Discriminating Rational Expectations Models with Non-Nested Hypothesis Testing: An Application to the Beef Industry

Satheesh V. Aradhyula and Stanley R. Johnson

Working Paper 87-WP 19
January 1987

Contents

Introduction

Price expectations are an integral part of agricultural decision making. Despite recognition of this fact, few changes have been made during the past two decades in the way price expectations appear in agricultural models. For example, the majority of existing supply response studies have assumed that price expectations are formed adaptively. The popular adaptive expectations hypothesis is, however, inadequate from an economic perspective. The inadequacy arises not because the adaptive expectations imply that the forecast of a particular variable is a distributed lag of its own past values, but because it implies that the distributed lag parameters are restricted in an ad hoc way. This is so because the parameter restrictions in the distributed lag are not the result of an optimization process.

The problem mentioned above can be partly overcome by assuming that expectations are rational. The concept of rational expectations provides a method of interpreting the use decisionmakers make of available information. This information consists of all available observations on the variable in question and on related variables at the time the forecast is made. According to Muth (1961), "Expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory."

Incorporating the rational expectations hypothesis (REH) raises the question of what is the appropriate production lag; that is, how many periods ahead of production the expectations are formed. The few models that incorporated rational expectations assumed some production lag in an ad hoc way without actually applying a test for it. For example, in their chicken sector model, Goodwin and Sheffrin (1982) assumed that expectations are formed two months prior to production. This is an unwarranted assumption of production lag in the face of the technologies that exist to produce the same output. In the beef industry, for example, the lag between placement and marketing is approximately one year for feedlot operators, while for cow-calf operators the lag is two to three years.

Imposing one lag or the other in formulating supply response is an ad hoc specification. One needs to exercise caution on the choice of production lag in modeling markets. This paper presents a method for systematically discriminating alternative production lags for modeling purposes.

In this paper we estimate three different models of supply and demand for the beef industry, using three different hypotheses about production lags, and under the assumption of rational expectations in the sense of Muth. Joint tests of the REH and the model specification for each of the three models are provided. These models are then discriminated with non-nested hypothesis tests to determine which of the production lag hypotheses the data supports the most. Our econometric procedure is related to the recent work of Wallis (1980) and Davidson and MacKinnon (1981) and combines time series analysis with traditional econometric estimation techniques. Under the assumptions of rational expectations, the model can be solved for the expected price as a function of the expected values of the exogenous variables. This function can then be substituted into the model leading to a specification which contains the original endogenous and exogenous variables plus the expected values of the exogenous variables. Time series analysis is used to generate the necessary forecasts of the exogenous variables. The suppliers are assumed to act as if they know both the underlying structure of the market and the stochastic process governing the exogenous variables, the two requirements of rational expectations. While the expected price enters only the supply equations of the model, it is necessary, in the econometric formulation, to specify the demand equation and others. By specifying the complete model, the additional structure imposed on the problem allows us to estimate the coefficients and test the implied restrictions. The model uses annual data over the sample period 1960-1982. The complete system of equations is estimated by nonlinear three stage least squares, and the constraints implied by REH are tested using Gallant and Jorgenson's chi-square test.

The models that pass this test are discriminated by employing recent developments in the econometrics literature on non-nested hypothesis testing (Davidson and MacKinnon, 1981). Unlike previous studies, we evaluate alternative rational expectation models using hypothesis tests based on a structural norm. Because of differences in production lags incorporated in the model, empirical specifications used for evaluating these hypotheses with available data are non-nested. This paper reports results from applying non-nested hypothesis tests to the evaluation of alternative rational expectation models.

