Can Options Be Used as a Hedging Instrument?

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ABSTRACT

We develop a portfolio choice model for farmers faced with both price and production uncertainty who can hedge this uncertainty using both options and futures contracts. We then simulate the decision process of a typical Iowa farmer and derive his or her optimal options and futures position.

INTRODUCTION

In a recent paper Lapan, Moschini, and Hanson (LMH, 1991) extended Sandmo's expected utility model to analyze production, hedging, and speculative decisions where futures and options markets exist. One important implication of this work is that when individuals perceive futures and options markets to be unbiased and where local cash prices are a linear function of futures prices, then there is no place for options as hedging instruments. By extension, individuals who use options markets under these conditions may always be classified as speculators.

If the LMH model accurately reflects existing market conditions, then the historic legislative bias against options may be understood. For example, Cox and Rubinstein (1985, 23) argue that the "popular misconception equating options with gambling has resulted in extensive government regulation with puts and calls at times considered illegal."

The key to the LMH result is that one can divide the price risk into a component due to changes in end of period futures prices, and an orthogonal component reflecting undiversifiable basis risk. Because the diversifiable risk is linear in futures prices, futures contracts (which are also linear in futures prices) dominate options contracts that in the LMH model are nonlinear in futures prices.[1]

A recent survey of Iowa farmers indicated that more producers use options to hedge than futures (Sapp, 1990). This is clearly in contrast with the LMH result and raises the question of the conditions under which producers may find it optimal to use options to hedge. In the context of the

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[1]This motivation against using options to hedge price risk seems counter-intuitive. For example, Cox and Rubinstein (p. 46) argue that a primary reason for the success of options markets is that they offer patterns of returns that cannot be obtained by ownership of the underlying asset (the futures contract in the LMH model). We can construct synthetic futures with suitable combinations of options. However, the opposite is not true; the set of returns possible with options includes the set of returns that is possible with futures. The obvious solution in the LMH model would be to exclude futures and include both put and call options. Here, the model would suggest that only synthetic futures positions be used for hedging purposes.

LMH result, this is equivalent to the conditions under which the risk faced by producers is nonlinear in futures prices. One way to introduce this nonlinearity is to introduce production uncertainty. For example, Iowa grain producers may believe that low individual grain yields (caused by drought) may be associated with high grain prices. If producers have sold more grain on the futures or forward markets than they obtain from on-farm production then they will be forced to purchase expensive grain to meet contractual commitments. Alternatively, if prices are low (due possibly to abundant rainfall in the Upper Midwest) and if farm production is greater than anticipated, then the producer may not have hedged enough production to eliminate all price risk. In the case where the expected correlation between local output and futures prices is zero, production risk leads to another nonlinear relationship because the effect of quantity uncertainty on profit is greater at higher prices.

The purpose of this paper is to introduce production uncertainty into the LMH model both theoretically and through simulation examples, and to show how options can be used to hedge against production uncertainty when output is uncertain. We focus on the case when investors believe that both futures and options are unbiased.

The rest of this paper is organized as follows. The model is set out under the assumption that local production variation does not affect the price of the commodity, and the optimal positions for futures and options are illustrated. The effect of production uncertainty on optimal hedging behavior is then emphasized. In the next section, the independence assumption is relaxed. As might be expected with random variables of prices and output, two financial instruments, and a truncated distribution, results for the general case require a lengthy and somewhat tedious derivation. One motivation for presenting these derivations is that they can be used as the basis for a more specific and richer analysis. This is demonstrated in the last section of the paper by simulating the decision process of an Iowa corn producer.

MODEL

It is possible to replicate the payoff of any combination of futures, puts, and calls with any two of these three assets. Our attention will therefore be focused on futures and put options. For simplicity, only one strike price for put options is considered and this is assumed to be the current futures price. Suppose that a producer makes hedging decisions after he or she decides input levels. Suppose for the time being that production variability in one region does not affect prices. The random profit at harvest can be written as:

$$\tilde{Y} = \tilde{P}\tilde{Q} + (F - \tilde{F})\, X + (R - \tilde{R})\, Z - C(\bar{Q}) \tag{1}$$

where, the diacritical marks of ~ and – denote a random variable and expected value, $\tilde{Q}$ is the random output, X and Z are the futures quantity and put options quantity sold by the producer, and $C(\bar{Q})$ is a cost function, F is the futures price at the time of a production decision, $\tilde{F}$ is a futures price at harvest time, $\tilde{P}$ is the cash price at harvest time, R is a put option price at decision time, and $\tilde{R}$ is the terminal value of a put option with:

$$\tilde{R} = (F - \tilde{F})L \tag{2}$$

where,

$$L = 1 \; if \; \tilde{F} \leq F$$
$$L = 0 \; if \; \tilde{F} \geq F.$$

Following Benninga, Eldor, and Zilcha (1983, 1984) and Lapan, Moschini, and Hanson (1991) the cash price is assumed to be a linear regressive function of the futures price:

$$\tilde{P} = \tau + \beta\tilde{F} + \tilde{\epsilon} \tag{3}$$

where $E[\tilde{\epsilon}] = 0$ and $\tilde{\epsilon}$ is assumed to be independent of $\tilde{F}$ and $\tilde{Q}$. Substituting (2) and (3) into (1), the random profit is:

$$\tilde{Y} = (\tau + \beta\tilde{F} + \tilde{\epsilon})\,\tilde{Q} + (F - \tilde{F})\,X + \{\,R - (F - \tilde{F})\,L\,\}\,Z - C(\tilde{Q}). \tag{4}$$

The producer is assumed to choose X^* and Z^* to maximize expected utility of the random profit. The first-order conditions can be written as:

$$\begin{aligned} E\,[\,u'(\tilde{Y})\,(\tilde{F} - \bar{F})\,] &= E\,[\,(\tilde{F} - \bar{F})\,E\,\{\,u'(\tilde{Y})\mid\tilde{F}\,\}\,] \\ &= E\,[\,g(\tilde{F})\,(\tilde{F} - \bar{F})\,] = 0 \end{aligned} \tag{5}$$

and

$$\begin{aligned} E\,[\,u'(\tilde{Y})\,\{\,\bar{F} - \tilde{F})\,L - \bar{R}\,\}\,] &= E\,[\,\{\,(\bar{F} - \tilde{F})\,L - \bar{R}\,\}\,E\,\{\,u'(\tilde{Y})\mid\tilde{F}\,\}\,] \\ &= E\,[\,g(\tilde{F})\,\{\,(\bar{F} - \tilde{F})\,L - \bar{R}\,\}\,] = 0. \end{aligned} \tag{6}$$

Given subjective distributions of prices and output, and a known utility function, the optimal futures and put options position, denoting X^* and Z^*, can be found by numerical optimization. However, because the true utility function and the distributions of prices and output are not known, results derived from particular examples may be misleading and lack generality.

The mean value theorem[2] can be used to obtain results for the general case where neither the utility function nor the price and output distributions is known. Using this theory, one can solve for both the sign and the relative size of X^* and Z^*. In the general case, it can be shown that options are almost always used to hedge production uncertainty in a way that makes intuitive sense.

The mean value theorem (Stein, 1968) is:

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[2] Refer to Stein(1968) or most calculus books.

Let g be a continuous function on [a,b] and have a derivative at all x in [a,b] except at perhaps x = a and x = b. Then there is at least one argument X such that a < X < b and

$$g'(x) = \frac{g(b) - g(a)}{b - a}$$

The conditions for using the mean value theorem are continuity and differentiability. With the existence of options, the price distribution is truncated at the strike price and thus $g(\tilde{F})$ is not differentiable at $\bar{F}$. That is, since L = 1 when $\tilde{F}$ approaches $\bar{F}$ from the left side and L = 0 when $\tilde{F}$ approaches $\bar{F}$ from the right side, the slopes of $g(\tilde{F})$ with respect to $\tilde{F}$ at $\bar{F}^+$ and $\bar{F}^-$ are

$$\lim_{\tilde{F}\to\bar{F}^+} \frac{\partial g(\tilde{F})}{\partial \tilde{F}} = E\ [\ u''(\tilde{Y})\ (\beta\tilde{Q}\ - X)\ |\ \tilde{F}\to\bar{F}^+]$$

and

$$\lim_{\tilde{F}\to\bar{F}^-} \frac{\partial g(\tilde{F})}{\partial \tilde{F}} = E\ [\ u''(\tilde{Y})\ (\beta\tilde{Q}\ - X + Z)\ |\ \tilde{F}\to\bar{F}^-\ ].$$

Thus,

$$if\ Z \neq 0,\ \lim_{\tilde{F}\to\bar{F}^+} \frac{\partial g(\tilde{F})}{\partial \tilde{F}} \neq \lim_{\tilde{F}\to\bar{F}^-} \frac{\partial g(\tilde{F})}{\partial \tilde{F}}\ .$$

When $\tilde{F}$ approaches $\bar{F}$ from the left side the slope of $g(\tilde{F})$, it is different from the slope of $g(\tilde{F})$ when $\tilde{F}$ approaches $\bar{F}$ from the right. Therefore, $g(\tilde{F})$ is not differentiable at $\bar{F}$ (although it is continuous). However, $g(\tilde{F})$ is differentiable over the interval $[0, \bar{F}]$ and $[\bar{F}, \infty]$ and thus the mean value theorem can be applied to $g(\tilde{F})$ in $[0, \bar{F}]$ and $[\bar{F}, \infty]$.

Before proceeding, note that the function $g(\tilde{F})$ is strictly convex in $\tilde{F}$ over the interval $[0, \bar{F}]$ or $[\bar{F}, \infty]$. This can be shown by differentiating $g(\tilde{F})$ with respect to $\tilde{F}$ as follows:
To characterize the sign of $\partial^2 g(\tilde{F})/\partial\tilde{F}^2$ we need to sign $u'''(\tilde{Y})$. First, consider that absolute risk

$$\frac{\partial^2 g(\tilde{F})}{\partial \tilde{F}^2} = E\ [u'''(\tilde{Y})\ (\beta\tilde{Q} - X + LZ)^2 \mid \tilde{F}\ ].$$

aversion is given by $A \equiv -u''(\tilde{Y})/u'(\tilde{Y})$. Then, nonincreasing absolute risk aversion means:

$$\frac{\partial A}{\partial \tilde{Y}} = -\frac{u'''(\tilde{Y})}{u'(\tilde{Y})} + [\ \frac{u''(\tilde{Y})}{u'(\tilde{Y})}\ ]^2 \leq 0.$$

It indicates that $u'''(\tilde{Y})$ must be positive. Therefore, under nonincreasing absolute risk aversion, the second derivative of $g(\tilde{F})$ with respect to $\tilde{F}$ is positive since both terms within the expectation are positive. Consequently, under nonincreasing absolute risk aversion $g(\tilde{F})$ is strictly convex and differentiable in futures price over the interval $[0, \bar{F}\ ]$ or $[\bar{F}\ , \infty]$.

Figure 1 shows how the mean variance theorem can be applied to $g(\tilde{F})$. Suppose that the shape of $g(\tilde{F})$ is DACBE in Figure 2. Applying the mean value theorem to $g(\bullet)$ over the interval $[\bar{F}\ , \infty]$, there is a futures price $\tilde{f}$ such that $g(\tilde{f})$ will have the same slope as line AB. Here, $\tilde{f}$ is unique given the strict convexity of $g(\tilde{F})$ in $[0, \bar{F}\ ]$ and $[\bar{F}\ , \infty]$. Equivalently, the slope of the line connecting $(\bar{F}\ , g(\bar{F}))$ and $(\tilde{F}\ , g(\tilde{F}))$ is the same as $g'(\tilde{f})$, i.e.,

$$\begin{aligned}\frac{g(\tilde{F}) - g(\bar{F})}{\tilde{F} - \bar{F}} = \frac{\partial g(\tilde{f})}{\partial \tilde{f}} &= E\ [\ u''(\tilde{y})\ (\beta\tilde{Q} - X + LZ) \mid \tilde{f}\ (\tilde{F})\ ] \\ &= E\ [u''(\tilde{y})\ (\beta\tilde{Q} - X + LZ\ ) \mid \tilde{F}\ ]\end{aligned} \tag{7}$$

where $\tilde{y} = (\tau + \beta\tilde{f} + \tilde{\epsilon})\tilde{Q} + (\bar{F} - \tilde{f})X + \{\bar{R} - (\bar{F} - \tilde{f})L\}Z - C(\tilde{Q})$ and $\tilde{f}$ is a monotonically increasing function of $\tilde{F}$ because $g(\tilde{F})$ is strictly convex in $[0, \bar{F}]$ and $[\bar{F}, \infty]$. The left side of equation (7) represents the slope of line AB and the right side represents the slope of $g(\tilde{f})$. This analysis can be conducted for all $\tilde{F}$ in $[\bar{F}\ , \infty]$ by connecting point A and any point on curve $g(\tilde{F})$. Applying the mean value theorem to $g(\tilde{F})$ over the interval $[0, \bar{F}]$ is similar to the explanation here.

