The Value of Information on Crop Response Function to Soil Salinity in a Farm-Level Optimization Model

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ABSTRACT

The study investigates the value of additional information on the response function to soil salinity of a given crop (potatoes), with regard to a stochastic long-run optimization model for utilization of saline water in a single farm framework. The analysis provides a conceptual and methodological framework for investigating the expected value of sample information (EVSI), as well as an efficient tool for empirical application. Although a few approximations have been used, the results provide an estimate of EVSI and indicate the need for additional information.

Key words: value of information, loss function, crop-response function, soil salinity, optimization model.

THE VALUE OF INFORMATION ON CROP RESPONSE FUNCTION TO SOIL SALINITY IN A FARM-LEVEL OPTIMIZATION MODEL

1. Introduction

The response function of a given crop yield to soil salinity is an important factor in every optimization model concerning irrigation with saline water. The true value of the response function parameters are usually unknown to the decision maker, and therefore he or she uses their estimates and may become a victim of a suboptimal solution. The damage may be measured by a loss function and the calculation of its expectation. The parameters' estimates (which are arguments in the loss function) are based on a priori information available to the decision maker. But he or she can generally acquire additional information which will decrease the uncertainty and reduce the loss expectation. The expected value of sample information (EVSI) is defined as the difference between the reduction of the expected value of the loss function due to the additional information and the cost of its acquisition. The optimal number of observations to be acquired is the one that maximizes EVSI.

A broad theoretical presentation of decision theory, value of information and the Bayesian approach can be found in the textbooks of Pratt et al. (1965) and DeGroot (1970). A number of studies deal with the value of information in farm management (Ryan and Perrin, 1974; Maddock, 1973, Bie and Ulph, 1972; Mjelde et al., 1988; Preckel et al., 1987; Antonovitz and Roe, 1984) and in management of water resources (Davis and Dvoranchik, 1972; Duckstein et al., 1977; Klemes, 1977). It should be pointed out that most of these articles did not deal explicitly with the optimal size of the additional information. Moreover, the articles that dealt with the management of irrigation systems ignored water quality.

This study focuses on the value of additional information on the response function to soil salinity of a given crop (potatoes), with regard to a stochastic long-run optimization model (hereafter referred to as SLRO-model) for utilization of saline water in a single farm framework. It is beyond the scope of this paper to present the detailed estimation technique of the response function parameters as well as the formulation of the SLRO-model, but they are described briefly.

The organization of the rest of the paper is as follows. Section 2 presents a switching regression model to estimate the response-function parameters and their statistical properties. Section 3 presents a brief review of the SLRO-model. Considering the optimum values of the SLRO-model and the response function parameters a loss function is formulated and its possible situations are presented in Section 4. The loss function is approximated by a Taylor series expansion. Then, in Section 5, the expected value of additional sample information--EVSI--is calculated and the optimal sample size is determined. The analysis in the section utilizes the empirical results of the SLRO-model and, following reasonable approximations, shows how the possible situations of the loss function may be reduced to only one. Empirical findings for potatoes are also presented in Section 5. Finally, Section 6 contains a brief summary and concluding comments.

2. Estimates of the Response-Function Parameters

An accepted theory among soil researchers states that crop yield is independent of average soil salinity below a certain critical threshold, and thereafter decreases linearly (Maas and Hoffman, 1977). On the basis of this specification, the following switching regression model is used (see Figure 1):

$$Y_i = \begin{cases} b_1 + aS_0 + U_{1i} & \text{if } S_i \le S_0 \\ b_1 + aS_i + U_{2i} & \text{if } S_i > S_0 \end{cases} \qquad i = 1,\ldots,T \tag{1}$$

where:

- Y --yield per hectare (in physical units);
- i --the observation index;
- T --the a priori number of available observations;
- S --average soil salinity level in the root zone during the growing season [meq Cl/l];
- S_0 --the critical threshold [meq Cl/l];
- U_1, U_2 --independent random variables normally distributed with zero means and variances equal to σ_1^2 , σ_2^2, respectively; and
- b_1,a,S_0--the (unknown) parameters of the response function.

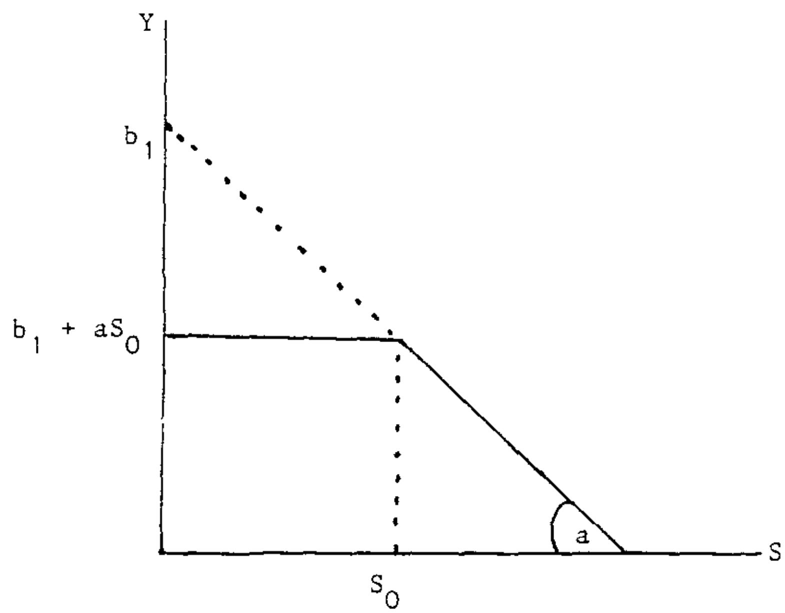

Figure 1. The Response Function

Let $\hat{\beta} = [\hat{S}_0, \hat{a}, \hat{b}_1]$ be the Maximum Likelihood (ML) estimate of $\beta = [S_0, a, b_1]$.

