# Stochastic Dominance and Uncertain Price Prospects

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## Abstract

This paper develops rules for ordering uncertain price prospects. For consumers with identical ordinal preferences, we propose stochastic dominance rules based on equivalent variation (EV). The second-degree stochastic dominance (SSD) rule on the induced distributions of EV yields a unanimous ranking among income risk averters. The SSD rule on consumer surplus or compensating variation provides a valid ranking for income risk averters if the income elasticity of demand is zero. Risk averse consumers with different ordinal preferences cannot have a unanimous ranking of price prospects. We delineate two classes of risk averse consumers that have opposing rankings of price prospects with the same mean.

STOCHASTIC DOMINANCE AND UNCERTAIN PRICE PROSPECTS

## 1. Introduction

Since the seminal and simultaneous publications of Hadar and Russell (1969) and Hanoch and Levy (1969) there has been a virtual explosion of papers investigating implications of stochastic dominance rules for decisions under uncertainty.[1] Dominance principles have important applications to portfolio choice, capital budgeting and financial intermediation. Stochastic dominance can also be applied to investment and production decision problems under uncertainty. In these areas, both overall and marginal impacts of uncertainty have received wide attention.[2]

The emphasis in consumer welfare analysis under uncertainty has been on the ranking of different price prospects. Ranking of price prospects is more complicated than that of income prospects for two related reasons. First, due to differences in ordinal preferences among consumers, the second degree stochastic dominance rule on price prospects does not yield a unanimous ranking among risk averse consumers. The literature has thus focused on the ranking of price prospects for a single consumer or consumers with identical ordinal preferences (e.g. Waugh, 1944; Massell, 1969; Samuelson, 1972). Second, there is no *a priori* reason for income risk averse consumers to exhibit aversion to fair price risks. Specifically, the indirect utility function of a risk averse consumer is neither concave nor convex in prices. Indirect ways of comparing consumer welfare under alternative price prospects — via expected consumer surplus, expected compensating variation or expected equivalent variation — have thus been investigated in the literature.

Expected consumer surplus (ECS) has been particularly popular for ranking certain and uncertain price prospects.[3] However, Turnovsky, Shalit and Schmitz (1980) showed that the ranking of price prospects based on ECS is generally inconsistent with the expected utility criterion. Specifically, they pointed out that ECS is an accurate measure of change in utility only if the marginal utility of income is independent of the random price, i.e., if the relative risk aversion index equals the income elasticity of demand. Thus, for instance, if the consumer is risk neutral and the income elasticity of demand is zero, ECS provides a correct ranking of certain and uncertain price prospects. Subsequently, Helms (1985a) demonstrated that expected compensating variation (ECV) yields a valid ranking of certain and uncertain price prospects, if the relative risk aversion index equals twice the income elasticity of demand.[4]

Since the consumer surplus approach imposes "stringent" restrictions on ordinal preferences, Turnovsky, Shalit and Schmitz (1980, p. 136) suggested that evaluation of certain or uncertain price prospects be based on consumer's expected indirect utility.[5] This paper provides two sets of rules for ordering uncertain price prospects that are consistent with expected utility. For consumers with identical ordinal preferences, we propose stochastic dominance rules based on the induced distributions of equivalent variation (EV). For given ordinal preferences, the SSD rule for the induced distributions of EV yields a unanimous ranking of induced income prospects among risk averters. Moreover, the SSD rule for the induced prospects of EV is the only criterion for risk averters that is consistent with the expected utility criterion. The SSD rule for consumer surplus (CS) or compensating variation (CV) provides a valid ranking of price prospects for income risk averters if and only if the income elasticity of demand is zero, in which case CV and CS coincide with EV. If the income elasticity of demand is not zero, then the SSD rule for the induced distributions of CS or CV yields a ranking of price prospects that is inconsistent with expected utility.

If consumers have different ordinal preference orderings on commodity bundles, they will demand different "equivalent" variations in income from a price change. Thus, induced distributions of EV cannot be used to compare price prospects for consumers with different ordinal preferences. Moreover, risk averse consumers with different ordinal preferences cannot have a unanimous ranking of price prospects. However, it is possible to have a unanimous ranking for some subsets of risk averse consumers. We delineate two classes of risk averse consumers that have opposite rankings of price prospects.

