GARCH Time Series Models: An Application to Retail Livestock Prices

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Working Paper 88-WP 29
June 1988

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Abstract

Traditional time series models assume a constant conditional variance. Realizing the implausibility of this assumption, Bollerslev proposed Generalized Autoregressive Conditional Heteroscedasticity (GARCH) processes, which are characterized by nonconstant conditional variances. In this paper, GARCH(1,1) processes were applied to model livestock prices. Results indicate that GARCH processes adequately describe retail meat price behavior.

Introduction

In recent years, agricultural economists have made extensive use of time series analysis to model economic data (Bessler and Brandt 1982; Shonkwiler and Spreen 1982; Harris and Leuthold 1985). Indeed, time series models, including univariate autoregressive and/or moving average processes, vector autoregressions, transfer functions, and dynamic regressions, have become fundamental tools of economic analysis. The considerable popularity of the time series approach can be attributed to a number of reasons. For instance, these models can be used to gain insights into the dynamic properties of complex systems (Bessler 1984; Brorsen, Chavas, and Grant 1985). In addition, time series analysis requires less subjective judgment on the part of the analyst; model identification and specification are obtained by exploiting systematic relationships in the data. But perhaps the most important reason for the widespread use of these models is their forecasting accuracy. Often, a parsimoniously specified univariate or multivariate time series model will yield better forecasts than more complex structural econometric models (Brandt and Bessler 1981).

There are several possible reasons for the enhanced forecasting performance of time series models, but the most likely is that these processes use past information optimally. For example, consider a standard first-order autoregressive (AR) process,

$$y_t = \beta_0 + \beta_1 y_{t-1} + \epsilon_t, \tag{1}$$

where y_t is a random variable drawn from a conditional density function $f(y_t|y_{t-1})$ and ϵ_t is white noise with mean zero and variance $V(\epsilon_t) = \sigma^2$. The forecast of today's value of y_t, conditioned on past information, is simply $E(y_t|y_{t-1}) = \beta_0 + \beta_1 y_{t-1}$. Likewise, the unconditional mean of y_t is $\beta_0/(1 - \beta_1)$.

The improved forecasting accuracy attributed to many time series models clearly derives from optimal use of past information. Oddly enough, these optimal forecasting properties have not, until recently, been extended to predictions of the variance.[1] So for real processes, one might expect more accurate forecast intervals if additional information on past observations of y_t were allowed to condition the forecast variance. A more general class of time series models seems desirable. Realizing this, Engle (1982) proposed a class of autoregressive processes better known as ARCH (Autoregressive Conditional Heteroscedasticity) models. The key feature of an ARCH process is that the forecast variance, h_t, is conditioned on past realizations of y_t.[2]

Although ARCH processes have been used successfully to model macroeconomic data by Engle (1982), Engle and Kraft (1983), and Weiss (1984), problems arise because of nonnegativity constraints associated with the parameter vector $\underline{\alpha}$ in the conditional variance equation. This has resulted in the use of rather arbitrary linear declining lag structures in the h_t equation to account for the long memory typically found in empirical work. Recognizing this, Bollerslev (1986) recently introduced a new class of conditional heteroscedastic models known as GARCH (Generalized Autoregressive Conditional Heteroscedasticity) processes. A chief advantage of GARCH processes over ARCH processes is that often a more flexible and parsimonious lag structure in the conditional variance equation can be obtained.[3]

There are a surprising number of areas in economics where GARCH models could be applied. For instance, portfolio models require information about price variances, and GARCH processes are a logical tool for generating proxy variables for risk premiums. Likewise, price and/or output risk variables are often included in aggregate supply equations (Just 1974; Antonovitz and Green 1986; Aradhyula and Holt 1987; and Seale and Shonkwiler 1987). Although ARIMA models are frequently used to predict the means included in these equations, *ad hoc* procedures are often employed to generate variance terms. GARCH models provide a natural framework for generating both conditional means and variances in these situations. There has also been considerable interest in modeling yields as stochastic processes (Bessler 1980). However, the variance associated with standard time series models is constant and consequently provides only limited information about higher-order moments.

The purpose of this study is to develop, estimate, and test GARCH models for the retail prices of beef, pork, and chicken. Retail meat prices seem reasonable to investigate, since they were relatively stable during the 1960s but experienced substantial volatility during the 1970s and early 1980s. The working hypothesis, then, is that GARCH models will yield more plausible forecast confidence intervals for these retail meat prices than will traditional time series models.

