Risk Behavior and Rational Expectations in the U.S. Broiler Market

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Abstract

This study examines the empirical implications of extending the rational expectations hypothesis (REH) to include price uncertainty. Unlike previous studies, a general estimation framework that incorporates both the restrictions on structural parameters and the variance-covariance terms is developed. A new time series approach known as GARCH processes is also used to generate time-varying expectations of both the means and the variances of exogenous variables in the REH model with risk.

The empirical application is with a quarterly model of the U.S. broiler industry; the results indicate that the rational expectation of price variance is an important determinant of broiler supply. Additionally, a formal test indicates that the restrictions implied by the REH cannot be rejected. The restricted model also compares favorably with an unrestricted version that uses instruments for the mean and the variance of expected prices.

Introduction

In recent years considerable research, both theoretical and applied, has been aimed at improving the specification and estimation of aggregate agricultural supply relationships (e.g., Eckstein 1985; Lee and Helmberger 1985; Choi and Johnson 1987). Although many issues have been investigated, two recurring themes have been the manner in which agents form expectations about future prices, and the effects of revenue or price uncertainty on production decisions. Previous studies, including those by Just (1974), Traill (1978), Hurt and Garcia (1982), and Brorsen, Chavas, and Grant (1987), have found that risk terms are important conditioning variables in aggregate supply equations. At the same time, the rational expectations hypothesis has emerged as a credible alternative to more traditional approaches based on naive expectations. Studies by Goodwin and Sheffrin (1982), Shonkwiler and Emerson (1982), Eckstein (1984), and Shonkwiler and Maddala (1985) have illustrated that the rational expectations approach is a valid option for modeling expectations in agricultural supply response equations.

Only in recent years have agricultural economists begun to examine the theoretical and empirical implications associated with extending the rational expectations hypothesis to a more general model that includes risk averse behavior. Empirical investigations of the effects of price uncertainty in a rational expectations setting have been reported by Antonovitz and Roe (1986), Antonovitz and Green (1987), and Seale and Shonkwiler (1987). Although these studies represent important contributions to the literature on agricultural supply analysis, several problems remain.

For instance, Antonovitz and Roe (1986) used an instrumental variable approach to generate expectations of the mean and the variance of price. While this approach serves as a useful first approximation, it does not use all information implied by rationality in the estimation. Consequently, formal tests of the rational expectations hypothesis cannot be conducted. Alternatively, Antonovitz and Green (1987) and Seale and Shonkwiler (1987) developed expressions for the rational expectation of price variance by using the underlying model's implied reduced form. The cross-equation restrictions resulting from the rational expectation of both the mean and the variance of the price distribution can then be imposed and tested in the usual manner. However, the models considered by both sets of authors are misspecified in that the variance-covariance terms associated with the model's error process are omitted from the reduced-form price and variance equation for purposes of estimation. The implication is that any tests of the restrictions resulting from the dual assumptions of rational expectations and risk averse behavior will have little empirical meaning (White 1982).

The objective of this paper is to examine the empirical implications associated with extending the rational expectations hypothesis to include price risk. A general approach for modeling price variance in a rational expectations framework is developed, and a maximum likelihood estimation procedure that does not entail omitting variance-covariance terms is described. Another important feature of this study is that a new time-series approach known as GARCH (Generalized Autoregressive Conditional Heteroscedastic) processes is used to generate time-varying predictions of the conditional forecast variances for exogenous variables. The GARCH approach represents the logical extension of Box-Jenkins methods, typically used to generate forecasts of exogenous variables in standard rational expectations models (Wallis 1980), to the case where price variance is incorporated. The application is with a quarterly model of the U.S. broiler industry.

The broiler industry seems promising for examining the effects of price uncertainty in a rational expectations setting for several reasons. First, previous studies have confirmed the importance of rational expectations in broiler supply response (Huntzinger 1979; Goodwin and Sheffrin 1982). Second, as Goodwin and Sheffrin (1982, p. 660) indicate, "The decision to supply broilers is, of course, made under uncertainty, and in principle, other moments of the probability distribution of prices besides the mean could affect behavior." Finally, the effects of price risk on production may be relatively easy to isolate, since output uncertainty is typically negligible (Lasley 1983).

General Framework

Consider a market model consisting of G linear equations where agents form expectations about the mean and the variance of H endogenous variables (G ≥ H). In matrix notation, the model can be written as

$$B\underline{y}_t + A_1\underline{y}_t^e + A_2\underline{y}_t^v + \Gamma_1\underline{x}_{1t} + \Gamma_2\underline{x}_{2t} = \underline{u}_t. \qquad (1)$$

Here B, A_1, A_2 are G x G parameter matrices; Γ_1 and Γ_2 are G x K_1 and G x K_2 parameter matrices, respectively; $\underline{y}_t$, $\underline{y}_t^e$, and $\underline{y}_t^v$ are G-dimensional vectors; $\underline{x}_{1t}$ is a K_1-dimensional vector of exogenous variables whose one-period-ahead values are known with certainty; and $\underline{x}_{2t}$ is a K_2-dimensional vector of exogenous variables whose future values are not known.[1] Also, $\underline{u}_t$ is a G x 1 vector of joint normally distributed error terms with mean vector $\underline{0}$ and positive definite variance-covariance matrix Σ. The endogenous variables $\underline{y}_t$ and exogenous variables $\underline{x}_{1t}$ and $\underline{x}_{2t}$ are observable. The vector $\underline{y}_t^e$ represents unobservable expectations, formed in period t - 1, about the means of H endogenous variables; and $\underline{y}_t^v$ denotes unobservable expectations, also formed in period t - 1, about the variances of H endogenous variables. Any lagged endogenous variables are included in the vector $\underline{x}_{1t}$.