Model Specification

The empirical specification is based on a simple model of intertemporal competitive equilibrium with rational expectations. The decision to supply cattle for slaughter is made under uncertainty; that is, prior to observing the ultimate slaughter steer price. Production is assumed to come from a large number of identical firms, each small relative to the market. Production requires n discrete periods with the production decision made at the start of the first period. The cost of producing an amount q_t is:

$$c(q_t) = \alpha_1 q_t^2 + \alpha_2 x_t q_t$$

with $\alpha_1 > 0$ and x_t price of input. In addition there are costs associated with making adjustments in output from period to period as follows:

$$A(q_t) = \gamma(q_t - q_{t-n})^2$$

where $A(q_t)$ represents adjustment costs and $\gamma > 0$ is an adjustment parameter. Conceptually, these costs are related to production inertia which can, in turn, be caused by physical plant limitations, credit restrictions, labor constraints, availability of feed supplies, limitations in management expertise, etc. The effects of these adjustment costs are to make the firm unwilling to alter q_t from previous level of output. Both cost functions are quadratic in nature. The cost of adjustments function, as specified above, implies that large changes in output will induce proportionally more adjustment costs than will relatively minor changes in production levels. It is also assumed that producers choose non-negative quantities q_1, q_2,..., etc. to maximize the expected present value of the profits:

$$Z_t = E\{\sum_{t=1}^{\infty} \rho^t[P_{t+n} q_t - \alpha_1 q_t^2 - \alpha_2 x_t q_t - \gamma(q_t - q_{t-n})^2]\}$$

where $0 < \rho < 1$, is the discount factor and P_{t+n} is the output price in period t+n. This is a quadratic programming problem with a unique maximum which satisfies the conditions for certainty equivalence. First order conditions are necessary and are:

$$P^e_{t+n} - 2\alpha_1 q_t - \alpha_2 x_t - 2\gamma(q_t - q_{t-n}) \leq 0 \text{ if } q_t > 0,\ t \geq 1$$

where the superscript e on P_{t+n} indicates expected value of P_{t+n} conditional on all the information available in period t. If the quantity actually produced differs from that planned by a stochastic component, v_t, which has an expected value of zero, then aggregate production satisifies:

$$q_t = a_1 \cdot P^e_{t+n} + a_2 x_t + a_3 q_{t-n} + v_t$$

with $a_1 = 1/2(\alpha_1 + \gamma)$, $a_2 = -\alpha_2/2(\alpha_1 + \gamma)$, and $a_3 = \gamma/(\alpha_1 + \gamma)$. Thus, the supply function of beef is specified as:

(1) $$PD_t = a_1\ {}_{t-n}WP^e_t + a_2FC_t + a_3PD_{t-n} + u_{1t}$$

where

PD_t = production of beef in period t, in billions of pounds,

${}_{t-n}WP^e_t$ = the expected real cattle slaughter price in period t viewed from period t-n, in dollars per hundred weight, and FC_t = real feed cost index, 1967 = 100.

The consumption demand for beef is assumed to depend on retail price index of beef, price of substitutes, and personal consumption expenditure. We specify that:

$$CN_t = b_0 + b_1 RP_t + b_2 PC_t + b_3 CE_t + b_4 SF_t + u_{2t} \quad (2)$$

where

CN_t = quantity of beef consumed in period t, in billions of pounds,
RP_t = real retail price index of beef, 1967=100,
CE_t = real personal expenditure, in millions of dollars,

PC_t = real retail price index of chicken, 1967=100, and
SF_t = a "tastes shift" variable = 0 for years < 1975, CE_t elsewhere.

The slaughter price of beef is assumed to be related to retail price by the following equation:

$$WP_t = c_1 RP_t + c_2 FU_t + u_{3t} \quad (3)$$

where

FU_t = fuel and utility price index, 1967=100.

Finally the model is closed by the assumption that supply equals consumption demand plus other demand

$$PD_t = CN_t + OT_t \quad (4)$$

where OT_t represents other demand, in billions of pounds. In this case, other demand equals net export demand plus net stock demand. Following the format of Wallis (1980), we can write the structural model in standard form as:

$$By_t + Ay_t^e + \Gamma X_t = U_t \quad (5)$$

$$E(U_t) = 0, \; E(U_t U_t') = \alpha_{t\ell}\Sigma, \; \alpha_{t\ell} = \begin{cases} 1 & t=\ell \\ 0 & t\neq\ell \end{cases}$$

where

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -b_1 & 0 & 1 \\ 0 & -c_1 & 1 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 0 & -a_1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$y_t' = \begin{bmatrix} PD_t \, RP_t \, WP_t \, CN_t \end{bmatrix} \qquad y_t^{e'} = \begin{bmatrix} PD_t^e \; RP_t^e \; WP_t^e \; CN_t^e \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} -a_2 & -a_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -b_o & -b_2 & -b_3 & -b_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -c_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$X_t' = [FC_t PD_{t-n} \; 1 \; PC_t CE_t SF_t FU_t OT_t]$$

