Input Price Uncertainty and Factor Demand

S. Devadoss and E. K. Choi

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Abstract

A simple two-input and one-output model is used to examine the effects of variable input price uncertainty on a quasi-fixed factor. These theoretical results, applied to a livestock firm, indicate that choice of the quasi-fixed factor depends upon the attitude of the farmer toward risk and whether the inputs are complements, substitutes, or independents.

Introduction

In the agricultural sector, firms frequently have to make input demand decisions before output prices are known. In certain instances, livestock firms in particular make decisions even before certain input prices are known. For example, livestock producers decide the number of animals to keep long before corn, the major livestock feed, is harvested and corn prices are known. So farmers are faced with choosing the number of animals before the uncertainty of their variable input is resolved. The choices of optimal demand for the fixed input, capital, depend on farmers' attitudes toward risk. This study examines the effects of input-price uncertainty on the fixed input for a risk-neutral and a risk-averse firm.

In investigating this issue the following assumptions are made: (a) output price and fixed input price are known with certainty; (b) variable input price is uncertain, and the firm has some subjective probability distribution regarding the input price before it is observed; and (c) capital input is quasi-fixed in the sense that it must be chosen before the input price uncertainty is resolved, and variable input is chosen after its price is observed. This third assumption is crucial because it considerably alters decisions facing the firm by allowing it to choose only the variable input after its price is observed rather than both fixed and variable inputs. The firm can then adjust the variable input after observing its price if the firm has made a poor decision in choosing the quasi-fixed factor. This ability to make adjustments would be missing if the firm chose both inputs before the uncertainty concerning the variable input price was resolved.

After a brief review of earlier studies on the effects of price uncertainty on factor demand, the theoretical model used in this paper is explained. An analysis for a risk-neutral firm is presented and the analysis is expanded to a risk-averse firm. The final section presents conclusions and implications of the theoretical results.

Review of Literature

The agricultural sector is faced with output price uncertainty more often than input price uncertainty, so it might seem more important to examine the effects of output uncertainty on capital than to examine the effects of input uncertainty on capital. Earlier studies have already examined the effects of output price uncertainty. For example, Hartman (1976) used a two-input and one-output model to show that product price uncertainty has significant effects on a quasi-fixed factor that must be chosen before product price is known. Hartman's analysis included both risk-neutral and risk-averse firms.

Following Hartman's study, Stewart (1978) investigated the effect of input price uncertainty on factor-proportions and concluded that risk-averse producers tend to substitute fixed factors of production for factors with prices subject to random fluctuation. The major drawback of Stewart's study is that, even though he assumes fixed inputs as quasi-fixed, when he solves the model for optimal input demands he considers simultaneously all the inputs to be chosen, contradicting his assumption.

Perrakis (1980), commenting on Stewart's paper, corrected this error, but Perrakis assumed in his model that output is fixed, which is not quite correct. If we let the firm choose the variable input after resolving any uncertainty concerning its price, the firm's output is very likely to change. Thus, results obtained by Perrakis are subject to criticism.

Wright (1984) recently reexamined the effects of output and input price uncertainty on the quasi-fixed factor, but limited his analysis to risk-neutral firms. Because it is important that risk-averse firms be included in this type of study, this analysis is extended to include them.

The Model

Consider a competitive price-taking firm producing one output, Q, by using two inputs, a variable input, X, and a quasi-fixed input, K. The firm's production is represented by the production function as

$$Q = F(X, K). \tag{1}$$

The marginal products of inputs X and K, respectively, are given as

$$F_x(X, K) > 0, \quad F_k(X, K) > 0. \tag{2}$$

The production function is strictly concave, which implies that

$$F_{xx}(X, K) < 0, \quad F_{kk}(X, K) < 0, \text{ and} \tag{3}$$
$$F_{xx}(X, K)F_{kk}(X, K) - F_{xk}^2(X, K) > 0.$$

