A Study of Productivity Changes on Individual Crops

Chang-Fu Wanglian and J. Arne Hallam

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Abstract

This paper is an original attempt to find ways to estimate total factor productivity change on individual crops especially when certain input quantities and cost and profit data are unavailable. The study also develops a method, which is a measure of change in absolute productivity, to allow comparison of productivity changes among crops.

Though many agricultural economists agree that technological change is one of the most important factors affecting crop production, few of them have estimated the magnitude of the effect of technological changes on individual crops. Most studies focus on the estimate of the effect of technological change for agriculture as a whole or on the effect of technological change on total crop production.

Table 1 shows that technological changes occur on different crops in different ways. For example, the production growth rate of soybeans over the last two decades was 5.8 percent while cotton was -0.3 percent. While these differences may be the result of different input usage on different crops, the difference also may reflect different levels of technological progress. If the data are adjusted to reflect acres planted, the growth of yield per acre of corn, for example, was 2.5 percent per year, while for cotton it was only 0.2 percent per year.

What are the actual growth rates for these crops when all inputs are considered? The answer to this question is one of the most neglected subjects in agricultural economics, and it is difficult to find the answer in the existing economic literature. Generally, the growth rate of output when all inputs remain unchanged is called the change rate of total factor productivity, or simply, the change rate of productivity. The first purpose of this paper is to present a method for analyzing the measurement of productivity changes on individual crops. The second purpose of the paper is to develop a method to allow comparison of productivity changes among crops.

Problems of Measuring Productivity Changes on Individual Crops

Many methods of measuring the total factor productivity of production are discussed in the economic literature, but most of the methods are inadequate when applied to the measure of productivity change on individual crops. While the theoretical measures are sound, the input allocation data for individual crops are rarely available in agricultural statistics. For example, input allocations for individual crops are usually not available for labor, capital, fertilizer, and irrigation. In addition, the total cost of production or the profit from a crop is not observable in agricultural statistics. Even at a micro level many farmers do not usually record input allocations, such as machinery, among crops. This study will present a model, based on some basic economic assumptions, that can overcome this shortage of data and be used to measure the total factor productivity of individual crops (see A Model to Measure Productivity Change on an Individual Crop).

Table 1. The growth rates of crops: 1961-1982 (average annual growth rate percent)*

	Wheat	Rice	Corn	Soybean	Cotton	Barley
Production	4.0	4.3	4.1	5.8	-0.3	0.4
Yield per acre	1.6	0.9	2.5	1.2	0.2	2.0

Source: Agricultural Statistics. USDA. Various years.

*The annual growth rate is calculated by estimating the regression of $\ln x = a + bt$, where x is the variable and t is the trend of time, b is the growth rate of x over time.

Problems of Comparing the Productivity Changes Among Crops

When comparing productivity changes among different products, the value of outputs and inputs must unavoidably be used in the measurement of total factor productivity. Physical productivity change, as mentioned above, is not a useful indicator in comparing productivity changes among crops, because the value and units of outputs are different. (Baumol and Wolff 1984).

In recent years, the most popular method of comparing productivity over time, different regions, or countries has been the Törnquist approximation to the Divisia index. But this index cannot be applied to comparisons of productivity changes among different products, the formula for the index for different products will result in taking the logarithm of zero, which is meaningless.[1]

Baumol and Wolff (1984) explored the meaning and possible significance of base-year measures and deflated indexes used to compare the absolute productivity between sectors. They concluded that although the base-year measure is a defensible measure of the growth of physical productivity in

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[1]The Törnquist productivity index (D) is the product of the ratio of the Törnquist output index (Q) to the Törnquist input index (X) and the scale factor (r). In logarithmic form, it is

$$\text{(1)} \qquad \ln D = \ln Q\ (p^{1},\ p^{k},\ y^{\ell},\ y^{k}) - \ln X\ (W^{\ell},\ w^{k},\ X^{\ell},\ X^{k}) + r\ (W^{\ell},\ W^{k},\ X^{1},\ X^{k},\ \varepsilon^{\ell},\ \varepsilon^{k}).$$

where Y^{s}, X^{s}, P^{s}, W^{s}, and ε^{s} are vectors of outputs, inputs, output prices, input prices, and the degree returns to scale, respectively, for s = 1 and k, which denote two different firms (countries, time periods, or regions). And ln Q, ln X, and ε are

$\ln Q\ (P^{1},\ P^{k},\ Y^{1},\ Y^{k})$

$$\text{(2)} \qquad = \frac{1}{2} \sum_{i=1}^{M} \left[\frac{P_i^{k} \cdot Y_i^{k}}{P^{k} \cdot Y^{k}} + \frac{P_i^{1} \cdot Y_i^{1}}{P^{1} \cdot P^{1}}\right] (\ln Y_i^{1} - \ln Y_i^{k}),$$