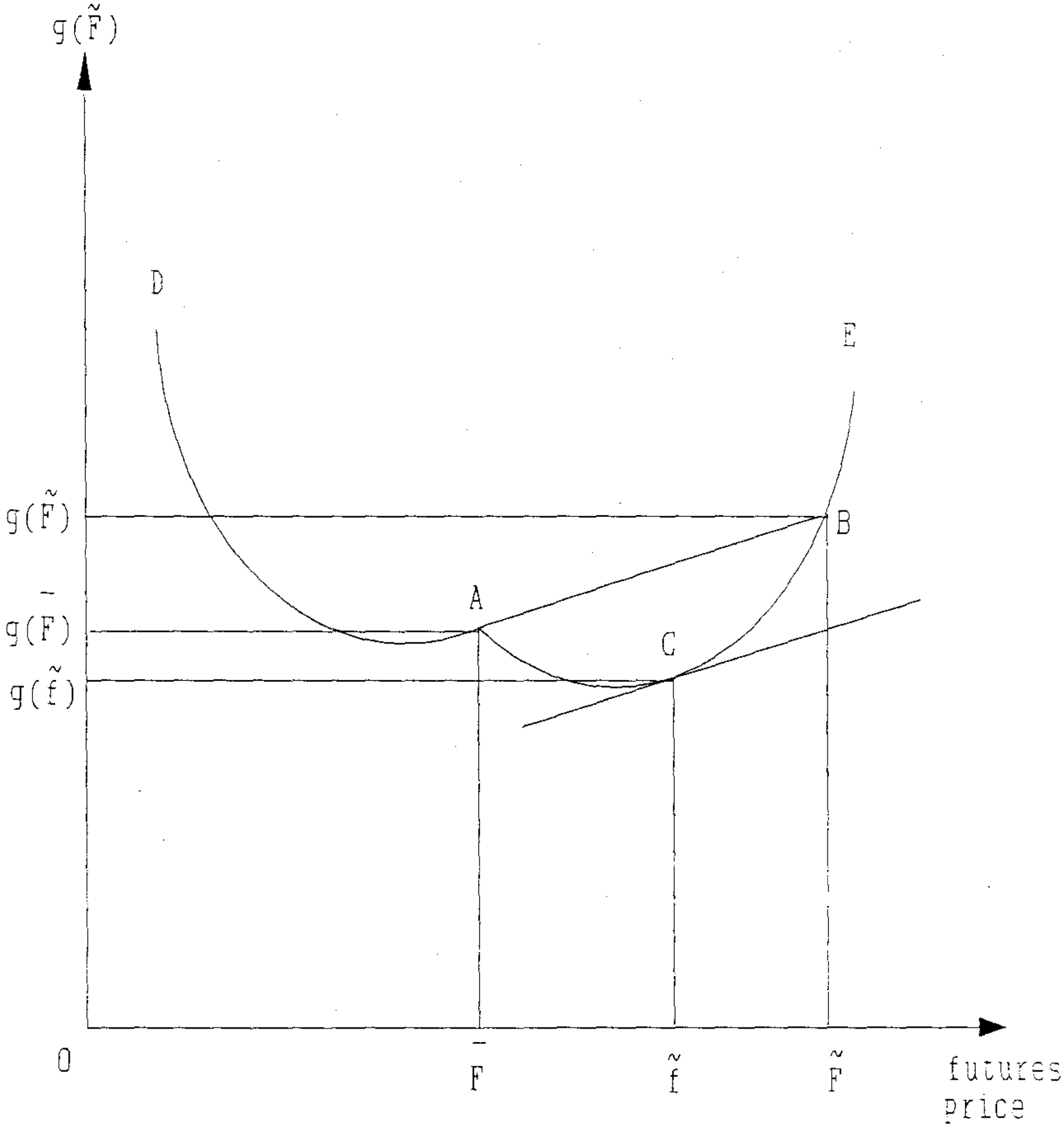

Figure 1. A schematic representation of how the expected marginal utility conditioned on futures price responds to the futures price using the mean value theorem

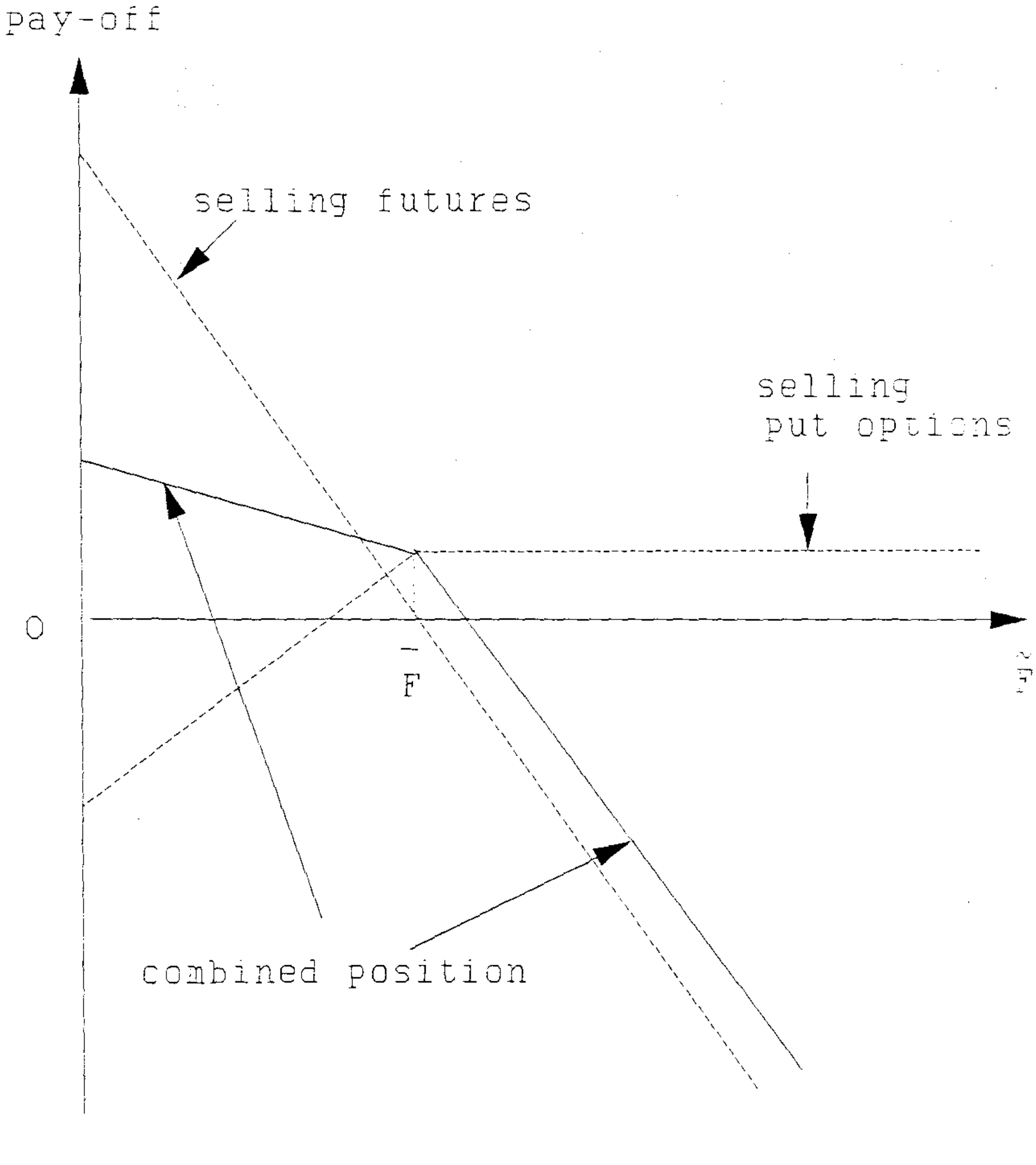

Figure 2. An example of the combined position of optimal futures and options

From (7),

$$g(\tilde{F}) = E\,[\,u'(\tilde{Y}) \mid \tilde{F}\,] \\ = g(\bar{F}) + (\tilde{F} - \bar{F})\,E\,[u''(\bar{y})\,(\beta\tilde{Q} - X + LZ) \mid \tilde{F}\,]. \tag{8}$$

Substituting (8) into (5) gives:

$$E\,[u'(\tilde{Y})\,(\tilde{F} - \bar{F})\,] = E\,[\,g\,(\tilde{F})\,(\tilde{F} - \bar{F})\,] \\ = E\,[\,(\tilde{F} - \bar{F})\,g(\bar{F})\,] + E\,[(\tilde{F} - \bar{F})^2 \\ \{\,E\,[\,u''(\bar{y})\,(\beta\tilde{Q} - X + LZ) \mid \tilde{F}\,]\,\}\,] = 0.$$

Since $g(\bar{F})$ is a fixed number and $E[E\{\bullet|\tilde{F}\}] = E[\bullet]$, this can be rewritten as:

$$E\,[\,(\tilde{F} - \bar{F})\,]\,g\,(\bar{F}) + E\,[u''(\bar{y})\,(\tilde{F} - \bar{F})^2\,(\beta\tilde{Q} - X + LZ)\,] = 0.$$

Therefore, under the unbiased assumption, i.e., $E[\tilde{F} - \bar{F}] = 0$, equation (5) can be rewritten as

$$E\,[u''(\bar{y})\,(\tilde{F} - \bar{F})^2\,(\beta\tilde{Q} - X + LZ)\,] = 0. \tag{9}$$

Equation (6) can be rewritten in a similar manner:

$$E\,[u'(\tilde{Y})\,\{\,L(\bar{F} - \tilde{F}) - \bar{R}\}\,] = 0 \\ = -\alpha E_1\,[\,g(\tilde{F})\,(\tilde{F} - \bar{F})\,] - \bar{R}E\,[u'(\tilde{Y})\,], \tag{10}$$

where $\alpha = \text{Prob}[\,\tilde{F} \leq \bar{F}\,]$ and the subscript 1 represents the conditional expectation on $\tilde{F} \leq \bar{F}$, i.e., $E_1[\bullet] = E[\bullet|\tilde{F} \leq \bar{F}\,]$. The first term on the right side of equation (10) can be written, substituting $g(\tilde{F})$ of (8), as:

$$-\alpha E_1\,[\,g(\tilde{F})\,(\tilde{F}-\bar{F})\,] = -\alpha E_1[(\tilde{F} - \bar{F})\,\{g(\bar{F}) \\ + (\tilde{F} - \bar{F})E\,[u''\,(\bar{y})\,(\beta\tilde{Q} - X + LZ)|\tilde{F}\,]\,\}\,]$$

Factoring out terms within $\{\bullet\}$ and using $E[E\{\bullet|\tilde{F}\}] = E[\bullet]$, it follows that

$$-\alpha E_1[g(\tilde{F})\,\tilde{F} - \bar{F}\,)] = -\alpha g\,(\bar{F})\,E_1\,[\tilde{F} - \bar{F}\,] \\ -\alpha E_1\,[\,u''(\bar{y})\,(\tilde{F} - \bar{F})^2\,(\beta\tilde{Q} - X + Z)\,].$$

Now using $\bar{R} = E[(\bar{F} - \tilde{F})L] = -\alpha E_1[\tilde{F} - \bar{F}\,]$ and $g(\bar{F}\,) = E\,[u'(\tilde{Y})|\,\tilde{F} = \bar{F}\,]$ yields:

$$-\alpha E_1 \ [g(\tilde{F}) \ (\tilde{F} - \bar{F}) \ ] = -\bar{R}E \ [\ u'(\tilde{Y}) \ | \ \tilde{F} = \bar{F}]$$
$$-\alpha E_1 \ [\ u''(\tilde{y}) \ (\tilde{F} - \bar{F})^2 \ (\beta\tilde{Q} - X + Z) \ ].$$