Before the model in (1) can be implemented, it is necessary to posit some method for determining values of the unobservable expectations vectors $\underline{y}_t^e$ and $\underline{y}_t^v$. The approach used is to assume that agents form expectations rationally. That is, the predictions made by agents regarding the unobservable means and variances of endogenous variables are consistent with the underlying model structure as depicted in (1) (Muth 1961). The implication is that the unobservable expectations represented by the mean vector $\underline{y}_t^e$ will equal the mathematical expectation of $\underline{y}_t$, implied by the the model in (1), conditional on the information set Ω_{t-1} available at time t - 1. That is,

$$\underline{y}_t^e = E_{t-1}(\underline{y}_t | \Omega_{t-1}), \qquad (2)$$

where the subscript t - 1 on the expectation operator denotes the period in which expectations are formed. The econometric implications of the assumption in (2) have been considered by Wallis (1980) and others. However, the models considered previously have not allowed for the possibility that agents exhibit risk avoiding behavior and, hence, that expectations of higher-order moments are also relevant. The rational expectation of the variance of endogenous variables can be defined in a manner analogous to that in (2). Specifically,

$$\begin{aligned} Y_t^v &= \text{diag } E_{t-1}[(Y_t - E_{t-1}\ (Y_t|\Omega_{t-1}))(Y_t - E_{t-1}(Y_t|\Omega_{t-1}))^1] \\ &= \text{var}_{t-1}\ (Y_t|\Omega_{t-1}), \end{aligned} \tag{3}$$

where Var is the variance operator. The expression in (3) defines the rational expectation of the variances of relevant endogenous variables.

The econometric implications of the assumptions in (2) and (3) can be examined by obtaining the reduced form

$$\underline{y}_t = -B^{-1}A_1\underline{y}_t^e - B^{-1}A_2\underline{y}_t^v - B^{-1}\Gamma_1\underline{x}_{1t} - B^{-1}\Gamma_2\underline{x}_{2t} + B^{-1}\underline{u}_t. \tag{4}$$

Taking the mathematical expectation of (4), conditional on the information set Ω_{t-1}, gives

$$\underline{y}_t^e = -B^{-1}A_1\underline{y}_t^e - B^{-1}A_2\underline{y}_t^v - B^{-1}\Gamma_1\underline{x}_{1t} - B^{-1}\Gamma_2\underline{x}_{2t}^e\ , \tag{5}$$

where $\underline{x}_{2t}^e$ is the expectation vector of unknown exogenous variables, $\underline{x}_{2t}$. The usual approach is to solve the system in (5) for $\underline{y}_t^e$ as a function of model parameters and the expectations of exogenous variables. The resulting expression for the rational predictor $\underline{y}_t^e$ is then substituted into the system in (1) and, given instruments for the expected values $\underline{x}_{2t}^e$, estimation proceeds by using a nonlinear full-information systems estimator. The procedure is more complicated in the present case, however, as illustrated by the presence of $\underline{y}_t^v$ in Equation 5. In other words, Equation 5 is only a partially reduced form, since the rational expectation of the variances of endogenous variables appears as a right-hand-side argument. The model be closed can only by deriving a suitable expression for the rational expectation $\underline{y}_t^v$.

Subtracting $\underline{y}_t^e$ from $\underline{y}_t$ gives

$$\underline{y}_t - \underline{y}_t^e = -B^{-1}\Gamma_2(\underline{x}_{2t} - \underline{x}_{2t}^e) + B^{-1}\underline{u}_t. \tag{6}$$

Multiplying both sides of Equation 6 with their respective transposes, taking conditional expectations, and assuming that $\underline{x}_{2t}$ and $\underline{u}_t$ are uncorrelated, gives

$$E_{t-1}[(\underline{y}_t - \underline{y}_t^e)(\underline{y}_t - \underline{y}_t^e)'] = B^{-1}\Gamma_2\psi_{2t}\Gamma_2'B^{-1'} + B^{-1}\Sigma B^{-1'}, \tag{7}$$

where ψ_{2t} is the variance-covariance matrix associated with $\underline{x}_{2t}$. Only the diagonal elements of Matrix Equation 7 are of interest, since covariance measures between endogenous variables are not considered. It is easily verified that the matrix defined in (7) is positive definite, since both the first and second terms are positive definite (Dhrymes 1974, p. 578).

Using (7), the rational expectation of the variance of endogenous variables $\underline{y}_t$ can be written as

$$\underline{y}_t^v = \mathrm{diag}(B^{-1}\Gamma_2\psi_{2t}\Gamma_2'B^{-1'} + B^{-1}\Sigma\, B^{-1'}). \tag{8}$$

Matrix Equation 8 illustrates that the rational predictor for the variances of endogenous variables is a function of model parameters, including elements in the error variance-covariance matrix, and the forecast variances of exogenous variables whose values are unknown at time t - 1. Expression 8 represents a marked departure from the way variance terms traditionally have been defined for empirical work. Price variance, for instance, is typically expressed as a weighted moving average of the squared deviations of price from its expected value (Just 1974; Brorsen, Chavas, and Grant 1987). Consequently, the degree of price variability defined in the model depends only on past price changes; no attempt is made to identify the underlying structure generating random prices. On the other hand, the expression in (8) specifies that in a linear system the forecast variances of endogenous variables are uniquely defined by structural parameters and the forecast variances of exogenous variables. Not only is this formulation consistent with the rational expectations hypothesis but it is also more appealing intuitively than previous definitions because a clear statement of the causality underlying the variance process is provided.