From the properties of ML estimates, under fairly general conditons (e.g., Zacks, 1971), $\hat{\beta}$ is asymptotically normally distributed with mean β and variance variance-covariance matrix $\Sigma_{\beta} = \left\{ E \left(- \frac{\partial^2 \ln L}{\partial \beta_i \partial \beta_j} \right)^{-1} \right\}$;

where ln L is the logarithm of the relevant likelihood function. By substituting the explicit partial derivatives, we get the following asymptotic results:

$$\Sigma_{\beta} = \begin{bmatrix} V(\hat{S}_0) & cov(\hat{S}_0, \hat{a}) & cov(\hat{S}_0, \hat{b}_1) \\ cov(\hat{a}, \hat{S}_0) & V(\hat{a}) & cov(\hat{a}, \hat{b}_1) \\ cov(\hat{b}_1, \hat{S}_0) & cov(\hat{b}_1, \hat{a}) & V(\hat{b}_1) \end{bmatrix} =$$

$$\begin{bmatrix} \frac{\sigma_2^2\left[(T-t)S_0^2 + \sum_j S_j^2 - 2S_0 \sum_j S_j\right]}{a^2 A} + \frac{\sigma_1^2}{ta^2}; & \frac{\sigma_2^2\left[\sum_j S_j - S_0(T-t)\right]}{a A}; & \frac{\sigma_2^2\left[S_0 \sum_j S_j - \sum_j S_j^2\right]}{a A} \\ \frac{\sigma_2^2\left[\sum_j S_j - S_0(T-t)\right]}{a A}; & \frac{\sigma_2^2 (T-t)}{A}; & \frac{-\sigma_2^2 \sum_j S_j}{A} \\ \frac{\sigma_2^2\left[S_0 \sum_j S_j - \sum S_j^2\right]}{a A}; & \frac{\sigma_2^2 \sum S_j}{A}; & \frac{\sigma_2^2 \sum S_j^2}{A} \end{bmatrix} \quad (2)$$

where t is the number of observations that satisfies $S_i \le S_0$; V is variance; and cov is covariance:

$$\sum_j \equiv \sum_{j=t+1}^{T} \; ; \text{ and } A \equiv \left[(T-t) \sum_j S_j^2 - (\sum_j S_j)^2 \right] \quad .$$

As mentioned in the introduction, the empirical findings relate to potatoes. The estimates of response-function parameters are based on experimental results of Sadan and Berglas (personal communication). Their one-year experiment conducted in the Northern Negev of Israel yielded a total of 17 observations (S_i, Y_i). By applying a switching regression technique (Quandt, 1960), the following ML estimates were derived:

$$\hat{S}_0 = 6.054 \text{ [meq Cl/l]} \; ; \; \hat{a} = -1.09 \left(\frac{\text{tons/ha}}{\text{meq Cl/l}}\right) \; ;$$

$\hat{b}_1 = 52.55$ [tons/ha]; $\hat{\sigma}_1^2 = 0.14$ [tons/ha]^2 ; $\hat{\sigma}_2^2 = 15.78$ [tons/ha]^2 ; and $\hat{t} = 3$.

Thus, the estimated response function is:

$$\hat{Y}_i = \begin{cases} 45.95 & \text{if } S_i \le 6.054 \\ 52.55 - 1.09\, S_i & \text{if } S_i > 6.054 \end{cases} \; ; \; R^2 = 0.776 \qquad (1')$$

By substituting these estimates into (2), a consistent estimator of $\Sigma_{\underline{\beta}}$ is achieved.

3. The Stochastic Long-Run (SLRO) Optimization Model -- A Brief Review

The planning model considers a single kibbutz farm in southern Israel and incorporates in one endogeneous system both physical/biological relationships (such as response functions and salt distribution in the soil profile) and economic relationships. The farm has three water sources, differing in availability, quality (salinity level), and price. Water from different sources may be mixed, providing for additional quality options. The farm has at its disposal five plots of land, differing in area and initial salinity level of the soil solution.

The cropping alternatives of the farm are as follows: fall potatoes, fall carrots, cotton, and a mature grapefruit grove. The yields of these crops (except cotton) are sensitive to soil salinity. The parameters for the yield response functions to salinity for potatoes and grapefruits were estimated by a switching regression approach whereas the estimated parameters for carrots were taken from Maas and Hoffman (1977). An irrigation season is defined as one year and is subdivided into two subseasons: spring/summer and autumn/winter.

The SLRO-model refers to the water-soil-crop-farm system over a sequence of four irrigation seasons and considers rainfall uncertainty. Conceptually, it is an extension of the two-stage LP model under uncertainty (Dantzig and Madansky, 1961; El Agizy, 1967). The objective function of the risk neutral farmer is to maximize the present value of the expected net profits from the crops' net returns over the time horizon subject to total water and land supplies, quotas for potatoes and carrots, and linear balance equations which describe the evolution of the soil-related state variables over time. The farm's decision variables include crop mix, quantities and qualities of irrigation water for the various crops, and quantities and qualities of leaching water for the soil plots. The results provide priorities in the allocation of water and soil plots of varying salinity levels as well as empirical estimates of the shadow prices and the rates of substitution among the limited sources.

A detailed description of the SLRO-model can be found in Feinerman and Yaron (1983).

4. The Loss Function and Its Possible Situations

Based on the results of the SLRO-model, the following loss function, h, is defined:

$$h\,(\hat{\underline{\beta}} - \underline{\beta}) = Z(\underline{\beta}/\underline{\beta}) - Z(\hat{\underline{\beta}}/\underline{\beta}) \tag{3}$$

where:

$Z(\underline{\beta}/\underline{\beta})$ = the optimal value of the objective function, given that the true values of the parameters--$\underline{\beta}$--are known to the decision makers with certainty; and

$Z(\underline{\beta}/\hat{\underline{\beta}})$ = the optimal value of the objective function when $\underline{\beta}$ is unknown and the decision makers use instead (in the SLRO-model) its ML estimates-$\hat{\beta}$.