Therefore, (10) can be rewritten as:

$$E \ [u' \ (\tilde{Y}) \ \{ \ L \ (\tilde{F} - \bar{F}) - \bar{R} \ \} \ ] = -\bar{R} \ \{ \ E \ [u' \ (\tilde{Y}) \ ] - E \ [u' \ (\bar{Y}) \ | \ \tilde{F} = \bar{F} \ ] \ \}$$
$$-\alpha E_1 \ [\ u'' \ (\tilde{y}) \ (\tilde{F} - \bar{F})^2 \ (\beta\tilde{Q} - X + Z) \ ] = 0. \tag{11}$$

Using (9) and (11), the first-order conditions are

$$E \ [\ u'' \ (\tilde{y}) \ (\tilde{F} - \bar{F})^2 \ \{ \ \beta\tilde{Q} - X + LZ \ \} \ ] = 0 \tag{12}$$

and

$$-\bar{R} \ \{ \ E \ [u' \ (\tilde{Y}) \ ] - E \ [\ u'(\tilde{Y}) \ | \ \tilde{F} = \bar{F} \ ] \ \}$$
$$- \ \alpha E_1 \ [\ u''(\tilde{y}) \ (\tilde{F} - \bar{F})^2 \ \{ \ \beta\tilde{Q} - X + Z \ \} \ ] = 0. \tag{13}$$

Factoring out terms in $\{\bullet\}$, at the optimal futures and put options positions equations (12) and (13) can be rewritten as follows:

$$\beta E \ [\ u'' \ (\tilde{y}) \ (\tilde{F} - \bar{F})^2 \ \tilde{Q} \ ] - X^* \ E \ [\ u'' \ (\tilde{y}) \ (\tilde{F} - \bar{F})^2 \ ]$$
$$+ \ Z^*\alpha E_1 \ [\ u'' \ (\tilde{y}) \ (\tilde{F} - \bar{F})^2 \ ] = 0. \tag{14}$$

and

$$- \ \bar{R} \ \{ \ E \ [\ u' \ (\tilde{Y}) \ ] - E \ [u' \ (\tilde{Y}) \ | \ \tilde{F} - \bar{F} \ ] \ \}$$
$$- \ \beta\alpha E_1 \ [\ u'' \ (\tilde{y}) \ \tilde{F} - \bar{F})^2 \ \tilde{Q} \ ]$$
$$+ \ X^*\alpha E_1 \ [u''(\tilde{y}) \ (\tilde{F} - \bar{F})^2 \ ]$$
$$- \ Z^*\alpha E_1 \ [\ u'' \ (\tilde{y}) \ (\tilde{F} - \bar{F})^2 \ ] = 0. \tag{15}$$

Consequently, equations (12) and (13) can be rearranged as

$$\begin{aligned} \mathcal{L}_{FF}X^{*} - \mathcal{L}_{FF1}Z^{*} &= \beta\alpha \\ -\mathcal{L}_{FF1}X^{*} + \mathcal{L}_{FF1}Z^{*} &= -\beta b + c \end{aligned} \tag{16}$$

where,

$$\begin{aligned} \mathcal{L}_{FF} &= E\,[\,u''(\tilde{y})\,(\tilde{F} - \bar{F})^2\,] < 0 \\ \mathcal{L}_{FF1} &= \alpha E_1\,[\,u''(\tilde{y})\,(\tilde{F} - \bar{F})^2\,] < 0 \\ \alpha &= E\,[\,u''(\tilde{y})\,\tilde{Q}\,(\tilde{F} - \bar{F})^2\,] < 0 \\ b &= \alpha E_1\,[\,u''(\tilde{y})\,\tilde{Q}\,(\tilde{F} - \bar{F})^2\,] < 0 \\ c &= -\bar{R}\,\{\,E\,[\,u'(\tilde{Y})\,] - E\,[\,u'(\tilde{Y})|\,\tilde{F} = \bar{F}\,]\,\} < 0 \end{aligned}$$

By assumption $u''(\tilde{y})$ is always negative for all $\tilde{y}$, and $\tilde{Q}$ and $(\tilde{F} - \bar{F})^2$ are positive so that $\mathcal{L}_{FF}$, $\mathcal{L}_{FF1}$, a and b are all negative. And $\mathcal{L}_{FF} < \mathcal{L}_{FF1} < 0$ and $a < b < 0$ since

$$\int_0^{\infty} E\,[\,h\,(\tilde{F},\tilde{Q})\mid\tilde{F}\,]\,\Gamma\,(\tilde{F})\,d\tilde{F} < \int_0^{\bar{F}} E\,[\,h\,(\tilde{F},\tilde{Q})\mid\tilde{F}\,]\,\Gamma\,(\tilde{F})\,d\tilde{F}$$

if $E\,[h(\tilde{F}, \tilde{Q})|\tilde{F}]$ is negative for all futures prices. Here $h(\bullet)$ is some functional form for $\tilde{F}$ and $\tilde{Q}$, and $\Gamma(\tilde{F})$ is the density function for $\tilde{F}$. The sign of c is less straightforward than for $\mathcal{L}_{FF}$, $\mathcal{L}_{FF1}$, a, and b. Using $c = -\bar{R}\;[E\{g(\tilde{F})\} - E\{g(\bar{F})\}] = -\bar{R}\;E[g(\tilde{F}) - g(\bar{F})]$ and substituting $g(\tilde{F}) - g(\bar{F}) = (\tilde{F} - \bar{F})E[u''(\tilde{y})(\beta\tilde{Q} - X + LZ)|\tilde{F}]$, then c can be rewritten

$$\begin{aligned} c &= -\bar{R}E\,[\,(\,\tilde{F} - \bar{F})\,E\,\{\,u''(\tilde{y})\,(\beta\tilde{Q} - X + LZ)\mid\tilde{F}\,\}\,] \\ &= -\bar{R}E\,[\,u''(\tilde{y})\,(\tilde{F} - \bar{F})\,(\beta\tilde{Q} - X + LZ)\,] \\ &= -\bar{R}Cov\,[\,\tilde{F},\,u''(\tilde{y})\,(\beta\tilde{Q} - X + LZ)\,]. \end{aligned} \tag{17}$$

The covariance term has the sign of $\partial[u''(\tilde{y})(\beta\tilde{Q} - X + LZ)]/\partial\tilde{F} = u'''(\tilde{y})(\beta\tilde{Q} - X + LZ)^2(\partial\tilde{f}/\partial\tilde{F})$, which is positive under nonincreasing absolute risk aversion. Therefore, c is also negative.

The optimal futures and put options amounts of X^* and Z^* can be expressed as a function of $\mathscr{L}_{FF}$, $\mathscr{L}_{FF1}$, a, b and c. In effect, these terms are also functions of X^* and Z^*. However, when X^* and Z^* are expressed in terms of $\mathscr{L}_{FF}$, $\mathscr{L}_{FF1}$, a, b, and c, we find the signs of X^* and Z^* and/or the relative size of X^* and Z^*. For example, suppose that $X^* = h_1(X^*, Z^*) > 0$, $Z^* = h_2(X^*, Z^*) > 0$ and $h_1(X^*, Z^*)$ are always greater than $h_2(X^*, Z^*)$, where h_1 and h_2 are some functional forms. Then, we can conclude that $X^* > Z^* > 0$ and that the combined position is decreasing in futures price as shown in Figure 2, which shows a standard "payoff" diagram. This represents profit or loss at harvest time in futures and options to the futures price on that date. If the producer has sold a futures contract, then profits from this portion of the portfolio fall as the futures price increases. This is true because he or she has promised to deliver at a fixed price an asset whose value is increasing. Selling put options, on the other hand, leads to a profit or payoff that is increasing in futures prices in the price range from 0 to the strike price ($\bar{F}$) and independent of the futures price in the region from $\bar{F}$ to infinity. If we know that the producer has sold more futures contracts than he or she has sold options, then we can describe how the total or net position responds to the futures price. The payoff line for the combined position is determined by adding the payoffs of two assets vertically for each futures price realized at harvest time. From Figure 2, at any point up to the strike price, the reduced profit in futures market from an increase in futures price is greater than the increased benefit (reduced loss) in the options market. At any point beyond the strike price, a price increase causes the loss in futures increase while the profit in put options does not affect changes in the futures price. Therefore, we can conclude that the net payoff is decreasing in futures price in all regions.

When $\mathscr{L}_{FF}$, $\mathscr{L}_{FF1}$, a, b and c are considered as fixed coefficients, X^* and Z^* can be obtained from (16) as:

$$X^* = \frac{\{\ \beta(a - b) + c\ \}\ \mathscr{L}_{FFI}}{\Delta} > 0 \qquad \textbf{(18)}$$

and

$$Z^* = \frac{\beta\,(\mathcal{L}_{FF1}\, a\ -\ \mathcal{L}_{FF}\, b)\ +\ \mathcal{L}_{FF}\, c}{\Delta} \tag{19}$$

where, $\Delta = \mathcal{L}_{FF}\mathcal{L}_{FF1} - \mathcal{L}_{FF1}^2 = \mathcal{L}_{FF1}\,(\mathcal{L}_{FF} - \mathcal{L}_{FF1}) > 0$ since $\mathcal{L}_{FF} < \mathcal{L}_{FF1} < 0$. Consequently, the producer always sells futures, i.e., $X^* > 0$. This occurs regardless of the level of X since (18) is always positive. On the other hand, whether the producer sells or buys put options is ambiguous.

Rather than analyze the relative positions described in equations (18) and (19), we now focus on the additional hedging caused by production uncertainty. It is useful to show that under nonstochastic output the optimal decision is to sell $\beta\bar{Q}$ on the futures market and stay out of the options market. To find the optimal stochastic position from the results reported below we simply add this nonstochastic position.

To emphasize production uncertainty, it is useful to substitute $\tilde{Q} \equiv \bar{Q} + (\tilde{Q} - \bar{Q})$ into a, b, and c in equations (16) and (17) so that production can be separated into a nonstochastic and a stochastic component. First, substituting $\tilde{Q} = \bar{Q} + (\tilde{Q} - \bar{Q})$ into c in (17) and rearranging, we obtain:

$$\begin{aligned}
c &= \bar{R}\ \{\beta E\ [\ u''(\tilde{y})\ \tilde{Q}\ (\tilde{F} - \bar{F})\ ]\ -\ X^* E\ [\ u''(\tilde{y})\ (\tilde{F} - \bar{F})\ ] \\
&\qquad +\ Z^*\ \alpha\ E_1\ [\ u''(\tilde{y})\ (\tilde{F} - \bar{F})\ ]\ \} \\
&= -\bar{R}\ \{\ \beta\bar{Q}E\ [\ u''(\tilde{y})\ (\tilde{F} - \bar{F})\ ]\ +\ \beta E\ [u''(\tilde{y})\ (\tilde{Q} - \bar{Q})\ (\tilde{F} - \bar{F})\ ] \\
&\qquad -X^* E\ [\ u''(\tilde{y})\ (\tilde{F} - \bar{F})\ ]\ +\ Z^*\alpha E_1\ [\ u''(\tilde{y})\ (\tilde{F} - \bar{F})\ ]\ \} \\
&= -\bar{R}\ [\ \beta\bar{Q}\mathcal{L}_F\ +\ \beta\mathcal{L}_{QF}\ -\ X^*\mathcal{L}_F\ +\ Z^*\mathcal{L}_{F1}\ ],
\end{aligned}$$

where $\mathcal{L}_F = E[u''(\tilde{y})(\tilde{F} - \bar{F})]$, $\mathcal{L}_{F1} = \alpha E_1[u''(\tilde{y})(\tilde{F} - \bar{F})]$ and $\mathcal{L}_{QF} = E[u''(\tilde{y})(\tilde{Q} - \bar{Q})(\tilde{F} - \bar{F})$. Therefore,

$$c = -\bar{R}\ [\ \beta\bar{Q}\mathcal{L}_F\ +\ \beta\mathcal{L}_{QF}\ -\ X^*\mathcal{L}_F\ +\ Z^*\mathcal{L}_{F1}\ ]. \tag{20}$$

Substituting $\tilde{Q} \equiv \bar{Q} + (\tilde{Q} - \bar{Q})$ into a and b, they can be rewritten as

$$a = E\,[\,u''(\tilde{y})\tilde{Q}(\tilde{F} - \bar{F})^2\,]$$
$$= \bar{Q}E\,[\,u''(\tilde{y})\,(\tilde{F} - \bar{F})^2\,] + E\,[\,u''(\tilde{y})\,(\tilde{Q} - \bar{Q})\,(\tilde{F} - \bar{F})^2\,]$$

so that

$$a = \bar{Q}\mathcal{L}_{FF} + \mathcal{L}_{QFF}$$

where $\mathcal{L}_{QFF} = E[u''(\tilde{y})(\tilde{Q} - \bar{Q})(\tilde{F} - \bar{F})^2]$.