The left-hand side of Matrix Equation 8 can be substituted for the vector $\underline{y}_t^v$ in (5). Assuming that $B + A_1$ is nonsingular, the resulting expression for the rational expectations vector $\underline{y}_t^e$ can be solved for. After collecting terms, it is

$$\begin{aligned}\underline{y}_t^e = &-(B + A_1)^{-1}\Gamma_1\underline{x}_{1t} - (B + A_1)^{-1}\Gamma_2\underline{x}_{2t}^e \\ &- (B + A_1)^{-1}A_2\mathrm{diag}(B^{-1}\Gamma_2\psi_{2t}\Gamma_2'B^{-1'} + B^{-1}\Sigma B^{-1'}).\end{aligned} \tag{9}$$

The vectors in (8) and (9) can be substituted for the expectations of the mean and of the variance in the original system, Equation 1, to obtain the following estimable form:

$$\begin{aligned}By_t &- A_1(B + A_1)^{-1}\Gamma_1\underline{x}_{1t} - A_1(B + A_1)^{-1}\Gamma_2\underline{x}_{2t}^e \\ &- A_1(B + A_1)^{-1}A_2\mathrm{diag}(B^{-1}\Gamma_2\psi_{2t}\Gamma_2'B^{-1'} + B^{-1}\Sigma B^{-1'}) \\ &+ A_2\mathrm{diag}(B^{-1}\Gamma_2\psi_{2t}\Gamma_2'B^{-1'} + B^{-1}\Sigma B^{-1'}) + \Gamma_1\underline{x}_{1t} + \Gamma_2\underline{x}_{2t} = \underline{u}_t.\end{aligned} \tag{10}$$

The system in (10) shows that the observed values of $\underline{y}_t$ are determined by the expected means and variances of unknown exogenous variables, the actual values of exogenous variables, and the variance-covariance terms associated with the model's error terms. The fourth and fifth terms in (10) are a direct result of incorporating rational expectations about the second moments of endogenous variables. In fact, if matrix A_2 vanishes, then the system in (10) reduces to the standard estimable form for a rational expectations model.

Model Estimation and Implementation

An important aspect of the system in (10) is that the variance-covariance matrix Σ enters as an explanatory component. Previous applications of the rational expectations hypothesis that have incorporated uncertainty have omitted the terms involving Σ for purposes of estimation (e.g., Antonovitz and Green 1987; Seale and

Shonkwiler 1987). However, this is inappropriate given the model specification. The implications are that (1) the implied values for expectations of both the means and the variances of endogenous variables will be biased by a constant additive factor, and (2) the resulting parameter estimates will be biased, thus clouding any interpretation of the results.[2]

In terms of estimation, the problem is that changes in Σ directly affect the values of the computed residuals associated with the system in (10), thus affecting the estimates of the remaining model parameters $\underline{\alpha}$.[3] In other words, the derivatives of the log likelihood function corresponding to the system in (10) with respect to $\underline{\alpha}$ and Σ cannot be solved independently of each other. The reason is that there are additional restrictions associated with the variance-covariance matrix Σ resulting from the rational expectations of the variances of endogenous variables. Consequently, standard software packages that employ a "concentrated" log likelihood function are not appropriate for estimating rational expectations models that include variance terms. This problem can be circumvented, however, by using an "unconcentrated" log likelihood function for estimation (Fair and Taylor 1983, p. 1183).[4] The resulting first-order conditions can be obtained analytically or numerically, and they resemble those obtained for disequilibrium models (Amemiya 1974). Numerical maximization procedures that use only first derivatives, such as the Davidon-Fletcher-Powell (DFP) method or the Brendt et al. (1974) method, can be used to obtain maximum likelihood estimates of the system in (10). Consequently, there is no need to delete terms involving Σ for purposes of econometric estimation.

Before estimation can proceed, some method is needed to obtain instruments for the expectations of the means and the variances of exogenous variables.[5] In most empirical applications of the rational expectations hypothesis, ARIMA models are estimated for the exogenous variables and are then used to generate instruments for the expected values of exogenous variables. While these methods are appropriate for the case where predictions of the variances of exogenous variables are not required, they are not suitable here. This is because standard time series models are specified so that both the conditional and unconditional forecast variances are constant over time (Engle, 1982). From Equation 8 it is obvious that if ψ_{2t} is constant, $\underline{y}_t^v$ will also be time invariant; thus it will be impossible to estimate the system in (10). Previous studies have used sequential updating of the parameter estimates associated with the ARIMA models of exogenous variables to obtain time-varying estimates of the forecast variances. An important limitation of this approach is that a relatively long period of presample data on the exogenous variables is required for implementation.

Fortunately, there is a reasonable alternative. In a series of recent articles, Bollerslev (1986, 1987) has examined the properties of autoregressive models with conditional heteroscedastic error processes. These are denoted as GARCH models. A chief advantage of these processes is that, unlike standard time series models, the conditional variance h_t of a real stochastic process x_t (the conditioning variables including but not being limited to lagged values of x_t) is nonstationary. In a GARCH model the conditional variance of the stochastic process is specified as a function of past innovations as well as lagged values of the conditional variance.

To illustrate, let ϵ_t be an innovation in a linear regression,

$$\epsilon_t = x_t - \underline{z}_t'\underline{b} \; , \tag{11}$$

where x_t is the dependent variable; $\underline{z}_t$ is a vector of observations on explanatory variables that include past realizations of x_t; and $\underline{b}$ is an unknown parameter vector. Furthermore, assume that $\epsilon_t \sim N(0,h_t)$, where

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \tag{12}$$

and where

$$\begin{array}{ll} p \geq 0, & q \geq 0 \\ \alpha_0 > 0, & \alpha_i \geq 0,\ i = 1, \ldots, q, \text{ and} \\ \beta_i \geq 0, & i = 1, \ldots, p. \end{array}$$

The conditional variance equation in (12) is a GARCH(p,q) process.[6] The nonnegativity constraints associated with the parameters in the h_t equation are necessary to satisfy certain regularity conditions associated with the estimated GARCH process. GARCH models are used in the empirical application to generate conditional forecasts for both the means, $\underline{x}_t^e$, and variances, ψ_{2t}, of exogenous variables.