The plan of this paper is as follows. Section 2 investigates two rules, based on induced distributions of EV, for ranking price prospects for consumers with identical ordinal preference orderings. Section 3 examines the SSD rule for induced distributions of CS and CV. Section 4 relaxes the assumption of identical ordinal preferences and delineates two classes of utility functions with conflicting rankings of price prospects. Section 5 contains a brief summary and concluding remarks.

## 2. Ranking of Price Prospects with Identical Ordinal Preferences

The literature has compared certain price and uncertain price prospects using consumer surplus of the representative consumer with the implicit assumption that consumers have identical ordinal preferences. Thus, we first compare price prospects for consumers with identical ordinal preferences and compare our results with the literature. The proposed rules for consumers with identical ordinal preferences are based on the induced distributions of equivalent variation (EV). Marshallian consumer surplus (CS) and Hicksian compensating variation (CV) also summarize ordinal preferences, and the rankings of price prospects based on induced distributions of CS and CV are considered in the next section.

Imagine a consumer who allocates a given income I between good X whose price is uncertain and a composite good Y whose price is certain, equal to unity. The numeraire good Y represents the consumption of all goods, exclusive of X. Let u(X,Y) be the direct utility function, and let X(p,I) and Y(p,I) denote the demand functions obtained by maximizing u(X,Y) subject to the budget constraint, I = pX + Y. The demand functions also generate an indirect utility function V(p,I) and an expenditure function e(p,u).

Consider a change in the price of X from s to p. The minimum amount of compensation w that the consumer with income I demands at price s, in order to maintain the same level of utility obtained at price p, is implicitly defined by

$$V(p,I) = V(s,I + w). \tag{1}$$

The "equivalent" variation in income w can be explicitly written

$$w(s,p,I) = e(s,V(p,I)) - I. \tag{2}$$

From the definition in (1), we note that equivalent variation (EV) provides a correct ranking of certain prices. That is, $V(p_1,I) \geq V(p_2,I)$ if and only if $w(s,p_1,I) \geq w(s,p_2,I)$ for some s. Moreover, the ranking on EV is independent of the reference price s, i.e., $w(s,p_1,I) \geq w(s,p_2,I)$ for some s implies $w(s,p_1,s) \geq w(s,p_2,I)$ for all s.[6] It should be noted that the EV function is derived from the expenditure function and hence reflects ordinal preferences. Thus, "equivalent" variations in income from a price change will be different between consumers with distinct ordinal preferences.

Note that if p is a random variable, so is the equivalent variation w(s,p,I). Let F(p) be a price distribution and **F(p)** be the class of all price distributions. Since the equivalent variation w(s,p,I) is a random variable, the price distribution

$F(p)$ induces a distribution of equivalent variation, $\hat{F}(w)$. Taking expectations of (1), we obtain

$$E_F V(p,I) \equiv E_F V(s,I + w(s,p,I)) \equiv E_{\hat{F}} V(s,I + w). \tag{3}$$

Let $\hat{\mathbf{F}}(w)$ be the class of all distributions of EV, and let a and b be constants such that $\hat{F}(a) = 0$ and $\hat{F}(b) = 1$ for all $\hat{F} \in \hat{\mathbf{F}}$.

Hadar and Russell (1969) and Hanoch and Levy (1969) defined two types of stochastic dominance for income prospects. To distinguish them from the dominance relations for price prospects, we reproduce definitions of First Degree Stochastic Dominance (FSD) and Second Degree Stochastic Dominance (SSD) using equivalent variation. Let

$$\hat{F}^*(w) = \int_a^w \hat{F}(t)dt, \qquad \hat{G}^*(w) = \int_a^w \hat{G}(t)dt.$$

Definition 1: $\hat{F}(w)\ D_1\ \hat{G}(w)$ if $\hat{F}(w) \leq \hat{G}(w)$ for all w and the strict inequality holds for some w.

Definition 2: $\hat{F}(w)\ D_2\ \hat{G}(w)$ if $\hat{F}^*(w) \leq \hat{G}^*(w)$ for all w and the strict inequality holds for some w.