The output price, P_q, and quasi-fixed input price, r, are known with certainty. Only the variable input price, P_x, is uncertain, and quasi-fixed factor must be chosen before P_x is observed. Because variable input X is chosen after determining capital input and after observing all prices, X is determined by maximizing the short-run profits:

$$P_qF(X, K) - P_xX. \tag{4}$$

The first-order condition for this maximization problem is

$$P_qF_x(X, K) = P_x \tag{5}$$

and can be solved to obtain the optimal level of X,

$$X^* = X^*(P_q, P_x, K). \tag{6}$$

The optimal level of variable input is substituted into (4) to obtain the short-run profit function or the maximum profit function,

$$\Pi^*(P_q, P_x, K) = P_qF[X^*(P_q, P_x, K), K] - P_xX^*(P_q, P_x, K), \tag{7}$$

which is a function of output price, variable input price, and capital.

The results of envelope theorems (8) and (9) are used for analysis in the following sections.

$$\frac{\partial\Pi^*(P_q, P_x, K)}{\partial P_x} = -X^*(P_q, P_x, K) \tag{8}$$

and

$$\frac{\partial\Pi^*(P_q, P_x, K)}{\partial K} = P_qF_k[X^*(P_q, P_x, K), K] \tag{9}$$

Differentiating the first-order condition (5) implicitly with respect to K, we get

$$\frac{\partial X^*}{\partial K}(P_q, P_x, K) = -F_{kx}/F_{xx}. \tag{10}$$

The following sections examine the optimal choice of a quasi-fixed factor for a risk-neutral and a risk-averse firm.

Risk-Neutral Firm

According to previous studies of uncertainty theory, a risk-neutral firm maximizes expected profits. Therefore, the optimal level of the quasi-fixed factor is determined by maximizing the expected value of long-run profits,

$$\begin{aligned} &E[\Pi^*(P_q, P_x, K) - rK] \\ &= E\{P_q F[X^*(P_q, P_x, K), K] - P_x X^*(P_q, P_x, K) - rK\}. \end{aligned} \tag{11}$$

The first-order condition for the preceding optimization problem is obtained by differentiating (11) with respect to K:

$$\begin{aligned} &E[P_q(F_x \frac{\partial X^*}{\partial K} + F_k) - P_x \frac{\partial X^*}{\partial K} - r] = 0 \\ &E[(P_q F_x - P_x) \frac{\partial X^*}{\partial K} + P_q F_k - r] = 0. \end{aligned} \tag{12}$$

At the optimum, $(P_q F_x - P_x)$ is equal to zero, so (12) can be simplified as

$$E[P_q F_k - r] = 0 \text{ or } E[P_q F_k] = r \text{ or using (9) } E(\frac{\partial \Pi^*}{\partial K}) = r. \tag{13}$$

The second-order condition is satisfied because of the strict concavity assumption of the production function. It can be shown, by differentiating (13) with respect to K and using (10), that the second-order condition

$$E[P_q F_{kk} + P_q F_{kx} \frac{\partial X^*}{\partial K}] \tag{14}$$

or

$$E[P_q(F_{kk}F_{xx} - F_{kx}^2)/F_{xx}]$$

is negative (also see Eq. 3).

Using these results, it can be shown graphically that the optimal level of capital K* is determined such that the price of capital r is equal to the expected value of the marginal product of capital (Figure 1).

Further, the effects of increased input price uncertainty on capital input can be examined. As in Rothschild and Stiglitz (1970), the increased input price uncertainty is used as a mean-preserving price spread. The notion of mean-preserving price spread implies that the input price has greater variation but still has the same mean. The Rothschild and Stiglitz results imply that if $\frac{\partial \Pi^*}{\partial K}$ is convex (concave) in P_x, the mean-preserving price spread increases (decreases) $E(\frac{\partial \Pi^*}{\partial K})$, leading to an increase (decrease) in the optimal capital input, K*.