The optimal solution determines the following values:

$\hat{S}^*_{ng}(i,k)$ = the average soil salinity level [in meq Cl/l] of soil plot g, associated with crop n, in year i with winter type(rainfall level) k; (n=1,...,N; g=1,...,G; i=1,...,I; k=1,...,K); and

$\hat{\Pi}^*_{ng}(i,k)$ = the net return [in U.S.$/ha] of crop n, associated with plot g, in year i with rainfall level k.

The net return function is given by:

$$\hat{\Pi}^*_{ng}(i,k) = \begin{cases} R_{1n}(a\,\hat{S}_{on} + b_1) - R_{2n} & \text{if} \quad \hat{S}^*_{ng}(i,k) \le \hat{S}_{on} \\ R_{1n}(a\,\hat{S}^*_{ng}(i,k) + b_1) - R_{2n} & \text{if} \quad \hat{S}^*_{ng}(i,k) > \hat{S}_{on} \end{cases} \tag{4}$$

where:

R_{1n} - net income (U.S.\$/ton) of crop n, as a function of the yield (revenue less yield-dependent variable costs such as harvesting, grading, packing and transportation);

R_{2n} = variable cost (U.S.\$/ha), independent of yield; and

$\hat{S}_{on}$ = ML estimate of crop n's average threshold-salinity level of the soil;

It is assumed that R_{1n}, R_{2n} and $\hat{S}_{on}$ are independent of g, i and k.

Let $S^*_{ng}(i,k)$ and $\Pi^*_{ng}(i,k)$ be the optimal values of the average soil salinity and the net income, respectively, given that the true values of parameters ($\underline{\beta}$) are known to the decision maker.

Let us now define the following sets (for convenience the index n is omitted from now on):

$$\hat{E}_1: \left\{ \hat{S}^*_g(i, k) \mid \hat{S}^*_g(i,k) > \hat{S}_0 \right\}$$

$$E_1: \left\{ S^*_g(i, k) \mid S^*_g(i,k) > S_0 \right\}$$

$$\hat{E}_2: \left\{ \hat{S}^*_g(i, k) \mid \hat{S}^*_g(i,k) \leq \hat{S}_0 \right\}$$

$$E_2: \left\{ S^*_g(i, k) \mid S^*_g(i,k) \leq S_0 \right\} .$$

One may distinguish among eight alternatives associated with the possible values of the loss function based on all possible combinations of the relationships between $\hat{S}_0$ and S_0, S^*_g and S_0, and $\hat{S}^*_g$ and $\hat{S}_0$:

$$\hat{S}_0 \leq S_0 \ , \ S^*_g(i, k) > S_0 \ , \ \hat{S}^*_g(i, k) > \hat{S}_0 \tag{5a}$$

$$\forall\ S_g^*(i,\ k)\ \epsilon\ E_1 \quad ; \quad \hat{S}_g^*(i,\ k)\ \epsilon\ \hat{E}_1 \quad ;$$

$$\hat{S}_0 > S_0\ ,\ S_g^*(i,\ k) > S_0\ ,\ \hat{S}_g^*(i,\ k) > \hat{S}_0 \tag{5b}$$

$$\forall\ S_g^*(i,\ k)\ \epsilon\ E_1 \quad ; \quad \hat{S}_g^*(i,\ k)\ \epsilon\ \hat{E}_1 \quad ;$$

$$\hat{S}_0 \le S_0\ ,\ S_g^*(i,\ k) \le S_0\ ,\ \hat{S}_g^*(i,\ k) \le \hat{S}_0 \tag{5c}$$

$$\forall\ S_g^*(i,\ k)\ \epsilon\ E_2 \quad ; \quad \hat{S}_g^*(i,\ k)\ \epsilon\ \hat{E}_2 \quad ;$$

$$\hat{S}_0 > S_0\ ,\ S_g^*(i,\ k) \le S_0\ ,\ \hat{S}_g^*(i,\ k) \le \hat{S}_0 \tag{5d}$$

$$\forall\ S_g^*(i,\ k)\ \epsilon\ E_2 \quad ; \quad \hat{S}_g^*(i,\ k)\ \epsilon\ \hat{E}_2\ .$$

There are 4 additional possibilities but they can be disregarded since the ML estimates $\hat{\underline{\beta}}$ are consistent and tend to $\underline{\beta}$, so that asymptotically:

$$Pr[(S_g^*(i,\ k) > S_0) \cap (\hat{S}_g^*(i,\ k) \le \hat{S}_0)] \to 0 \quad \text{and} \tag{6a}$$

$$\forall\ i,\ j,\ k$$

$$Pr[(S_g^*(i,\ k) \le S_0) \cap (\hat{S}_g^*(i,\ k) > \hat{S}_0)] \to 0 \tag{6b}$$

where Pr stands for probability.

Using indicator functions the loss function in (3) can be written as:

$$h(\hat{\underline{\beta}} - \underline{\beta}) = h_a(\hat{\underline{\beta}} - \underline{\beta})\ I_{\{a\}} + h_b(\hat{\underline{\beta}} - \underline{\beta})\ I_{\{b\}} + h_c(\hat{\underline{\beta}} - \underline{\beta})\ I_{\{c\}} \tag{7}$$
$$+ h_d(\hat{\underline{\beta}} - \underline{\beta})\ I_{\{d\}}$$

where I takes values of 1 or 0 as follows:

$$I_{\{l\}} = \begin{cases} 1 & \text{if } l \text{ holds} \\ 0 & \text{otherwise} \end{cases} \quad ; \quad l = a,b,c,d$$

Assume that:

(A1) $h_l(o) = 0$, $l = a,b,c,d$;

(A2) $h_l(\hat{\underline{\beta}} - \underline{\beta})$ is increasing function of $|\hat{\underline{\beta}} - \underline{\beta}|$; and

(A3) the first and the second, right and left derivatives of $h_l(\hat{\underline{\beta}} - \underline{\beta})$ exist at the point $\hat{\underline{\beta}} = \underline{\beta}$, (since $h_l(\hat{\underline{\beta}} - \underline{\beta})$ has a minimum at the point $\hat{\underline{\beta}} = \underline{\beta}$, the first derivatives are zero).