This study reviews the key assumptions underlying GARCH processes and fits them to beef, pork, and chicken prices. Empirical results are then evaluated and contrasted with standard autoregressive models. The final section examines the use of GARCH models to estimate conditional variances and reviews implications for future research.

The GARCH(p,q) Process

Let ϵ_t denote a real valued discrete-time stochastic process and Ω_t denote the set of all information available through time period t. The GARCH(p,q) process for a normal conditional distribution is then given by

$$\epsilon_t | \Omega_t \sim N(0, h_t), \tag{2}$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} , \tag{3}$$

where

$p \geq 0$, $q \geq 0$

$\alpha_0 > 0$, $\alpha_i \geq 0$, $i = 1, \ldots, q$, and

$\beta_i \geq 0$, $i = 1, \ldots, p$.

Note that for p = 0 the process reduces to an ARCH(q) process. Also, for p = q = 0, the conditional variance is constant, as in typical time series models, and the innovation ϵ_t simply reduces to white noise.

In the ARCH(q) process the conditional variance is specified as a linear function only of the past sample variances. Alternatively, the GARCH(p,q) process allows lagged values of the conditional variance to enter the h_t equation as well. This corresponds to the extension of an AR process to an ARMA process in traditional times series modeling and, consequently, implies some sort of adaptive learning mechanism.

The GARCH(p,q) regression model can be obtained by letting the ϵ_t's be innovations in a linear regression,

$$\epsilon_t = y_t - x_t'b, \tag{4}$$

where y_t is the dependent variable, x_t is a vector of observations on explanatory variables including past realizations of y_t, and b is a vector of unknown parameters to be estimated. If all roots of 1 - B(p) = 0 lie outside the unit circle, (3) can be respecified as a distributed lag of past squared innovations. That is,

$$\begin{aligned} h_t &= \alpha_0 (1 - B(1))^{-1} + A(L)(1 - B(L))^{-1} \epsilon_t^2 \\ &= \alpha_0 \Big(1 - \sum_{i=1}^{p} \beta_i\Big)^{-1} + \sum_{i=1}^{\infty} \delta_i \epsilon_{t-i}^2 , \end{aligned} \tag{5}$$

which, together with (2), implies an infinite-dimensional ARCH(∞) process. The δ_i's can be obtained from a power series expansion of $D(L) = A(L)(1 - B(L))^{-1}$, where

$$\begin{aligned} \delta_i &= \alpha_i + \sum_{j=1}^{n} \beta_j \delta_{i-j}, \qquad & i &= 1, \ldots, q, \\ &= \sum_{j=1}^{n} \beta_j \delta_{i-j}, & i &= q+1, \ldots, \end{aligned} \tag{6}$$

and $n = \min\{p, (i-1)\}$. Thus, if $D(1) < 1$, the GARCH(p,q) process can be approximated to any degree of accuracy by a stationary ARCH(q) process with a sufficiently large value of q.

But as an ARMA analogue, the GARCH process could be justified, through a Wald's decomposition type of argument, as a more parsimonious description. Bollerslev (1986) shows that a sufficient condition for the GARCH(p,q) process defined in (2) and (3) to be stationary is that $A(1) + B(1) < 1$. The unconditional mean and variance of the innovation ϵ_t are given by $E(\epsilon_t) = 0$ and $Var(\epsilon_t) = \alpha_0/(1 - A(1) - B(1))^{-1}$. Thus, in the GARCH(p,q) process, the unconditional variance is constant, while the conditional variance could change over time.

Of practical concern is the identification and diagnostic checking of the appropriate lag structure for the conditional variance equation in a GARCH process. Autocorrelation and partial autocorrelation functions of the innovation series are typically used when identifying and checking the time series behavior of ARMA models (Box and Jenkins 1976). Bollerslev (1986) shows that these same functions, as applied to the squared residual series, can be useful for identifying and checking the time series behavior of the conditional variance equation of the GARCH form.

Identification and diagnostic checking of a GARCH process proceed as follows. Let τ_n denote the n^{th} autocorrelation and ϕ_{kk} denote the k^{th} partial autocorrelation of ϵ_t^2, obtained by solving the GARCH analogue to the Yule-Walker equations. The usual interpretations apply. For an ARCH(q) process, ϕ_{kk} cuts off after the q^{th} lag. This is identical to the behavior of the partial autocorrelation function of the estimated residuals ϵ_t for an AR(q) process. Likewise, the partial autocorrelation function of ϵ_t^2 for a GARCH(p,q) process is, in general, nonzero, and it dampens slowly. In this manner, the autocorrelation and partial autocorrelation functions of the ϵ_t^2's can be used for identifying and checking the GARCH form.