$\ln X\ (W^{1},\ W^{k},\ X^{1},\ X^{k})$

$$\text{(3)} \qquad = \frac{1}{2} \sum_{j=1}^{N} \left[\frac{W_j^{k} \cdot X_j^{k}}{W^{k} \cdot X^{k}} + \frac{W_j^{1} \cdot X_j^{1}}{W^{1} \cdot X^{1}}\right] \left[\ln X_j^{1} - \ln X_j^{k}\right], \text{ and}$$

$$\text{(4)} \qquad \varepsilon^{s} = P^{s} \cdot Y^{s} / W^{s} \cdot X^{s} \quad \text{for } s = k,\ 1.$$

$r\ (W^{1},\ X^{1},\ X^{k},\ \varepsilon^{1},\ \varepsilon^{k})$ is the Törnquist input index with the shares multiplied by unity minus the degree of returns to scale. (Caves, Christensen, and Diewert 1982a, 1982b, Denny and Fuss 1983).

When using equation (1) to calculate the Törnquist productivity index, estimates of the Törnquist output index, the Törnquist input index, and the scale factor are needed. When the comparison is between two sectors which produce different groups of output, however, problems result. In equation (2), it is clear that y_i^{1} denotes product i produced in sector 1, and y_i^{k} is the same product i produced in sector k. Since the two sectors have no products in common, one of the variables Y_i^{1}, Y_i^{k}, is zero. Because the logarithm of zero is meaningless, the Törnquist productivity index can not be used in comparisons among sectors.

any sector of the economy, it can tell nothing about an industry's absolute productivity and it is useless for interindustry comparison. The main reason for their conclusion was that the price of the base year cannot represent the social value of that product. They also concluded that the deflated index is the "right" measure of a sector's economic productivity, although it does not tend to show substantial differences in absolute productivities among industries.

Because productivity changes do differ among different industries and because productivity comparisons among industries can provide valuable economic information, a new index, other than the deflated index, is needed to measure the absolute productivity of different sectors.

This study will present a model to overcome the two problems outlined above (see A Model to Compare Productivity Changes Among Crops). This model can analyze not only physical productivity changes but also current changes in the value of output and inputs. The model can also be used when some production data are unavailable.

A Model to Measure Productivity Change on an Individual Crop

Suppose the production process in crop i, which allows the efficiencies of capital, labor, and fertilizer to rise over time, is represented by

$$Y_{it} = G\,(A(t)K_{it},\ B(t)L_{it},\ C(t)F_{it},\ N_{it}), \tag{1}$$

where Y_{it} is the output level of crop i at time t; and K_{it}, L_{it}, F_{it}, and N_{it} are capital, labor, fertilizer, and land used in the production of Y_{it} it at time t. A(t), B(t), and C(t) are factor augmenting parameters that concern capital, labor, and fertilizer inputs into efficiency units. Assume that the production function is homogeneous of degree one in inputs, with positive but declining marginal products, and all factor and output markets are competitive, and therefore, the factors are paid their marginal products.

To find the rate of cahnge of technology equation (1) can be totally differentiated with respect to time t. This will yield

$$\frac{dY}{dt} = \frac{\partial G}{\partial (AK)}\, A \frac{dK}{dt} + \frac{\partial G}{\partial (AK)}\, K \frac{dA}{dt} + \frac{\partial G}{\partial (BL)}\, B \frac{dL}{dt} + \frac{\partial G}{\partial (BL)}\, B \frac{dB}{dt} + \frac{\partial G}{\partial (CF)}\, C \frac{dF}{dt} + \frac{\partial G}{\partial (CF)}\, F \frac{dC}{dt} + \frac{\partial G}{\partial N} \frac{dN}{dt} \tag{2}$$

Use of the chain rule implies that

$$\frac{\partial G}{\partial A} = \frac{\partial G}{\partial (AK)} K \qquad \frac{\partial G}{\partial K} = \frac{\partial G}{\partial (AK)} A$$
$$\frac{\partial G}{\partial B} = \frac{\partial G}{\partial (BL)} L \qquad \frac{\partial G}{\partial L} = \frac{\partial G}{\partial (BL)} B \tag{3}$$
$$\frac{\partial G}{\partial C} = \frac{\partial G}{\partial (CF)} F \qquad \frac{\partial G}{\partial F} = \frac{\partial G}{\partial (CF)} C$$

Substituting equation (3) into equation (2) yields

$$\frac{dY}{dt} = \frac{\partial G}{\partial K}\frac{dK}{dt} + \frac{\partial G}{\partial K}\frac{K}{A}\frac{dA}{dt} + \frac{\partial G}{\partial L}\frac{dL}{dt} + \frac{\partial G}{\partial L}\frac{L}{B}\frac{dB}{dt} + \frac{\partial G}{\partial F}\frac{dF}{dt} + \frac{\partial G}{\partial F}\frac{F}{C}\frac{dC}{dt} + \frac{\partial G}{\partial N}\frac{dN}{dt} \quad (4)$$