Note that the model allows for contemporaneous correlation of the errors across equations but not for autocorrelation. Of course, the model could be generalized to allow for autocorrelation, although this would further complicate the analysis. Solving (5) for y_t and taking conditional expectations (E_{t-n}) gives an expression for the rational predictor. Substituting this in the original model gives the following form that can be estimated:

$$By_t - A(B + A)^{-1} \Gamma E_{t-n}(x_t) + \Gamma X_t = U_t \tag{6}$$

Actual levels of quantity and prices in the market are determined by the actual and expected values of the exogenous variables and the structural disturbances. Note that although the underlying structural model in (5) is linear, the model in (6) which includes the rational predictor is nonlinear in the parameters. Consequently, nonlinear estimation techniques must be used to account for the nonlinear, cross-equation restrictions.

It still remains to specify the expectation of the exogenous variables, $E_{t-n}(X_t)$. Since expectations are formed n periods ahead, the information set, among other variables, includes PD_{t-n}. Thus PD_{t-n} is known at the time ${}_{t-n}WP_t^e$ is formed. We make the assumption that the intercept column of ones and the tastes shift variable in the demand equation are known with certainty. The fact that some exogenous variables are "forecasted" with certainty means that a subset of the elements of $E_{t-n}(X_t)$ and X_t are identical and equation (6) can not be directly estimated. To rewrite the structure in a form that allows estimation, we must rearrange and comformably partition Γ, X_t and $E_{t-n}(X_t)$:

$$\Gamma = (\Gamma_1 \,\vdots\, \Gamma_2) = \begin{bmatrix} 0 & -a_3 & \vdots & -a_2 & 0 & 0 & 0 & 0 & 0 \\ -b_o & 0 & \vdots & 0 & -b_2 & -b_3 & -b_4 & 0 & 0 \\ 0 & 0 & \vdots & 0 & 0 & 0 & 0 & -c_2 & 0 \\ 0 & 0 & \vdots & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$(7) \qquad X_t' = (X_{1t}' \,\vdots\, X'_{2t}) = (1 \;\; PD_{t-n} \,\vdots\, FC_t \, PC_t \, CE_t \, SF_t \, FU_t \, OT_t)$$

$$(8) \qquad E_{t-n}(X_t') = (X_{1t}' \,\vdots\, E_{t-n}(X_{2t}'))$$

By combining the coefficients of the common elements of X_t and $E_{t-n}(X_t)$, we have a structure which can be directly estimated by nonlinear three-stage least squares:

$$(9) \quad By_t + (-A(B + A)^{-1} \Gamma_1 + \Gamma_1)X_{1t} - A(B + A)^{-1}\Gamma_2 E_{t-n}(X_{2t}) + \Gamma_2 X_{2t} = U_t$$

Time series models are used to specify the stochastic process governing the remaining exogenous variables. Our econometric procedure also treats the time series forecasts as data, as if they are given to the producers by a forecasting service. Although simultaneous estimation of the structural and time series parameters yields consistent and efficient estimates, the procedure used here will still result in consistent estimates of the structural parameters. The system represented by (9) is estimated for three different values of n, n=1 (model 1), n=2 (model 2), and n=3 (model 3). Note that n represents the production lag or the number of years ahead of which expectations are formed. Models with production lags of more than three years are not considered because of the biological nature of beef production.