We now compare expected utilities under two price prospects, $F(p)$ and $G(p)$, using the induced distributions, $\hat{F}(w)$ and $\hat{G}(w)$. The difference in expected utilities is given by

$$E_{\hat{F}}V - E_{\hat{G}}V = \int_a^b V(s,I + w)d[\hat{F}(w) - \hat{G}(w)]. \tag{4}$$

It should be noted that here we are concerned with a conditional ranking of price prospects, $F(p)$ and $G(p)$, for a given income I. Since an increase in income changes demand and the expected utilities under the two price distributions, ordering of uncertain price prospects is affected by changes in income.

Integrating (4) by parts gives

$$E_{\hat{F}}V - E_{\hat{G}}V = V(s,I + b)[\hat{F}(b) - \hat{G}(b)] - \int_a^b V_I(s,I + w)[\hat{F}(w) - \hat{G}(w)]dw$$

$$= - \int_a^b V_I(s,I + w)[\hat{F}(w) - \hat{G}(w)]dw \tag{5}$$

since $\hat{F}(a) = \hat{G}(a) = 0$ and $\hat{F}(b) = \hat{G}(b) = 1$. Let $\mathbf{V_I} \equiv \{V(p,I + w): V_I > 0\}$ be the class of all indirect utility functions that are monotone increasing in income. Then

<u>PROPOSITION 1</u>: $\hat{F}(w)\ D_I\ \hat{G}(w)$ iff $E_{\hat{F}}V(s,I + w) \geq E_{\hat{G}}V(s,I + w)$ for all $V \in \mathbf{V_I}$.

Thus, the FSD rule holds for all individuals with indirect utility functions that are monotone increasing in income.[7] Note that differences in ordinal preferences are irrelevant for the ranking of the two prospects since the FSD rule is defined in terms of EVs which incorporate ordinal preferences. However, a distribution $\hat{F}(w)$ implies distinct price prospects for individuals with different ordinal preference orderings.

We now develop a rule for ordering uncertain price prospects for income risk averters, using the induced distributions of EV. Let

$$\hat{F}^*(w) = \int_a^w \hat{F}(t)dt, \qquad \hat{G}^*(w) = \int_a^w \hat{G}(t)dt.$$

Integrating (4) by parts yields

$$E_{\hat{F}}V - E_{\hat{G}}V = - V_I(s,I + b)[\hat{F}^*(b) - \hat{G}^*(b)] + \int_a^b V_{II}(s,I + w)[\hat{F}^*(w) - \hat{G}^*(w)]dw. \tag{6}$$

Let $V_{II} \equiv \{V(s,I + w): V_I > 0 \text{ and } V_{II} \leq 0\}$ be the class of all indirect utility functions that are monotone increasing and concave in income. Since the right side of (6) is a weighted sum of $[\hat{F}^*(w) - \hat{G}^*(w)]$, using $(-V_I)$ and $V_{II}$ as weights, the necessary and sufficient condition for $E_{\hat{F}}V \geq E_{\hat{G}}V$ is that $\hat{F}^*(w) \leq \hat{G}^*(w)$ for all w, as Hadar and Russell (1969) have shown. Thus, we have

<u>PROPOSITION 2</u>: $\hat{F}(w)\ D_2\ \hat{G}(w)$ iff $E_{\hat{F}}V(s,I + w) \geq E_{\hat{G}}V(s,I + w)$

for all $V \in V_{II}$.

A price variation is "equivalent" to a change in income. Recall that EVs are the same for consumers with identical ordinal preferences. Thus, it is possible to rank uncertain price prospects indirectly by comparing the induced distributions of EV. We have shown that the necessary and sufficient condition for all income risk averters with identical ordinal preferences to prefer one price prospect to another is that an SSD relationship holds between the induced distributions of EV.

Since Proposition 2 is stated for a given stable price s, which is used as a common reference price to obtain the induced distributions of EV, one might suspect that the ranking of price prospects depends on the choice of the stable reference price s. However, using the definition of expected utility in (3), $E_F V \geq E_G V$ implies $E_{\hat{F}}V(s,I + w) \geq E_{\hat{G}}V(s,I + w)$ for all s. Conversely, $E_{\hat{F}}V(s,I + w) \geq E_{\hat{G}}V(s,I + w)$ for some s implies $E_F V \geq E_G V$. Thus, the ranking of price prospects, F(p) and G(p), is independent of the choice of the stable reference price s. This implies that if $\hat{F}^*(w) \leq \hat{G}^*(w)$ for some s, then not only all risk averse consumers prefer $\hat{F}^*(w)$ to $\hat{G}^*(w)$ and hence F(p) to G(p), but the inequality $\hat{F}^*(w') \leq \hat{G}^*(w')$ also holds for any other reference price s′, where $w' = w(s',p,I)$. Thus, an alternative version of Proposition 2 can be written:

PROPOSITION 2': $\hat{F}(w)\ D_2\ \hat{G}(w)$ for all s, iff

$$E_F V(p,I) \geq E_G V(p,I) \quad \text{for all } V \in V_{II}.$$

We conclude this section by noting that ranking price prospects by FSD and SSD rules on the induced distributions of EV are as practical as ordering income prospects. The use consumer surplus in applied welfare analyses presupposes some knowledge of the properties of the demand functions (e.g. income and price elasticities) or expenditure functions. Once the demand or expenditure functions are known, compensated demand curves can be constructed from the Slutsky equation. Thus, stochastic dominance approach based on EV requires no more information than that using consumer surplus, which has been popular in applied welfare analyses.

## 3. Consumer Surplus, Compensating Variation and SSD Rules

We have shown that for given ordinal preferences an SSD relationship between two induced distributions of EV results in a unanimous preference among all income risk averters for one price prospect to another. In this section we investigate whether a similar SSD rule on the induced distributions of CS or CV is valid for ranking price prospects for all income risk averters with the same ordinal preferences.

If the price of X changes from s to p, the Marshallian consumer surplus from the price change is

$$A(s,p,I) = \int_s^p X(P,I)dP. \tag{7}$$

On the other hand, compensating variation C for this price change is implicitly defined by

$$V(p, I - C) = V(s, I).$$

More explicitly, compensating variation can be written

$$C(s,p,I) = e(p, V(p,I)) - I. \tag{8}$$

Assume that X is a normal good with a nonnegative income effect ($X_I \geq 0$). Figure 1 illustrates the relationship between three alternative measures of welfare change when the price falls from s to p. The Marshallian demand curve is denoted by X(p,I). If the income effect is positive, the compensated demand curve labeled $D_s$ — which preserves the utility level at V(s,I) — is steeper than X(p,I). Similarly, the compensated demand curve $D_p$ associated with a utility level V(p,I) is also steeper than X(p,I) and lies to the right of $D_s$.

The Marshallian consumer surplus from this price change is represented by area (samp). Compensating variation and equivalent variation from this price change are represented by area (sacp) and area (semp), respectively. Thus, if the price <u>falls</u> from s to p and $X_I \geq 0$, then

$$w(s,p,I) \geq A(s,p,I) \geq C(s,p,I) > 0, \qquad \text{for all } p < s. \tag{9}$$

Alternatively, if the price rises from s to p and $X_I \geq 0$, then

$$w(s,p,I) \leq A(s,p,I) \leq C(s,p,I) < 0, \qquad \text{for all } p > s. \tag{9'}$$

Since w, A and C are moving in the same direction, for given values of the stable price s and income I, equivalent variation can be written as w = w(p), and similarly, consumer surplus as A = A(p). Let p = h(A) be the conditional — on s and I — inverse consumer surplus function. Then