Similarly,

$$b = \alpha E_1\,[\,u''(\tilde{y})\,\tilde{Q}\,(\tilde{F} - \bar{F})^2\,]$$
$$= \bar{Q}\mathcal{L}_{FFI} + \mathcal{L}_{QFFI}$$

where $\mathcal{L}_{QFFI} = \alpha E_1[u''(\tilde{y})(\tilde{Q} - \bar{Q})(\tilde{F} - \bar{F})^2]$.

Substituting the a and b obtained here into (18), then optimal X^* can be rewritten as:

$$X^* = \frac{\mathcal{L}_{FFI}\{\,\beta(a - b) + c\,\}}{\Delta}$$
$$= \frac{\mathcal{L}_{FFI}\{\,\beta\,(\bar{Q}\mathcal{L}_{FF} + \mathcal{L}_{QFF} - \bar{Q}\mathcal{L}_{FFI} - \mathcal{L}_{QFFI}) + c\,\}}{\Delta}$$
$$= \frac{\beta\bar{Q}\mathcal{L}_{FFI}\,(\mathcal{L}_{FF} - \mathcal{L}_{FFI})}{\Delta} + \frac{\mathcal{L}_{FFI}\,\{\,\beta\,(\mathcal{L}_{QFF} - \mathcal{L}_{QFFI}) + c\,\}}{\Delta}.$$

Since $\Delta = \mathcal{L}_{FFI}(\mathcal{L}_{FF} - \mathcal{L}_{FFI})$, it follows that

$$X^* = \beta\bar{Q} + \frac{\mathcal{L}_{FFI}\,\{\,\beta\,(\mathcal{L}_{QFF} - \mathcal{L}_{QFFI}) + c\,\}}{\Delta}.$$

Substituting a and b into (19) again yields:

$$Z^* = \frac{\beta(\mathcal{L}_{FFI}\,a - \mathcal{L}_{FF}\,b) + \mathcal{L}_{FF}\,c}{\Delta}$$
$$= \frac{\beta\mathcal{L}_{FFI}\,(\bar{Q}\mathcal{L}_{FF} + \mathcal{L}_{QFF}) - \mathcal{L}_{FF}\,(\bar{Q}\mathcal{L}_{FFI} - \mathcal{L}_{QFFI}) + \mathcal{L}_{FF}\,c}{\Delta}$$

so that

$$Z^* = \frac{\beta\,(\mathcal{L}_{FFI}\mathcal{L}_{QFF} - \mathcal{L}_{FF}\mathcal{L}_{QFFI}) + \mathcal{L}_{FF}\,c}{\Delta}.$$

Therefore, X^* and Z^* can be rewritten as:

$$X^* = \beta\bar{Q} + \frac{\mathcal{L}_{FFI}\{\beta(\mathcal{L}_{QFF} - \mathcal{L}_{QFFI}) + c\}}{\Delta} \tag{21}$$

$$Z^* = 0 + \frac{\beta(\mathcal{L}_{FFI}\mathcal{L}_{QFF} - \mathcal{L}_{FF}\mathcal{L}_{QFFI}) + \mathcal{L}_{FF}\, c}{\Delta}. \tag{22}$$

Substituting X^* and Z^* from (21) and (22) into (20), we obtain:

$$c = -\bar{R}\,[\,\beta\bar{Q}\mathcal{L}_F + \beta\mathcal{L}_{QF} - \mathcal{L}_F\{\,\beta\bar{Q} + \frac{\mathcal{L}_{FFI}(\beta\mathcal{L}_{QFF} - \beta\mathcal{L}_{QFFI} + c)}{\Delta}\,\}$$
$$+ \mathcal{L}_{FI}\{\,\frac{\beta\mathcal{L}_{FFI}\mathcal{L}_{QFF} - \beta\mathcal{L}_{FF}\mathcal{L}_{QFFI} + \mathcal{L}_{FF}\, c}{\Delta}\,\}\,],$$

which can be rewritten as:

$$c = -\bar{R}\,[\,\beta\Delta\mathcal{L}_{QF} - \beta\mathcal{L}_F\mathcal{L}_{FFI}(\mathcal{L}_{QFF} - \mathcal{L}_{QFFI}) + \beta\mathcal{L}_{FI}\,(\mathcal{L}_{FFI}\mathcal{L}_{QFF} - \mathcal{L}_{FF}\mathcal{L}_{QFFI})\,]$$
$$[\,\frac{1}{\bar{R}\,(\mathcal{L}_F\mathcal{L}_{FFI} - \mathcal{L}_{FI}\mathcal{L}_{FF}) - \Delta}\,]. \tag{23}$$

When the production process is nonstochastic, $\mathcal{L}_{QF}$, $\mathcal{L}_{QFF}$ and $\mathcal{L}_{QFFI}$ are zero[3] and thus $c = 0$. The second terms of the right sides of equations (21) and (22) are all zero, so the optimal futures amount under the production certainty is $\beta\bar{Q}$ and options are redundant. Consequently, the right sides of (21) and (22) can be separated by two parts: the first term of the right side is optimal futures and put options sold by the producer under production certainty and the second term represents the additional futures and put options arising from production uncertainty.

Under the independence of price uncertainty and output uncertainty, $\mathcal{L}_{QFF} = E[u''(\tilde{y})(\tilde{Q} - \bar{Q})(\tilde{F} - \bar{F})^2] = E[(\tilde{F} - \bar{F})^2 \mathrm{Cov}\{u''(\tilde{y}), \tilde{Q} \mid \tilde{F}\}]$. The conditional covariance has the sign of $\partial u''(\tilde{y})/\partial\tilde{Q} = \beta u'''(\tilde{y})\tilde{f}(\partial\tilde{f}/\partial\tilde{F})$, which is positive since $u''' > 0$ and $\partial\tilde{f}/\partial\tilde{F} > 0$ with nonincreasing absolute risk aversion. The additional futures and put options due to production

[3] For example, $\mathcal{L}_{QF} = E[u''(\tilde{y})(\tilde{Q} - \bar{Q})(\tilde{F} - \bar{F})] = 0$ since $\tilde{Q} = \bar{Q}$ under nonstochastic production.

uncertainty are denoted as ΔX and ΔZ, respectively. Then,

$$\Delta X - \Delta Z = \frac{(\mathscr{L}_{FFI} - \mathscr{L}_{FF})\ (c - \beta \mathscr{L}_{QFFI})}{\Delta} < 0 \tag{24}$$

since $\mathscr{L}_{FFI} - \mathscr{L}_{FF} > 0$, $c < 0$ and $\mathscr{L}_{QFFI} > 0$.[4] Therefore, we can conclude that $\Delta Z > \Delta X$ even though their signs are unknown.

This result is the key for delineating the additional hedging position. Production uncertainty (independent of prices) causes producers to take options and futures positions that are different from those taken when production is certain. The precise nature of the additional positions will depend on the individual's utility function, and the subjective distributions of output and prices. Nevertheless, five possible outcomes are shown in Figure 3. The dotted lines represent payoff diagrams in futures and put options and the continuous line represents the payoff for the combined position. The underhedging against low futures prices is common to all five possibilities. Intuitively, this occurs because profit risk caused by output uncertainty is lowest at low prices.[5] In four of the five cases (b through e) the producer takes additional insurance when futures price is high. This can be explained by the positive correlation between futures price and profit risk. Case a is the only exception to this rule. The payoff diagram for the combined position of Case a is inversely V-shaped (hereafter, denoted as "Λ" shaped). Here the effect of production uncertainty is to hedge against small price changes and to accept losses when price changes are large. This situation might occur when the producer's subjective estimate of output variance is low and he or she perceives that there is little possibility of high prices so he or she is unconcerned about this possibility.

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[4] Similar to $\mathscr{L}_{QFF} > 0$, $\mathscr{L}_{QFFI}$ is also positive.

[5] For simplicity, suppose that $\tilde{F}=\tilde{P}$ and that $\tilde{Y}_0$ is the unhedged profit. The conditional variance of unhedged profit on the futures price is $Var[\tilde{Y}_0 \mid \tilde{F}]Var(\tilde{Q})$. Therefore, the profit variation conditioned on the futures price increases as the futures price goes up.

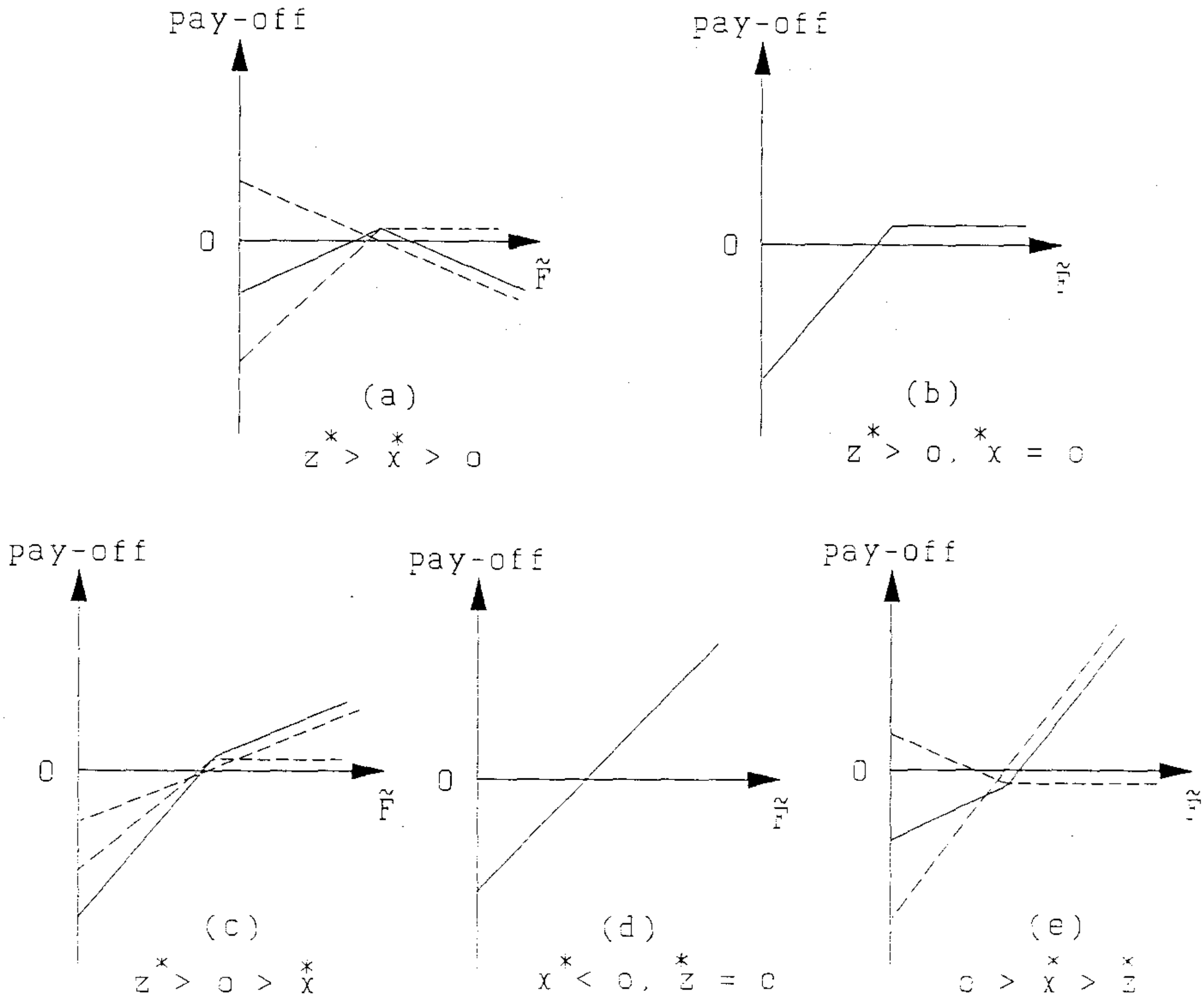

Figure 3. Additional hedging positions required due to production uncertainty

Interestingly, there is one possibility (d) where no options are purchased (Figure 3). In this case, however, the number of futures contracts is different from that in the LMH model (i.e., $X^* = \beta\bar{Q} + \Delta X$, where $\Delta X < 0$ since $\Delta X < \Delta Z = 0$). This leads to the perhaps unsurprising conclusion that production uncertainty creates hedging decisions that are different in the LMH model regardless of the functional form of the utility, or the expected price distribution. Additionally, McKinnon (1967) and Losq (1982) showed that in the absence of options markets production uncertainty causes producers to hedge less than would otherwise be the case ($\Delta X < 0$).[6,7] When the options market is considered with the futures market, our results indicate that ΔX may be positive (a in Figure 3) or negative.