Approximating equation (7) by a second-order Taylor series expansion around $h_l(0)$ yields:

$$h_l(\hat{\underline{\beta}} - \underline{\beta}) \approx h_l(0) - \sum_{j=1}^{3} \frac{\partial h_l(\ )}{\partial \hat{\beta}_j / \hat{\underline{\beta}}=\underline{\beta}} (\hat{\beta}_j - \beta_j) + \frac{1}{2} \sum_{j=1}^{3} \frac{\partial^2 h_l(\ )}{\partial \hat{\beta}^2 / \hat{\underline{\beta}}=\underline{\beta}_j} (\hat{\beta}_j - \beta_j)^2$$

$$+ \frac{\partial^2 h_l(\ )}{\partial \hat{S}_0\, \partial \hat{a} / \hat{\underline{\beta}}=\underline{\beta}} (\hat{S}_0 - S_0)(\hat{a} - a) + \frac{\partial^2 h_l(\ )}{\partial \hat{S}_0\, \partial \hat{b}_1 / \hat{\underline{\beta}}=\underline{\beta}} (\hat{S}_0 - S_0)(\hat{b}_1 - b_1) \quad (8)$$

$$+ \frac{\partial^2 h_l(\ )}{\partial \hat{a} \partial \hat{b}_1 \ / \hat{\underline{\beta}}=\underline{\beta}} (\hat{a} - a)(\hat{b}_1 - b_1) \ .$$

Under assumptions A1-A3, equation (8) reduces to

$$h_l(\hat{\underline{\beta}} - \underline{\beta}) \approx \frac{1}{2} \frac{\partial^2 h_l(\ )}{\partial \hat{S}_0^2 / \hat{\underline{\beta}}=\underline{\beta}} (\hat{S}_0 - S_0)^2 + \frac{1}{2} \frac{\partial^2 h_l(\ )}{\partial \hat{a}^2 / \hat{\underline{\beta}}=\underline{\beta}} (\hat{a} - a)^2 \quad (9)$$

$$+ \frac{1}{2} \frac{\partial^2 h_l(\ )}{\partial \hat{b}_1^2 / \hat{\underline{\beta}}=\underline{\beta}} (\hat{b}_1 - b_1)^2 + \frac{\partial^2 h_l(\ )}{\partial \hat{S}_0 \partial \hat{a} / \hat{\underline{\beta}}=\underline{\beta}} (\hat{S}_0 - S_0)(\hat{a} - a)$$

$$+ \frac{\partial^2 h_1(\ )}{\partial\hat{S}_0 \partial\hat{b}_1 / \hat{\underline{\beta}}=\underline{\beta}} (\hat{S}_0 - S_0)(\hat{b}_1 - b_1) + \frac{\partial^2 h_1(\ )}{\partial\hat{a}\partial\hat{b}_1 / \hat{\underline{\beta}}=\underline{\beta}} (\hat{a} - a)(\hat{b}_1 - b_1) \equiv TAI(h_1) .$$

The observations Y_i are normally distributed and the approximation loss function (7) is proportional to the squared errors. It is therefore asymptotically true that Bayes estimates (i.e., the parameter estimates that minimize the expected loss) are equivalent to the ML estimates (e.g., Bickel and Yahav, 1969).

As $\hat{\underline{\beta}} = (\hat{S}_0, \hat{a}, \hat{b}_1)$ is a random vector, the loss function is also random. For given values of $\underline{\beta}$, σ_1^2, σ_2^2 and a given scatter $\tilde{S}C(T)$ of the T observations S_1, $S_2, \ldots, S_T$, the conditional expectation of the loss function is:

$$E[h(\hat{\underline{\beta}} - \underline{\beta})/\underline{\beta}, \sigma_1^2, \sigma_2^2] \equiv H[T, \tilde{S}C(T)] \tag{10}$$

$$\approx E[TAI(h_a) . I_{\{a\}} /\underline{\beta}, \sigma_1^2, \sigma_2^2] + E[TAI(h_b) . I_{\{b\}}/ \underline{\beta}, \sigma_1^2, \sigma_2^2]$$

$$+ E[TAI(h_c) . I_{\{c\}}/ \underline{\beta}, \sigma_1^2, \sigma_2^2] + E[TAI(h_d) I_{\{d\}}/\underline{\beta}, \sigma_1^2, \sigma_2^2] .$$

Since the true parameter values of $(\underline{\beta}, \sigma_1^2, \sigma_2^2)$ are unknown, their ML estimates $(\hat{\underline{\beta}}, \hat{\sigma}_1^2, \hat{\sigma}_2^2)$ are used instead ,respectively. Thus the best attainable estimate of (10) is :

$$\hat{E} [h(\hat{\underline{\beta}} - \underline{\beta})/\hat{\underline{\beta}}, \hat{\sigma}_1^2, \hat{\sigma}_2^2) \equiv \hat{H} [T, \tilde{S}C(T)]. \tag{11}$$

Since $(\hat{\underline{\beta}}, \hat{\sigma}_1^2, \hat{\sigma}_2^2)$ tend asymptotically to $(\underline{\beta}, \sigma_1^2, \sigma_2^2)$, $\hat{H}[\cdot]$ might be regarded as a good approximation to $H[\cdot]$ if the number of observations, T, is large enough.

5. The Value of Additional Information

In this section the profitability of acquiring additional observations (S_i, Y_i) for potatoes is calculated. Let n be the number of additional observations and let C(n) be the cost of their acquisition. Finding the optimal spread of the additional observations is a complicated statistical problem whose analytical solution is beyond the scope of this paper. Following personal communication with statisticians, the spread of additional n observations $\tilde{S}C^*(T + n)$, which was used in the empirical application of the analysis, is uniformly scattered in a given interval around $\hat{S}_0(T)$ (the ML estimate of S_0, which is based on the a priori T observations).