$$w = w(h(A)) \equiv W(A), \; W'(A) > 0, \tag{10}$$

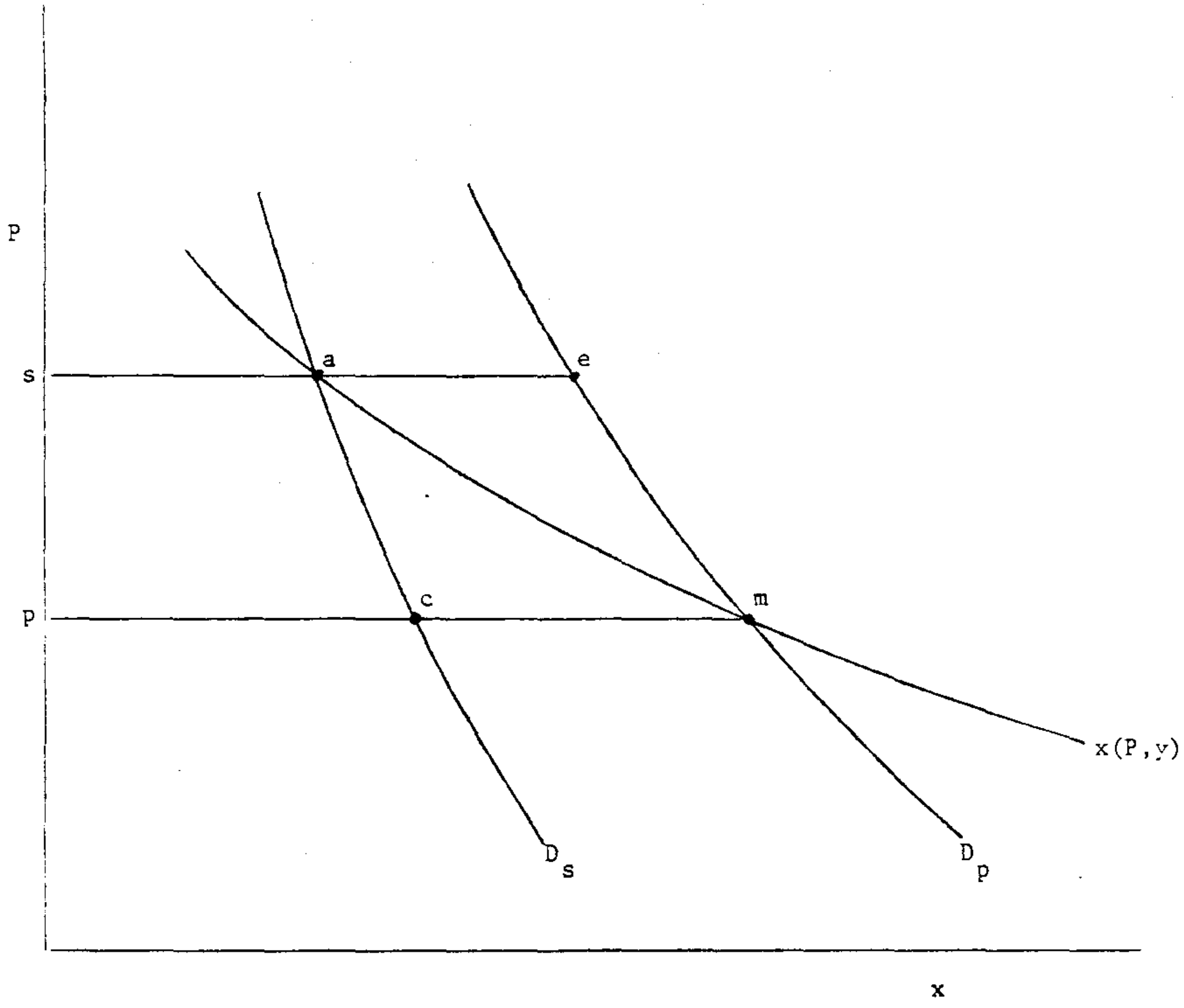


Figure 1. Relationship between three alternative measures of welfare change when price falls from s to p

where the arguments s and I are held constant and hence are suppressed for notational convenience. Since w is decreasing in p, which in turn is decreasing in A, W(A) is monotone increasing in A. Thus, the indirect utility function

$$V(s, I + W(A))$$

is monotone increasing in A, but is in general neither convex nor concave in A.

Let $V_{AA} \equiv \{V(s, I + W(A)) \colon V_A > 0 \text{ and } V_{AA} \leq 0\}$ be the class of all indirect utility functions that are monotone increasing and concave in A. Let $\hat{F}(A)$ and $\hat{G}(A)$ be the induced distributions of CS, derived from F(p) and G(p), respectively. Let

$$\hat{F}^*(A) = \int_c^A \hat{F}(t)dt, \qquad \hat{G}^*(A) = \int_c^A \hat{G}(t)dt,$$

where c is a positive constant such that the cumulative distribution of consumer surplus at c, $\hat{F}(c)$ and $\hat{G}(c)$, are both zero. Then a straightforward application of Hadar and Russell's (1969) SSD rule implies

$$\hat{F}(A)\ D_2\ \hat{G}(A) \quad \text{iff} \quad E_{\hat{F}}V(s, I + W(A)) \geq E_{\hat{G}}V(s, I + W(A)) \quad \text{for all } V \in V_{AA}. \tag{11}$$