Estimation of the GARCH regression model can be achieved by using standard maximum likelihood (ML) methods. Let z'_t =

$(1, \epsilon^2_{t-1}, \ldots, \epsilon^2_{t-q}; h_{t-1}, \ldots, h_{t-p})$, $w' = (\alpha_0, \alpha_1, \ldots, \alpha_q; \beta_1, \ldots, \beta_p)$, and $\theta = (b', w')$. The GARCH model in (2), (3), and (4) may then be rewritten as

$$\epsilon_t = y_t - x'_t b, \tag{7}$$

$$\epsilon_t | \Omega_t \sim N(0, h_t),$$

$$h_t = z'_t w.$$

Apart from a constant term, the log likelihood function for a sample of T observations is

$$L_T = T^{-1} \sum_{t=1}^{T} \ell_t(\Theta), \tag{8}$$

$$\ell_t(\Theta) = -.5 \log h_t - .5\, \epsilon_t^2\, h_t^{-1}.$$

The first and second derivatives of the log likelihood function in (8) with respect to Θ are outlined in Bollerslev (1986; pp. 315-16).

A convenient feature of the GARCH model is that the off-diagonal blocks of the information matrix associated with the $\delta \ell_t \,/\, \delta b\, \delta w'$ terms can be shown to be zero. Because of this asymptotic independence, w can be consistently estimated using initial consistent (OLS) estimates of b. This is a useful property, since initial consistent estimates of b and w can be easily obtained for starting the ML iterative estimation. Finally, as with ARMA models, the derivatives of (8) contain recursive terms. To start the recursion, we need presample estimates for both ϵ_t and h_t, $t \leq 0$. In this instance, the sample analogue $T^{-1}\ \epsilon'\epsilon$ is used to obtain consistent estimates for the presample values of ϵ_t and h_t.

Empirical Results

The estimates of GARCH models for three retail price series--beef, pork, and chicken--are reported here, along with the estimates of standard AR models as applied to each series.

The retail prices of beef, pork, and chicken were used because they have been associated with varying degrees of volatility over the past 20 years. During the 1960s and early 1970s, meat prices were relatively stable. However, large shocks in the price of feed grains, high inflation rates in the general economy, price controls, and the subsequent breeding herd liquidations that occurred in the mid and late 1970s resulted in volatile meat prices during this period. Also, there is evidence that structural change has occurred in the demand for red meats in recent years (Chavas 1983), possibly adding a further dimension of uncertainty to the forecasts of retail meat prices. These casual observations suggest that it is reasonable to believe that forecast variances associated with these prices would not have remained constant during this period. Consequently, an improved model specification would allow the conditional variance term to reflect this increased volatility.

The estimated GARCH and AR models were obtained using quarterly data, from the first quarter of 1967 through the last quarter of 1986, from various published USDA sources. The ML estimates of the model parameters were obtained by following procedures outlined in the previous section and by using the Davidon-Fletcher-Powell algorithm. In addition, the inequality and nonnegativity constraints associated with the parameter vector w in the GARCH model were enforced explicitly by using a penalty function in the estimation (Judge et al. 1982, pp. 655-57).

Estimation results for the autoregressive models are presented in Table 1. The roots of all three estimated AR models are outside the unit circle, thus satisfying the usual stationarity requirements. The Box-Pierce Q statistics, along with the MAPEs (mean absolute percent errors) and R^2, indicate that the conditional means of the fitted models do a good job of tracking actual levels. The first 20 autocorrelations for the ϵ_t's were examined for all three models and none was found to be significantly different from zero at the 5 percent level. However, the autocorrelations and partial autocorrelations of the squared residual series presented a different picture. In all instances, there were spikes in the autocorrelation function that exceeded two standard deviations. In addition,

