The Forward-Looking Competitive Firm Under Uncertainty

Sergio H. Lence and Dermot J. Hayes

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Center for Agricultural and Rural Development
Iowa State University
Ames, Iowa 50011-1070

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Sergio H. Lence is a CARD postdoctoral research associate; and Dermot J. Hayes is an associate professor of economics and head of the Trade and Agricultural Policy Division of CARD.

Abstract

This study of the firm under uncertainty relaxes the standard single production cycle assumption. Under realistic circumstances, a forward-looking risk-averse firm will produce more than a risk-neutral firm, and an increase in the mean-preserving price spread will increase the risk-averse firm's production. These results depend on firms realizing that the prices of inputs required for production in subsequent periods are correlated with the prices of current output.

There are two important implications of this work. First, empirical work should not assume, nor should it find a monotonic relationship between output and the level of risk or risk aversion. Secondly, one can rationalize previously unexplained real-world behavior such as the relative insensitivity of production and sales to current prices, and the spreading of sales over time.

Keywords: Uncertainty, forward-looking decision making, production, storage.

Few studies have relaxed Sandmo's (1971) implicit assumption that the firm plans to end production at the end of the current period. However, this assumption is crucial for some of the most important results of the theory of the firm under uncertainty. As we show later, under certain realistic circumstances a forward-looking risk-averse firm will produce more than a risk-neutral one, and an increase in the mean-preserving price spread will increase the risk-averse firm's production.

There is at least one reason to expect the assumption of a single production cycle to matter. Consider an industry for which input and output prices are positively correlated and firms remain in production for several production cycles. Here, the firm's end-of-period cash flow includes the costs required to initiate production in the subsequent period.[1] Therefore, the positive effect of high output prices may be expected to be offset by higher input prices. In addition, having output to sell, even if produced at a loss, can act as a hedge against input prices. Firms operating in this environment will be concerned about revenue and cost risks at several points in time and may choose to offset risk in one period against risk in another and will have an opportunity to diversify risk across time.

Most of the existing literature on the topic assumes that firms are concerned only with current period production and risk. But by assuming that firms behave myopically in this manner, the literature excludes by assumption the intertemporal diversification of risk that drives the results of this paper. Existing nonmyopic models have generally restricted utility to be intertemporally additive and prices to be independently distributed across time (Newbery and Stiglitz 1981, Hey 1987). These are strong assumptions because intertemporal additivity implies perfect substitution among single-period utilities, and price independence is not supported by empirical research. Other work exists upon which we can build. Zabel (1971) uses a constant absolute risk-averse intertemporal utility function but assumes intertemporal price independence. Chavas (1988) presents a forward-looking mean-variance model of speculative storage. However, using the

[1] Note that there is no difference in real time between the end of one period and the beginning of the following period.

mean-variance paradigm in this setting is difficult to justify because the random storage function has a truncated distribution.

In the next section, we introduce a forward-looking firm whose only productive activity is speculative storage (or asset holding). In this case, the correlation between input and output prices is most obvious and leads to a straightforward analysis of the firm's behavior. We also show that Sandmo-type behavior is nested within the more general model by restricting the firm to be myopic. Then, we allow the firm to be involved in a more standard productive activity and derive some propositions. The results for this more general case are derived at the cost of some additional assumptions about the technology set.

I. A Speculative Storing Competitive Firm

Consider a competitive firm with a twice continuously differentiable von Neumann-Morgenstern utility function and assume that utility is strictly concave in its argument terminal wealth $[U(W_T), U'(W_T) > 0, U''(W_T) < 0]$.[2] Terminal wealth is

$$(1.1) \quad W_T = r_{-1}\, r_0\, r_1 \ldots r_{T-1}\, W_{-1} + r_0\, r_1 \ldots r_{T-1}\, \pi_0 + r_1 \ldots r_{T-1}\, \pi_1 + \ldots + r_{T-1}\, \pi_{T-1} + \pi_T$$

where W_t denotes monetary wealth at end of trading date t, π_t is cash flow at time t, and r_t equals one plus the one-period interest rate prevailing at t. Interest rate need not be constant over time, but it is restricted to be nonrandom. At each trading date $0 \leq t < T$ the firm can borrow and lend unlimited amounts of money for one period at the prevailing interest rate.

It will become clear later that input price randomness plays a key role in the forward-looking scenario, so that we want to account for it explicitly. But allowing for input price randomness renders the model intractable, as suggested by the related literature (Batra and Ullah

[2]As noted by Katz (1983), the proper argument of utility is wealth and not profits, although the latter approach has been widely (and incorrectly) used.

1974, Hartman 1975, Ratti and Ullah 1976, Wright 1984, Stewart 1978). One way to tackle this problem is to postulate a *speculative* firm whose only activity is storing a product (or asset) to profit from its resale. In this instance, the relevant cash flow at any date is represented by

$$(1.2) \quad \pi_t = p_t P_t - i(I_t - P_t) \qquad \text{s.t.} \quad I_t = I_{t-1} - P_{t-1} \geq 0$$

where P_t represents the quantity sold at date t, p_t denotes the price at t, $i(\cdot)$ is a convex storage cost function such that $i'(\cdot) > 0$,[3] and I_t is beginning inventory at date t. Positive sales means that the firm sells from beginning stocks, whereas negative sales means that the firm buys to store and sell at a later date. Sales cannot exceed beginning inventory (i.e., $P_t \leq I_t$). The amount $(I_t - P_t)$ is the unsold beginning inventory at date t, which is carried over at nonrandom storage cost $i(I_t - P_t)$ to become beginning inventory at time t+1 (I_{t+1}). This kind of cash flow reduces the problem to its essentials and is generalized later.

Assume that at any moment t the firm chooses current product sales (P_t) to maximize expected utility of terminal wealth, given available information. The optimal sales level at the current date t = 0 is obtained by solving the following set of recursive equations

$$(1.3) \quad M_T(r_{T-1} W_{T-1}, I_T; \mathbf{p}_T) = \max_{P_T \leq I_T} U[r_{T-1} W_{T-1} + p_T P_T - i(I_T - P_T)]$$

$$(1.4) \quad M_t(r_{t-1} \ldots r_{T-1} W_{t-1}, I_t; \mathbf{p}_t) = \max_{P_t \leq I_t} E_t[M_{t+1}(r_t \ldots r_{T-1} W_t, I_{t+1}; \mathbf{p}_{t+1})], \; t = 0, 1, \ldots, T\text{-}1$$

where $E_t(\cdot)$ denotes the expectation operator based on information available at t, $\mathbf{p}_t \equiv (p_0, \ldots, p_t)$ is a vector containing past and current prices, and terminal wealth and cash flows are given by (1.1)

[3]For a risk-averse firm, $i''(\cdot) = 0$ yields a bounded solution. This is important because $i''(\cdot) \equiv 0$ is a quite common situation in the real world (for example, gold and common stock are most likely carried over at constant marginal storage cost). In contrast, for a risk-neutral firm we need $i''(\cdot) > 0$ for the solution to be bounded.

and (1.2), respectively. The solution to the problem summarized by expressions (1.3) and (1.4) can be obtained by recursively solving the Lagrangian functions

(1.5) $£_T = U[r_{T-1} W_{T-1} + p_T P_T - i(I_T - P_T)] + \eta_T (I_T - P_T)$

(1.6) $£_t = E_t[M_{t+1}(r_t \dots r_{T-1} W_t, I_{t+1}; p_{t+1})] + \eta_t (I_t - P_t), t = 0, 1, \dots, T-1$

where η_t is the Lagrangian multiplier.

The first-order conditions (FOCs) corresponding to the terminal date T are

(1.7) $\frac{\partial £_T}{\partial P_T} = (p_T + i') M_T' - \eta_T = 0$

(1.8) $\frac{\partial £_T}{\partial \eta_T} = I_T - P_T \geq 0, \eta_T \geq 0, \eta_T (I_T - P_T) = 0$

where M_T' represents U' evaluated at the optimum. The first term of the derivative of the Lagrangian function with respect to sales is positive, hence the Lagrangian multiplier (η_T) is also positive to satisfy (1.7). Because $\eta_T > 0$, $\partial £_T/\partial \eta_T$ must equal zero to avoid violating the Kuhn-Tucker condition (1.8). Therefore, the optimal sales policy at T is to liquidate the inventories (e.g., $P_T = I_T$), the optimal cash flow reduces to $\pi_T = p_T I_T$, and the value function is

(1.9) $M_T(r_{T-1} W_{T-1}, I_T; p_T) = U(r_{T-1} W_{T-1} + p_T I_T)$

For all dates prior to the terminal date ($0 \leq t < T$), the FOCs are (see Appendix A):

(1.10) $\frac{\partial £_t}{\partial P_t} = r_{t+1} \dots r_{T-1} [r_t (p_t + i') M_t' - E_t(p_{t+1} M_{t+1}')] - \eta_t = 0$

(1.11) $\frac{\partial £_t}{\partial \eta_t} = I_t - P_t \geq 0, \eta_t \geq 0, \eta_t (I_t - P_t) = 0$

where $M_t' = E_t(M_{t+1}')$ evaluated at the optimum corresponding to date t (note that $M_t' > 0$). The solution to (1.10) and (1.11) is a unique absolute constrained maximum because the objective function is strictly concave and the constraint set is convex.[4] Expression (1.9) together with FOCs (1.10) and (1.11) provide the framework needed to analyze the forward-looking risk-averse firm.

A myopic firm is defined as one whose *planning* horizon is the same as its *decision* horizon, which is equal to one period.[5] This definition of myopia leaves two possibilities: the firm is either at terminal date T, or at time T-1. At T the firm faces no risk; therefore, we focus on the behavior of the myopic firm at date T-1. In contrast, a forward-looking (or nonmyopic) firm is characterized by a planning horizon that consists of at least two decision horizons. This means that a forward-looking firm is one optimizing at any date before T-1.

Because we will compare the risk-averse firm with an otherwise identical risk-neutral firm, we need to know the optimal behavior of the latter. It is straightforward to show that the risk-neutral FOCs for any date preceding the terminal time T are

$$(1.12)\quad \frac{\partial \pounds_t}{\partial P_t} = r_{t+1} \cdots r_{T-1} \left[r_t (p_t + i') - E_t(p_{t+1})\right] - \eta_t = 0$$

$$(1.13)\quad \frac{\partial \pounds_t}{\partial \eta_t} = I_t - P_t \geq 0,\ \eta_t \geq 0,\ \eta_t (I_t - P_t) = 0$$

and that the optimal sales policy at T is given by $P_T = I_T$ (see Appendix B). It follows immediately from (1.12) and (1.13) that, in the risk-neutral context, optimal myopic and forward-looking sales are identical, therefore we will not distinguish between myopic and nonmyopic risk-neutral behavior.

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[4]We will assume for the remainder of the analysis that the solution to (1.3) and (1.4) exists. The conditions for this are given in Bertsekas (1976, p. 375).