However, our main interest is ranking price prospects for the class of income risk averters, $V_{II}$, and not for the class $V_{AA}$. Differentiating (11) with respect to A twice gives

$$V_{AA} = V_{II}(W')^2 + V_I W''.$$

The necessary and sufficient condition for $V_{AA}$ and $V_{II}$ to be the identical class is that $V_{AA}$ must be nonpositive for all $V_{II} \leq 0$. That is, $W''(A)$ must be zero, and hence W(A) must be a linear function, i.e.,

$$W(A) = \alpha + \beta A$$

for some constants, $\alpha$ and $\beta$. Since $W(A)$ is monotone increasing in A, $\beta$ must be positive. Moreover, $w(s,p,I) = A(s,p,I) = C(s,p,I) = 0$ for all I, if $p = s$. This implies that $\alpha$ is zero.

We now show that $\beta$ must be unity. From (9), we obtain

$$w(s,p,I) = \beta A(s,p,I) \geq A(s,p,I) \quad \text{for all } p < s, \tag{12}$$

which implies $\beta \geq 1$. Similarly, from (9'),

$$w(s,p,I) = \beta A(s,p,I) \leq A(s,p,I) \quad \text{for all } p > s, \tag{12'}$$

which implies $\beta \leq 1$. Thus, $\alpha = 0$ and $\beta = 1$ are necessary and sufficient conditions for $V_{AA} = V_{II}$. That is, the two classes are identical if and only if the income effect, $X_I$, is zero. In this case, EV coincides with CS and CV. Similar arguments can be made to show that income effect $X_I$ must be zero for $V_{II} = V_{CC} \equiv \{V(s,I + W(C)): V_C \geq 0 \text{ and } V_{CC} \leq 0\}$.

<u>PROPOSITION 3</u>: The SSD rules for the induced distributions of CS and CV can be used to rank uncertain price prospects for all income risk averters with identical ordinal preferences, if and only if the income effect $X_I$ is zero.

A comparison of our result with the existing literature can be made for the case of zero income effect. Let $R \equiv - IV_{II}/V_I$ be the Arrow-Pratt relative risk aversion index. If the consumer is risk neutral, then $R = \eta$ (= 0) and hence the marginal utility of income is invariant with respect to the random price. In this case, expected consumer surplus (ECS) can be used to rank uncertain price prospects, a result predicted by Turnovsky, Shalit and Schmitz (1980). Moreover, since $R = 2\eta$ (= 0), expected compensating variation (ECS) can be used not only to rank a stable price and an uncertain price prospect — which requires $R = 2\eta$ — but also to rank any nondegenerate price prospects, as shown by Helms (1984). If the consumer is risk averse (and the income effect is zero), then the SSD rules for the induced distributions of CS or CV yield a correct ranking of uncertain price prospects.

If the income elasticity of demand is not zero, then only the SSD rule for the induced distributions of EV provides a correct ranking of uncertain price prospects for all income risk averse consumers. The only situation in which the SSD rules for CS or CV yield a correct ranking of uncertain price prospects is when the income effect is zero, i.e., when CS and CV coincide with EV.

## 5. Ranking of Price Prospects with Heterogenous Ordinal Preferences

The ranking of price prospects via the induced distributions of EV is difficult when consumers have different ordinal preferences. The functional form of w(s,p,I) depends on the ordinal preferences or the shape demand function X(p,I). Since a given price prospect yields different distributions of equivalent variations for consumers with different ordinal preferences, a unanimous ranking of price prospects is generally infeasible. However, it is possible to obtain a unanimous ranking of price prospects for some proper subsets of risk averse consumers with different ordinal preferences. Let

$$F^*(p) = \int_0^p F(t)dt, \qquad\qquad G^*(p) = \int_0^p G(t)dt.$$

To facilitate the ranking of price prospects we define first degree stochastic price dominance (FSPD) and second degree stochastic price dominance (SSPD):

Definition 1': $F(p)\ D_1\ G(p)$ if $F(p) \geq G(p)$ for all p and the strict inequality holds for some p.