Table 1. Maximum likelihood estimates of autoregressive models fitted

Price of beef (PB_t)

$$\underset{}{(1} - \underset{(0.006)}{1.073\,B} + \underset{(0.012)}{0.304\,B^2} - \underset{(0.012)}{0.470\,B^3} + \underset{(0.006)}{0.257\,B^4})PB_t$$

$$= \underset{(3.334)}{4.611} + \epsilon_{1t}$$

$$h_{1t} = var(\epsilon_{1t}) = \underset{(30.660)}{48.207}$$

$Q = 14.84 \quad MAPE = 3.07 \quad R^2 = 0.98 \quad \chi^2_{.05}(15) = 25.00$

Price of pork (PP_t)

$$(1 - \underset{(0.005)}{1.144\,B} + \underset{(0.008)}{0.301\,B^2} - \underset{(0.009)}{0.289\,B^4} + \underset{(0.013)}{0.824\,B^5} - \underset{(0.006)}{0.602\,B^6}$$

$$- \underset{(0.001)}{0.070\,B^{12}})PP_t = \underset{(5.132)}{5.600} + \epsilon_{2t}$$

$$h_{2t} = var\ (\epsilon_{2t}) = \underset{(25.181)}{41.380}$$

$Q = 15.79 \quad MAPE = 3.64 \quad R^2 = 0.98 \quad \chi^2_{.05}(13) = 22.36$

Price of chicken (PC_t)

$$(1 - \underset{(0.006)}{0.764\,B} - \underset{(0.012)}{0.163\,B^2} + \underset{(0.012)}{0.128\,B^3} - \underset{(0.007)}{0.165\,B^4})\ PC_t$$

$$= \underset{(2.652)}{3.037} + \epsilon_{3t}$$

$$h_{3t} = var\ (\epsilon_{3t}) = \underset{(5.581)}{20.567}$$

$Q = 20.26 \quad MAPE = 4.82 \quad R^2 = 0.91 \quad \chi^2_{.05}(15) = 25.00$

Notes: B is the lag operator, $B^s x_t = x_{t-s}$. Figures in parentheses are approximate standard errors. All prices are nominal retail prices in cents per pound.

the partial autocorrelations were positive and exhibited dampening behavior, suggesting that retail meat prices might be better represented as GARCH processes.

The ML estimates of GARCH(1,1) regression models for beef, pork, and broiler prices are reported in Table 2. GARCH(1,1) processes were used because, as Bollerslev suggests, they are parsimonious and are often the most likely candidates in applied analysis. The results indicate that stationarity conditions for both the conditional mean and variance of the estimated GARCH models are satisfied.[4] The reported MAPEs and R^2 values also indicate that the estimated parameters associated with the conditional means do a good job of explaining historical movements, although these results do not indicate any improvement in explanatory power relative to the AR models presented in Table 1. The implication is that GARCH processes will not necessarily improve upon the forecast performance of the mean of the stochastic process and, in fact, there is no reason to believe they should. But GARCH models will provide more information about the precision of these forecasts.

To illustrate, confidence intervals (99 percent) for the one-period-ahead forecasts associated with beef prices were computed. The 99 percent confidence intervals for beef, along with the actual price series, are shown in Figure 1. As indicated, retail beef prices were volatile during the mid-1970s, as reflected by the wider confidence intervals associated with the GARCH forecasts during this period. By comparison, the 1960s and early 1970s were characterized by relatively stable and trending beef prices. The results in Figure 1 indicate that the confidence intervals associated with the one-step-ahead forecasts during this period are much smaller than those for the mid-1970s. Again, traditional time series models do not give such intuitively appealing results since the width of the confidence interval (conditional forecast variance) would be constant.

Although the estimated GARCH models result in confidence intervals more intuitively appealing than those of the AR models, this is no guarantee that the GARCH process is a statistically valid improvement over the AR process. In other words, it is desirable to have a formal test of the GARCH

Table 2. Maximum likelihood estimates of GARCH models fitted

<u>Price of beef</u> (PB_t)

$$\underset{}{(1} - \underset{(0.011)}{1.060}\,B + \underset{(0.017)}{0.605}\,B^2 - \underset{(0.018)}{0.959}\,B^3 + \underset{(0.011)}{0.428}\,B^4)\,PB_t$$

$$= \underset{(1.476)}{3.549} + \epsilon_{1t}$$

$$h_{1t} = \underset{(6.347)}{4.185} + \underset{(0.022)}{0.518}\,\epsilon^2_{1t-1} + \underset{(0.019)}{0.465}\,h_{1t-1}$$