Model Specification

The empirical model is based on a three-equation supply-demand structure of the U.S. broiler market. The assumptions are that broiler production and prices are determined in a competitive environment, that producers form expectations about future values of endogenous variables in a manner consistent with the rational expectations hypothesis, and that producers are risk averse, with the major source of uncertainty being with respect to product price. Also, the biological production cycle for broilers is approximately eight weeks, with an additional one to three weeks required for cleanup between batches. Consequently, a quarterly time frame is suitable for investigating supply response in this industry (Chavas and Johnson 1982).

Previous research has found that broiler producers form price expectations in a manner consistent with the rational expectations hypothesis. Both the studies by Huntzinger (1979) and by Goodwin and Sheffrin (1982) used a rational expectations approach to model expectations in the broiler industry. In both instances, reasonable estimates of supply parameters, as measured by elasticities, signs, and statistical significance, were obtained. In addition, Goodwin and Sheffrin (1982) were unable to reject the cross-equation restrictions implied by rationality.

While studies have confirmed the relevance of rational expectations for modeling the expected price in broiler supply equations, previous studies have not used a rational expectations framework to model price uncertainty in this industry. Although real broiler prices have declined steadily over the past 30 years--primarily because of rapid technological advancement--broiler prices can still exhibit substantial short-run fluctuations. As Lasley (1983) indicates, short-term price volatility reflects in part the ability of producers to adjust production rapidly in response to changing profit conditions. For instance, producers can adjust the number of chicks started per square foot of housing space, alter the length of the growout period, or change the number of batches raised per unit of time. This suggests that price expectations are important in determining ultimate production levels and, if growers collectively exhibit risk averse behavior, that some measure of precision or confidence associated with the mean expectation also may be important.

The broiler supply function is specified as

$$QBP_t = a_1D_{1t} + a_2D_{2t} + a_3D_{3t} + a_4D_{4t} + a_5WPB_t^e + a_6WPB_t^v \qquad (13)$$
$$+ a_7PBF_{t-1} + a_8HATCH_{t-1} + a_9QBP_{t-4} + u_{1t},$$

where

QBP_t = broiler production in period t, million pounds,
D_{jt} = quarterly dummy variable, j = 1,..., 4,
WPB_t^e = the expected real wholesale price of broilers in time t, viewed from period t - 1, dollars per cwt,
WPB_t^v = the expected variance of the wholesale price of broilers in time t, viewed in time t - 1,
PBF_{t-1} = real price of broiler feed in period t - 1, cents per pound, and
$HATCH_t$ = hatch of broiler-type chicks in commercial hatcheries in period t - 1, thousand head.

Since the biological production lag for broilers is approximately two months, it follows that current quarter production depends on the expectations formed by producers in the previous quarter. Wholesale prices are used in place of farm prices because there is a high level of vertical integration in the broiler industry and because the farm-wholesale price spread tends to be relatively stable (Lasley 1983). The only input price included is for feed, PBF_{t-1}, which was determined as a weighted average of the prices of corn and soybean meal.[7] As Rogers (1979) indicates, feed costs have historically accounted for 64-73 percent of total broiler production costs. Consequently, the feed cost variable, lagged one quarter, should reflect the important changes in short-run production costs. All prices in the supply equation were deflated by the Consumer Price Index (CPI).

In the short run, broiler production also depends on the number of broiler-type chicks available. This is reflected in the supply equation by including the HATCH variable, lagged one quarter. Broiler producers also may not be able to fully adjust production to a desired level during any given quarter. This could be due to capacity constraints, adjustment costs, and fixed contract periods. To account for this inertia in production, a lagged dependent variable was included in the supply equation.[8] On the basis of previous research (Chavas and Johnson 1982), a four-quarter lag on the dependent variable was used.

The demand equation for broilers is specified in price dependent form and is a function of the quantity of broilers demanded, the price of substitutes (including beef, pork, and turkey), and the expenditures for food. As before, all prices are deflated by the CPI. Seasonal dummy variables were also included to account for seasonal price patterns. The demand equation is given by

$$WPB_t = b_1D_{1t} + b_2D_{2t} + b_3D_{3t} + b_4D_{4t} + b_5QBD_t \qquad (14)$$

$$+ b_6RPB_t + b_7RPP_t + b_8RPT_t + b_9FEXP_t + u_{2t} ,$$

where

WPB_t = wholesale price of broilers, dollars per cwt,
QBD_t = quantity of broilers consumed, ready to cook, billion pounds,
RPB_t = retail price of beef, dollars per cwt,
RPP_t = retail price of pork, dollars per cwt,
RPT_t = retail price of turkey, dollars per cwt,
$FEXP_t$ = real food expenditures, dollars.

Note that the inverse demand function in (14) does not represent a consumer demand curve per se. The essential difference between (14) and a representative consumer demand curve is that the wholesale price of broilers has been used in place of the retail price. Granger causality tests were subsequently used to verify this price determination assumption.[9] The results confirm those obtained by Chavas (1978) and suggest that price determination in the broiler market does occur at the wholesale level.

represent a fractional part of the total market for broilers. A complete description of the derivation of the estimable form of the model for the broiler market is provided in the appendix.