Definition 2': $F(p)\ D_2\ G(p)$ if $F^*(p) \geq G^*(p)$ for all p and the strict inequality holds for some p.

Since consumers with different ordinal preferences have different EV functions, we compare price prospects directly. Let **F(p)** be the set of all price prospects. To avoid using different symbols, let a and b now denote respectively the lower and upper bounds of the range of the price prospect, i.e., $F(a) = 0$ and $F(b) = 1$ for all $F(p) \in \mathbf{F(p)}$. Expected utility of a price prospect F(p) is given by

$$E_F V(p,I) = \int_a^b V(p,I)dF(p).$$

The difference in expected utilities between two prospects, F(p) and G(p), is

$$E_F V(p,I) - E_G V(p,I) = - \int_a^b V_p(p,I)[F(p) - G(p)] \tag{13}$$

since $F(b) = G(b) = 1$. Let $\mathbf{V_p} \equiv \{V(p,I): V_p < 0\}$. From Roy's identity, $V_p = - V_I X$, $V_p$ is negative for all indirect utility functions $V \in \mathbf{V_I}$, and hence $\mathbf{V_I} \equiv \mathbf{V_p}$. Thus, we have the following FSPD rule:

PROPOSITION 4: $F(p)\ D_1\ G(p)$ iff $E_F V(p,I) \geq E_G V(p,I)$ for all $V \in \mathbf{V_I}$.

Next, let $\mathbf{V_{pp}} \equiv \{V(p,I): V_p < 0, V_{pp} \geq 0\}$ be the class of indirect utility functions that are monotone decreasing and convex in p. Integrating (13) by parts yields

$$E_F V(p,I) - E_G V(p,I) = - V_p(b,I)[F(b) - G(b)] + \int_a^p V_{pp}(p,I)[F^*(p) - G^*(p)]dp.$$

PROPOSITION 5: $F(p)\ D_2\ G(p)$ iff $E_F V(p,I) \geq E_G V(p,I)$ for all $V \in \mathbf{V_{pp}}$.

Although $V_p \equiv V_I$, the classes $V_{pp}$ and $V_{II}$ are different. Using $V_{pI} = - V_I X_I - V_{II} X$, we obtain a result, originally due Turnovsky, Shalit and Schmitz (1980):

$$V_{pp} = (\eta + \epsilon/s - R)V_I X^2/I.$$

where $\eta \equiv (\partial X/\partial I)(I/X)$ is the income elasticity of demand, $\epsilon \equiv - (\partial X/\partial p)(p/X)$ is the price elasticity of demand and $s \equiv pX/I$ is the budget share of commodity X. Thus, the class $V_{pp}$ excludes consumers that are too risk averse ($R \geq \eta + \epsilon/s$) and includes mildly risk averse consumers ($R \leq \eta + \epsilon/s$). Moreover, it also includes risk neutral as well as risk loving consumers ($R \leq 0$).

We now restrict our attention to two proper subsets of risk averse consumers. Observe that $V_{II}$ is the union of three sets, **A**, **B** and **C** where

$$\mathbf{A} \equiv \{V(p,I)\colon V \in V_I,\ R \leq \eta + \epsilon/s\},$$

$$\mathbf{B} \equiv \{V(p,I)\colon V \in V_I,\ R \geq \eta + \epsilon/s\},$$

$$\mathbf{C} \equiv \{V(p,I)\colon V \in V_I,\ R > \eta + \epsilon/s \text{ for some } p,\ R < \eta + \epsilon/s \text{ for some } p'\}.$$

<u>PROPOSITION 5'</u>: If $F(p)\ D_2\ G(p)$, then $E_F V(p,I) \geq E_G V(p,I)$ for all $V \in \mathbf{A}$.