[5]According to Merton (1982, p. 656), the planning horizon "is the maximum length of time for which the investor gives any weight in his utility function," and the decision horizon is "the length of time between which the investor makes successive decisions, and is the minimum time between which he would take any action."

II. Myopic versus Forward-Looking Speculative Storage Behavior

We can obtain comparative statics corresponding to the risk-averse firm by total differentiation of FOCs (1.10) and (1.11). The myopic and forward-looking responses of sales and storage to current price, beginning inventories, the degree of absolute risk aversion, the interest rate, and the parameters of the distribution of next-date price are summarized in Propositions 1 and 2, respectively. Note that we use the acronyms CARA, DARA, and IARA to denote constant, decreasing, and increasing absolute risk aversion, respectively.

PROPOSITION 1: MYOPIC STORAGE AND SALES BEHAVIOR. *For any positive amount stored, a myopic risk-averse firm behaves as follows:*

a) Sales:

$$\frac{\partial P_{0=T-1}}{\partial p_0} \begin{cases} > 0 \text{ if } CARA;\text{ or } DARA \text{ and } P_0 \leq 0;\text{ or } IARA \text{ and } P_0 \geq 0 \\ \gtrless 0 \text{ if } DARA \text{ and } P_0 > 0;\text{ or } IARA \text{ and } P_0 < 0 \end{cases}$$

$$\frac{\partial P_{0=T-1}}{\partial r_0} \begin{cases} > 0 \text{ if } CARA;\text{ or } DARA \text{ and } p_0 P_0 \leq i;\text{ or } IARA \text{ and } p_0 P_0 \geq i \\ \gtrless 0 \text{ if } DARA \text{ and } p_0 P_0 > i;\text{ or } IARA \text{ and } p_0 P_0 < i \end{cases}$$

$$\frac{\partial P_{0=T-1}}{\partial I_0} \begin{cases} < 1 \text{ if } DARA \\ = 1 \text{ if } CARA \\ > 1 \text{ if } IARA \end{cases}$$

$$\left.\frac{\partial P_{0=T-1}}{\partial \mu_{0;1}}\right|_{\sigma=1} \begin{cases} < 0 \text{ if } DARA \text{ or } CARA \\ \gtrless 0 \text{ if } IARA \end{cases}$$

$$\left.\frac{\partial P_{0=T-1}}{\partial \sigma_{0;1}}\right|_{\sigma=1} \begin{cases} > 0 \text{ if } DARA \text{ or } CARA \\ \gtrless 0 \text{ if } IARA \end{cases}$$

$$\frac{\partial P_{0=T-1}}{\partial \lambda} > 0$$

b) Storage:

$$\frac{\partial I_{1=T}}{\partial p_0} \begin{cases} < 0 \text{ if } CARA;\text{ or } DARA \text{ and } P_0 \leq 0;\text{ or } IARA \text{ and } P_0 \geq 0 \\ \gtrless 0 \text{ if } DARA \text{ and } P_0 > 0;\text{ or } IARA \text{ and } P_0 < 0 \end{cases}$$

$$\frac{\partial I_{1=T}}{\partial r_0} \begin{cases} < 0 \text{ if } CARA; \text{ or } DARA \text{ and } p_0 P_0 \leq i; \text{ or } IARA \text{ and } p_0 P_0 \geq i \\ \gtrless 0 \text{ if } DARA \text{ and } p_0 P_0 > i; \text{ or } IARA \text{ and } p_0 P_0 < i \end{cases}$$

$$\frac{\partial I_{1=T}}{\partial I_0} \begin{cases} > 0 \text{ if } DARA \\ = 0 \text{ if } CARA \\ < 0 \text{ if } IARA \end{cases}$$

$$\left.\frac{\partial I_{1=T}}{\partial \mu_{0;1}}\right|_{\sigma=1} \begin{cases} > 0 \text{ if } DARA \text{ or } CARA \\ \gtrless 0 \text{ if } IARA \end{cases}$$

$$\left.\frac{\partial I_{1=T}}{\partial \sigma_{0;1}}\right|_{\sigma=1} \begin{cases} < 0 \text{ if } DARA \text{ or } CARA \\ \gtrless 0 \text{ if } IARA \end{cases}$$

$$\frac{\partial I_{1=T}}{\partial \lambda} < 0$$

where: $p_1 = \sigma_{0,1}\, p_1 + (1 - \sigma_{0,1})\, \mu_{0,1}$, $\sigma_{0,1}$ = constant, $\mu_{0,1} = E_0(p_1)$

λ = Arrow-Pratt coefficient of absolute risk aversion ($\lambda = - M_t''/M_t'$)

Proof: See Appendix C.

Because storage (I_1) is the "productive" activity of this speculative firm, Proposition 1 reiterates the findings of the standard literature on the firm under uncertainty for the case of speculative storage. If the myopic firm is DARA or CARA, storage increases with higher expected price or smaller Rothschild-Stiglitz mean-preserving price spread. Also, myopic storage is negatively related to the firm's degree of absolute risk aversion.

From Proposition 1, beginning stocks have a positive (negative) effect on storage if the firm is DARA (IARA). This result is to be expected: a *ceteris paribus* increase in beginning stocks makes the firm wealthier and therefore less absolute risk averse if DARA. But we know that storage is negatively associated to the degree of absolute risk aversion, so that storage grows when beginning stocks increase for a DARA firm.

The ambiguous response to current price in Proposition 1 seems counterintuitive. One would expect current price to affect storage negatively because current price may be considered an input price for storage. But a current price change also causes a wealth change and, consequently, a change in the degree of absolute risk aversion unless the firm is CARA. For the CARA firm, the degree of absolute risk aversion does not depend on current price and the storage response to current price is unambiguously negative. Also, the DARA firm that buys good to store (i.e., $P_0 \leq 0$) reduces storage as current price increases. Otherwise, DARA storage bears an ambiguous relationship with current price. A similar explanation can be given to the result regarding the interest rate in Proposition 1.

When the firm is CARA the degree of absolute risk aversion is not affected by changes in exogenous variables. Therefore, the CARA firm's response to a change in a specific exogenous variable does not include the indirect effect of that variable on the degree of absolute risk aversion. For non-CARA firms, this indirect effect may be of opposite direction and sufficiently large so as to outweigh the exogenous variable direct effect, which is the reason for the ambiguities that arise from DARA or IARA attitudes in Proposition 1. Because of these ambiguities, non-CARA forward-looking behavior cannot be characterized without imposing more restrictions (see Proposition 2).

PROPOSITION 2: FORWARD-LOOKING STORAGE AND SALES BEHAVIOR. *For any positive amount stored, the sales and storage responses of a forward-looking risk-averse firm to changes in current price, interest rate, beginning inventories, expected next-date price, mean-preserving price spread, and degree of absolute risk aversion are ambiguous in general. If the firm is CARA, it behaves as follows:*

a) Sales:

$$\frac{\partial P_{0<T-1}}{\partial p_0} > 0, \ \frac{\partial P_{0<T-1}}{\partial r_0} > 0, \ \frac{\partial P_{0<T-1}}{\partial I_0} = 1,$$

$$\left.\frac{\partial P_{0<T-1}}{\partial \mu_{0;1}}\right|_{\sigma=1} \gtrless 0,\ \left.\frac{\partial P_{0<T-1}}{\partial \sigma_{0;1}}\right|_{\sigma=1} \gtrless 0,\ \frac{\partial P_{0<T-1}}{\partial \lambda} \gtrless 0$$

b) Storage:

$$\frac{\partial I_{1<T}}{\partial p_0} < 0,\ \frac{\partial I_{1<T}}{\partial r_0} < 0,\ \frac{\partial I_{1<T}}{\partial I_0} = 0,\ \left.\frac{\partial I_{1<T}}{\partial \mu_{0;1}}\right|_{\sigma=1} \gtrless 0,\ \left.\frac{\partial I_{1<T}}{\partial \sigma_{0;1}}\right|_{\sigma=1} \gtrless 0,\ \frac{\partial I_{1<T}}{\partial \lambda} \gtrless 0$$

Proof: See Appendix C.

When we constrain the forward-looking firm to be CARA, we obtain unambiguous responses to current price, interest rate, and beginning inventory. Moreover, these responses are qualitatively the same as for the myopic CARA firm. But the effect of next-date expected price, next-date Rothschild-Stiglitz mean-preserving price spread, and the coefficient of absolute risk aversion on forward-looking sales and storage are ambiguous even for CARA. This result is counterintuitive and stands in contrast with what was found for the myopic case. This finding merits a more careful analysis because it challenges some of the main conclusions of the standard theory of the competitive firm under uncertainty.

We can show that the ambiguous forward-looking response is a plausible characterization of real-world firm behavior. To explain this behavior, it is helpful to rewrite FOC (1.10) in terms of the covariance between prices and marginal utility.[6] If the firm stores something at the present date (e.g., $I_1 = I_0 - P_0 > 0$), the Lagrangian multiplier must equal zero ($\eta_0 = 0$) and we can express (1.10) as

$$(2.1) \quad E_0(p_1) + \frac{\mathrm{Cov}(p_1; M_1')}{M_0'} = r_0\,[p_0 + i'(I_1)]$$

[6]Recall that for any pair of random variables x and y, $E(x\,y) = E(x)\,E(y) + \mathrm{Cov}(x, y)$.

On the other hand, if the firm stores nothing ($I_1 = I_0 - P_0 = 0$), the Lagrangian multiplier is positive ($\eta_0 \geq 0$), and instead of (2.1) we have

$$(2.2)\quad E_0(p_1) + \frac{Cov(p_1; M_1')}{M_0'} \leq r_0\,[p_0 + i'(0)]$$

M_0' is always positive, and we can infer the sign of $Cov(p_1, M_1')$ in expressions (2.1) and (2.2) by examining the response of M_1' to changes in p_1, i.e.,

$$(2.3)\quad \frac{\partial M_1'}{\partial p_1} = r_1 \ldots r_{T-1}\, I_1\, M_1'' - r_1 \ldots r_{T-1}\, I_2\, M_1'' + \frac{\partial M_1'}{\partial P_1}\frac{\partial P_1}{\partial p_1} + \int_{p_2} M_2'\, \frac{\partial p_2(p_2 | p_1)}{\partial p_1}\, dp_2$$

$$+ \ldots + \{ \int_{p_2} [\int_{p_3} \ldots \int_{p_{T-1}} (\int_{p_T} M_T'\, \frac{\partial p_T(p_T | p_{T-1})}{\partial p_1}\, dp_T)\, p_{T-1}(p_{T-1} | p_{T-2})\, dp_{T-1} \ldots]\, p_2(p_2 | p_1)\, dp_2 \}$$

where $M_T'' = U''$ and $M_t'' = E_t(M_{t+1}'')$, evaluated at the optimum, and $p_{t+1}(p_{t+1} | p_t)$ is the conditional density function of p_{t+1}, given p_t. The term $(r_1 \ldots r_{T-1}\, I_1\, M_1'')$ reflects the impact of current storage and is nonpositive. The term $(- r_1 \ldots r_{T-1}\, I_2\, M_1'')$ is due to the effect of next-date storage and is nonnegative. The third term in the right-hand side of (2.3) captures the impact of changes in absolute risk aversion and vanishes for a CARA decision maker. Finally, the integration terms represent the effect of next-date price attributable to its effect on posterior prices.