Empirical Results

Maximum likelihood methods were used to estimate GARCH time series models for the retail prices of beef, pork, and turkey; food expenditures; and exogenous other demand. The estimation is based on sample data covering the 80-quarter period beginning with the first quarter of 1967 through the last quarter of 1986. In all cases, the models were fitted initially as GARCH(1,1) processes. In several instances, the estimated autocorrelation functions associated with the squared innovations indicated that higher-order GARCH processes were called for. A penalty function was used in the estimation so that the inequality constraints associated with the parameters in the conditional variance equations were imposed directly. The estimated GARCH models are reported in Table 1, along with several measures of fit including standard errors, R^2, and mean absolute error (MAPE). Note that the estimated coefficients associated with the conditional variance equations in the GARCH models are all significant. The implication is that the one-step-ahead forecast errors associated with these variables are time-varying. This is potentially important if producers do indeed behave rationally and if they exhibit risk averse behavior.

The estimated GARCH models in Table 1 were used to predict the unknow means and variances of exogenous variables that, in turn, were used as data for estimating the rational expectations model. Maximum likelihood estimates of the structural system were obtained by using the DFP algorithm with sample data from the 1967-1986 time period. All cross-equation restrictions implied by the rational expectations hypothesis, including restrictions on the variance-covariance matrix, were incorporated directly into the estimation. The maximum likelihood estimates of the broiler mode with rational expectations of both the mean and the variance of price are reported in the column headed REH in Table 2.

All estimated parameters have theoretically correct signs except for the retail price of beef, RPB_t, in the demand equation. Importantly, the sign on the estimated coefficient for expected price, WPB_t^e, is positive, while the sign on the estimated coefficient for expected price variance, WPB_t^v, is negative. The estimated coefficient for expected price is also significant at all usual α levels, while a one-tailed test on the risk coefficient indicates that it is significant at the 0.10 level. All other estimated coefficients associated with economic variables are significant at conventional levels.[10]

The implied short-run elasticities of supply with respect to the expected mean and variance of broiler price are 0.305 and -0.045, respectively. The supply elasticity with respect to feed price is -0.058. The estimated supply elasticities with respect to the expected price and feed costs seem reasonable and are well within the range of previously reported estimates (e.g., Goodwin and Sheffrin 1982; Chavas and Johnson 1982). Unfortunately, no comparisons are available for the estimated risk elasticity.

In a systems framework, common measures of individual equation explanatory power, such as R^2, have little meaning. An overall indication of explanatory power of the entire system can be obtained from the "generalized R^2," originally proposed by Baxter and Cragg (1970). The generalized R^2 is defined as

$$\tilde{R}^2 = \{1 - \exp[2(L_o - L_{max})/K]\},$$

where L_o is the value of the log likelihood function obtained when all parameters, excluding the seasonal dummy variables and the variance-covariance terms, are constrained to zero; L_{max} is the maximized value of the log likelihood function obtained when all parameters are allowed to vary; and K is the total number of observations. The $\tilde{R}^2$ coefficient for the estimated system in Table 2 was 0.996, indicating that the goodness of fit is extremely high.

Additional insight into the validity of the estimated model can be gained by testing the restrictions implied by rationality. A likelihood ratio test was used to evaluate the restrictions implied by rational expectations (Hoffman and Schmidt 1981). This requires estimating an unconstrained model and then comparing these results

Table 1. Maximum likelihood estimates of GARCH models

Price of Beef (RPB_t)

$$(1 - \underset{(0.003)}{0.972}\,B - \underset{(0.012)}{0.229}\,B^4 + \underset{(0.009)}{0.264}\,B^5)RPB_t = \underset{(13.439)}{5.443} + \epsilon_{1t}$$

$$h_{1t} = \underset{(0.512)}{1.835} + \underset{(0.011)}{0.156}\,\epsilon^2_{1t-1} + \underset{(0.009)}{0.740}\,h_{1t-1}$$

MAPE = 3.039 $\quad$ R^2 = 0.840

Price of Pork (RPP_t)

$$(1 - \underset{(0.004)}{1.137}\,B + \underset{(0.016)}{0.462}\,B^3 - \underset{(0.015)}{0.417}\,B^4 + \underset{(0.007)}{0.219}\,B^5)RPP_t = \underset{(6.144)}{8.017}$$

$$h_{2t} = \underset{(0.496)}{1.502} + \underset{(0.011)}{0.178}\,\epsilon^2_{2t-1} + \underset{(0.072)}{0.743}\,h_{2t-1}$$

MAPE = 4.384 $\quad$ R^2 = 0.850

Price of Turkey (RPT_t)

$$(1 - \underset{(0.190)}{1.198}\,B + \underset{(0.042)}{0.185}\,B^2 + \underset{(0.020)}{0.279}\,B^3 - \underset{(0.010)}{0.326}\,B^4 + \underset{(0.025)}{0.200}\,B^5$$

$$- \underset{(0.008)}{0.098}\,B^6)RPT_t = \underset{(0.531)}{1.252} + \epsilon_{3t}$$

$$h_{3t} = \underset{(0.201)}{0.833} + \underset{(0.150)}{0.937}\,\epsilon^2_{3t-1} + \underset{(0.002)}{0.021}\,\epsilon^2_{3t-2} + \underset{(0.001)}{0.016}\,h_{3t-1}$$

MAPE = 3.290 $\quad$ R^2 = 0.912

Food Expenditures ($FEXP_t$)

$$(1 - \underset{(0.021)}{1.203}\,B + \underset{(0.144)}{0.328}\,B^2 - \underset{(0.221)}{0.375}\,B^3 - \underset{(0.060)}{0.254}\,B^4)FEXP_t = \underset{(5.544)}{0.964} + \epsilon_{4t}$$

$$h_{4t} = \underset{(0.181)}{1.793} + \underset{(0.034)}{0.072}\,\epsilon^2_{4t-1} + \underset{(0.006)}{0.037}\,h_{4t-1}$$