With $\hat{H}[T, \tilde{S}C(T)]$ (equation (11) describing the situation a priori, the expected value of additional information from n observations with spread $\tilde{S}C^*(T + n)$ is:

$$EVSI(n^*) = \max_n EVSI(n) \qquad (12)$$

As mentioned, the empirical computation of EVSI(n) relates to potatoes. It was found from the empirical application of the SLRO-model that only 5.6 hectares of potatoes belongs to $\hat{E}_2(\hat{S}^*_g(i,k) \leq \hat{S}_0 = 6.054)$; which is only 1.4 of the total area of potatoes (400 ha) during the 4-year planning horizon. Hence, it is assumed that $I_{\{c\}} = I_{\{d\}} = 0$ (see equation (5)). Thus equation (7) can be rewritten as:

$$h(\hat{\underline{\beta}} - \underline{\beta}) = h_a(\hat{\underline{\beta}} - \underline{\beta})\ I_{\{\hat{S}^*_g(i,k)\epsilon\hat{E}_1\}} \cdot I_{\{S^*_g(i,k)\epsilon E_1\}} \cdot I_{\{\hat{S}_0 \leq S_0\}} \qquad (13)$$

$$+ h_b(\hat{\underline{\beta}} - \underline{\beta})\ I_{\{\hat{S}^*_g(i,k)\epsilon\hat{E}_1\}} \cdot I_{\{S^*_g(i,k)\epsilon E_1\}} \cdot I_{\{\hat{S}_0 > S_0\}} \quad .$$

But this expression can be further simplified based on the following arguments: Since the ML estimates are consistent it is asymptotically true that

$$P_r[S^*_g(i,k) > So) \cap (S^*_g(i,k) > \hat{S}_0)] \rightarrow 1 \text{ for every } i,g \text{ and } k. \qquad (14a)$$

Hence, the multiplication $I_{\{\hat{S}^*_g(i,k)\epsilon\hat{E}_1\}} \cdot I_{\{S^*_g(i,k)\epsilon E_1\}}$ can be replaced by the single indicator $I_{\{\hat{S}^*_g(i,k)\epsilon\hat{E}_1\}}$.

$$P_r[\hat{S}_0 \leq S_0) \cap (\hat{S}_0 > S_0)] = 0, \text{ i.e., } I_{\{\hat{S}_0 \leq S_0\}} + I_{(\hat{S}_0 > S_0)} = 1 \ . \qquad (14b)$$

$$h_a(\hat{\underline{\beta}} - \underline{\beta}) \cdot I_{\{\hat{S}^*_g(i,k)\epsilon\hat{E}_1\}} = h_b(\hat{\underline{\beta}} - \underline{\beta}) \cdot I_{\{\hat{S}^*_g(i,k)\epsilon\hat{E}_1\}} \ . \qquad (14c)$$

This has to be true because both cases, a and b, are related only to the linear decreasing segment of the response function. Under the specific empirical results of the SLRO-model it can be assumed that $I_{\{\hat{S}^*_g(i,k)\epsilon\hat{E}_1\}} = 1$, for every i,j,k. Hence, the possible situations of the loss function reduce to one and equation (11) can be written as (the letter T in parentheses represents the number of observations used to estimate $\underline{\beta}$, σ_1^2 and σ_2^2):

$$\hat{E}\left[h(\hat{\underline{\beta}} - \underline{\beta}) \;/\; \hat{\underline{\beta}}, \hat{\sigma}_1^2 \;,\; \hat{\sigma}_2^2\right] \simeq E\left[TAI(h) \;/\; \hat{\underline{\beta}}(T) \;,\; \hat{\sigma}_1^2(T) \;,\; \hat{\sigma}_2^2(T)\right]$$

$$= \frac{1}{2} \frac{\partial^2 h(\;)}{\partial \hat{S}_0^2 / \hat{\underline{\beta}} = \underline{\beta}} E(\hat{S}_0 - S_0)^2 + \frac{1}{2} \frac{\partial^2 h(\;)}{\partial \hat{a}^2 / \hat{\underline{\beta}} = \underline{\beta}} E(\hat{a} - a)^2 + \frac{1}{2} \frac{\partial^2 h(\;)}{\partial \hat{b}_1^2 / \hat{\underline{\beta}} = \underline{\beta}} E(\hat{b}_1 - b_1)^2$$

$$+ \frac{\partial^2 h(\;)}{\partial \hat{S}_0 \partial \hat{a} / \hat{\underline{\beta}} = \underline{\beta}} E(\hat{S}_0 - S_0)(\hat{a} - a) + \frac{\partial^2 h(\;)}{\partial \hat{S}_0 \partial \hat{b}_1 / \hat{\underline{\beta}} = \underline{\beta}} E(\hat{S}_0 - S_0)(\hat{b}_1 - b_1)$$

$$+ \frac{\partial^2 h(\;)}{\partial \hat{a} \partial \hat{b}_1 / \hat{\underline{\beta}} = \underline{\beta}} E(\hat{a} - a)(\hat{b}_1 - b_1) \;. \qquad (15)$$

Noting that $E(\hat{\beta}_i - \beta_i)^2 = V(\hat{\beta}_i)$ and $E(\hat{\beta}_i - \beta_i)(\hat{\beta}_j - \beta_j) = Cov(\hat{\beta}_i, \hat{\beta}_j)$, equation (15) can be rewritten as:

$$\hat{H}[T, \bar{S}c[T]] = \hat{E}\{h(\hat{\underline{\beta}} - \underline{\beta}) / \hat{\underline{\beta}} \;,\; \hat{\sigma}_1^2, \; \hat{\sigma}_2^2] = \frac{1}{2} \frac{\partial^2 h(\;)}{\partial \hat{S}_0^2 / \underline{\beta} = \underline{\beta}} \cdot [V(\hat{S}_0(T)) / \hat{\underline{\beta}}(T) \ldots]$$

$$+ \frac{1}{2} \frac{\partial^2 h(\;)}{\partial \hat{a}^2 / \hat{\underline{\beta}} = \underline{\beta}} [V(\hat{a}(T)) / \hat{\underline{\beta}}(T) \ldots] + \frac{1}{2} \frac{\partial^2 h(\;)}{\partial \hat{b}_1^2 / \hat{\underline{\beta}} = \underline{\beta}} [V(\hat{b}_1(T)) / \hat{\underline{\beta}}(T) \ldots]$$