To summarize, in the absence of an anticipated correlation between the individual's output and prices, the effect of production uncertainty on profit risk is greatest near the mean price or at high prices. The producer will hedge against this additional risk by creating payoff schemes that create losses at low prices and generate profits near the mean or at high prices. The producer's hedging position depends on the individual's utility function and distribution of output and prices.[8]

INTRODUCING DEPENDENCE BETWEEN PRICES AND OUTPUT

Consider a circumstance where local production changes are correlated with price changes. Following Losq (1982), the aggregate demand and random output ($\tilde{Q}$) faced by the individual

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[6] Losq (p. 69) argued that the optimal hedge in the forward market should be less than the firm's expected output as long as the third derivative of the utility function is positive if price and output are stochastically independent. In his model, the additional futures selling is $(X - \bar{Q})$ since he considered forward market instead of futures market, i.e., $\tilde{P} = \tilde{F}$.

[7] In our model, if only the futures market is considered, $\Delta X < \Delta Z = 0$ so that we can get the same result with McKinnon and Losq.

[8] The additional hedging position is sensitive to the utility function. It is shown in Appendix A that the additional hedging position arising from the production uncertainty under $u''' > 0$ is exactly opposite the one under $u''' < 0$.

producer are:

$$\tilde{Q}^d = Q^d(\tilde{P})$$
$$\tilde{Q} = K(\tilde{Q}^s; \tilde{\kappa}) \tag{25}$$

where, $\tilde{Q}^d$ and $\tilde{Q}^s$ are aggregate demand and aggregate supply, respectively. Here, $\tilde{\kappa}$ represents the component of the firm-specific production uncertainty, which does not influence aggregate supply and price. At equilibrium, $\tilde{Q}^s = \tilde{Q}^d$ so that the random output of the producer is

$$\tilde{Q} = K[Q^d(\tilde{P}); \tilde{\kappa}]. \tag{26}$$

The first derivative of $\tilde{Q}$ with respect to $\tilde{P}$ is

$$\frac{d\tilde{Q}}{d\tilde{P}} = \frac{\partial K}{\partial Q^d}\frac{\partial Q^d}{\partial \tilde{P}}.$$

Multiplying by $\tilde{P}/K$ on both sides and arranging, the following relation holds:

$$\tilde{\eta} = \tilde{\eta}_1\tilde{\eta}_2$$

where,

$$\tilde{\eta} = \frac{d\tilde{Q}}{d\tilde{P}}\frac{\tilde{P}}{\tilde{Q}}, \; \tilde{\eta}_1 = \frac{\partial K}{\partial Q^d}\frac{Q^d}{K}$$

and

$$\tilde{\eta}_2 = \frac{\partial Q^d}{\partial \tilde{P}}\frac{\tilde{P}}{Q^d}.$$

In this case, $\tilde{\eta}$ is the product of the elasticity of local production with respect to aggregate supply

$(\tilde{\eta}_1)$ and the elasticity of the aggregate demand with respect to price $(\tilde{\eta}_2)$.. Assume that $\tilde{Q}^d$ and $\tilde{P}$ are negatively correlated and that η is constant and greater than -1 so that $-1 < \eta < 0$.[9]

If the producer expects the price he or she perceives to be correlated with production, then the derivative of $\tilde{p}\,\tilde{q}$ with respect to $\tilde{f}$ is:

$$\frac{d\tilde{p}\tilde{q}}{d\tilde{f}} = \tilde{q}\left\{ 1 + \frac{\tilde{p}}{\tilde{q}}\frac{\partial \tilde{q}}{\partial \tilde{p}} \right\}\frac{\partial \tilde{p}}{\partial \tilde{f}}$$

where $\tilde{q} = Q(\tilde{f})$ and $\tilde{p} = (\tau + \beta\tilde{f} + \tilde{\epsilon})$. Since $(\partial\tilde{p}/\partial\tilde{f}) = \beta$, it follows that

$$\frac{d\tilde{p}\tilde{q}}{d\tilde{f}} = \beta\tilde{q}\ (1+\eta)$$

where η evaluated at $\tilde{P}$ is equal to η evaluated at $\tilde{p}$ because η is assumed to be constant.

From (26), $\tilde{Q}$ is a function of $\tilde{P}$ (in turn, $\tilde{P}$ is a function of $\tilde{F}$ and $\tilde{\epsilon}$) and $\tilde{\kappa}$, i.e., $\tilde{Q} = K(\tilde{Q}^d(\tilde{F}, \tilde{\epsilon}), \tilde{\kappa}) = K(\tilde{F}, \tilde{\epsilon}, \tilde{\kappa})$. Here, $\tilde{F}$, $\tilde{\epsilon}$, $\tilde{\kappa}$ are independent of each other, so the joint density function of $\tilde{F}$, $\tilde{\epsilon}$, and $\tilde{\kappa}$ is the product of each density function. Therefore, the first-order conditions derived from the maximization of the expected utility function have the same form as (5) and (6).

The derivative of $g(\tilde{f})$ with respect to $\tilde{f}$ is:

$$\frac{dg(\tilde{f})}{d\tilde{f}} = E\left[\ u''(\tilde{y})\ \{\ \beta\ (1+\eta)\ \tilde{q} - X + LZ\ \}\ \mid\ \tilde{F}\ \right].$$

Thus,

$$g(\tilde{F}) = g(\bar{F}) + (\tilde{F} - \bar{F})\ E\left[\ u''(\tilde{y})\ \{\ \beta\ (1+\eta)\ \tilde{q} - X + LZ\ \}\ \mid\ \tilde{F}\ \right]. \tag{27}$$

The only difference between (8) and (27) is that $\tilde{q}\ (1+\eta)$ is replaced with $\tilde{Q}$. Therefore, the optimal futures and put options amount sold by the producer under the dependence assumptions are:

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[9] The producer might believe that farm yields are correlated with regional yields and that changes in regional yields can cause price changes.

$$X^* = \frac{\mathcal{L}_{FF1}\ \{\ \beta(1\ +\ \eta)\ (a\ -\ b)\ +\ c\ \}}{\Delta}$$

$$Z^* = \frac{\beta\ (1+\eta)\ (\mathcal{L}_{FF1}a\ -\ \mathcal{L}_{FF}b)\ +\ \mathcal{L}_{FF}\ c}{\Delta} \qquad \textbf{(28)}$$

where $\tilde{Q}$ in a, b, $\mathcal{L}_{FF}$ and $\mathcal{L}_{FF1}$ are replaced with $\tilde{q}$ compared with previous notations in the independence case. Thus, $\mathcal{L}_{FF} < \mathcal{L}_{FF1} < 0$ and $a < b < 0$.

The procedure to find the sign of c is almost the same as the previous one. The second derivative of $g(\tilde{F})$ with respect to $\tilde{F}$ is:

$$\frac{\partial^2 g(\tilde{F})}{\partial \tilde{F}^2} = E\ [\ u'''(\tilde{Y})\ \{\ \beta\tilde{Q}\ (1\ +\ \eta)\ -\ X\ +\ LZ\ \}^2\ |\ \tilde{F}]$$

$$+\ E\ [\ u''(\tilde{Y})\ \beta\ (1\ +\ \eta)\ \frac{\partial \tilde{Q}}{\partial \tilde{F}}\ |\ \tilde{F}\ ].$$

Under the assumptions of nonincreasing absolute risk aversion and $-1 < \eta < 0$, the second derivative of $g(\tilde{F})$ with respect to $\tilde{F}$ is positive and thus $g(\tilde{F})$ is convex in $[0, \bar{F}\ ]$ or $[\bar{F}\ , \infty]$.

Also, c can be rewritten as

$$c = -\bar{R}E\ [\ u''(\tilde{y})\ (\tilde{F}\ -\ \bar{F})\ \{\ \beta\tilde{q}\ (1\ +\ \eta)\ -\ X\ +\ LZ\ \}\ ]$$
$$= \bar{R}Cov\ [\ \tilde{F},\ u''(\tilde{y})\ \{\ \beta\tilde{q}\ (1\ +\ \eta)\ -\ X\ +\ LZ\ \}\ ].$$

The covariance has a sign of $\partial[u''(\tilde{y})\{\beta\tilde{q}\ (1 + \eta) - X + LZ\}]/\partial\tilde{F} = u'''(\tilde{y})\{\beta\tilde{q}\ (1 + \eta) - X + LZ\}^2 + u''(\tilde{y})\beta(1 + \eta)(\partial\tilde{q}/\partial\tilde{f})$, which is positive under $u''' > 0$ and $-1 < \eta < 0$. Therefore, c is also negative.

One obvious fact is that the producer always sells futures. Similar to the previous analysis, the possible shapes are presented in Figure 4. In part a, $Z^* > X^* > 0$ where the producer will hedge when the realized futures price is near the mean price. Parts b through e demonstrate that the payoff diagram for the combined position is decreasing in futures price.

To understand the intuition here, assume that $\tilde{P} = \tilde{F}$ correlated with price. Since $\partial(\tilde{P}\tilde{Q})/\partial\tilde{P} = \tilde{Q}\ (1 + \eta) > 0$ and $\partial^2(\tilde{P}\tilde{Q})/\partial\tilde{P}^2 = (1 + \eta)(\partial\tilde{Q}/\partial\tilde{P}) < 0$ if $-1 < \eta < 0$, the unhedged

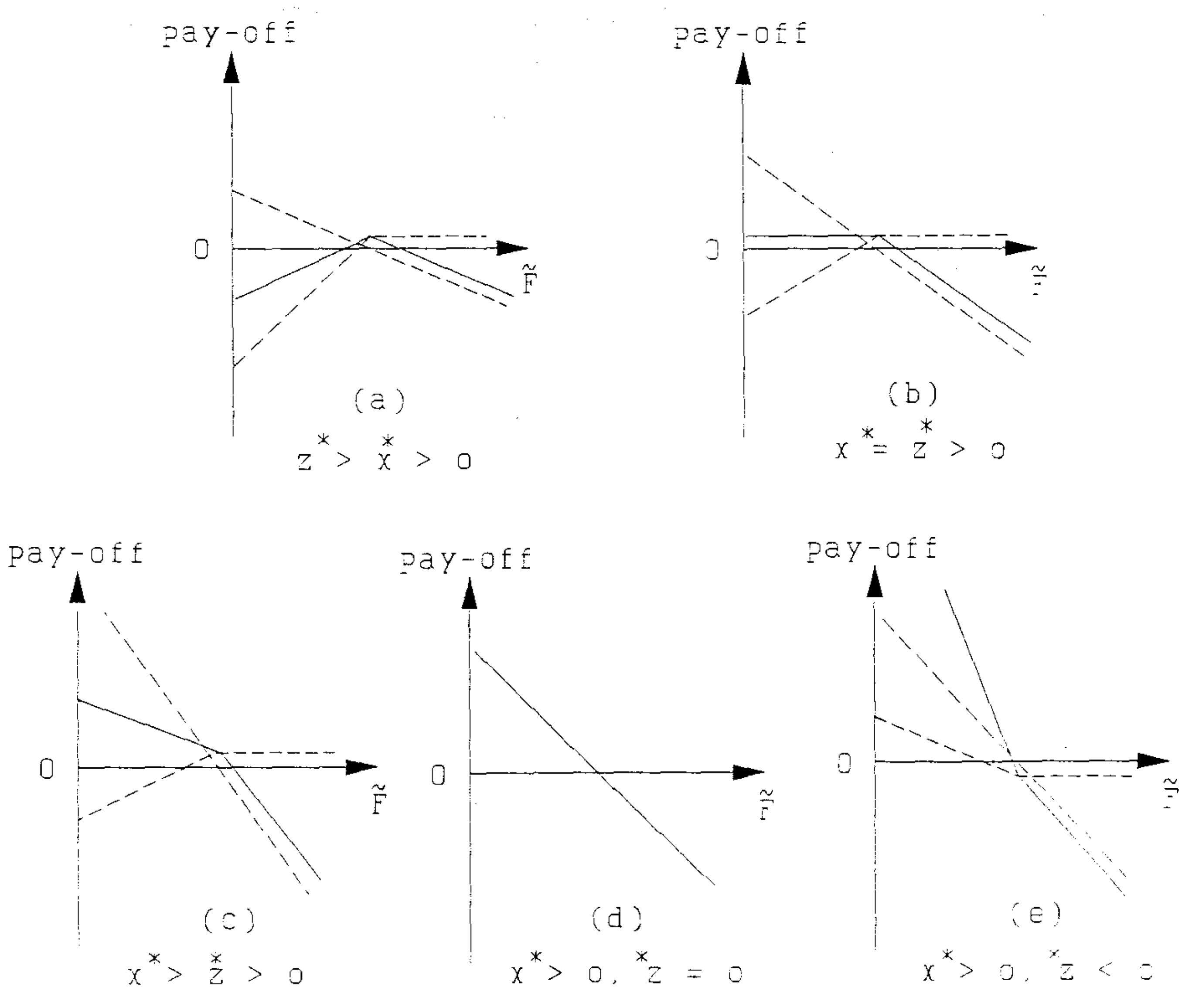

Figure 4. The combined futures and options position

random revenue ($\tilde{P}\tilde{Q}$) is concave in the realized price and its slope is always positive. Options are required due to such nonlinearity and the producer will not place a hedge against high futures prices.