MAPE = 3.31 $\quad R^2 = 0.98$

<u>Price of pork</u> (PP_t)

$$(1 - \underset{(0.011)}{1.019}\,B + \underset{(0.011)}{0.211}\,B^2 - \underset{(0.010)}{0.220}\,B^4 + \underset{(0.110)}{0.726}\,B^5 - \underset{(0.040)}{0.537}\,B^6$$

$$- \underset{(0.001)}{0.160}\,B^{12})\,PP_t = \underset{(11.260)}{3.377} + \epsilon_{2t}$$

$$h_{2t} = \underset{(67.704)}{12.101} + \underset{(0.096)}{0.813}\,\epsilon^2_{2t-1} + \underset{(0.011)}{0.163}\,h_{2t-1}$$

MAPE = 3.55 $\quad R^2 = 0.98$

<u>Price of chicken</u> (PC_t)

$$(1 - \underset{(0.010)}{0.838}\,B - \underset{(0.008)}{0.115}\,B^2 + \underset{(0.004)}{0.080}\,B^3 - \underset{(0.006)}{0.116}\,B^4)\,PC_t$$

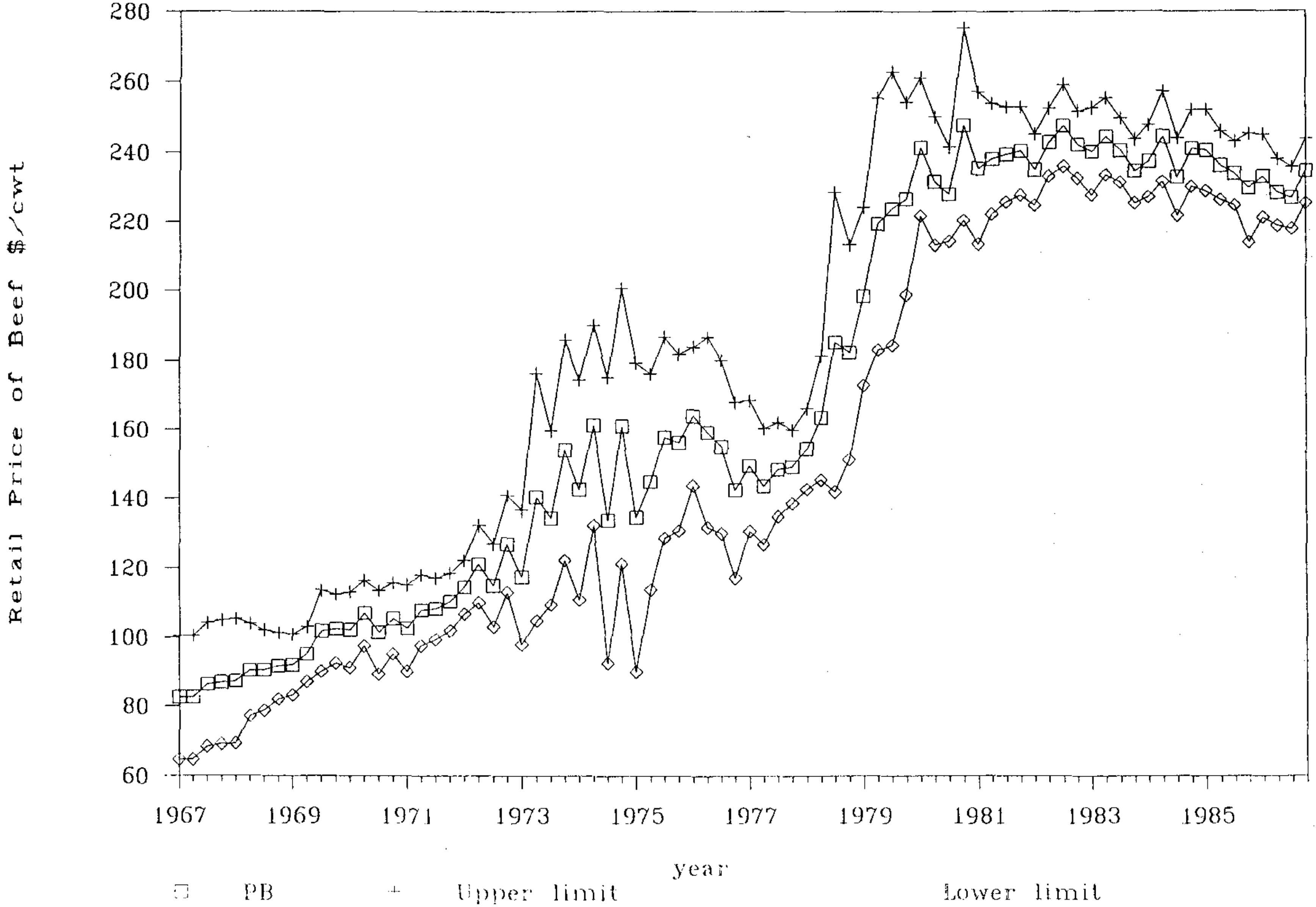
$$= \underset{(1.567)}{1.331} + \epsilon_{3t}$$

$$h_{3t} = \underset{(4.317)}{12.719} + \underset{(0.013)}{0.323}\,\epsilon^2_{3t-1} + \underset{(0.007)}{0.043}\,h_{3t-1}$$

