A TATONNEMENT MODEL FOR DETERMINING
FUTURE MARKET PRICES AND QUANTITIES
FOR SOME U.S. CROPS

by

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People concerned with planning and analysis of the agricultural

sector are faced with two general problems which in turn gives rise

for two general types of models. One problem involves positive or

predictive models which attempt to predict the "real world" as it

actually exists based on response functions. Models directed towards

these types of predictions are usually based on time series observa-

tions and use statistical estimation techniques such as regression

equations. The second problem involves normative models, which ask

the question: what conditions could prevail if certain conditions

and goals were to be met? Frequently, these conditions have never

prevailed in the past and time series observations do not exist.

Problems of this type cannot be handled by time-series regression

models but more nearly involve some type of operations research methods.

Specific techniques in the set of possibilities include mathematical

programming and systems simulation. Mathematical programming models

lend themselves to great detail on spatial characteristics of agriculture

that cannot be accomplished with time-series regression models.

The focus of this study is on the development of a model that

determines future equilibrium prices and quantities for agricultural

commodities given conditions such as yields and resource constraints.

Since the study is concerned partly with predicting the "real world" and

partly with analyzing alternative future conditions, a model incorporating

both positive and normative techniques is developed.

The next section provides a discussion of several methodologies available. This section is followed by a description of the methodology chosen. Results for two applications are then presented. The last section presents the conclusions about the limitations of the methodology chosen, the model used and the results obtained.

Model Methodologies

Since Samuelson established the desired formal equivalence between the equilibrium of interregional trade and a maximum problem, spatial programming models have been used to examine how the agricultural sector works and to analyze the implications of a range of policy actions. Spatial programming models have been formulated in several ways, however, linear models have enjoyed widespread use because of the powerful algorithm available to obtain their solutions.

Either prices or quantities must be assumed fixed in linear programming. Both cannot be solved for by the linear programming model. Linear programming can determine the optimal pattern of production including resource use, production location, transportation flows, and supply prices given fixed quantities of demand. Or, given a fixed level of prices, the supply quantities can be determined along with the resource use, production location, and transportation flows.

The assumption of fixed demands in linear programming models is restrictive, limiting the usefulness of the results. The importance of the restriction depends on the nature and purpose of the study. If the

changes being analyzed caused food prices to increase sufficiently, demand quantities would not stay fixed and modifications in agricultural production and resource allocation would occur. To remedy this situation, consumer demand functions can be incorporated into the programming models.

Some early linear programming studies used an iterative solution process with changing quantities of demand to obtain the equilibrium price and quantity relationships. The iterative process was proposed by Fox and further explored by Judge and Wallace and Schrader and King. Their results were consistent with the competitive equilibrium solution. However, the rationale for the method was not firmly based in mathematics or in economic theory. In addition, the iterative procedure was both expensive and time consuming. In 1964, Takayama and Judge developed an extension of the Samuelson maximization approach which solved the equilibrium problem by means of concave programming. Plessner and Heady and Stoecker applied a quadratic programming model, a form of concave programming, to the U.S. agricultural economy. Quadratic programming models are usually solved using a much smaller set of production activities than the linear programming models contain because the solution algorithms are much more expensive.

Because of the high cost of quadratic programming, separable programming was developed and refined using linear approximations of the nonlinear functions to solve the nonlinear model. Separable programming has been used by Yaron and Heady, Duloy and Norton and Huang and

Hogg to solve nonlinear programming models. Separable programming models have the disadvantage that the results are sensitive to changes in the segments used to linearize the nonlinear function. Also, the optimality conditions for a competitive equilibrium are only approximately satisfied because separable programming used linearized functions to approximate the nonlinear functions.

In the study reported here, an iterative technique used to solve a spatial linear programming model for equilibrium prices and quantities is examined. The iterative process is based on the economic theory of tatonnement. In the past, iterative processes have been avoided because of the computer expense and time required to adjust the demand levels. Today, advances in computer software make this technique attractive from both a cost and flexibility point of view. The technique can use the spatial linear programming model with little modificiation of the coefficient matrix. The adjustments in demand levels and the determination of the approximate equilibrium point can be done in a single computer run by using the appropriate software.

The iterative process is based on the tatonnement theory of market adjustments. Netishi (p. 191) defines tatonnement as a trial and error process representing the market mechanism under free competition. The tatonnement model involves an iterative process paralleling the market mechanism as it adjusts to equilibrium. In effect, it works in this manner: An auctioneer sets a price for each good. Consumers specify how much they will take at these prices and the producers indicate how much they will produce and sell at these prices. If the two sets

of quantities are equal, the markets are cleared and equilibrium exists.
If the quantities are not equal, the auctioneer adjusts the prices to
eliminate surplus demand or surplus supply. In the model application,
statistically estimated demand functions are used to denote the demands
while a linear programming model is used to denote the supplies.

As Ginsburgh and Waelbroeck (p. 48) state, the main advantage of
converting to a linear programming format from a nonlinear format is
to permit the use of existing highly sophisticated softwares. These
softwares are more highly developed than nonlinear programming packages
which do not make as skillful use of the properties of sparse matrices.

The stability of tatonnement procedures has been extensively
studied in economic theory. The conclusions reached have been disap-
pointing and have given rise to the widespread belief that tatonne-
ment procedures do not provide a sound approach to the computation of
equilibria. However, Ginsburgh and Waelbroeck have stated, "We feel
that this belief is not entirely justified, because it overlooks essen-
tial differences between the economic process which economists wished
to represent and the way in which a model-builder solves a model"
(p. 107). They then give three reasons to support their beliefs. These
reasons can be summarized as follows:

1. The modeller understands the properties of his model and uses
    the knowledge to decide how computation should be set up.