NUMERICAL SIMULATIONS

The results obtained from the previous sections are now supported numerically. This section also analyzes the effect of several elements on hedging behavior: the degree of absolute risk aversion, output variation, the shape of price distribution, and the size of η (see Appendix A for the effect of the utility function on hedging behavior). First, the method to find the optimal futures and options amounts is explained briefly. Then the optimal futures and options position is calculated (a) when the production uncertainty is independent of prices and (b) when it is dependent on prices. In all cases we assume that $\tilde{P} = \tilde{F}$ and that the producer has a constant absolute risk aversion, i.e., $u(\tilde{Y}) = -\exp[-A\tilde{Y}]$ where A is a constant absolute risk aversion coefficient.

Data and Method

The mean and variance of the output for typical Iowa corn producers were calculated from *Iowa Farm Costs and Returns* (for 1970 through 1989). The coefficient of variation of Iowa corn production per farmer was 0.158113.[10] Average 1989 corn production is used to represent mean production approximately 20,000 bushels, and thus the variance of corn production is assumed to be 1×10^7.[11]

Corn is assumed to be planted the second week of May and harvested the second week of September. Assume that the September corn futures price in September is the mean price for the year. The deviation of the futures price from the mean is calculated as a difference between the

[10] The coefficient of variation of output is defined as $\{\mathrm{Var}[\tilde{Q}/\bar{Q}]\}^{1/2}$.

[11] When the expected production is 20,000 bushels and the coefficient of variation is 0.026165, the variance of production is calculated as $(0.158113 \times 20{,}000)^2 = 10{,}466{,}000$.

September corn futures prices in May and in September. Years considered here for corn futures price are 1974 to 1989 and the coefficient of variation of the futures price is 0.173205. The September corn futures price in the second week of September 1989 was 2.92, which is set as the mean price for the year, and thus variance of the futures price is assumed to be 0.255792.

The first step of the optimization procedure is to establish for X an interval of $\pm$ 10,000 around a starting point and to divide this interval into 19 evenly spaced segments so that the number of Xs considered for calculation is 20. For example, suppose that the starting point of X is 0. Then, the values for X are ($-10{,}000$, ..., $-2{,}000$, $-1{,}000$, 0, 1,000, 2,000, ..., 10,000).

The values for Z are obtained the same way. In the X-Z plane there is now a grid of 400 points. The expected utility level at each point on the grid is calculated. The second step is to choose the point, say (X_1, Z_1), on the grid, where the expected utility function is greatest. If the point is an interior solution, then the first step is repeated within an interval of $\pm$ 1,000 around (X_1, Z_1). If the point is a corner solution, then the first step within an interval of $\pm$ 10,000 around (X_1, Z_1) is repeated. This procedure is repeated until an interior solution is found and the first step will then be repeated within an interval of $\pm$ 1,000 around the point on the grid where the expected utility is greatest. The point is regarded as an optimum point. Strictly speaking, the point may not be an optimum point. However, we are trying to find the shape of the combined position for futures and options and these two steps are enough to find it. Moreover, when the minimum contract size in futures market and options markets is considered,[12] it may be acceptable that the maximum deviation of our optimal amounts from the true optimal amounts is $\pm$ 100. We use a CARA (constant absolute risk aversion) utility function so that the second-order condition is always satisfied (see Appendix B). Therefore, this search procedure allows us to avoid problems caused by local optimum.

[12] The contract size for corn in futures is 1,000 bushels at the MidAmerica Commodity Exchange and 5,000 bushels at the Chicago Board of Trade.

The numerical integration becomes accurate as the domain of random variables is divided as many times as possible. However, the more accurate calculation requires more cost. The method used trades cost for accuracy. In this research, the random variables are divided into 70, 100, 150, or 200 according to the step. We choose more segments for the higher step.[13]

Independence Case

Assume that price and output are normally distributed as follows:

$\tilde{F} \sim N\ (2.92,\ 0.255792)$

$\tilde{Q} \sim N\ (20{,}000,\ 1x10^7).$

Normal distribution has a domain from $-\infty$ to $+\infty$. In this research, the bounds of random variables are set such that the probability of the truncated area is almost zero.

Table 1 represents the producer's hedging behaviors in the various situations considered here. In Case 1.6 and Case 1.7, the distribution of price is truncated from the mean price. The conditional density function for half normal distribution is the product of both density function of normal

Table 1. Possible hedging position under independence between prices and output

Condition	Shape	Figure 3
$Z^* > X^* > 0$	"Λ" shaped	(a)
$Z^* = X^* > 0$	sell call	(b)
$X^* > Z^* > 0$	(−) sloped	(c)
$X^* > 0,\ Z^* = 0$	sell futures	(d)
$X^* > 0 > Z^*$	(−) sloped	(e)

distribution and 1/0.5. The conditional means of futures price are obtained from integrating the product of $\tilde{F}$ and a conditional density function of half normal distribution with respect to $\tilde{F}$, whose

[13] Seventy is chosen for the first step and 150 or 200 is generally chosen for the final result. In effect, dividing the random variables by the segments of 150 or 200 was enough, according to our simulation.

domain is 0 to $\bar{F}$ in LHND and $\bar{F}$ to 7 in RHND. These can be easily calculated from intquad1 in GAUSS program. The conditional means of futures price are:

$$E\,[\,\tilde{F}\mid 0 \le \tilde{F} \le 2.92\,] = 2.5164628 \qquad \textit{for LHND and}$$

$$E\,[\,\tilde{F}\mid 2.91 \le \tilde{F} \le 7\,] = 3.3235372 \qquad \textit{for RHND.}$$

Under the independence between output and price and $\tilde{F} = \tilde{P}$, the results indicate that the producer will always sell $\bar{Q}$ futures under production certainty and will use additional futures and options for hedging the unequal variation of profit caused by production uncertainty. The combined position of futures and options is simply a sum of selling futures by $\bar{Q}$ and the additional hedging position due to production uncertainty. Therefore, the combined position is automatically determined if the additional position is found. Therefore, we examine only the additional hedging position here.

1. Cases 1.1 through 1.3: Different Absolute Risk Aversion Coefficients

Figure 5 demonstrates the effect on hedging behavior of the degree of absolute risk aversion. The producer sells futures of expected output, $\bar{Q}$, and he or she hedges the variation in profit from production uncertainty, which increases as the realized futures price is higher. If producer A is more risk averse than producer B, producer A will hedge more the high variation range in profit (thus higher price range) and less the low variation range than producer B. Therefore, in Figure 5 the slope of the payoff line for producer A is greater in $[\bar{F}, \infty]$ and less in $[0, \bar{F}]$ than that for producer B.

2. Cases 1.1, 1.4 and 1.5: Different Variance of Output

The hedging behavior according to the different variances of output can be interpreted from Cases 1.1, 1.4, and 1.5, as shown in Figure 6. The combined positions of additional futures and options are increasing in futures price in Case 1.1 and "Λ"-shaped in Cases 1.4 and 1.5. The higher variation in output causes the producer to hedge more the high price range than the lower variation.