MAPE = 4.63 $\quad R^2 = 0.91$

Notes: B is the lag operator, $B^s x_t = x_{t-s}$. Figures in parentheses are approximate standard errors. All prices are nominal retail prices in cents per pound.

Figure 1. 99 percent confidence intervals for one-step-ahead forecasts of beef retail price.



hypothesis that conditional forecast variances are nonconstant. This can be accomplished by performing a standard likelihood ratio test where, under the null hypothesis, the parameters α_1 and β_1 are constrained to zero (the standard AR representation). The alternative hypothesis is that the model follows a GARCH form. The appropriate statistic is twice the difference of the maximized values of the log likelihood functions for the unconstrained and constrained models, respectively, which will have a chi-square distribution with p + q degrees of freedom under the null hypothesis.

The results of the likelihood ratio tests are presented in Table 3, and for chicken and beef, the null hypothesis that conditional forecast variances are constant could be rejected at all usual levels of significance. Results for pork are not quite as strong, since the AR model could be rejected only at the 1 percent level. The results in Table 3 are encouraging and lend support to the contention that the conditional forecast variances of retail meat prices have been nonstationary during the past 20 years.

Summary

Traditional time series models assume a constant one-period-ahead forecast variance. In recent years, the implausibility of this assumption has been recognized and several new classes of stochastic processes have been postulated. These include the ARCH process (Engle 1982) and the GARCH process (Bollerslev 1986). These are mean zero, serially uncorrelated processes with nonconstant variances that are conditioned on past information. The GARCH and ARCH processes represent an important advance in time series modeling since much of the forecasting accuracy associated with traditional time series models derives from optimal use of past information. These same optimality conditions now can be used to generate time-varying predictions of the conditional forecast variance.

In this study, GARCH(1,1) processes were applied to retail meat prices. The estimated models replicated historical movements in these price series adequately; confidence

Table 3. Results of likelihood ratio tests

Variable	Value of Log Likelihood Function: AR Model	Value of Log Likelihood Function: GARCH Model	Value of the Test Statistic (χ^2)	Result of test
Price of beef (PB_t)	-370.44	-347.71	45.46	Reject AR
Price of pork (PP_t)	-321.15	-316.93	8.44	Reject AR
Price of chicken (PC_t)	-305.70	-289.18	33.04	Reject AR

Notes: Value of likelihood function reported here is up to an additive constant. The value of χ^2 at two degrees of freedom and at 5 percent (1 percent) level of significance is 5.99(9.21).

intervals, derived from the conditional forecast variances, changed substantially over the sample period, highlighting the potential importance of the GARCH process. A formal test of the joint significance of the α_1 and β_1 parameters in the conditional variance equations in the GARCH models revealed that the constant variance assumption associated with the estimated AR models could be rejected.

The results of this study indicate that recent advances in econometrics literature may be fruitfully applied to agricultural data. There are many instances where additional knowledge about forecast variances derived from a GARCH process could be beneficial. In addition, the normality assumption associated with conditional distribution does not present a limitation; other distributions could be used as well (Bollerslev 1987). The empirical examples presented here should encourage a wider acceptance of GARCH models in applied time series modeling.

Endnotes

1. To see this, note that the conditional variance of y_t in (1) is σ^2, while the unconditional variance is $\sigma^2/(1 - \beta_0^2)$. Thus, the conditional variance is constant and does not use information pertaining to past realizations of y_t.

2. For instance, the conditional variance of a first-order ARCH process can be written as $h_t = \alpha_0 + \alpha_1\epsilon^2_{t-1}$. More generally, the variance function can be expressed as $h_t = h(y_{t-1}, \ldots, y_{t-p}; \underline{\alpha})$, where p is the order of the ARCH process.

3. The extension of the ARCH process to a GARCH process bears a striking resemblance to the extension of the standard AR process to a more general ARMA process.

4. The stationarity conditions associated with the h_t equation are imposed.

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