$$+ \frac{\partial^2 h(\;)}{\partial \hat{S}_0 \partial \hat{a} / \hat{\underline{\beta}} = \underline{\beta}} [Cov(\hat{S}_0(T), \hat{a}(T)) / \hat{\underline{\beta}}(T) \ldots] + \frac{\partial^2 h}{\partial \hat{S}_0 \partial \hat{b}_1 \,|\, \hat{\underline{\beta}} = \underline{\beta}} [Cov(\hat{S}_0(T), \hat{b}_1(T)) / \hat{\underline{\beta}}(T) \ldots]$$

$$+ \frac{\partial^2 h}{\partial \hat{a} \partial \hat{b}_1 / \hat{\underline{\beta}} = \underline{\beta}} Cov(\hat{a}(T), \hat{b}_1(T)) / \hat{\underline{\beta}}(T) \ldots] \;. \qquad (16)$$

The term $\hat{H}(T, \bar{S}C(T)$ is an a priori point of reference necessary for the computation of EVSI(n) (see equation (12)). The values of the variances and the covariances of equation (16) can be calculated from equation (2).

6. Empirical Results

As mentioned, the empirical calculations of EVSI(n) were performed with respect to potatoes only. For T = 17 and C(n) = \$130n (based on personal communication with soil researchers from the Institute of Soil and Water, the Volcani Center, Bet Dagan) we get (all the partial derivatives are computed at the point $\hat{\underline{\beta}} = \underline{\beta}$):

$$
\begin{aligned}
EVSI(n) = \; & \frac{1}{2}\frac{\partial^2 h(\;)}{\partial \hat{S}_0^2}\,[V(\hat{S}_0(17)) - V(\hat{S}_0(17+n))/\hat{\beta}(17)\ldots] \qquad (17) \\
& + \frac{1}{2}\frac{\partial^2 h(\;)}{\partial \hat{a}^2}\,[V(\hat{a}(17)) - V(\hat{a}(17+n))/\hat{\beta}(17)\ldots] \\
& + \frac{1}{2}\frac{\partial^2 h(\;)}{\partial \hat{b}_1^2}\,[V(\hat{b}_1(17)) - V(\hat{b}_1(17+n))\;/\hat{\underline{\beta}}(17)\ldots] \\
& + \frac{\partial^2 h(\;)}{\partial \hat{S}_0 \partial \hat{a}}\,[Cov(\hat{S}_0(17),\;\hat{a}(17)) - Cov(\hat{S}_0(17+n),\hat{a}(17+n))/\hat{\underline{\beta}}(17)\ldots] \\
& + \frac{\partial^2 h(\;)}{\partial \hat{S}_0 \partial \hat{b}_1}[Cov(\hat{S}_0(17),\hat{b}_1(17)) - Cov(\hat{S}_0(17+n),\hat{b}_1(17+n))/\hat{\underline{\beta}}(17)\ldots] \\
& + \frac{\partial^2 h(\;)}{\partial \hat{a} \partial \hat{b}_1}[Cov(\hat{a}(17),\hat{b}_1(17)) - Cov(\hat{a}\;(17+n),\hat{b}_1(17+n))/\hat{\underline{\beta}}(17)\ldots] - 130n.
\end{aligned}
$$

The partial derivatives of equation (17) were approximated and calculated numerically. The computation of each approximated derivative involves a few runs of the SLRO-model and assumptions concerning the marginal increments of the relevant parameters. The operative goal of the analysis is to provide, with a reasonable level of accuracy, an estimation of EVSI, and indicate the need for additional observations. Since considerable computer time and memory space is needed in order to calculate the partial derivatives, it was decided to compare only three combinations of marginal increments which yield three values of EVSI(n). Each of the derivatives was calculated three times; they differ from one another in the marginal increments of the relevant parameters. The derivative approximations and the values of the marginal increments are left for the Appendix. It was found that the differences among the three runs are relatively small (see Table 1) so there is no significant difference in the operative conclusions.

Based on the results of Table 1, about 30 additional observation (S_i, Y_i) on potatoes should be made. The expected value of the additional information is in the range of $10,000 to $14,000. The magnitude of EVSI(n) is only about 1% of the total expected present value of the linear SLRO-model's objective function. However, the additional field experiments may contribute toward the reduction of the expectation of the loss function for more than a single farm in the experimental region and therefore, to substantially increase the profitability of additional sampling.

It is important to note that since a few approximations were used in the analysis (second-order Taylor expansion, approximations of the partial derivatives [see Appendix], and the use of asymptotical statistical theory with the results based on a medium sized sample), the results must be regarded as approximate. Their main value is that they enable us to learn the order of magnitude of EVSI(n) and to draw operative conclusions about additional sampling.

Table 1: The three computations of EVSI(n) for the response function of potatoes, with regard to the linear SLRO-model

n	EVSI(n) - in U.S. dollars computation I	computation II	computation III
2	2338	3078	3104
4	4325	5701	5766
6	5766	7623	7714
8	6838	9071	9182
10	7649	10188	10311
12	8272	11058	11194
14	8812	11747	11890
16	9240	12286	12442
18	9390	12721	12877
20	9597	13052	13221
22	9747	13318	13494
24	9844	13519	13708
26	9909	13675	13864
28	|9942|	13786	13974
30	9935	13864	14052
32	9909	|13916|	14097
34	9864	13909	|14162|
36	9740	13876	14110
38	9721	13701	14084
40	9630	13610	13981

7. Summary

The estimation of the response function of a given crop to soil salinity and the calculation of the expected value of additional information on the parameters of this function are important steps in the process of decision making regarding irrigation with saline water under conditions of uncertainty.

Considering the optimum value of the linear SLRO-model and the piecewise linear response-function parameters, a loss function was constructed and its possible states were defined. The loss function was approximated by a second-order Taylor expansion (after some suitable assumptions) and its approximated expectation was derived. Then, the expected value of additional information on the response function parameters and the optimal sample size were calculated for potatoes.