MAPE = 0.716 $\quad$ R^2 = 0.989

Other Demand (QOD_t)

$$(1 - \underset{(0.018)}{0.737}\,B - \underset{(0.023)}{0.329}\,B^2 + \underset{(0.085)}{0.352}\,B^3 - \underset{(0.045)}{0.175}\,B^4)QOD_t = \underset{(6.157)}{13.908} + \epsilon_{5t}$$

$$h_{5t} = \underset{(83.498)}{45.389} + \underset{(0.004)}{0.056}\,\epsilon^2_{5t-1} + \underset{(0.007)}{0.148}\,\epsilon^2_{5t-2} + \underset{(0.080)}{0.667}\,h_{5t-1}$$

MAPE = 29.055 $\quad$ R^2 = 0.911

Note: B is a lag operator such that $B^s x_t = x_{t-s}$. Figures in parentheses are approximate standard errors. All prices and the food expenditures are deflated by the CPI.

Table 2. Maximum likelihood estimates of a supply-demand model for the U.S. broiler industry

Equation	Parameter	Variable	REH Model	GARCH Model	Augmented Model
Supply	a_1	D_1	-146.103 (25.411)	-89.525 (12.543)	-157.370 (24.517)
	a_2	D_2	-151.989 (26.504)	-95.151 (13.426)	-165.096 (25.682)
	a_3	D_3	-169.958 (27.478)	-110.215 (14.136)	-182.980 (26.507)
	a_4	D_4	-149.645 (24.834)	-100.579 (13.581)	-163.870 (24.180)
	a_5	WPB_t^e	3.481 (0.774)	1.725 (0.390)	3.941 (0.747)
	a_6	WPB_t^v	-2.439 (1.756)	-0.235 (0.136)	-2.489 (1.177)
	a_7	PBF_{t-1}	-5.196 (2.159)	-2.791 (1.432)	-4.894 (1.920)
	a_8	$HATCH_{t-1}$	2.430 (0.225)	2.117 (0.182)	2.403 (0.200)
	a_9	QBP_{t-4}	0.523 (0.059)	0.505 (0.050)	0.546 (0.053)
	α		———	———	0.785 (0.100)
	β		———	———	0.885 (0.079)
Demand	b_1	D_1	5.794 (4.105)	1.024 (4.376)	5.119 (4.212)
	b_2	D_2	7.551 (4.113)	2.606 (4.361)	6.757 (4.196)
	b_3	D_3	7.882 (4.115)	2.858 (4.361)	7.107 (4.209)

Table 2. (continued)

Equation	Parameter	Variable	REH Model	GARCH Model	Augmented Model
	b_4	D_4	4.386	-0.378	3.721
			(4.102)	(4.356)	(4.208)
	b_5	QBD_t	-0.069	-0.057	-0.062
			(0.010)	(0.014)	(0.011)
	b_6	RPB_t	-0.050	-0.043	-0.048
			(0.023)	(0.029)	(0.021)
	b_7	RPP_t	0.113	0.140	0.104
			(0.029)	(0.039)	(0.029)
	b_8	RPT_t	0.230	0.267	0.267
			(0.050)	(0.061)	(0.050)
	b_9	$FEXP_t$	0.150	0.137	0.136
			(0.036)	(0.048)	(0.036)
Variance-Covariance					
	σ_{11}		22.945	38.654	21.831
			(3.830)	(6.437)	(1.342)
	σ_{22}		3.684	3.774	3.697
			(0.607)	(0.624)	(0.100)
	σ_{12}		-1.427	-6.217	-2.486
			(1.264)	(1.689)	(0.0 79)
Log likelihood			-243.68	-253.65	-239.83

Note: Figures in parentheses are asymptotic standard errors.

with those obtained under the null hypothesis that the restrictions implied by rational expectations are true.[11] Formally, the restrictions implied by rational expectations of both the mean and the variance of the price distribution reduce the dimension of the parameter space by eight. The calculated test statistic was 8.34, which is well below the tabled values of the chi-square statistic with eight degrees of freedom at all conventional levels of significance. This result provides more evidence that the rational expectations hypothesis is suitable for modeling producer expectations of both the mean and the variance of the output price distribution in the U.S. broiler market.

The above test results and corresponding measures of fit provide important information about the empirical validity of the rational expectations hypothesis with regard to the first and second moments of price in the broiler market. However, the results reported do not give any indication about the strength of the rational expectations hypothesis relative to other expectations mechanisms. One obvious alternative to the rational expectations hypothesis is to generate instruments for the expected mean and variance of price by using a GARCH process. This approach is similar in spirit to the instrumental variables method employed by Antonovitz and Roe (1986) and others for estimating unrestricted rational expectations models.

To measure the relative performance of each expectations hypothesis, an approach similar to that described by Shonkwiler (1982) is adopted. That is, an augmented version of the rational expectations model is estimated where the expectations of both the mean and the variance are modeled as convex combinations of the rational predictors and the corresponding price and variance instruments from an estimated GARCH model. Specifically, the supply equation with augmented expectations takes the form

$$QBP_t = \sum_{j=1}^{4} a_j D_{jt} + a_5[\alpha WPB^v_{rt} + (1-\alpha)WPB^e_{gt}] + a_6[\beta WPB^v_{rt} + (1-\beta)WPB^v_{gt}] + a_7 PBF_{t-1} + a_8 HATCH_{t-1} + a_9 QBP_{t-4} + u_{1t}, \tag{15}$$

where α and β are mixing parameters and the subscripts r and g denote rational predictor and GARCH predictor, respectively.