Expressions (1.12) through (2.3) together with the following characterization of prices

$$(2.4)\quad p_t = \alpha + \beta\, p_{t-1} + e_t,\ 0 \leq \beta \leq 1,\ e_t \text{ i.i.d. random variable}$$

give us the elements to derive our second set of results, which are summarized in Propositions 3 and 4, and their respective Corollaries.

PROPOSITION 3: MYOPIC RESERVATION PRICE FOR STORAGE. *The reservation price above which a myopic risk-averse firm does not store is equal to the reservation price for the risk-*

neutral firm. A myopic risk-averse firm will store at a level at which discounted expected next-date price is higher than current price plus marginal storage cost.

Proof: The risk-neutral reservation price is $p^{rn}_{0<T} = E_0(p_1)/r_0 - i'(0)$; the proof is trivial from FOCs (1.12) and (1.13).

For a myopic firm, $I_2 = I_{T+1} = 0$, and the right-hand side of (2.3) reduces to $I_T\, M_T'' \leq 0$. Therefore,

$$(2.5) \quad Cov(p_T, M_T') \begin{cases} < 0 \text{ if } I_T > 0 \\ = 0 \text{ if } I_T = 0 \end{cases}$$

because p_T is monotonically increasing in p_T, and M_T' is monotonically nonincreasing in p_T.[7] Applying expression (2.5) to (2.1) and (2.2) we get

$$(2.6) \quad E_{T-1}(p_T) \begin{cases} > r_{T-1}\, [p_{T-1} + i'(I_T)] \text{ if } I_T > 0 \\ \leq r_{T-1}\, [p_{T-1} + i'(I_T)] \text{ if } I_T = 0 \end{cases}$$

Hence, the myopic risk-averse reservation price is $p^{ra}_{0=T-1} = E_0(p_1)/r_0 - i'(0) = p^{rn}_{0<T}$. Q.E.D.

COROLLARY TO PROPOSITION 3. *The myopic risk-averse firm stores less than does the risk-neutral firm.*

PROPOSITION 4: FORWARD-LOOKING RESERVATION PRICE FOR STORAGE. *(1) The reservation price above which a forward-looking risk-averse firm does not store is generally different from the risk-neutral or the myopic risk-averse reservation price. Moreover, this firm does not necessarily store an amount at which discounted expected next-date price is higher than current price plus marginal storage cost.*

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[7]This result is obtained by employing Theorem 43 in Hardy, Littlewood, and Pólya (1967).

(2) If the firm is CARA and prices behave as shown in expression (2.4), then the forward-looking reservation price is higher than the risk-neutral or the myopic risk-averse reservation price.

Proof: We will only show part (2) because it implies part (1). If the firm is CARA, $\partial M_1'/\partial P_1 = \partial M_1/\partial P_1 = 0$ by FOC. Therefore, with prices characterized by (2.4), expression (2.3) becomes

$$(2.7)\quad \frac{\partial M_1'}{\partial p_1} = r_1 \ldots r_{T-1} I_1 M_1'' - r_2 \ldots r_{T-1} (r_1 - \beta) I_2 M_1'' - r_3 \ldots r_{T-1} \beta (r_2 - \beta) E_1(I_3 M_2'')$$

$$- \ldots - r_{t+1} \ldots r_{T-1} \beta^{t-1} (r_t - \beta) E_1(I_{t+1} M_t'') - \ldots - \beta^{T-2} (r_{T-1} - \beta) E_1(I_T M_{T-1}'')$$

But $1 \geq \beta \geq 0$, $(r_t - \beta) > 0$, $M_t'' < 0$, and $I_t \geq 0$. Therefore, if $I_1 = 0$, we will have $\partial M_1'/\partial p_1 = 0$ when $I_2 = 0$ and $\beta = 0$, and $\partial M_1'/\partial p_1 > 0$ otherwise. It follows that $Cov(p_1, M_1') > 0$ unless $E_0(I_2) = 0$ and $\beta = 0$, and the forward-looking CARA reservation price $p^{CARA}_{0<T-1}$ is such that $p^{CARA}_{0<T-1} > E_0(p_1)/r_0 - i'(0) = p^{ra}_{0=T-1} = p^{rn}_{0<T}$. Q.E.D.

COROLLARY TO PROPOSITION 4. *If the firm is CARA and prices behave as shown in expression (2.4), there exists a range of current prices over which the forward-looking CARA firm stores more than the risk-neutral firm stores.*

Proposition 3 and its Corollary extend well-known results from the theory of the firm under uncertainty to the myopic speculative storage scenario. Proposition 4 and its Corollary contain some of the key findings of this paper and provide the intuition for the seemingly paradoxical results of Proposition 2. Comparison of Propositions 3 and 4 (and their respective Corollaries) reveals that relaxing the myopic assumption under uncertainty yields nontrivial differences in speculative storing behavior.

Our results hold not only in the unrealistic case of independent prices (i.e., $\beta = 0$) but also in the realistic cases of random walk ($\beta = 1$) and stationary autoregressive processes of order 1

$(0 < \beta < 1)$. Moreover, the price behavior assumed [i.e., expression (2.4)] can be easily relaxed and still obtain the main results of Proposition 4 and its Corollary, i.e., that there exists a range of current prices over which the forward-looking CARA firm stores more than the risk-neutral firm stores. In the interest of space, however, we will not pursue further the characterization of price processes that lead to the main results of Proposition 4 and its Corollary.

In passing, it is worth noting that in the forward-looking scenario we cannot use normally distributed prices to justify mean-variance analysis because terminal beginning inventory is random but not normally distributed. From Proposition 3, it follows that $I_T = 0$ when the current price is greater than the myopic reservation price [i.e., when $p_{T-1} > E_{T-1}(p_T)/r_{T-1} - i'(0)$]. This result truncates the density function of terminal wealth, making it a non-normal density.

In Figures 1 and 2, we illustrate the key findings reported in Propositions 1 through 4. Figure 1 is drawn in storage-current price space, whereas Figure 2 is done in sales-current price space. In each figure, we depict the curves "myopic CARA," "forward-looking CARA," and "risk-neutral" to represent the hypothetical behavior of three firms assumed identical except for their planning horizons and risk attitudes. The slope of the storage curves for the CARA firms is negative (see Propositions 1 and 2). Also, the CARA storage curves are steeper than the risk-neutral curve, as inferred from the equations giving the storage response to current price (see Appendix C), i.e.,

$$(2.8) \quad \left|\frac{\partial I_{0<T}^{CARA}}{\partial p_0}\right| = \frac{1}{i'' - r_0 \ldots r_{T-1} E_0\{[(p_0 + i') - p_1/r_0]^2 M_1''\}/M_0'} < \frac{1}{i''} = \left|\frac{\partial I_{0<T}^{rn}}{\partial p_0}\right|$$

for $I_{0<T}^{CARA} = I_{0<T}^{rn}$

where |x| represents the absolute value of x, and the superscripts CARA and rn stand for CARA and risk-neutral firms, respectively.

As stated in Proposition 3, the risk-neutral and myopic CARA reservation prices are identical in Figures 1 and 2. Also, risk-neutral storage is always above myopic CARA storage.

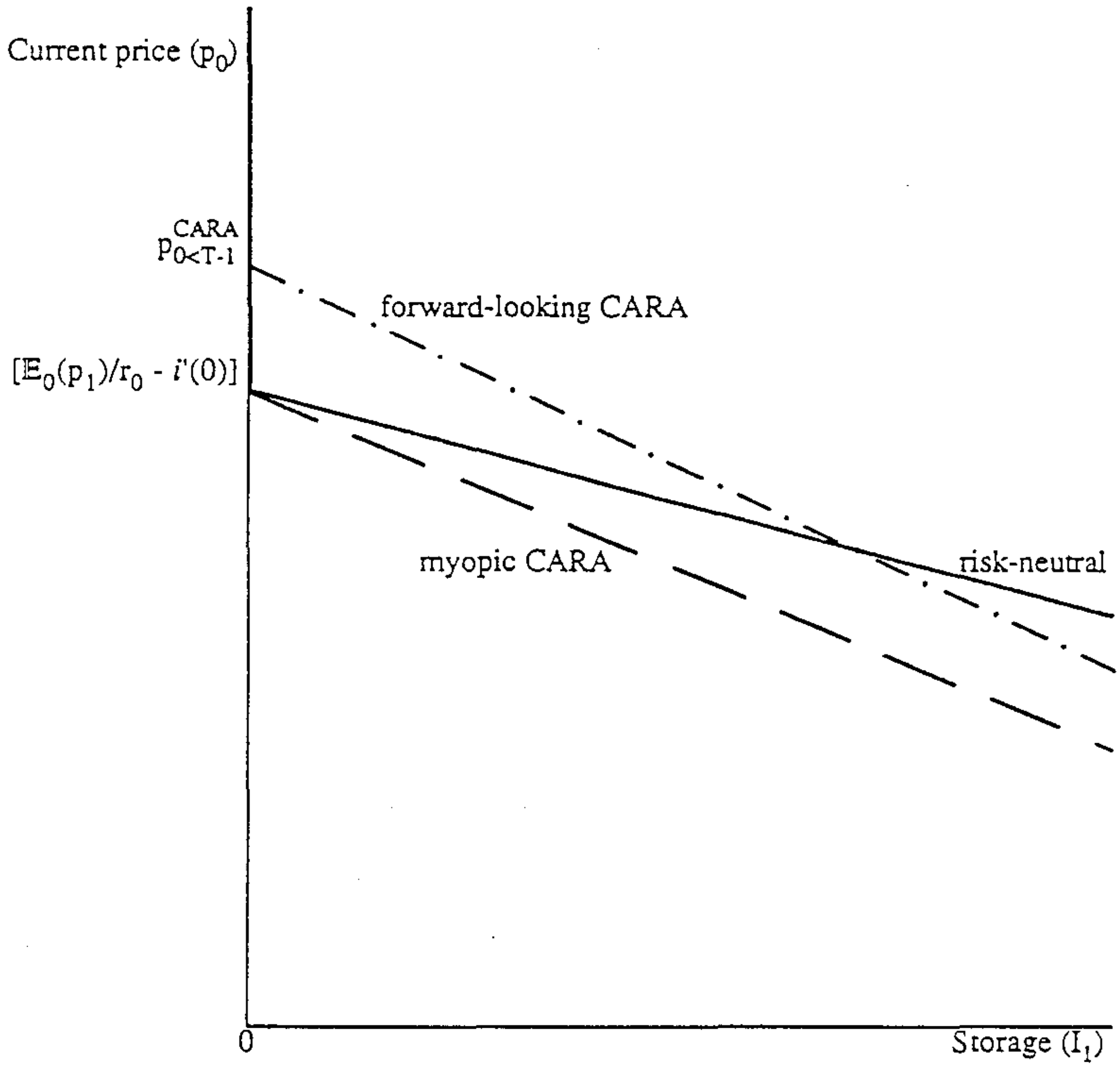


Figure 1. Storage behavior of risk-neutral, myopic CARA, and nonmyopic CARA firms