Conflicting preferences among risk averse consumers are revealed when comparing two price prospects with the same mean. Note that F(p) and G(p) have the same mean if and only if $F^*(b) = G^*(b)$. Given the same mean, the difference in expected utilities reduces to

$$E_F V(p,I) - E_G V(p,I) = \int_a^p V_{pp}(p,I)[F^*(p) - G^*(p)]dp.$$

This implies that consumers with class **A** utility functions and those with class **B** utility functions will have opposing rankings of price prospects with the same mean:[8]

PROPOSITION 6: Assume that F(p) and G(p) have the same mean. Then

$$F(p)\ D_2\ G(p) \quad \text{implies} \quad E_F V(p,I) \geq E_G V(p,I) \text{ for all } V \in \mathbf{A}, \text{ and}$$
$$E_F V(p,I) \leq E_G V(p,I) \text{ for all } V \in \mathbf{B}.$$

If two price prospects have the same mean, individuals with class **A** utility functions unanimously prefer the price prospect that is second degree dominant, while those in class **B** prefer the dominated price prospect. This proposition indicates that some restrictions must still be placed on ordinal properties (e.g. on the magnitudes of price and income elasticities and budget shares) to rank price prospects for consumers with different tastes.

## 5. Concluding Remarks

This paper investigates how stochastic dominance rules can be applied to consumer welfare analysis. Using the demand curve of the representative consumer the emphasis in the literature to date has been on comparisons of Marshallian consumer surplus for certain and uncertain price prospects. Recently, Turnovsky, Shalit and Schmitz (1980) demonstrated that a stringent restriction must be placed on ordinal preferences for expected consumer surplus to yield a ranking of price prospects consistent with that based on expected utility. Thus, alternative operational rules for ordering price prospects must be developed to evaluate the benefits of price stabilization policies and other commodity programs that alter price distributions. For instance, stabilization programs in the United States and the European Economic Community not only eliminate downside price risks but also shift the price distribution via supply response (Gardner, 1987).

There are two main obstacles to ranking price prospects for consumers exhibiting income risk aversion, one arising from differences in ordinal preferences and the other from nonconvexity of the indirect utility function in prices. The second degree stochastic price dominance (SSPD) rule does not yield a unanimous ranking of price prospects for risk averse consumers with different ordinal preferences. We have shown that if two price prospects have the same mean price and second degree dominance holds, the risk averse consumers will be split between two classes with conflicting rankings of price prospects.

For many policy problems in applied welfare analyses, expected consumer surplus has been used to evaluate the benefits from price stabilization. With the implicit assumption of identical ordinal preferences, the demand curve of the representative consumer is often used to estimate consumer surplus. We have shown that the SSD rule on the induced distributions of EV is a necessary and sufficient condition for all income risk averters with identical ordinal preferences to achieve a unanimous ranking of price prospects. The SSD rules on the induced distributions of CS and CV, however, yield a valid ranking of price prospects only if the income elasticity of demand is zero. Since EV, CS and EV are all derived from the same demand curve, the SSD rule on EV requires no more information than the SSD rule on consumer surplus or the expected consumer surplus criterion. Moreover, the SSD rule on EV is valid for all income risk averters without imposing restrictions on the income elasticity of demand. Insofar as the form of the utility function is not known, these two attractive features of the SSD rule on EV makes it an operationally useful rule for comparing consumer welfare under alternative price stabilization policies.

ENDNOTES

1. See Whitmore (1970) for third degree stochastic dominance rule, which can be used to rank income prospects for a subset of risk averters with positive third derivatives of the utility function.

2. For instance, see Baron (1970) and Sandmo (1971) for the impacts of price uncertainty on the behavior of the competitive firm. For general applications of stochastic dominance on the theory of the firm, see Hadar and Russell (1978).

3. This is particularly due to Willig (1976) who showed that if income effect is small CS fairly well approximates both CV and EV.

4. Helms (1984) also shows that the relative risk aversion index and the income elasticity of demand must be both zero for ECV to yield a valid ranking of nondegenerate price prospects. The ECV criterion is more restrictive than the ECS criterion, since the latter only requires $R = \eta$, not necessarily equal to zero.

5. See Anderson (1979) and Helms (1985b) for the use of *ex ante* compensating and equivalent variations. Since they are implicitly defined using expected utility, they are also consistent with the latter criterion.

6. Since the indirect utility function is monotone decreasing in price for all y, the ranking of EVs under price certainty is also independent of income.

7. Since the indirect utility function is monotone decreasing in price, first degree stochastic dominance (FSD) rule can be directly applied to price distributions. However, we include the FSD rule for the induced distributions of EV in order to develop the SSD rule for income risk averse consumers shortly.

8. Analytically, this case for SSPD with the same mean price resembles the mean preserving spread developed by Rothschild and Stiglitz (1970), except the former applies to price prospects.

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