The results obtained by using the GARCH model forecasts as instruments and augmented expectations are also presented in Table 2 in the columns headed GARCH Model and Augmented Model, respectively. In the augmented model, the mixing parameters α and β are estimated simultaneously along with the other structural parameters. The results obtained ($\alpha = 0.785$ and $\beta = 0.885$) indicate that the largest weights are associated with the rational predictors of both the mean and the variance. A test of the joint hypothesis that $\alpha = \beta = 1$ yielded a chi-square statistic of 7.70 with two degrees of freedom, which implies that the null hypothesis can be accepted only at the 0.01 level. Alternatively, a test of the joint hypothesis that $\alpha = \beta = 0$ resulted in a chi-square test statistic of 27.64 with two degrees of freedom. So, the hypothesis that GARCH forecasts adequately represent the expectations variables in the supply equation clearly can be rejected. In addition, the reason for rejecting the hypothesis $\alpha = \beta = 1$ is apparently because α is significantly different from one, not because β is significantly different from unity. While these results do not provide conclusive evidence that rational expectations dominate GARCH expectations in the broiler industry, they are encouraging in that larger weights are associated with the rational predictors. The rational expectation of price variance also seems to dominate the same expectation as generated by a GARCH model.

Conclusions

The primary goal of this study has been to extend the rational expectations framework to include price uncertainty. With several exceptions, previous studies of aggregate supply response have used ad hoc representations of the risk variables. The most common approach is to approximate risk terms with a distributed lag relationship. In marked contrast, the rational expectations specification assumes that producers use all currently available information to form expectations about both the mean and the variance of price. The implication is that risk variables in a rational expectations model are ultimately determined by the forecasts variances of exogenous variables and structural parameters, and by knowledge of the model's error process.

The few studies that have used a rational expectations approach to model risk have relied on misspecified model structures. Because the elements in the variance-covariance matrix have been ignored in these studies, any empirical interpretations are suspect. In addition, these studies have used ad hoc procedures to generate time-varying expectations of the forecast variances of exogenous variables.

In this paper, we have shown that the correct estimator for a rational expectations model that includes risk is based on an unconcentrated log likelihood function. This is because the terms in the variance-covariance matrix must be estimated simultaneously with other model parameters. We have argued that new time series procedures known as GARCH processes should be used to generate the expectations of the means and the variances of exogenous variables in rational expectations models with risk. This is because these processes allow the conditional variance to change systematically on the basis of past information.

The dual assumptions of risk averse behavior and of rational expectations were subsequently examined in a model of the U.S. broiler industry, with the results indicating that price variance is an important determinant of broiler supply. A formal test also indicated that the restrictions implied by rationality could not be rejected. This result is not trivial because the assumption of risk averse behavior gives rise to even more nonlinearities and cross-equation restrictions (including restrictions on the variance-covariance matrix) relative to standard rational expectations models. In addition, the results suggest that using a rational expectations approach is, at the very least, no worse than using an instrumental variables method for approximating unobservable expectations in supply models. In summary, the results obtained here are encouraging since it seems that more sophisticated approaches to rational expectations modeling can be pursued successfully.

APPENDIX: Derivation of the Estimable Form of the Model for the Broiler Market

The rational predictors for the mean and the variance of wholesale broiler price can be obtained as follows. Substituting Equations 13 and 15 into 14 gives

$$\begin{aligned} WPB_t = &\sum_{j=1}^{4} b_j D_{jt} + b_5\{\sum_{j=1}^{4} a_j D_{jt} + a_6 WBP_t^v + a_7 PBF_{t-1} \\ &+ a_8\ HATCH_{t-1} + a_9 QBP_{t-4} + u_{1t} - QOD_t\} + b_6 RPB_t \\ &+ b_7 RPP_t + b_8 RPT_t + b_9 FEXP_t + u_{2t}\ , \end{aligned} \tag{A.1}$$

which is the reduced form for broiler price. Taking the variance operator through Equation A.1, and assuming that forecast variances between exogenous variables are zero, yields an expression for the rational expectation of price variance:

$$\begin{aligned} WPB_t^v = &\ b_5^2\sigma_{11} + b_5^2 QOD_t^v + b_6^2 RPB_t^v + b_7^2 RPP_t^v + b_8^2 RPT_t^v \\ &+ b_9 FEXP_t^v + \sigma_{22} + 2b_5\sigma_{12}. \end{aligned} \tag{A.2}$$

Equation A.2 shows that the rational expectation of price variance is a function of forecast variances of unknown exogenous variables, denoted by a superscript v, and the structural parameters, including the variance and covariance terms associated with the model's additive error structure.

The rational price predictor can be obtained by taking the expectation operator through the reduced form in A.1. Collecting terms and substituting the expression in A.2 for WPB_t^v gives

$$\begin{aligned} WPB_t^e = &\ (1 - a_5 b_5)^{-1}[\sum_{j=1}^{4} b_j D_{jt} + b_5\{\sum_{j=1}^{4} a_j D_{jt} + a_6(b_5^2\sigma_{11} + b_5^2 QOD_t^v \\ &+ b_6^2 RPB_t^v + b_7^2 RPP_t^v + b_8^2 RPT_t^v + b_9^2 FEXP_t^v + \sigma_{22} + 2b_5\sigma_{12}) \\ &+ a_7 PBF_{t-1} + a_8 HATCH_{t-1} + a_9 QBP_{t-4} - QOD_t^e\} + b_6 RPB_t^e \\ &+ b_7 RPP_t^e + b_8 RPT_t^e + b_9 FEXP_t^e], \end{aligned} \tag{A.3}$$

which is the rational expectation of price. The expression for expected price in A.3 depends on predetermined exogenous variables, the expectations of current exogenous variables, the forecast variances of current exogenous variables, and the variance-covariance terms associated with the behavioral equations.