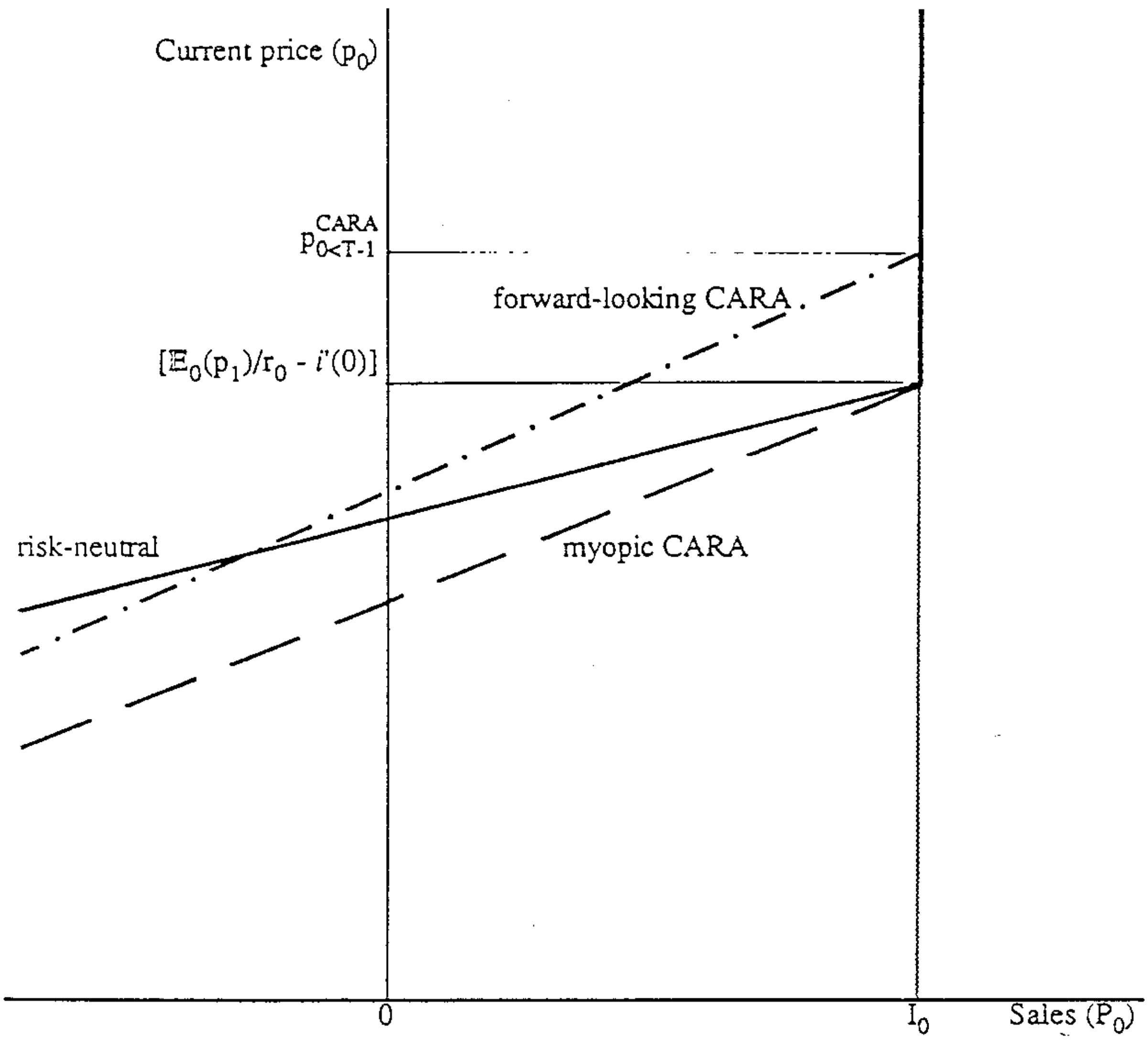


Figure 2. Sales behavior of risk-neutral, myopic CARA, and nonmyopic CARA firms

Figures 1 and 2 represent the case in which conditions (a), (b), or (c) of Proposition 4 are met, so that the forward-looking CARA reservation price is greater than the risk-neutral reservation price. Because storage curves are negatively sloped, the forward-looking CARA firm stores more than a risk-neutral firm when current price is between the risk-neutral and the forward-looking reservation prices (i.e., $I_{0<T-1}^{CARA} > I_{0<T}^{rn} = 0$ if $p_{0<T-1}^{CARA} > p_0 > p_{0<T}^{rn}$). Moreover, if storage cost is a strictly convex function (as depicted), forward-looking CARA storage will also exceed risk-neutral storage for some range of current prices less than the risk-neutral reservation price (i.e., $I_{0<T-1}^{CARA} > I_{0<T}^{rn} > 0$ for some $p_0 < p_{0<T}^{rn}$).

When current price is between the forward-looking CARA and the risk-neutral reservation prices, we observe a decrease in forward-looking CARA storage as we reduce the coefficient of absolute risk aversion from some positive value to zero (i.e., as firms become risk neutral). This is the reason why forward-looking CARA storage may increase with the degree of absolute risk aversion. We can apply a similar reasoning to explain the ambiguous effect of next-date expected price and next-date Rothschild-Stiglitz mean-preserving spread on forward-looking CARA storage.

From the proof of Proposition 4, it is clear that if current storage is sufficiently high we will have $\text{Cov}(p_1, M_1') < 0$ because the first term in the right-hand side of (2.3) will outweigh the other terms. Therefore, for sufficiently large current storage, we will have risk-neutral storage exceeding forward-looking CARA storage. Also, because of inequality (2.8), the forward-looking and risk-neutral curves intersect at a unique point. These observations are illustrated in Figure 1.

We can readily explain the *apparent* irrationality of a nonmyopic CARA firm holding inventories when current price is greater than discounted expected next-date price minus storage cost. Let t_0, t_1, and t_2 be three arbitrary successive *calendar* times, and write the terminal cash flow at dates t_0 and t_1 in the following way:

$$(2.9)\quad r_{t_0}\,\pi_{t_0} + \pi_{t_1} = r_{t_0}\,[p_{t_0}\,P_{t_0} - i(I_{t_0} - P_{t_0})] + [p_{t_1}\,P_{t_1} - i(I_{t_1} - P_{t_1})]$$

$$= r_{t_0}\,[p_{t_0}\,I_{t_0} - p_{t_0}\,I_{t_1} - i(I_{t_1})] + [p_{t_1}\,I_{t_1} - p_{t_1}\,I_{t_2} - i(I_{t_2})]$$

At time t_0, the planning horizon for the myopic firm ends at next date t_1, so that $T = t_1$. The myopic firm at date $T-1 = t_0$ plans to sell its entire current storage at date $T = t_1$. Therefore, at time t_0 the myopic firm only cares about revenue risk at t_1 (i.e., $p_{t_1} I_{t_1}$). In contrast, the forward-looking firm's planning horizon ends after the next date, i.e., $T > t_1$. Hence, the forward-looking firm generally expects to store something at t_1 [i.e., $E_{t_0}(I_{t_2}) > 0$], in which case it faces cost risk [i.e., $p_{t_1} I_{t_2} + i(I_{t_2})$] in addition to revenue risk from its activities at t_1. But revenue and input cost risks are related to each other and to current storage. In particular, current storage increases revenue risk but reduces input cost risk. The two opposing effects of current storage mean that the forward-looking firm may derive utility from holding some inventory, even when the one-period expected return from storage is negative. In a sense, the forward-looking firm diversifies assets intertemporally.

The results in this section apply not only to firms speculating with commodity storage, but also to speculative holders of stocks, bonds, and other nontransformable assets. These results are compatible with the findings of the standard theory of the firm under uncertainty because the standard results apply when the forward-looking firm stores a sufficiently large amount. But our model explains real-world facts that are incompatible with the standard model of the firm under uncertainty. For example, firms practice sequential marketing hold output and/or input reserves, and spread transactions over time to reduce risk (Robison and Barry, 1987, p. 65).

To illustrate the preceding findings, consider the following example. Assume that the forward-looking CARA firm is at decision date T-2, which corresponds to calendar time t_0. Prices at calendar times t_1 and t_2 (i.e., decision dates T-1 and T, respectively) have the following stationary discrete distribution:

$$(2.10)\quad p_t = \begin{cases} 13 \text{ with probability } 0.25 \\ 10 \text{ with probability } 0.50 \\ 7 \text{ with probability } 0.25 \end{cases}$$

Therefore, $E_{t_0}(p_{t_1}) = E_{t_0}(p_{t_2}) = E_{t_1}(p_{t_2}) = 10$, and $Var_{t_0}(p_{t_1}) = Var_{t_0}(p_{t_2}) = Var_{t_1}(p_{t_2}) = 4.5$. The coefficient of absolute risk aversion (λ) equals 0.001, and the storage cost function is quadratic

(2.11) $i(I_{t+1}) = 1\ I_{t+1} + 0.001\ I_{t+1}^2$

In addition, assume that the firm has no beginning inventories ($I_{t_0} = 0$) and that the interest rate is zero (i.e., $r_{t_0} = r_{t_1} = 1$).