To show under what conditions the function $\frac{\partial \Pi^*}{\partial K}$ is convex or concave in P_x, we have to differentiate $\frac{\partial \Pi^*}{\partial K}$ twice with respect to P_x. The first differentiation, using the envelope theorem, gives

$$\frac{\partial^2 \Pi^*}{\partial K \partial P_x} = \frac{\partial}{\partial K}\left(\frac{\partial \Pi^*}{\partial P_x}\right) = \frac{\partial(-X^*(P_q, P_x, K))}{\partial K} = \frac{-\partial X^*(P_q, P_x, K)}{\partial K}. \tag{15}$$

Further differentiating the above function with respect to P_x gives

$$\frac{\partial^3 \Pi^*}{\partial K \partial^2 P_x} = \frac{-\partial^2 X^*(P_q, P_x, K)}{\partial K \partial P_x} \tag{16}$$

$$\frac{\partial^3 \Pi^*}{\partial K \partial^2 P_x} \gtrless 0 \text{ if } \frac{-\partial^2 X^*(P_q, P_x, K)}{\partial K \partial P_x} \gtrless 0. \tag{17}$$

Clearly, an increase in input price uncertainty will increase (decrease) $E(\frac{\partial \Pi^*}{\partial K})$ and will lead to an increase (decrease) in optimal capital input demand only if $\frac{\partial^2 X^*(P_q, P_x, K)}{\partial K \partial P_x} < 0 \ (> 0)$.

Risk-Averse Firm

According to the economic theory of uncertainty, a risk-averse firm will maximize expected utility instead of expected profits. Thus, a risk-averse firm will determine optimal capital input by maximizing the expected utility function

$$EU(\Pi) = EU\{P_q F[X^*(P_q, P_x, K), K] - P_x X^*(P_q, P_x, K) - rK\}. \quad (18)$$

For a risk-averse firm, the utility function is concave, with

$$U'(\Pi) > 0 \text{ and } U''(\Pi) < 0. \quad (19)$$

The first-order condition for the preceding maximization problem is

$$E\{U' \cdot [P_q(F_x \frac{\partial X^*}{\partial K} + F_k) - P_x \frac{\partial X^*}{\partial K} - r]\} = 0 \quad (20)$$

$$= E[U' \cdot (P_q F_k - r)] = 0 \quad (20')$$

$$= EU' \cdot E(P_q F_k - r) + \text{cov}(U', P_q F_k - r) = 0$$

$$= EU' \cdot E(P_q F_k - r) + \text{cov}(U', P_q F_k) = 0, \quad (21)$$

where the functions are evaluated at the optimal level and $\text{cov}(U', P_q F_k)$ is the covariance between U' and $P_q F_k$. To determine the optimal level of capital input K^*, we have to solve equation (21) for K^*, which means we have to know the sign of $\text{cov}(U', P_q F_k)$. This can be found by examining the changes in U' and $P_q F_k$ for changes in P_x. Using the results of envelope theorem (8) gives

$$\frac{\partial U'}{\partial P_x} = \frac{\partial U'}{\partial \Pi} \frac{\partial \Pi}{\partial P_x} = U'' \cdot (-X^*) > 0, \text{ and} \quad (22)$$

$$\frac{\partial P_q F_k}{\partial P_x} = P_q \frac{\partial F_k}{\partial X^*} \frac{\partial X^*}{\partial P_x} = P_q F_{xk} \frac{\partial X^*}{\partial P_x} \lessgtr 0 \text{ if } F_{xk} \gtrless 0. \quad (23)$$

The results of (22) and (23) indicate that U' increases for an increase in P_x, and changes in $P_q F_k$ depend on whether X and K are substitutes ($F_{xk} < 0$), complements ($F_{xk} > 0$), or independents ($F_{xk} = 0$). If X and K are complements, $P_q F_k$ decreases for an increase in P_x and the $\text{cov}(U', P_q F_k)$ term is negative.