Now the problem can be reparameterized such that the output elasticities are constants, i.e.,

$$\frac{\partial G}{\partial K}\frac{K}{Y} = a, \quad \frac{\partial G}{\partial L}\frac{L}{Y} = b, \quad \frac{\partial G}{\partial F}\frac{F}{Y} = c, \quad \frac{\partial G}{\partial N}\frac{N}{Y} = d$$

If the time derivative of a variable X is denoted by

$$\frac{dX}{dt} = \dot{X} \text{ (for } X = Y, K, L, F, N, A, B, \text{ and } C)$$

then by substituting these expressions in equation (4) and rearranging the following expression is obtained.

$$\dot{Y} = a(\tfrac{Y}{K})\dot{K} + a(\tfrac{Y}{A})\dot{A} + b(\tfrac{Y}{L})\dot{L} + b(\tfrac{Y}{B})\dot{B} + c(\tfrac{Y}{F})\dot{F} + c(\tfrac{Y}{C})\dot{C} + d(\tfrac{Y}{N})\dot{N} \quad (5)$$

Dividing both sides of equation (5) by Y yields

$$\frac{\dot{Y}}{Y} = a(\tfrac{\dot{K}}{K}) + b(\tfrac{\dot{L}}{L}) + c(\tfrac{\dot{F}}{F}) + d(\tfrac{\dot{N}}{N}) + a(\tfrac{\dot{A}}{A}) + b(\tfrac{\dot{B}}{B}) + c(\tfrac{\dot{C}}{C}) \quad (6)$$

Equation (6) states that the rate of growth of output is influenced not only by the rates of increase of the factor inputs but also by the rates of increase of efficiencies of capital, labor, and fertilizer weighted by their respective shares. Rearranging equation (6) gives the change in total factor productivity ($\dot{T}/T$).

$$\begin{aligned} \frac{\dot{T}}{T} &= a(\tfrac{\dot{A}}{A}) + b(\tfrac{\dot{B}}{B}) + c(\tfrac{\dot{C}}{C}) \\ &= \frac{\dot{Y}}{Y} - a(\tfrac{\dot{K}}{K}) - b(\tfrac{\dot{K}}{K}) - c(\tfrac{\dot{F}}{F}) - d(\tfrac{\dot{N}}{N}) \end{aligned} \quad (7)$$

If the shares of the inputs and the quantities used are known, then by using equation (7), it is easy to calculate the total factor productivity of a crop. Unfortunately, except for land, data on inputs for individual crops are unavailable, as are the shares of those inputs.

One way to avoid this shortage of data is to assume that factor-augmenting parameters are the same for all inputs. That is, to assume that the technilogical change is disembodied from factors. With this assumption, equation (1) can be rewritten as

$$Y = G\,(K, L, F, N, T), \quad (8)$$

where T denotes the level of technology. By the assumption of perfect competition in input markets, we have

$$P \cdot \frac{\partial Y}{\partial K} = P \cdot G_K\ (K,\ L,\ F,\ N,\ T) = r, \tag{9}$$

$$P \cdot \frac{\partial Y}{\partial L} = P \cdot G_L\ (K,\ L,\ F,\ N,\ T) = w, \text{ and} \tag{10}$$

$$P \cdot \frac{\partial Y}{\partial F} = P \cdot G_F\ (K,\ L,\ F,\ N,\ T) = f, \tag{11}$$

where P, r, w, and f are prices of Y, K, L, and F, respectively. Equation 8 through 11 contain four equations and three unknown variables (K, L, and F). By the implicit function theorem, we can drop three equations and three unknown variables and get

$$Y = H\ (\frac{P}{r},\ \frac{P}{w},\ \frac{P}{f},\ N,\ T). \tag{12}$$

By estimating equation (12) and substituting its results into equations (8), (9), (10), and (11), the shares a, b, c, and d and the unknown variables K, N, L can be calculated. For example, assume the production function is Cobb-Douglas:

$$Y = A_0 K^a\ L^b\ F^c\ N^d\ e^{\rho t} \tag{13}$$

where ρ is the rate of technological change, t is the time trend, and $A_0 e^{\rho t}$ represents the variable level of technology. Assuming markets are perfectly competitive, one obtains

$$\begin{aligned} P \cdot \frac{\partial Y}{\partial K} &= a \cdot \frac{P \cdot Y}{K} = r, \\ P \cdot \frac{\partial Y}{\partial L} &= b \cdot \frac{P \cdot Y}{L} = w, \text{ and} \\ P \cdot \frac{\partial Y}{\partial F} &= c \cdot \frac{P \cdot Y}{F} = f. \end{aligned} \tag{14}$$