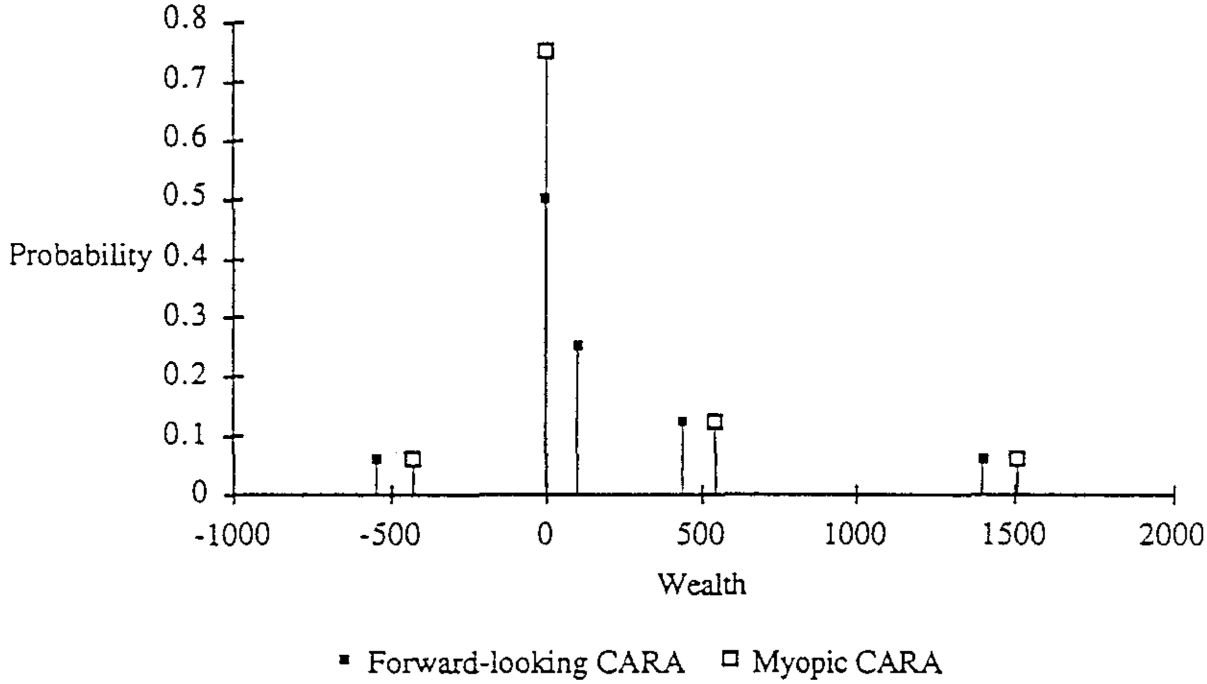
The utility-maximizing storage level was obtained by numerical maximization of the expected value of utility.[8] Table 1 and Figures 3 through 5 summarize the key results. It can be observed in Table 1 that the forward-looking CARA reservation price is above the myopic CARA and the risk-neutral reservation prices. Also, forward-looking CARA optimum storage equals 35.48 units when the current price is $p_{t_0} = 9$ (i.e., the myopic CARA reservation price). The expected profit from storing 35.48 units from date t_0 to date t_1 is -1.3 (i.e., a loss of 1.3). However, the forward-looking CARA expected utility at $I_{t_1} = 35.48$ equals -0.92875, which is greater than the expected utility at $I_{t_1} = 0$ (expected utility in this instance is -1). The intuition for this result can be found in Figure 3, which shows the probability distribution of wealth at date t_2 for the forward-looking and myopic CARA firms, assuming that the current price is $p_{t_0} = 9$.[9] Given this current price, the myopic firm stores nothing at t_0. When date t_1 arrives, if the myopic firm stays in business until date t_2 it will store when $p_{t_1} = 7$ (a 0.25 probability), and it will not store when $p_{t_1} = 10$ or 13 (a 0.75 probability). If the myopic firm stores at t_1, there are three possible wealth outcomes: (i) a profit of 1,514 if $p_{t_2} = 13$, which has a 0.0625 (0.25×0.25) unconditional probability; (ii) a profit of 543 if $p_{t_2} = 10$ (a 0.125 unconditional probability); and (iii) a loss of 429 if $p_{t_2} = 7$ (a 0.0625 unconditional probability). In contrast, the forward-looking firm stores 35.48 units at t_0 and behaves myopically at t_1. Therefore, there are five possible wealth outcomes. The most likely outcome is a loss of 1.3 (a 0.5 probability), which occurs when $p_{t_1} = 10$. There is also a 0.25 probability that the firm will make a profit of 105 (when $p_{t_1} = 13$). Finally, if $p_{t_1} = 7$ (a 0.25 probability), the forward-looking firm will lose 108 on its t_0 storage but

[8]We used the utility function $U = -\exp(-\lambda W_T)$ so that utility values range between -1 and 0.

[9]To obtain the distribution of wealth at date t_2, we assumed that all three firms behave myopically at date t_1 and that they store nothing at date t_2.

Table 1. Example of a forward-looking CARA firm that stores more than do myopic CARA and risk-neutral firms

	Forward-looking CARA Firm	Myopic CARA Firm	Risk-neutral Firm
Reservation price	$p_{t_0} = 9.2177$	$p_{t_0} = 9$	$p_{t_0} = 9$
Optimum storage when $p_{t_0} = 9$	$I_{t_1} = 35.48$	$I_{t_1} = 0$	$I_{t_1} = 0$
Maximum expected utility when $p_{t_0} = 9$	-0.92875 (at $I_{t_1} = 35.48$)	-1 (at $I_{t_1} = 0$)	-1 (at $I_{t_1} = 0$)
Expected utility when $p_{t_0} = 9$ at $I_{t_1} = 0$	-1	-1	-1



	Average wealth at date t_2	St. deviation of wealth at date t_2
Forward-looking CARA	134.5	387.1
Myopic CARA	135.7	416.2

Figure 3. Probability distribution of wealth at date t_2, assuming that current price is $p_{t_0} = 9$

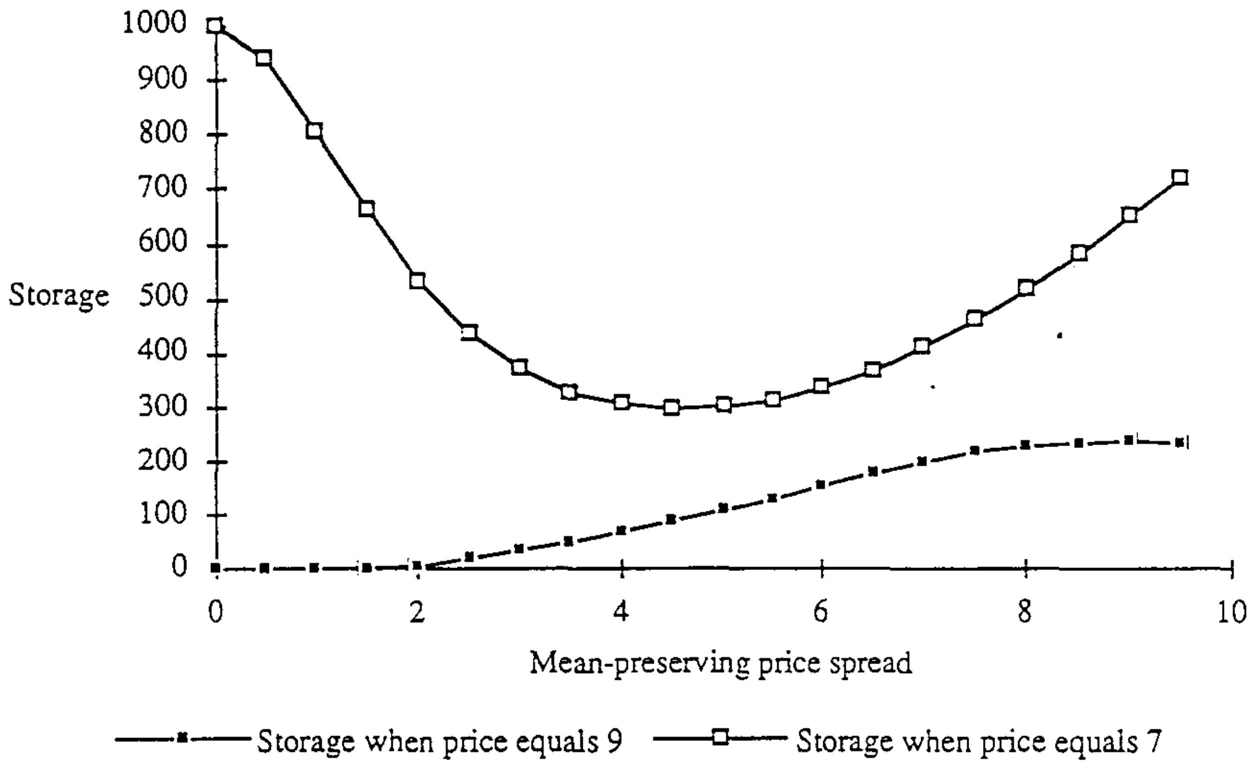


Figure 4. Effect of mean-preserving price spread at date t_1 on optimal forward-looking CARA storage for current price levels $p_{t_0} = 9$ and $p_{t_0} = 7$

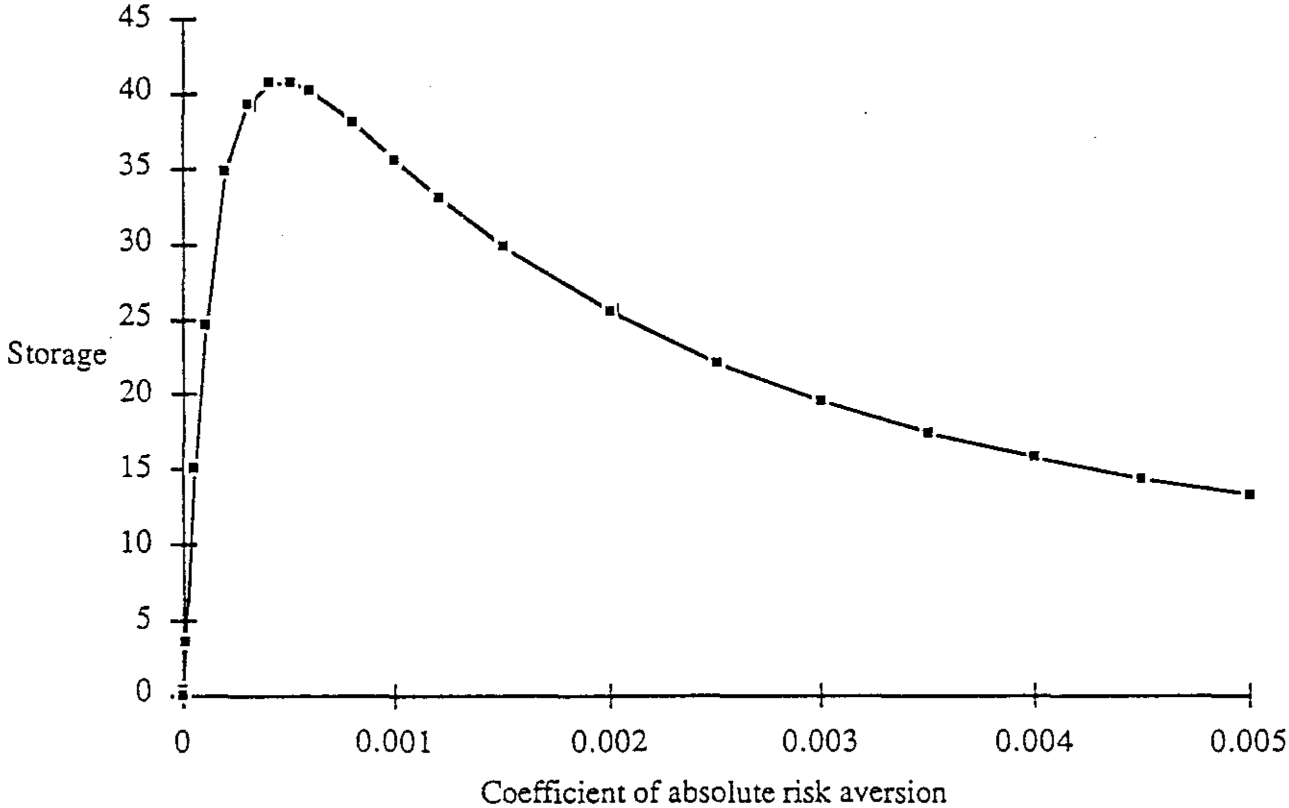


Figure 5. Effect of coefficient of absolute risk aversion on optimal forward-looking CARA storage, assuming that current price is $p_{t_0} = 9$

it will store at t_1, which may yield high or medium profits or a loss. The data in Figure 3 reveal that average wealth at t_2 is slightly lower for the forward-looking CARA compared to the myopic CARA firm, but the standard deviation for the latter is considerably higher than for the former.