Econometric Estimates and Validation of the Model

The econometric procedure first requires forecasts of the exogenous variables. The time series estimates, along with the associated Box-Pierce "Q" statistics appear in Table 1. These statistics indicate that the fitted models do a reasonably good job of explaining their respective historical series. These fitted time series models are used to generate $E_{t-1}(X_{2t})$, $E_{t-2}(X_{2t})$ and $E_{t-3}(X_{2t})$ needed for models 1, 2 and 3 respectively. The nonlinear three-stage least square estimates of the three models are given in Table 2. With the exception of feed cost, all the coefficient estimates are statistically significant and have signs that are consistant with *a priori* judgments. Calculating the elasticities at 1982 values in the sample gives the following results:

	Model 1	Model 2	Model 3
beef supply			
expected slaughter price	0.38	0.49	0.44
feed cost	-0.05	-0.01	-0.03
beef consumption demand			
retail price of beef	-0.90	-0.95	-0.98
retail price of chicken	0.21	0.23	0.30
income	1.21	1.28	1.44

The income and own price elasticities of demand seem to be slightly higher than estimates reported elsewhere. It should also be noted that this is not a per capita demand function and the equation is estimated in price dependent form. All the equations have acceptable R-squares while Durbin-Watson statistics generally indicate that there are only marginal problems with autocorrelations. The dynamic simulation statistics of the models are given in Table 3. These statistics indicate that models 1 and 2 are very good. The Theil's decomposition coefficient for bias in Model 3 is high, indicating that the model is consistently overpredicting or underpredicting. A more careful review of the equation by equation results is left to interested readers.

Table 1. Time series models fitted for the exogenous variables

Feed Cost (FC)

$$(1 + 0.445B + 0.277B^2)\Delta FC_t = \varepsilon_{1t}$$

(2.02) (1.26)

$Q = 3.10$

$\chi^2_{.05}(12) \approx 21.03$

Price of Chicken (PC)

$$(1 - 0.621B)\ PC_t = 0.986 + \varepsilon_{2t}$$

(-3.01) (17.90)

$Q = 3.71$

$\chi^2_{.05}(13) \approx 22.36$

Personal Expenditure (CE)

$$(1 - 0.987B)\ CE_t = 464.820 + \varepsilon_{3t}$$

(-23.38) (1.211)

$Q = 14.55$

$\chi^2_{.05}(13) \approx 22.36$

Feed and Utility index (FU)

$$(1 - 0.964B)FU_t = 1.169 + \varepsilon_{4t}$$

(-10.96) (5.98)

$Q = 12.43$

$\chi^2_{.05}(13) \approx 22.36$

Other Demand (OT)

$$(1 - 0.834B)OT_t = 1221.029 + \varepsilon_{5t}$$

(-7.07) (4.34)

$Q = 6.07$

$\chi^2_{.05}(13) \approx 22.36$

Notes: B is the lag operator, $B^s x_t = x_{t-s}$. $\Delta X = (1-B)X$. Figures in parenthesis are approximate T statistics.

Table 2. Nonlinear three-stage least square estimates

Model 1 (n=1)

$$PD_t = 0.374\,{}_{t-1}WP^e_t - 1.491\ FC_t + 0.642\ PD_{t-1} \qquad R^2 = 0.77$$

(3.05) (-0.84) (3.77) DW = 2.40

$$RP_t = 0.538 - 0.043\ CN_t + 0.329\ PC_t + 0.0024\ CE_t - 0.00029\ SF_t \qquad R^2 = 0.86$$

(3.59) (-5.20) (3.10) (7.99) (-4.32) DW = 2.61

$$WP_t = 32.128\ RP_t - 6.547\ FU_t \qquad R^2 = 0.92$$

(17.07) (-3.51) DW = 2.62

Model 2 (n=2)

$$PD_t = 0.478\,{}_{t-2}\ WP^e_t - 0.254\ FC_t + 0.455\ PD_{t-2} \qquad R^2 = 0.96$$

(2.94) (-0.10) (2.03) DW = 1.90

$$RP_t = 0.529 - 0.041\ CN_t\ 0.327\ PC_t + 0.0024\ CE_t - 0.00028\ SF_t \qquad R^2 = 0.93$$