The main advantage in the analysis provided in this paper is that it provides providing a conceptual and methodological framework with which to investigate the value of sample information in a long-run farm-level analysis as well as creating an efficient tool for empirical application. Although some approximations were used, the results provide an estimate of EVSI and indicate the need for additional observations.

Appendix: Approximations of the Partial Derivatives of Equation (14)

This appendix presents the numerical calculation technique of the second order partial derivatives of equation (14), with regard to two parameters -- $\hat{\beta}_1$ and $\hat{\beta}_2$. For the sake of simplicity, let $Z(\underline{\beta}/\underline{\beta}) \equiv Z(\underline{\beta})$ and $Z(\hat{\underline{\beta}}/\underline{\beta}) \equiv Z(\hat{\underline{\beta}})$. Hence, the loss function is (see (9)): $h(\hat{\underline{\beta}}-\underline{\beta}) = Z(\underline{\beta}) - Z(\hat{\underline{\beta}})$.

(I) Approximation of the derivatives $\left.\frac{\partial^2 h(\hat{\underline{\beta}}-\underline{\beta})}{\partial \hat{\beta}_i^2}\right/\hat{\underline{\beta}} = \underline{\beta}$ $\quad i=1,2$

since $Z(\underline{\beta})$ is constant,

$$\left.\frac{\partial^2 h(\hat{\underline{\beta}}-\underline{\beta})}{\partial \hat{\beta}_i^2}\right/\hat{\underline{\beta}}=\underline{\beta} = - \left.\frac{\partial^2 Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_i^2}\right/\hat{\underline{\beta}}=\underline{\beta} .$$

By a third-order Taylor expansion (all the following derivatives are computed at the point $\hat{\underline{\beta}}=\underline{\beta}$) we get:

$$\text{(a)} \qquad Z(\hat{\beta}_1 + \Delta, \hat{\beta}_2) \approx Z(\hat{\underline{\beta}}) + \Delta \cdot \frac{\partial Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_1} + \frac{\Delta^2}{2} \cdot \frac{\partial^2 Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_1^2} + \frac{\Delta^3}{3!} \cdot \frac{\partial^3 Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_1^3}$$

$$\text{(b)} \qquad Z(\hat{\beta}_1 - \Delta, \hat{\beta}_2) \approx Z(\hat{\underline{\beta}}) - \Delta \cdot \frac{\partial Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_1} + \frac{\Delta^2}{2} \cdot \frac{\partial^2 Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_1^2} - \frac{\Delta^3}{3!} \cdot \frac{\partial^3 Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_1^3} .$$

Summation of (a) and (b) yields

$$- \frac{\partial^2 Z(\hat{\beta})}{\partial \hat{\beta}_1^2} = \frac{\partial^2 h(\hat{\underline{\beta}}-\underline{\beta})}{\partial \hat{\beta}_1^2} \approx \frac{2Z(\hat{\underline{\beta}}) - [Z(\hat{\beta}_1 + \Delta, \hat{\beta}_2) + Z(\hat{\beta}_1 - \Delta, \hat{\beta}_2)]}{\Delta^2} ,$$

and equivalently, with regard to $\hat{\beta}_2$:

$$\frac{\partial^2 h(\hat{\underline{\beta}}-\underline{\beta})}{\partial \hat{\beta}_2^2} \approx \frac{2Z(\hat{\underline{\beta}}) - [Z(\hat{\beta}_1, \hat{\beta}_2 + \delta) + Z(\hat{\beta}_1, \hat{\beta}_2 - \delta)]}{\delta^2} .$$

(II) Approximation of the derivatives $\frac{\partial^2 h(\hat{\underline{\beta}}-\underline{\beta})}{\partial \hat{\beta}_1 \, \partial \hat{\beta}_2} \Big/ \hat{\underline{\beta}} = \underline{\beta}$

$$\frac{\partial^2 h(\hat{\underline{\beta}}-\underline{\beta})}{\partial \hat{\beta}_1 \, \partial \hat{\beta}_2} \Big/ \hat{\underline{\beta}}=\underline{\beta} = - \frac{\partial^2 Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_1 \, \partial \hat{\beta}_2} \Big/ \hat{\underline{\beta}}=\underline{\beta} .$$

By Taylor expansion:

(c)
$$Z(\hat{\beta}_1 + \Delta, \hat{\beta}_2 + \delta) \approx Z(\hat{\beta}) + (\Delta \frac{\partial Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_1} + \delta \frac{\partial Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_2})$$
$$+ \frac{1}{2} [\Delta^2 \frac{\partial^2 Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_1^2} + 2\Delta\delta \cdot \frac{\partial^2 Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_1 \partial \hat{\beta}_2} + \delta^2 \cdot \frac{\partial^2 Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_2^2}] + \frac{1}{3!} [\Delta^3 \cdot \frac{\partial^3 Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_1^3}$$
$$+ 3\Delta^2\delta \frac{\partial^3 Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_1^2 \partial \hat{\beta}_2} + 3\Delta\delta^2 \frac{\partial^3 Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_1 \partial \hat{\beta}_2^2} + \delta^3 \frac{\partial^3 Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_2^3}] ;$$

(d)
$$Z(\hat{\beta}_1 - \Delta, \hat{\beta}_2 - \delta) \approx Z(\hat{\underline{\beta}}) - (\Delta \frac{\partial Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_1} + \delta \frac{\partial Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_2}) + \frac{1}{2} [\Delta^2 \cdot \frac{\partial^2 Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_1^2}$$
$$+ 2\Delta\delta \frac{\partial^2 Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_1 \partial \hat{\beta}_2} + \delta^2 \cdot \frac{\partial^2 Z(\underline{\beta})}{\partial \hat{\beta}_2^2}] - \frac{1}{3!} [\Delta^3 \cdot \frac{\partial^3 Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_1^3} + 3\Delta^2\delta \cdot \frac{\partial^3 Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_1^2 \partial \hat{\beta}_2}$$
$$+ 3\Delta\delta^2 \cdot \frac{\partial^3 Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_1 \partial \hat{\beta}_2^2} + \delta^3 \cdot \frac{\partial^3 Z(\hat{\underline{\beta}})}{\partial \hat{\beta}_2^3}] .$$