If the covariance term is negative, then it follows from (21) that

$$E(P_q F_k - r) \text{ is positive, i.e.,} \quad (24)$$

$$E(P_q F_k) > r.$$

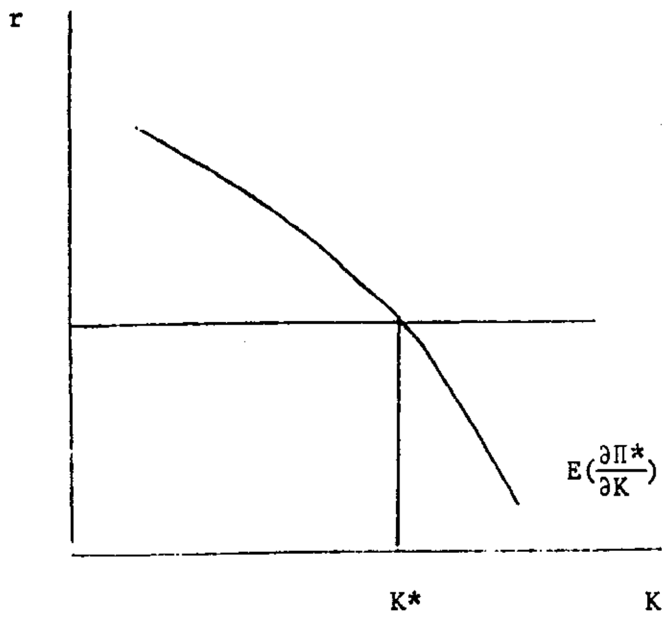

Figure 1. The Case of Risk-Neutral Firm

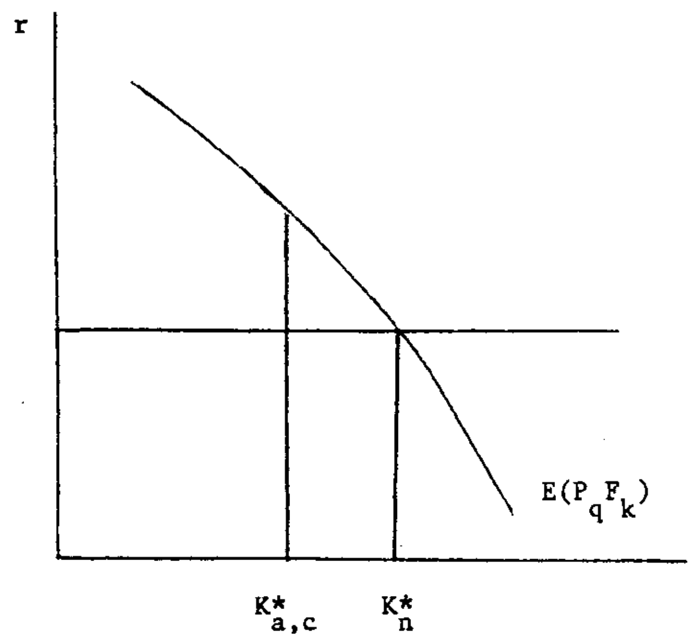

Figure 2. The Case of Risk-Averse Firm with Complement Inputs

The optimal level of capital input is shown graphically in Figure 2, where $K^*_{a,c}$ is the optimal capital input for a risk-averse firm with X and K as complementary inputs at the optimum. K^*_n is the optimal capital input for a risk-neutral firm, also shown in Figure 1. A risk-averse firm with complementary inputs will invest less in capital than will a risk-neutral firm, because in condition (24), $E(P_q F_k)$ is greater than r for the risk-averse firm, whereas $E(P_q F_k)$ is equal to r for a risk-neutral firm.

If K and X are substitutes ($F_{xk} < 0$), then the covariance term will be positive, because U' and $P_q F_{xk}$ move in the same direction for changes in P_x. This implies that

$$E(P_q F_k - r) < 0 \qquad \text{or} \qquad E(P_q F_k) < r. \tag{25}$$

The optimal capital, $K^*_{a,s}$, of the risk-averse firm with substitute inputs at the optimum is shown in Figure 3.

It is clear that the risk-averse firm will employ more capital than will the risk-neutral firm. For independent inputs, the covariance term will be zero, and

$$E(P_q F_k - r) = 0 \qquad \text{or} \qquad E(P_q F_k) = r, \tag{26}$$

the same as that of the risk-neutral firm. Thus, the optimal capital input demand is the same as that of the risk-neutral firm.