Figure 4 depicts the relationship between the mean-preserving price spread at date t_1 and the optimal forward-looking CARA storage.[10] The simulation shows the ambiguous effect of risk on storage. When $p_{t_0} = 7$, the firm expects to profit from storage and its response to risk is negative up to a mean-preserving spread of 4.5, i.e., it behaves like a myopic firm. The positive response to risk when the mean-preserving spread increases beyond 4.5 occurs because, by storing more from t_0 to t_1, the firm can take advantage of a high t_1 price. This advantage is not symmetrically offset by the losses resulting from a low p_{t_1} because a low t_1 price creates profit opportunities to store from t_1 to t_2 (remember that the mean-preserving price spread change occurs only at t_1 prices, which means that the t_2 price vector is 13, 10, and 7). When the t_0 price is 9, the forward-looking firm stores more as risk increases up to a mean-preserving price spread of 9 and stores less thereafter. By increasing storage in response to an increase in the mean-preserving price spread, the firm effectively reduces the standard deviation of wealth at t_2 compared to the situation in which storage is either unchanged or reduced (see Figure 3). This example provides intuition for the positive correlation between storage and risk.

Finally, Figure 5 shows that storage bears an ambiguous relationship to the coefficient of absolute risk aversion. Risk-neutral firms store nothing at $p_{t_0} = 9$. If we introduce a small degree of risk aversion, then storage is optimal because it reduces the variance of terminal wealth (and reduces expected terminal wealth as well). However, there is a point at which additional storage increases risk; beyond this point, increased absolute risk aversion reduces storage in the traditional manner.

[10]For example, a mean-preserving price spread of x units implies that p_{t_1} equals (10 + x) with probability 0.25, 10 with probability 0.50, and (10 - x) with probability 0.25.

III. The Case of a Productive Competitive Firm

The main results discussed in the preceding section were obtained by assuming the cash flow presented in (1.2) and are attributable to the contemporaneous relationship between revenue and input cost at each date. In this section, we will show that similar conclusions apply to firms characterized by less restrictive cash flows. The complications that arise from allowing for random input prices in a nonmyopic context are attributable to the possibilities of stochastic production and/or input substitution. Hence, we can apply our basic model to other types of cash flows by constraining the production function to be nonstochastic and such that inputs with random prices cannot be substituted.

Consider a competitive firm with a Leontief short-run production function,

$$(3.1) \quad Q_t = \min[Q_t^s/\Phi, q(V_t)]$$

where Q_t denotes output at date t, $Q_t \geq 0$, Q_t^s represents material input use, Φ is a fixed input-output coefficient ($\Phi > 0$), V_t is a vector of nonmaterial inputs, and $q(\cdot)$ is a concave production function. Output Q_t becomes available for sale at date t+1, i.e., the production process starts at time t but output cannot be sold until date t+1.

According to (3.1), adding Φ units of material input increases production by one unit over the range in which the vector of nonmaterial inputs does not constrain production. If enough units of material input are added, the set of nonmaterial inputs eventually becomes binding and production cannot increase. The fact that there is no substitutability between material input and $q(\cdot)$ does not mean that substitution among the nonmaterial inputs in vector V_t is prevented. For example, it may be feasible to substitute capital for labor in wheat milling, even though the substitutability of wheat for either of these two nonmaterial inputs combined or alone is negligible for all practical purposes. Note also that material input becomes nonbinding as Φ tends to zero, resulting in a standard production function $q(\cdot)$. In other words, the standard production function is nested in (3.1).

For our purposes, it is essential that the Leontief function (3.1) is nonstochastic and that there is no substitution between material and nonmaterial inputs. This allows us to examine the situation for which material input price is random without complications arising from input substitution or stochastic output. Storage, transportation, refining and/or purifying of raw materials (e.g., oil, sugar, and metals), grain milling (e.g., wheat and rice), oilseed crushing, alloy preparation, energy generation, meat packing, and livestock production are examples of processes that comply with this type of production function.

Diewert (1971) has shown that the cost function dual to (3.1) is

$$(3.2) \quad C = \Phi\, s_t\, Q_t - c(Q_t; v_t)$$

where C is variable cost, s_t is material input price, $c(\cdot)$ is a convex nonmaterial cost function such that $c'(\cdot) > 0$, and v_t is a vector of nonmaterial input prices. We will assume that nonmaterial input prices are constant, and we will simply write $c(Q_t)$ instead of $c(Q_t; v_t)$ because we will not be concerned with nonmaterial input prices. Assuming that material input price is stochastic while nonmaterial input prices are constant can be justified because in many situations the largest share of variable cost is due to the material input. In addition, nonmaterial input prices are generally less volatile, and substitutability among nonmaterial inputs should cause variable cost changes far less pronounced than those due to material input price changes. Hence, the cash flow corresponding to a nonstoring firm with the Leontief production function (3.1) can be represented by

$$(3.3) \quad \pi_t = p_t\, Q_{t-1} - \Phi\, s_t\, Q_t - c(Q_t) \quad \text{s.t.} \qquad Q_t \geq 0$$

Comparing (3.3) with (1.2) reveals that the latter is a special case of the former, in which $\Phi = 1$, $s_t = p_t$, and $I_{t+1} = Q_t = Q_t^s/\Phi$.

With random final product and material input prices, optimal production at the current date t = 0 is the Q_0 that yields $M_0[r_0 \ldots r_{T-1} (r_{-1} W_{-1} + p_0 Q_{-1}); s_0, p_0]$ in the following set of recursive equations[11]

$$(3.4) \quad M_t[r_t \ldots r_{T-1} (r_{t-1} W_{t-1} + p_t Q_{t-1}); p_t, s_t] = \max_{Q_t \geq 0} £_t$$

where: $£_T = U[r_{T-1} W_{T-1} + p_T Q_{T-1} - \Phi s_T Q_T - c(Q_T)]$

$$£_t = \int_{p_{t+1}} \int_{s_{t+1}} M_{t+1}[r_{t+1} \ldots r_{T-1} (r_t W_t + p_{t+1} Q_t); p_{t+1}, s_{t+1}] \, g_{t+1}(p_{t+1}, s_{t+1} | p_t, s_t) \, ds_{t+1} \, dp_{t+1}, \quad 0 \leq t < T-1$$

The cash flows in expression (3.4) are as shown in (3.3), $s_t \equiv (s_0, \ldots, s_t)$ is a vector containing past and current prices of material input, and $g_{t+1}(p_{t+1}, s_{t+1} | p_t, s_t)$ is the conditional density function of s_{t+1} and p_{t+1}, given (p_t, s_t). It is clear that optimal production at the terminal date T is zero ($Q_T = 0$) and that the Kuhn-Tucker condition corresponding to any previous date is

$$(3.5) \quad \frac{\partial £_t}{\partial Q_t} = r_{t+1} \ldots r_{T-1} [E_t(p_{t+1} M_{t+1}') - r_t (\Phi s_t + c') M_t'] \leq 0, \; Q_t \geq 0, \; Q_t \frac{\partial £_t}{\partial Q_t} = 0$$

With this basic setting, we can now derive Propositions 5 and 6, which are analogous to Propositions 3 and 4 for the productive firm. Note that in Proposition 6 we use the following expression regarding the relationship between output and material input prices:

$$(3.6) \quad s_t = \gamma + \delta p_t + u_t, \; \delta \geq \beta/\Phi, \; u_t \text{ i.i.d. random variable}$$

where β is the coefficient corresponding to the lagged output price in equation (2.4).

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[11] Actually, this firm at date t must decide how much to sell from what was produced at the preceding date (Q_{t-1}), how much to produce for sale at next date (Q_t), and how much material input to use (Q_t^s). But the firm will always sell all beginning stocks so long as current output price is positive and the firm cannot store, and optimal material input use is given by $Q_t^s = \Phi Q_t$. Hence, the decision variable set reduces to Q_t.

PROPOSITION 5: MYOPIC RESERVATION PRICE WITH PRODUCTION. *The reservation price above which a myopic risk-averse firm does not produce is equal to the risk-neutral reservation price. A myopic risk-averse firm will produce at a level at which discounted expected next-date output price is higher than weighted current material input price plus marginal production cost.*

Proof: See Appendix D.

COROLLARY TO PROPOSITION 5. *The myopic risk-averse firm produces less than the risk-neutral firm.*

PROPOSITION 6: FORWARD-LOOKING RESERVATION PRICE WITH PRODUCTION. *(1) If output and material input prices are positively related, the reservation price above which a forward-looking risk-averse firm does not produce is generally different from the risk-neutral or the myopic risk-averse reservation price. Moreover, such a firm does not necessarily produce an amount at which discounted expected next-date output price is higher than weighted current material input price plus marginal production cost.*

(2) If the firm is CARA and that output and material input prices behave as in (2.4) and (3.6), the forward-looking reservation price is higher than the risk-neutral or the myopic risk-averse reservation price.

Proof: See Appendix D.

COROLLARY TO PROPOSITION 6. *If output and material input prices are represented by (2.4) and (3.6), there exists a range of current prices over which the CARA forward-looking firm produces more than the risk-neutral firm produces.*

The intuition for Propositions 5 and 6 is the same as that for the speculative storing firm. Again, our findings extend and qualify the standard results of the firm under uncertainty. For example, Proposition 6 explains the observed fact that in many instances firms continue producing even if they expect not to recover their variable costs over short periods of time.

The key to the behavioral hypotheses derived for the forward-looking CARA firm is the positive contemporaneous relationship between output and material input prices. The obvious question that arises is how strong and of what sign is that relationship in real-world situations. To this end, we report in Table 2 the correlation coefficients for seven pairs of contemporaneous output and material input prices belonging to the U.S. agricultural marketing and farm sectors. It can be seen that in all cases output and input prices bear a positive relationship. Table 2 also shows that the output-material input price relationship may be too strong to be neglected *a priori* when analyzing the firm under uncertainty.