Summation of (c) and (d) yields

$$- \frac{\partial^2 Z(\hat{\underline{\beta}})}{\partial\hat{\beta}_1 \partial\hat{\beta}_1} = \frac{\partial^2 h(\hat{\underline{\beta}} - \underline{\beta})}{\partial\hat{\beta}_1 \partial\hat{\beta}_2} \approx \frac{Z(\hat{\underline{\beta}})}{\Delta\delta} + \frac{\Delta}{2\delta} \cdot \frac{\partial^2 Z(\hat{\underline{\beta}})}{\partial\hat{\beta}_1^2} + \frac{\delta}{2\Delta} \cdot \frac{\partial^2 Z(\hat{\underline{\beta}})}{\partial\hat{\beta}_2^2}$$

$$- \left[\frac{Z(\hat{\beta}_1 + \Delta, \hat{\beta}_2 + \delta) + Z(\hat{\beta}_1 - \Delta, \hat{\beta}_2 - \delta)}{2\Delta\delta} \right] .$$

Let $\underline{X}^*(\hat{\underline{\beta}})$ be the optimal solution vector of the linear SLRO-model, and $\underline{C}(\hat{\underline{\beta}})$ be the vector of the activities' expected net returns (in present value) in U.S.$/ha. Thus

$$Z(\hat{\underline{\beta}}) = \underline{C}(\hat{\underline{\beta}}) \cdot \underline{X}^*(\hat{\underline{\beta}}) \ ;$$

$$Z(\hat{\beta}_1 \pm \Delta, \hat{\beta}_2) = \underline{C}(\hat{\underline{\beta}}) \cdot \underline{X}^*(\hat{\beta}_1 \pm \Delta, \hat{\beta}_2) \ ;$$

$$Z(\hat{\beta}_1, \hat{\beta}_2 \pm \delta) = \underline{C}(\hat{\underline{\beta}}) \cdot \underline{X}^*(\hat{\beta}_1, \hat{\beta}_2 \pm \delta) \ ; \text{ and}$$

$$Z(\beta_1 \pm \Delta, \beta_2 \pm \delta) = \underline{C}(\hat{\underline{\beta}}) \cdot \underline{X}^*(\hat{\beta}_1 \pm \Delta, \hat{\beta}_2 \pm \delta) \ .$$

As mentioned, the partial derivatives were calculated three times, according to the following marginal increments of the parameters:

Alternative I : $\Delta\hat{S}_0 = 0.5$; $\Delta\hat{a} = 0.2$; $\Delta\hat{b}_1 = 4$

Alternative II: $\Delta\hat{S}_0 = 0.25$; $\Delta\hat{a} = 0.1$; $\Delta\hat{b}_1 = 2$

Alternative III: $\Delta\hat{S}_0 = 0.25$; $\Delta\hat{a} = 0.5$; $\Delta\hat{b}_1 = 2$.

REFERENCES

Antonovitz, F. and Roe, T., (1984), "The Value of Rational Expectations Forecast in a Risky Market: A Theoretical and Empirical Approach," Amer. J. Agric. Econ., 66:717-723.

Bickel, P.I. and Yahav, J.A., (1969), "Some Contributions to the Asymptotic Theory of Bayes Solution," Z. Wahrscheinlichkeitstheorie Verw., Geb. II, p. 257-276.

Bie, S.W. and Ulph. A., (1972), "The Economic Value of Soil Survey Information," J. Agron. Econ., 23: 285-295.

Dantzig, G.B. and Madansky, A., (1961), "On the Solution of Two-Stage Linear Programs Under Uncertainty," Proc. of Fourth Berkeley Symp. on Math. Stat. and Prob., Held at the University of California, Berkeley. Vol. 1.

Davis, D.R. and Dvoranchik, W.M., (1971), "Evaluation of the Worth of Additional Data," Water Res. Bul., 7: 700-707.

DeGroot, M.H., (1970), Optimal Statistical Decisions, McGraw-Hill, Carnegie Mellon Univeristy, New York.

Duckstein, L., Szidarorszky, F. and Yakowitz, S., (1977), "Bayes Design of Reservoir Under Random Sediment Yield," Water Res. Res., 13(5): 713-718.

El Agizy, M., (1967), "Two-Stage Programming Under Uncertainty with Discrete Distribution Function," Operations Research, 15: 55-70.

Feinerman, E. and Yaron, D., (1983), "Economics of Irrigation Water Mixing Within a Farm Framework," Water Res. Res., 19(2): 337-345.

Klemes, V., (1977), "Value of Information in Reservoir Optimization," Water Res. Res., 13(): 837-850.

Maas, E.V. and Hoffman G.J., (1977), "Crop Salt Tolerance-Current Assessment," ASCE, J. of the Irrigation and Drainage Division, IR2, 103: 115-134.

Maddock, T., (1973), "Management Model as a Tool for Studying the Worth of Data," Water Res. Res., 9: 270-280.

Mjelde, J.W., Sonka, S.T., Dixon, B.L. and Lamb, P.J., (1988), "Valuing Forecast Characteristics in a Dynamic Agricultural Production System," Amer. J. Agric. Econ. : 674-684.

Pratt, J.W., Raiffa, H. and Schlaifer, R., (1965), Introduction to Statistical Decision Theory, McGraw-Hill, New York.

Preckel, P.V., Loehman, E.T. and Kaylen, M.S., (1987), "The Value of Public Information for Microeconomic Production Decisions," Western J. of Agric. Econ., 12: 193-197.

Quandt, R.E., (1972), "A New Approach to Estimating Switching Regressions," Journal of the American Statistical Association, 67: 306-310.

Ryan, J.G. and Perrin, R.K., (1974), "Fertilizer Response Information and Income Gains: The Case of Potatoes in Peru," Amer. J. Agr. Econ., 56: 337-43.

Sadan, D. and Berglas, M., (1980), Unpublished Data Set, The Extension Service, Ministry of Agriculture, Israel.

Zacks, S., (1971), The Theory of Statistical Inference, John Wiley, New York.