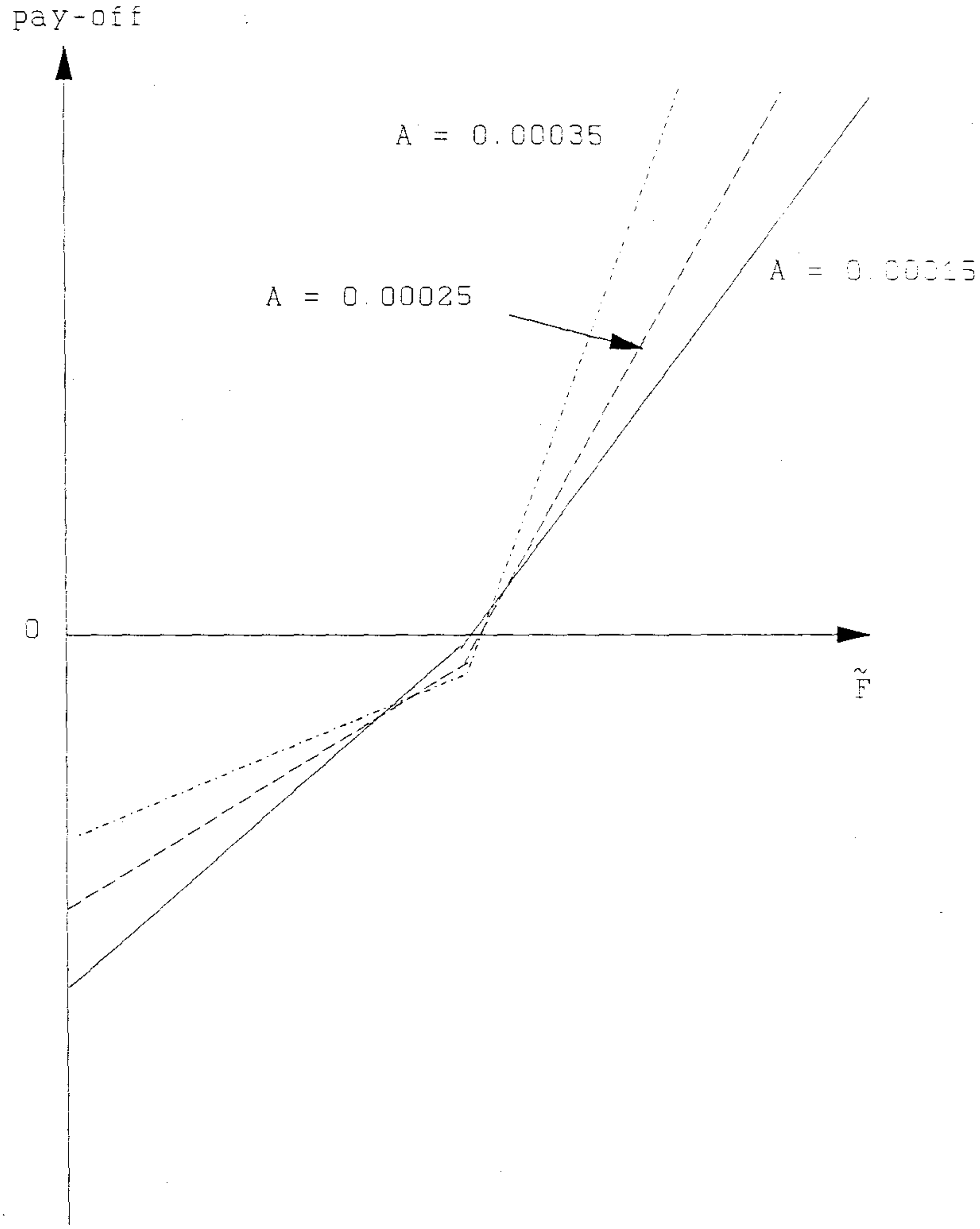

Figure 5. The effect of the degree of absolute risk aversion on hedging behavior

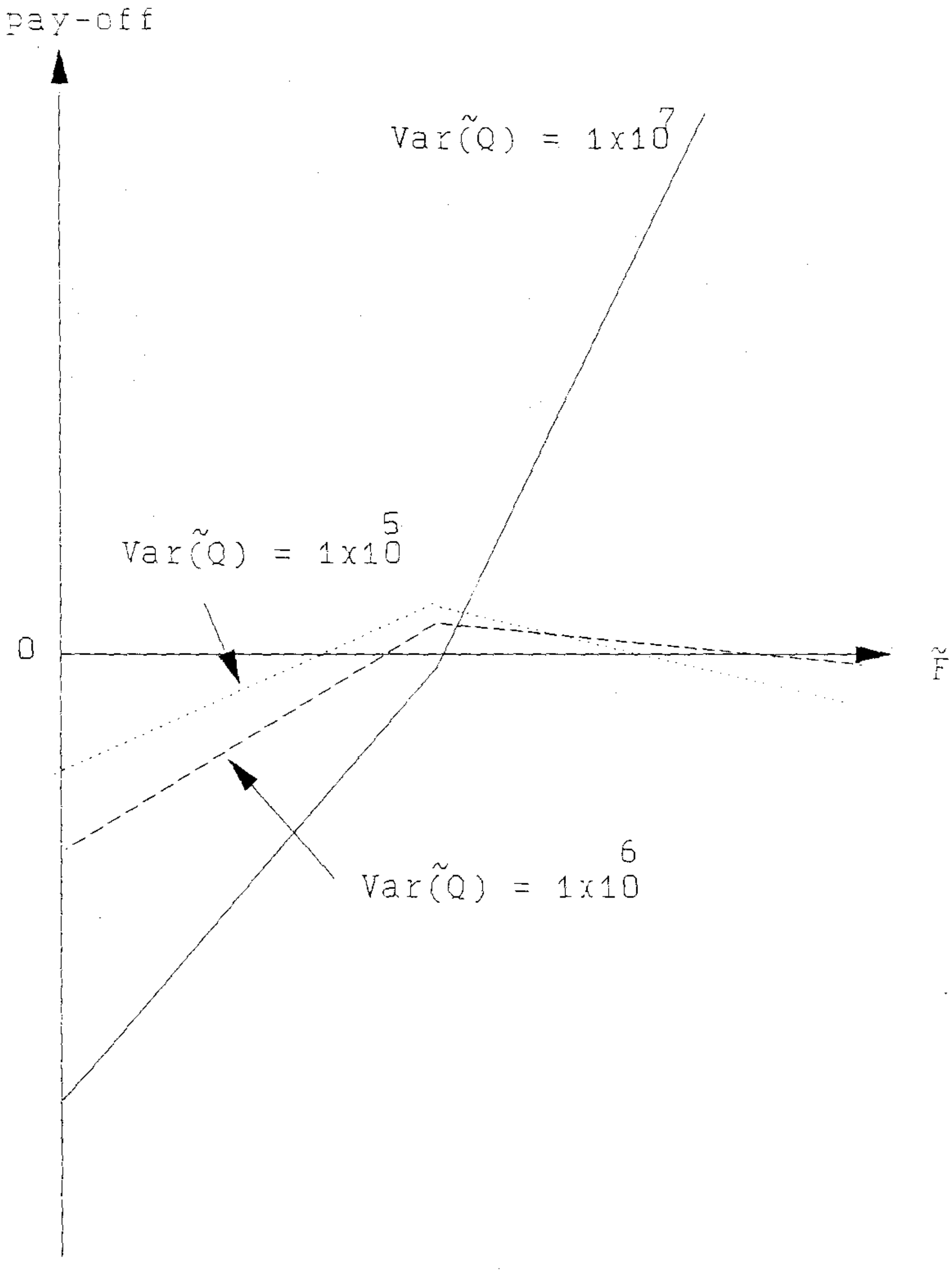

Figure 6. The effect of the magnitude of output variance on hedging behavior

This is interpreted from $Var[\tilde{F}\tilde{Q} - C(\bar{Q}) \mid \tilde{F}] = \tilde{F}^2 Var(\tilde{Q})$. When the realized futures price increases, the variance of unhedged profit increases more rapidly in the high variance of $\tilde{Q}$ than in its low variance. Therefore, the producer will hedge more on the high price range when Var($\tilde{Q}$) is large. Cases 1.4 and 1.5 have "Λ"-shaped positions, which can result from the interaction between the output variation and the shape of the price distribution. That is, if the probability of high price is low and output variation is small, the producer will be worried more about the high probability range with low profit risk than about the low probability range with high profit risk. Consequently, given absolute risk aversion, whether the producer will take increasing position in futures price or "Λ"-shaped position depends upon the distributional shape of price and output.

3. Cases 1.6 and 1.7: Nonsymmetric Price Distribution

Two extremely truncated price distributions are considered here: right half normal distribution (RHND) and left half normal distribution (LHND). Case 1.6 presents the LHND, where the producer gives more weight to the high price range for hedging decisions than to the low price range. On the other hand, Case 1.7 is the RHND case, where the producer allocates more weight to the low price range for hedging decisions. As shown in Figure 7, the additional hedging position of the producer increases with the futures price. However, the producer hedges more the high price range in the LHND case than in RHND. This emphasizes the effect of the price distributional shape on hedging behavior. In addition, the payoff diagram of Case 1.6 shows that the additional position is increasing in futures price even though the most weight is given to the low price range.

Consequently, in any of Cases 1.1 through 1.7, the producer will place a hedge position against the high price range or against the mean price for additional hedging due to production uncertainty. The precise position depends on several elements, including the utility function and distributions of output and prices.

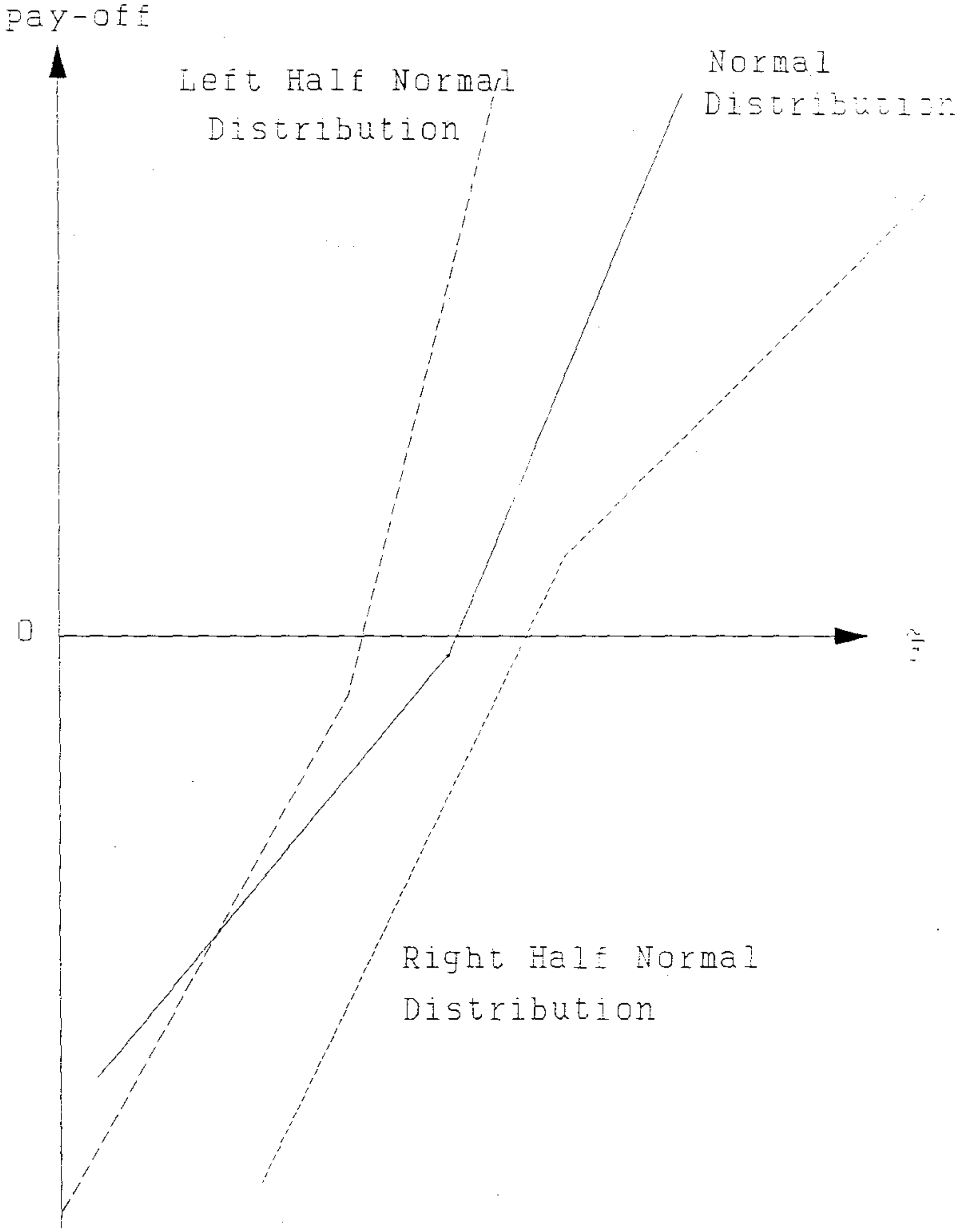

Figure 7. The effect of the shape of price distribution on hedging behavior

Independence Case

The effect of production uncertainty on hedging behavior has already been analyzed. In addition to $\tilde{F} = \tilde{P}$, we assume that production is perfectly correlated with price so that the elasticity of local production with respect to aggregate supply is one, $\eta_1 = 1$. Suppose that aggregate demand is constantly elastic with respect to price, $\tilde{Q} = \gamma\tilde{P}^{\eta}$ where η is the price elasticity and γ is some constant coefficient. The mean is 20,000 so that $E[\tilde{Q}] = \gamma E[\tilde{P}^{\eta}] = 20{,}000$. If price distribution and η are given, we can calculate the value of γ. The price distribution is assumed to be normally distributed with mean of 2.92 and variance of 0.255792. The optimal hedging behavior is simulated here with several values of η as presented in Table 2. The results are shown in Figure 8, which indicates that the producer who believes in a high price elasticity of demand will hedge more in the high price range. This can be illustrated from Figure 9. The unhedged profit becomes horizontal as η approaches -1 and inversely, as η increases from the -1, its curvature increases. In order to remove profit variation the producer's hedging position will take the inverse shape with the unhedged profit. Therefore, the producer will hedge more the high price range and less the low price range under high η than under low η.

There is another situation (Case 2.6), where firm-specific production uncertainty is considered. The simplest hypothesis for this is that the firm-specific production uncertainty $(\tilde{\kappa})$ is additive risk:

$$\tilde{Q} = \gamma\tilde{P}^{\eta} + \tilde{\kappa} ,$$

where $\tilde{\kappa}$ is assumed to be normally distributed with mean 0 and variance 7×10^6. Suppose that $\tilde{P}$ has right half normal distribution. The optimal futures and options obtained from the numerical simulation are 1,000 and 1,700, respectively, with a combined position that is "Λ"-shaped.

