amer. J. Agr. Econ.* 71(August 1989): 661-669.
- Cheung, Y. W. "Long Memory in Foreign-Exchange Rates." *J. Bus. and Econ. Stat.* 11(January 1993): 93-101.
- Cheung, Y. W., and K. S. Lai. "A Fractional Cointegration Analysis of Purchasing Power Parity." *J. Bus. and Econ. Stat.* 11(January 1993): 103-112.
- Dickey, D. A., A. W. R. Bell, and R. B. Miller. "Unit Roots in Time Series Models: Tests and Implications." *The Amer. Statistician* (February 1986): 12-26.
- Diebold, F. X., and G. D. Rudebusch. "Long Memory and Persistence in Aggregate Output." *J. Monetary. Econ.* 24(September 1989): 189-209.
- Engle, R. F. and C. W. J Granger. "Cointegration and Error Correction : Representation, Estimation, and Testing." *Econometrica* 55(March 1987): 251-276.
- Geweke, J., and S. Porter-Hudak. "The Estimation and Application of Long Memory Time Series Models." *J. Time Series Analysis* 4(1983): 221-238.
- Goodwin, B. K. "Multivariate Cointegration Tests and the Law of One Price in International Wheat Market." *Rev. Agr. Econ.* 12(January 1992): 117-24.
- Goodwin, B. K., T. Greenes, and M. Wohlgenant. "Testing Law of One Price When Trade Takes Time." *J. Inter. Money and Finance* 9(March 1990): 21-40.
- Granger, C. W. J. "Developments in the Study of Cointegrated Economic Variables." *Oxford Bulletin Econ. and Statist.* 48(August 1986): 213-228.
- Granger, C. W. J and R. Joyeux. "An Introduction to Long-Memory Time Series Models and Fractional Differencing." *J. Time Series Analysis* 1(1980): 15-39.
- Hosking, J. R. M. "Fractional Differencing." *Biometrika* 68(April 1981): 165-176.
- International Monetary Fund. *International Financial Statistics*, January 1967 to August 1994.
- Johansen, S. "Statistical Analysis of Cointegration Vectors." *J. Econ. Dynamics and Control* (March 1988):231-254.

- Johansen, S., K. Juselius. "Maximum Likelihood Estimation and Inference on Cointegration: with Applications to the Demand Theory of Money." *Oxford Bulletin of Econ. and Statist.* (May 1990):169-210.
- Miller, S. M., and F. S. Russek. "Co-Integration and Error-Correction Models: The Temporal Causality Between Government Taxes and Spending." *S. Econ. J.* 57(July 1990): 221-229.
- Officer, L .H. "The Purchasing-Power Theory of Exchange Rates: A review Article." *I. M. F. Staff Paper* 23(1976): 1-60.
- Stock, J. H. "Asymptotic Properties of Least Squares Estimators of Cointegrating Vectors." *Econometrica* 55(September 1987): 1035-56.
- Sowell, F. "The Fractional Unit Root Distribution." *Econometrica* 58(March 1990): 495-505.