The second-order condition for the utility maximization of the risk-averse firm should be negative. We can verify the second-order condition by differentiating the first-order condition (20') with respect to K and using (10),

$$\begin{aligned} \frac{\partial^2 EU}{\partial^2 K} &= E[U'' \cdot (P_q F_k - r)^2 + U' \cdot \frac{\partial(P_q F_k - r)}{\partial K}] \\ &= E[U'' \cdot (P_q F_k - r)^2] + E[U' \cdot \frac{\partial(P_q F_k - r)}{\partial K}] \\ &= E[U'' \cdot (P_q F_k - r)^2] + E\{U' \cdot [P_q (F_{kk} F_{xx} - F^2_{xk})/F_{xx}]\}. \end{aligned} \tag{27}$$

The second-order condition (27) is certainly satisfied since $U'' < 0$ and $(F_{kk} F_{xx} - F^2_{xk})/F_{xx}$ is negative.

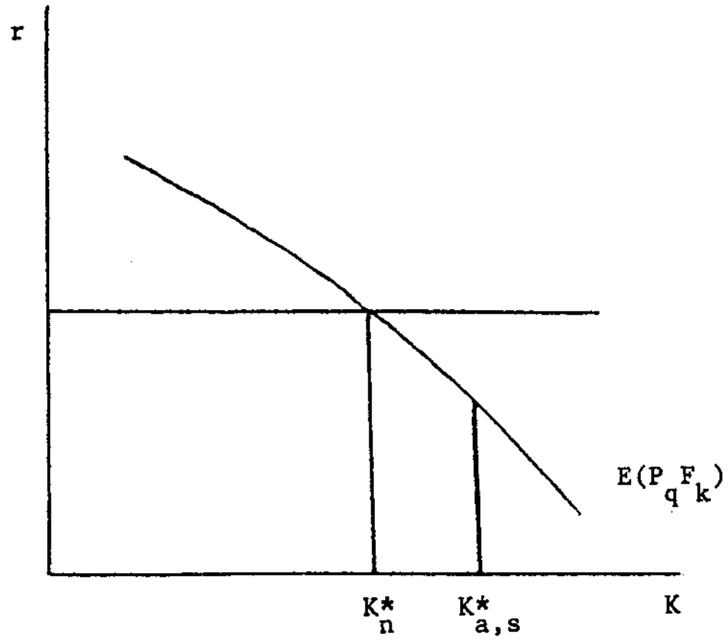

Figure 3. The Case of Risk-Averse Firm with Substitute Inputs

The effect of increased input price uncertainty on the quasi-fixed factor is not examined for the risk-averse firm because of the complicated nature of the mathematics.

As a corollary to these results, we can derive further results for a risk-loving firm. In this case, the second derivative of the utility function with respect to profit is positive; i.e.,

$$U'(\Pi) > 0 \text{ and } U''(\Pi) > 0. \tag{28}$$

Given that $U''(\Pi)$ is positive, it is easy to verify that, if inputs are complements (substitutes) at the optimum, the optimal level of capital for the risk-loving firm will be higher (lower) than that of the risk-neutral firm.

Conclusions and Implications

A simple two-input and one-output theoretical model has been used to examine the effects of variable input price uncertainty on capital input for a risk-neutral and a risk-averse firm. Capital input is considered quasi-fixed in the sense that it must be chosen before the uncertainty concerning the variable input price is resolved. Variable input is chosen after capital input has been determined and after input price has been observed. The following results were obtained:

1. For a risk-neutral firm, optimal capital is determined by equating expected marginal value product with the price of capital. A mean-preserving price spread in variable input price will increase (decrease) optimal capital if $\frac{\partial^2 X^*(P_q, P_x, K)}{\partial K \partial P_x}$ is negative (positive).
2. For a risk-averse firm, optimal capital is less (greater) than that of a risk-neutral firm if the two inputs are complements (substitutes) at the optimum. The exact opposite results hold for a risk-loving firm. If the inputs are independents, optimal capital use for both risk-averse and risk-loving firms is exactly equal to that of a risk-neutral firm.

For a livestock firm, the variable input is corn and the quasi-fixed input is the number of animals. When a farmer decides how many animals to keep for feeding, he does not know the price of corn, which depends upon, among other things, corn production the next season. Thus, the optimal number of animals to be chosen by a farmer depends on the attitude of that farmer toward risk, and whether the inputs--corn and livestock animals--are complements, substitutes, or independents.

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