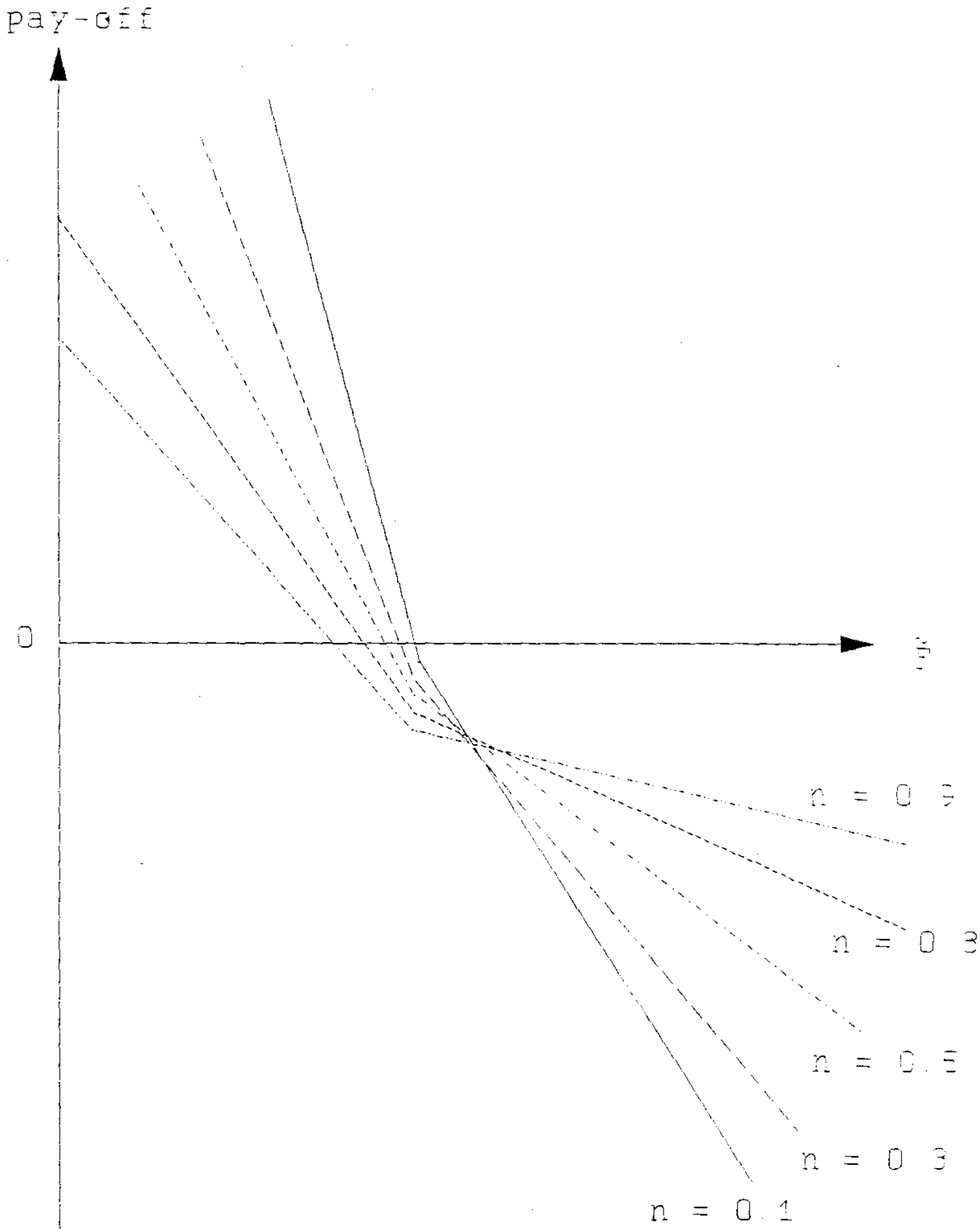

Figure 8. The effect of η on hedging behavior

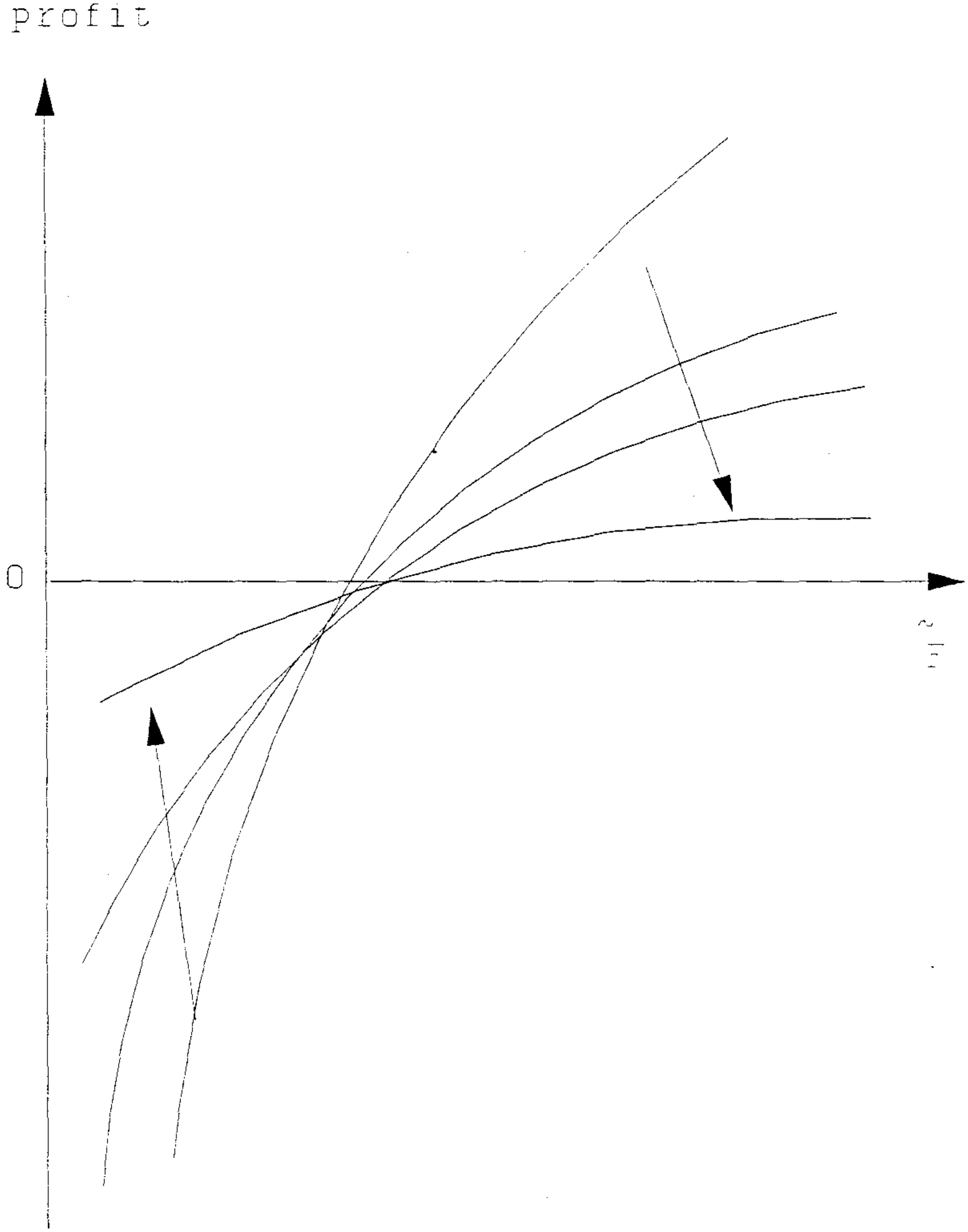

Figure 9. The shape of the unhedged profit curve under the different size of η

Table 2. Possible hedging position under dependence between prices and output

Condition	Shape	Figure 4
$Z^* > X^* > 0$	"Λ" shaped	(a)
$Z^* > 0, X^* = 0$	sell put	(b)
$Z^* > 0 > X^*$	(+) sloped	(c)
$X^* < 0, Z^* = 0$	buy futures	(d)
$0 > X^* > Z^*$	(+) sloped	(e)

CONCLUSIONS

The producer uses options as well as futures as hedging instruments under the assumption that futures and options prices are unbiased. Therefore, cash price is a linear function of futures price because the variance of unhedged profit increases as realized futures price increases. Thus, unhedged profit is nonlinear in futures prices, so the producer always sells futures. The precise position for futures and options depends on the utility function and the distribution of prices and output.

APPENDIX A. COMPARISONS OF ADDITIONAL HEDGING BEHAVIORS ACCORDING TO UTILITY FUNDTIONAL FORMS

The importance of the utility functional form was stressed earlier. To interpret this, we compare the additional hedging components under $u''' > 0$, $u''' = 0$ and $u''' < 0$. If the production process is nonstochastic, the risk of futures price uncertainty is removed so that the shape of the utility function does not affect hedging behavior.

In effect, many economists have used the mean-variance method to analyze the producer's optimal hedging decision. The mean variance method cannot be used due to a truncation of price distribution when an options market is considered unless the producer has quadratic utility function, i.e., $u''' = 0$.

With the quadratic utility function and independent assumption between prices and output, u'' is constant and therefore $\mathcal{L}_{QF}$, $\mathcal{L}_{QFF1}$, $\mathcal{L}_{QFF}$ and c are zero.[14] The second term of the right side of (17) is zero and thus the additional futures and put options are zero. Therefore, if we use the mean variance method to analyze hedging behavior with an options market, the optimal futures and options amounts under production certainty are equal to those under production uncertainty if price uncertainty and output uncertainty are independent.

Suppose that $u''' < 0$, then $\mathcal{L}_{QFF1} < 0$, $\mathcal{L}_{QFF} < 0$, $c > 0$ and thus $\Delta X > \Delta Z$. Therefore, if the additional futures and put options amounts under $u''' > 0$ denote ΔXX and ΔZZ respectively, then $\Delta XX = -\Delta X$ and $\Delta ZZ = -\Delta Z$ from (21) and (22). These are exactly opposite for results under $u''' > 0$. In effect, if $u''' < 0$, the producer would not hedge high profit variation since higher hedge return in the low variation increases his or her expected utility level.

[14]For example $\mathcal{L}_{QF} = E[u''(\tilde{y})(\tilde{Q} - \bar{Q})(\tilde{F} - \bar{F})^2] = u''E[(\tilde{Q} - \bar{Q})(\tilde{F} - \bar{F})^2]$ (since u'' is constant) $= u''E[\tilde{Q} - Q]E[\tilde{F} - \bar{F}]$ (since independent assumption) $= 0$.

APPENDIX B. THE SECOND-ORDER CONDITION UNDER A CARA UTILITY FUNCTION

Under the existence of futures and options markets, the second-order condition is

$$S.O.C. = E\,[\, u''(\tilde{Y})\,(\bar{F} - \tilde{F})^2\,]\; E\,[\, u''(\tilde{Y})\,\{\, \bar{R} - (\bar{F} - \tilde{F})\,L\,\}^2\,]$$
$$-\,\{\, E\,[\, u''(\tilde{Y})\,(\bar{F} - \tilde{F})\,\{\, \bar{R} - (\bar{F} - \tilde{F})\,L\,\}\,]\,\}^2$$

where

$$E[\, u''(\tilde{Y})\,\{\bar{R} - (\bar{F} - \tilde{F})\,L\,\}^2]$$
$$= \bar{R}^2 E[\, u''(\tilde{Y})\,] - 2\bar{R}\alpha E_1[\, u''(\tilde{Y})\,(\tilde{F} - \bar{F})\,] + \alpha E_1[\, u''(\tilde{Y})\,(\tilde{F} - \bar{F})^2\,].$$

$$E[u''(\tilde{Y})(\bar{F} - \tilde{F})\{\bar{R} - (\bar{F} - \tilde{F})L\}]$$
$$= -\,\bar{R}E[u''(\tilde{Y})(\tilde{F}-\bar{F})] - \alpha E_1[u''(\tilde{Y})(\tilde{F} - \bar{F})^2]$$

Under a CARA utility function, i.e., $E[u''(\tilde{Y})(\tilde{F}-\bar{F})] = 0$, the second-order condition above can be rearranged as follows:

$$S.O.C. = E\,[\, u''(\tilde{Y})\,(\tilde{F} - \bar{F})^2\,]\,\{\, \bar{R}^2 E\,[\, u''(\tilde{Y})\,] - 2\bar{R}\alpha E_1\,[\, u''(\tilde{Y})\,(\tilde{F} - \bar{F})\,\}$$
$$+\,\alpha E_1\,[\, u''(\tilde{Y})\,(\tilde{F} - \bar{F})^2\,]\,\{\, E\,[u''(\tilde{Y})\,(\tilde{F} - \bar{F})^2 - \alpha E_1\,[\, u''(\tilde{Y})\,(\tilde{F} - \bar{F})^2\,]\,\},$$

which is always positive since $E[u''(\tilde{Y})(\tilde{F}-\bar{F})^2] < \alpha E_1[u''(\tilde{Y})(\tilde{F}-\bar{F})^2 < 0$ *and* $E_1[u''(\tilde{Y})(\tilde{F}-\bar{F})] > 0.$.

Consequently, under a CARA utility function, the second-order condition is always satisfied.

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