Substituting the rational expectations of price and of variance in Equations A.2 and A.3 for WPB_t^v and WPB_t^e, respectively, in Equation 11 yields the estimable form of the supply equation:

$$
\begin{aligned}
QBP_t = {} & \sum_{j=1}^{4} a_j D_{jt} + a_5(1-a_5b_5)^{-1}[\sum_{j=1}^{4} b_j D_{jt} + b_5\{\sum_{j=1}^{4} a_j D_{jt} \\
& + a_6(b_5^2\sigma_{11} + b_5^2 QOD_t^v + b_6^2 RPB_t^v + b_7^2 RPP_t^v + b_8^2 RPT_t^v \\
& + b_9^2 FEXP_t^v + \sigma_{22} + b_6^2 2b_5\sigma_{12}) + a_7 PBF_{t-1} + a_8 HATCH_{t-1} \\
& + a_9 QBP_{t-4} - QOD_t^e\} + b_6 RPB_t^e + b_7 RPP_t^e + b_8 RPT_t^e \\
& + b_9 FEXP_t^e] + a_6(b_5^2\sigma_{11} + b_5^2 QOD_t^v + b_6^2 RPB_t^v + b_7^2 RPP_t^v \\
& + b_8 RPT_t^v + b_9^2 FEXP_t^v + \sigma_{22} + 2b_5\sigma_{12}) + a_7 PRB_{t-1} \\
& + a_8 HATCH_{t-1} + a_9 QBP_{t-4} + u_{1t}.
\end{aligned}
\tag{A.4}
$$

Finally, it can be easily verified if the variance-covariance matrix ψ_{2t} is diagonal, as it is in the current case, that the identification conditions as given by Wegge and Feldman (1983) are satisfied. That is, the number of imperfectly forecasted exogenous variables, K_2, must exceed the number of equations, G.

Endnotes

1. Although the number of expectations variables, H, clearly can be less than the number of endogenous variables, G, we have assumed that the parameter matrices A_1 and A_2 are also (G x G). This assumption is maintained primarily for notational convenience. Operationally, A_1 and A_2 can be augmented with rows of zeros in the case where $H < G$.
2. The direction of the bias is clearly negative for the expectation of the variance because the second term in Matrix Equation 8 is positive definite. However, the direction of the bias with respect to the mean expectations vector $\underline{y}_t^e$ cannot be inferred a priori and will depend on assumed parameter values.
3. In the present context, $\underline{\alpha}$ is a parameter vector that contains the nonzero elements of the matrices B, A_1, A_2, Γ_1, and Γ_2.
4. Apart from a constant, the unconcentrated log likelihood function can be written as

$$L = \sum_{t=1}^{T} \log|J_t| - \frac{T}{2}\log|\Sigma| - \frac{1}{2}\sum_{t=1}^{T} \underline{u}_t'\Sigma^{-1}\underline{u}_t ,$$

where J_t represents the Jacobian of the transformation from $\underline{u}_t$ to $\underline{y}_t$.
5. In principle, the parameters of the stochastic process used to derive the expectations of unknown exogenous variables are estimated simultaneously with the other parameters in the system (Wallis 1980). This results in further cross-equation restrictions as implied by the rationality assumption. In practice, however, the parameters of the stochastic processes used to forecast the exogenous variables are seldom estimated simultaneously with the structural parameters because of the additional burdens placed on the estimation. The usual approach is to estimate the parameters of the forecasting equations separately and then to use the resulting predictions as instruments (e.g., Goodwin and Sheffrin 1982; Shonkwiler and Emerson 1982). While this approach sacrifices some of the

informational content of the rational expectations hypothesis, it greatly reduces computational costs.

6. If the model is constrained so that $\beta_i = 0$ for all i, then the process reduces to Engle's (1982) ARCH(q) process.
7. The weights used are 0.70 for corn price and 0.30 for soybean meal price. These are identical to the ones reported by Chavas and Johnson (1982).
8. As Kennan (1979) has shown, using a partial adjustment framework in a rational expectations model is consistent with the notion that agents posses a quadratic criterion function that includes both disequilibrium and adjustment costs.
9. Using a four-period lag, and quarterly data from 1967 through 1986, we obtained an F-statistic of 5.30 for the test that wholesale prices cause retail prices. Since the critical value at the 5 percent level is 2.52, we conclude that wholesale prices do cause retail prices. Alternatively, the test that retail prices cause wholesale prices yielded an F-statistic of 2.18, which is not significant at the 5 percent level.
10. The results also indicate that a 2.43-pound broiler is produced for each egg hatched in commercial hatcheries. When a 72 percent dressing rate is assumed, this implies a broiler slaughter weight of approximately 3.4 pounds, which is slightly less than the reported average of 3.8 pounds.
11. The likelihood ratio test statistic is determined by

$$-2 \ln \lambda = -2[\ln L(\underline{\Theta}^*) - \ln L(\bar{\underline{\Theta}})],$$

where $\underline{\Theta}^*$ represents the restricted maximum likelihood estimates of the parameter vector $\underline{\Theta}$ and $\bar{\underline{\Theta}}$ denotes the corresponding unrestricted maximum likelihood estimates. Asymptotically, $-2 \ln \lambda$ is distributed as chi-square with J degrees of freedom (J equaling the number of independent restrictions being tested) under the null hypothesis that $\underline{\Theta}^*$ is true.

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