(3.84) (-6.09) (3.32) (9.35) (-4.55) DW = 2.01

$$WP_t = 31.460\ RP_t - 6.037\ FU_t \qquad R^2 = 0.89$$

(22.35) (-4.32) DW = 2.61

Model 3 (n=3)

$$PD_t = 0.391\,{}_{t-3}\ WP^e_t - 0.876\ FC_t + 0.587\ PD_{t-3} \qquad R^2 = 0.89$$

(1.90) (-0.35) (2.21) DW = 1.88

$$RP_t = 0.453 - 0.031\ CN_t + 0.332\ PC_t + 0.0021\ CE_t - 0.00026\ SF_t \qquad R^2 = 0.94$$

(3.11) (-5.62) (3.19) (8.94) (3.94) DW = 1.97

$$WP_t = 30.738\ RP_t - 5.227\ FU_t \qquad R^2 = 0.76$$

(20.55) (-3.51) DW = 2.32

Note: Figure in parentheses are approximate t statistics

Table 3. Dynamic simulation statistics of the model

Variable	Root Mean Square Percent Error	Theil's Forecast Error Measures: Decomposition Bias	Regress	Disturbance	Inequality Coefficient
Model 1					
PD	5.90	0.03	0.34	0.63	0.0027
CN	5.49	0.03	0.33	0.64	0.0023
RP	5.50	0.01	0.23	0.76	0.0501
WP	6.50	0.01	0.18	0.81	0.0024
Model 2					
PD	5.52	0.12	0.34	0.54	0.0026
CN	5.20	0.12	0.37	0.51	0.0023
RP	5.18	0.13	0.31	0.56	0.0469
WP	6.12	0.15	0.17	0.68	0.0021
Model 3					
PD	7.77	0.51	0.17	0.32	0.0035
CN	7.34	0.51	0.17	0.32	0.0032
RP	6.76	0.29	0.14	0.57	0.0615
WP	8.13	0.24	0.10	0.66	0.0029

Tests of the REH

Gallant and Jorgenson's chi-square test (GJ chi-square test) has been used to test the across-the-equations restrictions implied by the REH. Gallant and Jorgenson show that the change in the least-squares function can be used as an asymptotically valid chi-square test statistic. To perform the GJ chi-square test, an unconstrained model is first estimated:

$$By_t + W_1X_{1t} + W_2E_{t-n}(X_{2t}) + \Gamma_2X_{2t} = U_t$$

$$W_1 = \begin{bmatrix} w_1 & w_2 \\ w_3 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad W_2 = \begin{bmatrix} w_4 & w_5 & w_6 & w_7 & w_8 & w_9 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The unconstrained model contained 16 parameters. Formally the restrictions tested were:

$$W_1 = -A(B + A)^{-1}\,\Gamma_1 + \Gamma_1$$

$$W_2 = - A(B + A)^{-1} \Gamma_2$$

These restrictions reduced the dimension of parameter space by six. This test is conducted separately for each of the three models. The calculated chi-square values are 6.29, 7.74, and 23.27 for models 1, 2, and 3, respectively. The appropriate χ^2 value for six degrees of freedom at the 5 percent confidence level is 12.59. Thus, for models 1 and 2, the test results do not lead to rejection of the null hypothesis that the nonlinear restrictions implied by REH are valid. The restrictions implied by model 3 are, however, rejected. Thus, models 1 and 2 pass the test while model 3 is rejected. Note that this is a joint test of the REH and model specification.

Non-nested Hypothesis Tests

The Davidson and MacKinnon J-test has been used to carry out the non-nested hypothesis tests. These tests allow one to evaluate the "truth" of the specification of one model relative to the specification(s) of one or more alternative non-nested models. Equation (9) can be written as

(10) $$Y_t = D_1 X_{1t} + D_2 E_{t-n} (X_{2t}) + D_3 X_{2t} + V_t$$

or

(11)* $$Y_t = f_t \left(X^n_{1t}, E_{t-n} (X_{2t}), X_{2t}, D^n\right) + V_t$$

The models of interest are the ones for n=1 and n=2 i.e.,

(12) $$Y_t = h_t \left(X^1_{1t}, E_{t-1} (X_{2t}), X_{21}, D^1\right) + V_{1t}$$

and

(13) $$Y_t = g_t\left(X^2_{1t}, E_{t-2} (X_{2t}), X_{2t}, D^2\right) + V_{2t}$$

To carry out the J-test with (12) as the null and (13) as the alternative hypothesis the following auxiliary model is estimated and tested for α=0.