Our results have implications for empirical work. First, the usual technique of *a priori* restricting the firm's production response to risk to be of the same sign at all production levels may be inappropriate. In fact, doing so may bias empirical results toward rejection of the hypothesis that risk affects firm behavior. This observation is supported by empirical studies reporting that output price variance has a relatively low impact on production (e.g., Brorsen et al. 1985, Antonovitz and Roe 1986, Aradhyula and Holt 1989, Antonovitz and Green 1990), and that material input price has a relatively greater effect on production than does the expected output price (e.g., Antonovitz and Roe, Antonovitz and Green). Second, relaxing the myopic constraint seems relevant, given the recent developments toward allowing for both rational expectations and risk aversion (Aradhyula and Holt, Antonovitz and Green). Even though forward-looking behavior is not synonymous to rational expectations, the concept of rational expectations seems much more consistent with forward-looking than with myopic behavior.

Our findings are also relevant for the study of business cycles. Forward-looking CARA firms tend to produce less than do risk-neutral ones at high output levels but more at low levels of production. Forward-looking CARA firms will therefore dampen the effects of business cycles.

Table 2. Coefficients of correlation between contemporaneous output and material input prices, 1976:1-1987:12

Output	Material Input	Coefficient of Correlation[a]
Wholesale beef	Live beef	0.985[b]
Meal	Soybeans	0.942[b]
Oil	Soybeans	0.859[b]
Slaughter steers	Feeder steers	0.908[b]
Eggs	Egg feed	0.751[b]
Hogs	Corn	0.368[b]
Broilers	Broiler grower feed	0.550[b]

[a]The coefficients were estimated by using monthly data deflated by the Producer Price Index.

[b]Significant at 1 percent.

Sources: Wholesale beef and live beef: Average prices of choice yield grade 3 steers at leading marketing areas (USDA 1989).

Soybeans: Price of No. 1 Yellow, Illinois processors (USDA various issues).

Meal: Price of 44 percent protein, bulk, FOB Decatur (USDA various issues).

Oil: Price of crude, tanks, FOB Decatur (USDA various issues).

Slaughter steers: Price of choice slaughter steers, 900 to 1,100 pounds, Omaha (USDA 1989).

Feeder steers: Price of medium-frame number one feeder steers, 600 to 700 pounds, Kansas City (USDA 1989).

Eggs, hogs, and broilers: Prices received by farmers (Weimar and Cromer 1990).

Egg feed: Egg feed costs (Weimar and Cromer 1990).

Corn: Prices calculated from the hog-corn ratio (Weimar and Cromer 1990).

Broiler grower feed: Prices paid by farmers (Weimar and Cromer 1990).

IV. Conclusions

A well-known result from the theory of the firm under uncertainty is that a myopic risk-averse firm produces less than does an otherwise identical risk-neutral firm. Our analysis reveals, however, that this conclusion is due to the assumption of myopic behavior and/or lack of correlation between output and material input prices. If output and material input prices are contemporaneously correlated, a risk-averse forward-looking firm may produce more or less than will a risk-neutral firm.

We show that there are realistic circumstances under which the forward-looking constant absolute risk-averse (CARA) firm will produce more than will a risk-neutral firm. In such instances, forward-looking CARA production exceeds risk-neutral output at low levels of production and the opposite is true at high production levels.

The behavioral differences between myopic risk-averse and forward-looking CARA firms are attributable to the fact that the former cares only about revenue risk, whereas the latter is concerned about both revenue and input cost risks. The myopic firm acts as if it intends to exit the market at the end of the current cycle and therefore disregards future costs. For such a firm, the only risk effect of production is to increase revenue risk. In contrast, the forward-looking firm plans to stay in the market after the current production cycle and therefore it takes into account future costs. Hence, for the forward-looking firm, current production not only increases revenue risk but also lowers cost risk if output and material input prices are positively related. Therefore, the forward-looking CARA firm may be willing to produce even if the one-period expected return from doing so is negative.

The model introduced in this paper provides a rationale for stylized facts in microeconomics. For example, it explains why firms continue producing (or storing) in the short run even at an expected loss and why farmers spread sales over time as a means to reduce risk. Our findings may also explain why empirical studies have found that the variance of output price has a relatively small impact on production.

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Appendix A: Derivation of FOC (1.10)

The FOCs corresponding to the Lagrangian function for $0 \leq t < T$ are

$$\text{(A1)} \quad \frac{\partial \pounds_t}{\partial P_t} = E_t\left[\frac{\partial M_{t+1}(r_t \cdots r_{T-1} W_t, I_{t+1}; p_{t+1})}{\partial I_{t+1}} \frac{\partial I_{t+1}}{\partial P_t}\right]$$

$$+ r_t \cdots r_{T-1} [p_t + i'(\cdot)] M_t'(r_{t-1} \cdots r_{T-1} W_{t-1}, I_t; p_t) - \eta_t = 0$$

plus (1.11). Note that

$$\text{(A2)} \quad \frac{\partial I_{t+1}}{\partial P_t} = -1$$

$$\text{(A3)} \quad \frac{\partial I_{t+1}}{\partial I_t} = 1$$

Also,

$$\text{(A4)} \quad \frac{\partial M_t(r_{t-1} \cdots r_{T-1} W_{t-1}, I_t; p_t)}{\partial I_t} = E_t\left[\frac{\partial M_{t+1}(r_t \cdots r_{T-1} W_t, I_{t+1}; p_{t+1})}{\partial I_{t+1}} \frac{\partial I_{t+1}}{\partial I_t}\right]$$

$$- r_t \cdots r_{T-1} i'(\cdot) M_t'(r_{t-1} \cdots r_{T-1} W_{t-1}, I_t; p_t) + \eta_t$$

$$\text{(A4')} \quad = r_t \cdots r_{T-1} p_t M_t'(r_{t-1} \cdots r_{T-1} W_{t-1}, I_t; p_t)$$

where (A4') is obtained by using expressions (A1) through (A3). It follows from (A4') that

$$\text{(A5)} \quad \frac{\partial M_{t+1}(r_t \cdots r_{T-1} W_t, I_{t+1}; p_{t+1})}{\partial I_{t+1}} = r_{t+1} \cdots r_{T-1} p_{t+1} M_{t+1}'(r_t \cdots r_{T-1} W_t, I_{t+1}; p_{t+1})$$

Substituting (A2) and (A5) into FOC (A1) and rearranging yields expression (1.10).

Appendix B: Derivation of FOC (1.12)

For the risk-neutral firm, the set of recursive equations is analogous to (1.3) and (1.4) but with W_T instead of $U(W_T)$. Therefore, the FOCs corresponding to the terminal date T are

$$\text{(B1)}\quad \frac{\partial £_T}{\partial P_T} = (p_T + i') - \eta_T = 0$$

$$\text{(B2)}\quad \frac{\partial £_T}{\partial \eta_T} = I_T - P_T \geq 0,\ \eta_T \geq 0,\ \eta_T \frac{\partial £_T}{\partial \eta_T} = 0$$

and optimal terminal sales are $P_T = I_T$. For any period $t < T$, FOCs are as follows:

$$\text{(B3)}\quad \frac{\partial £_t}{\partial P_t} = E_t\left[\frac{\partial M_{t+1}(r_t \ldots r_{T-1} W_t, I_{t+1}; p_{t+1})}{\partial I_{t+1}} \frac{\partial I_{t+1}}{\partial P_t}\right] + r_t \ldots r_{T-1} [p_t + i'(\cdot)] - \eta_t = 0$$

plus (1.13). But (A2) and (A3) still apply, and the expressions analogous to (A4), (A4'), and (A5) are, respectively,

$$\text{(B4)}\quad \frac{\partial M_t(r_{t-1} \ldots r_{T-1} W_{t-1}, I_t; p_t)}{\partial I_t} = E_t\left[\frac{\partial M_{t+1}(r_t \ldots r_{T-1} W_t, I_{t+1}; p_{t+1})}{\partial I_{t+1}} \frac{\partial I_{t+1}}{\partial I_t}\right] - r_t \ldots r_{T-1}\, i'(\cdot) + \eta_t$$

$$\text{(B4')}\quad = r_t \ldots r_{T-1}\, p_t$$

$$\text{(B5)}\quad \frac{\partial M_{t+1}(r_t \ldots r_{T-1} W_t, I_{t+1}; p_{t+1})}{\partial I_{t+1}} = r_{t+1} \ldots r_{T-1}\, p_{t+1}$$

Expression (1.12) is obtained by replacing (A2) and (B5) into FOC (B3).

Appendix C: Proof of Propositions 1 and 2

The Lagrangian multiplier vanishes ($\eta_0 = 0$) if storage is positive ($I_1 = I_0 - P_0 > 0$). Comparative statics can be obtained by totally differentiating FOC (1.10) as expressed in (C1)[12]

$$\text{(C1)} \quad £_P = E_0\{[r_0\,(p_0 + i'(I_0 - P_0)) - p_1]\, M_1'(r_0\, W_0, I_1;\, p_1)\} = 0$$

and then calculating the effect of any variable x on sales as $\frac{\partial P_0}{\partial x} = -\frac{£_{Px}}{£_{PP}}$. The derivatives of $£_P$ are:[13]

$$\text{(C2)} \quad £_{PP} = r_1 \ldots r_{T-1}\, E_0\{[r_0\,(p_0 + i') - p_1]^2\, M_1''\} - r_0\, i''\, M_0' < 0$$

$$\text{(C3)} \quad £_{Pp} = r_0 \ldots r_{T-1}\, P_0\, E_0\{[r_0\,(p_0 + i') - p_1]\, M_1''\} + r_0\, M_0' \lessgtr 0$$

$$\text{(C4)} \quad £_{Pr} = r_1 \ldots r_{T-1}\, (p_0\, P_0 - i)\, E_0\{[r_0\,(p_0 + i') - p_1]\, M_1''\} + (p_0 + i')\, M_0' \lessgtr 0$$

$$\text{(C5)} \quad £_{PI} = r_0 \ldots r_{T-1}\, p_0\, E_0\{[r_0\,(p_0 + i') - p_1]\, M_1''\} - £_{PP} \gtrless 0$$

$$\text{(C6)} \quad £_{P\mu} = r_1 \ldots r_{T-1}\, E_0\{[r_0\,(p_0 + i') - p_1]\, P_1\, M_1''\} - M_0' \lessgtr 0$$

$$\text{(C7)} \quad £_{P\sigma} = r_1 \ldots r_{T-1}\, E_0\{[r_0\,(p_0 + i') - p_1]\, P_1\, (p_1 - \mu_{0,1})\, M_1''\} - E_0\{(p_1 - \mu_{0,1})\, M_1'\} \lessgtr 0$$

where $M_T'' = U''$ and $M_t'' = E_t(M_{t+1}'')$, evaluated at the optimum.

The sign of $E_0\{[r_0\,(p_0 + i') - p_1]\, M_1''\}$ is ambiguous in general, but for CARA or myopic firms it can be inferred. For CARA firms we have

[12]To make notation less cumbersome, in this section we use $£_P$ to denote $\partial £_t/\partial P_t$. The meaning of the remaining derivatives should be clear from the context.

[13]To simplify notation, whenever we refer to $£_{P\mu}$ and $£_{P\sigma}$ we assume they are evaluated at $\sigma_{0,1} = 1$.