Rearranging them will yield

$$K = a \cdot \frac{P \cdot Y}{r}, \tag{15}$$

$$L = b \cdot \frac{P \cdot Y}{w}, \text{ and} \tag{16}$$

$$F = c \cdot \frac{P \cdot Y}{f}. \tag{17}$$

Substituting (15), (16), and (17) into equation (13) yields

$$Y = A_0\ a^a\ b^b\ c^c\ (\frac{P \cdot Y}{r})^a\ (\frac{P \cdot Y}{w})^b\ (\frac{P \cdot Y}{f})^c\ N^d\ e^{\rho t}. \tag{18}$$

In equation (18) all the variables are observable, so by estimating it, one can get estimates of a, b, c, d, and ρ. Substituting these estimates into (15), (16), and (17), the estimates of K, L, and F can be easily calculated. Using these estimates together in equation (7), the total factor productivity can also be calculated.

A Model to Compare Productivity Change Among Crops

As in the previous section, assuming a Cobb-Douglas production function with constant return to scale and Hick's neutral technical change, by Euler's theorem, the total output is just equal to the sum of all inputs times their marginal product.

(19) $$Y = \frac{\partial Y}{\partial K} K + \frac{\partial Y}{\partial L} L + \frac{\partial Y}{\partial F} F + \frac{\partial Y}{\partial N} N$$

or

(19') $$p \cdot Y = r \cdot K + w \cdot L + f \cdot F + i \cdot N$$

where r, w, f, and i are prices of K, L, F, and N,

and $r = p \cdot \frac{\partial Y}{\partial K}$, $w = p \cdot \frac{\partial Y}{\partial L}$, $f = p \cdot \frac{\partial Y}{\partial F}$, and $i = p \cdot \frac{\partial Y}{\partial N}$.

Under the assumptions of perfect markets and a production function that is homogeneous of degree one, one may assume that all the increase in output due to technological change is shared proportionally by the inputs which have embodied technological change. In other words, the realized input price includes the shadow price of that input and the price of efficiency progress. Here, the shadow price of input is defined as the price if there were no embodied technological progress. If it is assumed that technological change only happens on capital, labor, and fertilizer, as in equation (1), the shadow prices of K, L, and F are

(20) $$r_s = p \cdot \frac{\partial G}{\partial (AK)} = \frac{1}{A} r, \quad w_s = p \cdot \frac{\partial G}{\partial (BL)} = \frac{1}{B} w, \text{ and} \quad f_s = p \cdot \frac{\partial G}{\partial (CF)} = \frac{1}{C} f,$$

where r_s, w_s, and f_s are shadow prices of K, L and F respectively.

Assuming Hick's neutral technological change, that is, assuming A =B = C, equation (13) becomes

(21) $$Y = A_0 K^a L^b F^c N^d C^{(a+b+c)}$$

Since equations (13) and (21) are the same function, one obtains

$$C^{(a+b+c)} = C^{1-d} = e^{\rho t},$$

or

(22)) $$C = e^{(\rho/1-d)t}.$$

Substituting (22) into (20),

$$r_s = e^{(\rho/1-d)t} \cdot r,$$

(23) $$w_s = e^{(\rho/1-d)t} \cdot w, \text{ and}$$

$$f_s = e^{(\rho/1-d)t} \cdot f.$$

Using equation (22) and the estimated inputs K, L and F, which are calculated from equations (15), (16), and (17), the opportunity cost of production (0) is

(24) $$0 = r_s K + W_s L + f_s F + nN.$$

0 is the cost of production assuming there is no technological change. It can also represent the total social revenue of the product when there is no technological progress and market is in equilibrium.

The ratio of total revenue and opportunity cost 0 then can be looked at as an indicator of absolute productivity in a crop. That is

(25) $$D = \frac{PY}{0} = \frac{PY}{PY - [(r-r_s)K + (w-w_f)L + (f-f_s)F)}.$$

The reason that D represents the levels of absolute productivity is that the denominator of this ratio is the social cost or social revenue without technological change, while its numerator denotes the social revenue with technological change. So this ratio gives the change in productivity between periods.

Conclusions

This paper is an attempt to find ways to estimate total factor productivity change on individual crops and to compare productivity among crops. Because of the lack of input allocation data, some assumptions must be made to analyze the issues. To be able to substitute input prices for unobservable inputs, perfect competition in input markets was assumed. For estimating the shares of inputs and inputs quantity, the assumption of Hick's neutral technological progress is made. In order to compare the opportunity cost of production without technological change and the social revenue of the production, the production function is assumed to have constant return to scale. Because the approach uses some duality methods, the production functions used in empirical analysis should be the forms that are either self-dual or can be explicitly derived from one another. Although these assumptions are common in economic literature, any study that further relaxes the assumptions would be desirable.

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