(14) $$Y_t = h_t (\underset{\sim}{1} - \underset{\sim}{\alpha}) + g_t \underset{\sim}{\alpha} + V_t; \underset{\sim}{\alpha} = (\alpha\ 0\ 0\ 0)'$$

The test statistics is 1.03, and the appropriate t statistic with 5 percent confidence level and 15 degrees of freedom is 1.75. Thus, we fail to reject the specification in (12). By itself, this is not enough to rule out the model in (13), since the J test is not symmetric. To carry out another test with (13) as the null and (12) as the alternative, another auxiliary model with h_t and g_t interchanged in (14) is estimated. The test statistics is 4.84, thus,

---

* $X_{1t} = X^n_{1t} = (1, PD_{t-n})'$; $X^1_{1t} = (1, PD_{t-1})'$; $X^2_{2t} = (1, PD_{t-2})'$.

indicating a rejection of the null hypothesis that (13) is the "true" model. On the basis of both tests, we can conclude that the rational expectations model with one period production lag dominates the model with two periods production lag.

Conclusion

Much of the supply response analysis in agriculture is conducted with ad hoc model specifications. Until recently, economic theory has had little to say about the expectation formation, and researchers have had to rely on intuition to guide them in model specification. There are, however, some recent contributions to econometrics that could offer help to the researcher. These relatively simple tests could be used effectively in model specification.

Using these non-nested tests, the present study has demonstrated a method for systematically discriminating alternative model specifications. This work may be viewed as an indication that recent contributions of Wallis, Davidson, and McKinnon to econometrics can be usefully applied to many agricultural markets. Using a similar approach to model output response in other agricultural markets should prove to beneficial.

References

Chavas, J. and S.R. Johnson. 1983. "Rational Expectations in Econometric Models." New Directions in Econometric Modeling and Forecasting in U.S. Agriculture, edited by G.R. Rausser, New-York: Elsevier-North Holland.

Davidson, R. and J.G. MacKinnon. 1981. "Several Tests for Model Specification in the Presense of Alternative Hypothesis." Econometrica 49:781-793.

Fomby, Thomas B., R.C. Hill, and S.R. Johnson. 1984. Advanced Econometric Methods, New York: Springer-Verlag.

Gallant, Ronald A. and D.W. Jorgenson. 1979. "Statistical Inference for a System of Simultaneous, Non-linear, Implicit Equations in the Context of Instrumental Variable Estimation." Journal of Econometrics 11:275-302.

Goodwin, Thomas H. and Steven M. Sheffrin. 1982. "Testing the Rational Expectations Hypothesis in an Agricultural Market." Review of Economics and Statistics 64:658-667.

Hoffman, Dennis L. and Peter Schmidt. 1981. "Testing the Restrictions Implied by the Rational Expectations Hypothesis." Journal of Econometrics 15:265-287.

Huntzinger, LeVar R. 1979. "Market Analysis with Rational Expectations." Journal of Econometrics 10:127-165.

MacKinnon, J. 1983. "Model Specification Tests Against Non-nested Alternatives." Econometric Review. 2:85-110.

MacKinnon, James G., H. White, and R. Davidson. 1983. "Tests for Model Specification in the Presence of Alternative Hypothesis." Journal of Econometrics 21:53-70.

Muth, John F. 1961. "Rational Expectations and the Theory of Price Movements." Econometrica 29:315-335.

Nelson, Charles. 1975. "Rational Expectations and the Predictive Efficiency of Economic Models." Journal of Business 48:331-43.

Revankar, Nagesh S. 1980. "Testing of the Rational Expectations Hypothesis." Econometrica 48:1347-63.

Wallis, Kenneth F. 1980. "Econometric Implications of the Rational Expectations Hypothesis." Econometrica 48:49-73.