(C8) $E_0\{[r_0 (p_0 + i') - p_1] M_1''\} = - \lambda E_0\{[r_0 (p_0 + i') - p_1] M_1'\} = 0$

by FOC. Hence, $£_{Pp} = r_0 M_0' > 0$, $£_{Pr} = (p_0 + i') M_0' > 0$, and $£_{PI} = - £_{PP} > 0$. Also, from $£_{Pp} > 0$ it follows that $\partial P_1/\partial p_1 \geq 0$ for CARA. Then

(C9) $E_0\{[r_0 (p_0 + i') - p_1] P_1 M_1''\} = \lambda E_0\{[r_0 (p_0 + i') - p_1] (P_k - P_1) M_1'\} \geq 0$

where P_k is a constant equal to P_1 when p_1 equals $[r_0 (p_0 + i')]$. Expression (C9) is nonnegative because $M_1' > 0$, and $(P_k - P_1)$ is positive (negative) whenever $[r_0 (p_0 + i') - p_1]$ is positive (negative). Therefore, $£_{P\mu} \gtrless 0$ even for the CARA firm. The sign of $£_{P\sigma}$ is also ambiguous in general; for example, $E_0[(p_1 - \mu_{0,1}) M_1'] = Cov(p_1, M_1')$ may be positive or negative for CARA forward-looking firms, as inferred from the proof of Proposition 4.

For myopic firms, we have date 0 = T-1 and $P_1 = I_T = I_1$. Therefore,

(C10) $E_0\{[r_0 (p_0 + i') - p_1] M_1''\} = E_0\{[r_0 (p_0 + i') - p_1] (\lambda_k - \lambda) M_1'\} \begin{cases} < 0 \text{ if DARA} \\ = 0 \text{ if CARA} \\ > 0 \text{ if IARA} \end{cases}$

(C11) $E_0\{[r_0 (p_0 + i') - p_1] P_1 M_1''\} = I_1 E_0\{[r_0 (p_0 + i') - p_1] M_1''\} \begin{cases} < 0 \text{ if DARA} \\ = 0 \text{ if CARA} \\ > 0 \text{ if IARA} \end{cases}$

(C12) $E_0\{[r_0 (p_0 + i') - p_1] P_1 (p_1 - \mu_{0,1}) M_1''\} = - I_1 E_0\{[r_0 (p_0 + i') - p_1]^2 M_1''\}$

$$- I_1 [\mu_{0,1} - r_0 (p_0 + i')] E_0\{[r_0 (p_0 + i') - p_1] M_1''\} \begin{cases} > 0 \text{ if DARA or CARA} \\ \gtrless 0 \text{ if IARA} \end{cases}$$

where λ_k is a constant equal to λ when p_1 equals $[r_0 (p_0 + i')]$. The sign of (C10) follows because $M_1' > 0$, and for DARA $(\lambda_k - \lambda)$ is positive if $[r_0 (p_0 + i') - p_1]$ is negative, and vice versa. For IARA we have $(\lambda_k - \lambda)$ positive (negative) when $[r_0 (p_0 + i') - p_1]$ is positive (negative). Therefore, for a myopic firm we have

(C13) $\pounds_{P_P} \begin{cases} > 0 \text{ if DARA and } P_0 \leq 0\text{; or if IARA and } P_0 \geq 0 \\ \gtrless 0 \text{ if DARA and } P_0 > 0\text{; or if IARA and } P_0 < 0 \end{cases}$

(C14) $\pounds_{P_r} \begin{cases} > 0 \text{ if DARA and } p_0\, P_0 \leq i\text{; or if IARA and } p_0\, P_0 \geq i \\ \gtrless 0 \text{ if DARA and } p_0\, P_0 > i\text{; or if IARA and } p_0\, P_0 < i \end{cases}$

(C15) $\pounds_{PI} \begin{cases} < - \pounds_{PP} \text{ if DARA} \\ > - \pounds_{PP} > 0 \text{ if IARA} \end{cases}$

(C16) $\pounds_{P\mu} \begin{cases} < 0 \text{ if DARA or CARA} \\ \gtrless 0 \text{ if IARA} \end{cases}$

(C17) $\pounds_{P\sigma} \begin{cases} > 0 \text{ if DARA or CARA} \\ \gtrless 0 \text{ if IARA} \end{cases}$

To show the effect of the degree of absolute risk aversion on the myopic firm, take two firms A and B such that $\lambda_A(W) > \lambda_B(W)$ for all W.[14] Rewrite FOC corresponding to firm A as:

$$\text{(C18)} \quad \int_0^{r_0(p_0+i')} \frac{M_{1A}'}{U_A'(W_k)} [r_0\,(p_0 + i') - p_1]\, p_1(p_1)\, dp_1 + \int_{r_0(p_0+i')}^{\infty} \frac{M_{1A}'}{U_A'(W_k)} [r_0\,(p_0 + i') - p_1]\, p_1(p_1)\, dp_1 = 0$$

where W_k is terminal wealth corresponding to $p_1 = r_0\,(p_0 + i')$ and $p_1(p_1)$ is the density function of p_1. Equality (C18) is satisfied at firm A's optimum sales level (P_{0A}). For firm B, a similar expression to (C18) evaluated at P_{0A} is

$$\text{(C19)} \quad \int_0^{r_0(p_0+i')} \frac{U_B'(P_0 = P_{0A})}{U_B'(W_k)} [r_0\,(p_0 + i') - p_1]\, p_1(p_1)\, dp_1 + \int_{r_0(p_0+i')}^{\infty} \frac{U_B'(P_0 = P_{0A})}{U_B'(W_k)} [r_0\,(p_0 + i') - p_1]\, p_1(p_1)\, dp_1 < 0$$

[14]This proof follows Holthausen's methodology (1979).

Negativity of expression (C19) can be proven as follows. Subtract (C18) from (C19) to get

$$\text{(C20)} \quad \int_{0}^{r_0(p_0+i')} \left[\frac{U_B'(P_0 = P_{0A})}{U_B'(W_k)} - \frac{M_{1A}'}{U_A'(W_k)}\right] [r_0\,(p_0 + i') - p_1]\, p_1(p_1)\, dp_1$$

$$+ \int_{r_0(p_0+i')}^{\infty} \left[\frac{U_B'(P_0 = P_{0A})}{U_B'(W_k)} - \frac{M_{1A}'}{U_A'(W_k)}\right] [r_0\,(p_0 + i') - p_1]\, p_1(p_1)\, dp_1$$

The greater the next-date price, the greater terminal wealth because the firm is myopic. Hence, $W_k > W_T$ in the first integral and $W_k < W_T$ in the second integral. Applying inequality (20) in Pratt (1964, p. 129), it follows that the term involving ratios is negative in the first integral and positive in the second. Also, the term $[r_0\,(p_0 + i') - p_1]$ is positive in the first integral and negative in the second. Therefore, both integrals are negative, (C20) is negative, and (C19) must be negative. We conclude that firm B's optimum sales (P_{0B}) must be lower than P_{0A} because firm B's FOC is negative when evaluated at P_{0A}. Hence, for myopic firms we have $P_{0A} > P_{0B}$ if $\lambda_A > \lambda_B$.

The degree of absolute risk aversion has an ambiguous effect on forward-looking sales, even for CARA firms. This is shown in the simulations at the end of this section (e.g., Figure 5).

Comparative statics for storage follow by applying the identity $I_{t+1} = I_t - P_t$.

Appendix D: Proof of Propositions 5 and 6

We will outline the proofs of Propositions 5 and 6 because they can be performed by employing the same techniques we used to show Propositions 3 and 4, respectively.

We may express FOC (3.5) in covariance terms as

$$\text{(D1)} \quad E_0(p_1) + \frac{\mathrm{Cov}[p_1; E_0(M_1'| p_1)]}{M_0'} \begin{cases} = r_0\, [\Phi\ s_0 + c'(Q_0)] \text{ if } Q_0 > 0 \\ \leq r_0\, [\Phi\ s_0 + c'(0)] \text{ if } Q_0 = 0 \end{cases}$$

where: $E_0(M_1'| p_1) = \int_{s_1} M_1'[r_1 \ldots r_{T-1}\, (r_0\, W_0 + p_1\, Q_0);\, s_1,\, p_1]\; s_1(s_1| s_0, p_1)\; ds_1 > 0$

$E_0(M_1'| p_1)$ is the conditional expectation of M_1', given p_1, and $s_1(s_1| s_0, p_1)$ denotes the conditional density function of s_1, given (s_0, p_1). Expression (D1) is analogous to (2.1) and (2.2) for the risk-averse productive firm. Similarly, a risk-neutral productive firm is characterized by:

$$\text{(D2)} \quad E_0(p_1) \begin{cases} = r_0\, [\Phi\ s_0 + c'(Q_0)] \text{ if } Q_0 > 0 \\ \leq r_0\, [\Phi\ s_0 + c'(0)] \text{ if } Q_0 = 0 \end{cases}$$

For a myopic risk-averse decision maker, we have $E_0(M_1'| p_1) = M_T'$, and

$$\text{(D3)} \quad \frac{\partial E_0(M_1'| p_1)}{\partial p_1} = Q_{T-1}\, M_T'' \begin{cases} < 0 \text{ if } Q_{T-1} > 0 \\ = 0 \text{ if } Q_{T-1} = 0 \end{cases}$$

Therefore,

$$\text{(D4)} \quad \mathrm{Cov}(p_T, M_T') \begin{cases} < 0 \text{ if } Q_{T-1} > 0 \\ = 0 \text{ if } Q_{T-1} = 0 \end{cases}$$

Proposition 5 follows immediately by noting that (D4) is analogous to (2.5).

For a nonmyopic CARA firm with output and material input prices that behave as in (2.4) and (3.6), we have

$$(D5)\quad \frac{\partial(M_1'|p_1)}{\partial p_1} = r_1 \dots r_{T-1}\, Q_0\, E_0(M_1''|\, p_1)$$

$$- r_2 \dots r_{T-1}\,(r_1\, \Phi\, \delta - \beta)\, E_0(Q_1\, M_1''|\, p_1) - r_3 \dots r_{T-1}\, \beta\, (r_2\, \Phi\, \delta - \beta)\, E_0(Q_2\, M_2''|\, p_1)$$

$$- \dots - r_{t+1} \dots r_{T-1}\, \beta^{t-1}\,(r_t\, \Phi\, \delta - \beta)\, E_0(Q_t\, M_t''|\, p_1) - \dots - \beta^{T-2}\,(r_{T-1}\, \Phi\, \delta - \beta)\, E_0(Q_{T-1}\, M_{T-1}''|\, p_1)$$

The proof of Proposition 6 is straightforward because $[r_1 \dots r_{T-1}\, Q_0\, E_0(M_1''|\, p_1)]$ is negative if $Q_0 > 0$ and zero if $Q_0 = 0$, whereas the other terms in the right-hand side of (D5) are nonnegative.