2. The modeller has at least qualitative knowledge of the struc-
    ture of the Jacobian of the excess demand functions from

study of the model and examination of the iterations of trial calculations. This knowledge provides guidance in designing the adjustment procedure.

3. While in theoretical work there is little freedom in selecting the form of the adjustment process, there are no such restrictions in computations.

For a discussion of the uniqueness, stability, convergence, and speed of adjustment of tatonnement procedures see chapters 7 and 8 of Ginsburgh and Waelbroeck.

Tatonnement Model Description

The demand equations are estimated econometrically using time series data. A linear programming model is used to simulate the supply equations. The demand and supply interaction of five crops-- barley, corn for grain, oats, sorghum for grain, soybeans, and wheat-- is considered in this analysis. Soybean demand and supply is in the form of soybean meal.

Demand sector

The demand equations for the model are estimated on a national basis. The quantity demanded is disaggregated into food, feed, and export components. Soybean meal has no food component. The demand for soybean oil is considered to be insignificant in determining the soybean equilibrium market.

A total of 17 equations are estimated -- six feed, six net export, and five food demand -- using data for the years 1950 to 1979, The equations are estimated using seemingly unrelated regression. The net export equations are estimated as functions of own price, government exports of the commodities, competing crop prices, time and some dummy variables representing changes in government crop program policies. The feed equations are estimated as functions of own price, quantities of production of various livestock commodities, competing crop prices and the dummy variables representing changes in government crop program policies. The food equations are estimated as functions of own price, competing crop prices, population, and price indices of other food and non food items. The estimated equations can be found in Schatzer or Schatzer and Heady.

Supply model

An interregional linear programming model is used to simulate the supply side of the tatonnement model. The linear programming model is constructed for the year 2000 and is based upon models previously developed at the Center for Agricultural and Rural Development (Turhollow, Short, and Heady and English, Alt, and Heady).

The linear programming model is a regionalized, one land group model covering the geographical area of the continental United States. The 48 states are divided into 105 producing areas (PAs) as shown in Figure 1. The PAs are based on the Water Resource Council's aggregate
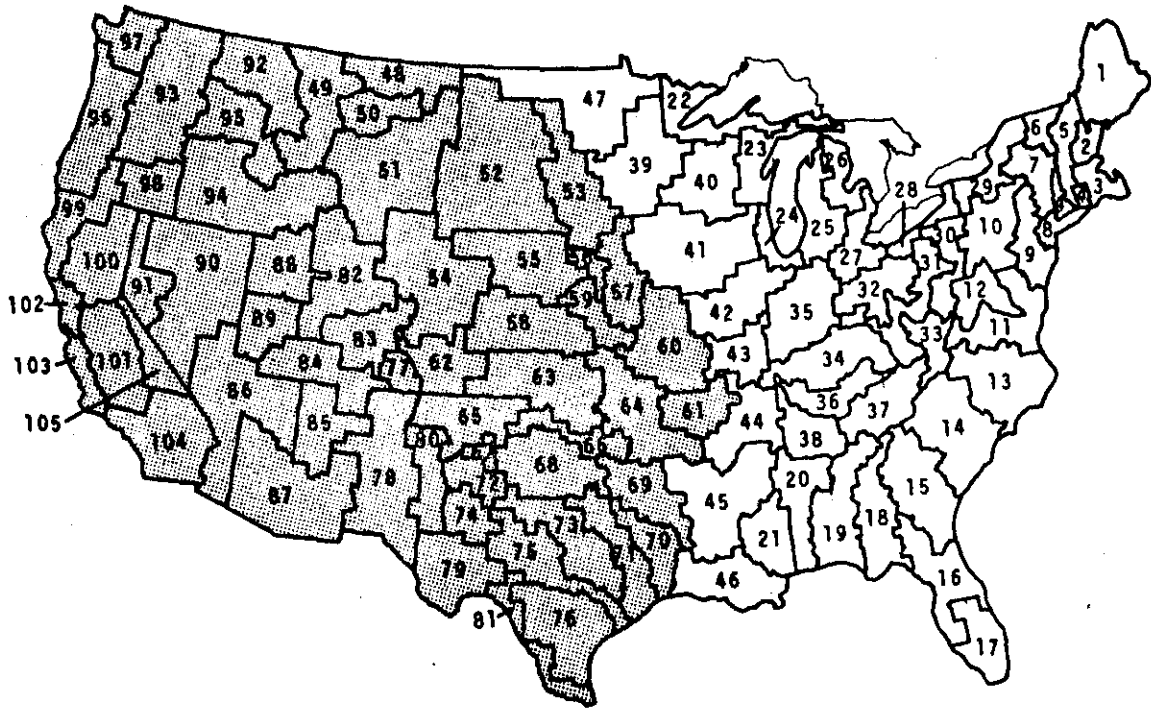
Figure 1. The 105 producing areas with the irrigated producing areas shaded.

subarea. PAs 48 to 105 serve as water supply regions and are the only PAs where irrigation is allowed. The PAs are aggregated into 28 market regions (MRs) as shown in Figure 2. The MRs serve as the smallest breakdown for commodity demands and transportation activities.

The objective function of the linear programming model minimizes the total cost of crop production and transportation. The costs include labor, machinery, pesticides, fertilizers, water, energy, and transportation from the location of the production centers to the location of the consumption centers. The costs are in terms of 1975 farm input prices except for energy which has been adjusted to 1980 prices.

Restraints in the model are defined for land, water, and commodity demands. The driving force in the linear programming model is the restraints on the minimum levels of the commodity demands at the market region level as determined by the demand equations. The land and water restraints are defined at the PA level. The cropland available in each area is adjusted for exogenous crop requirements and nonagricultural uses. The amount of land available is based on the 1977 National Resource Inventory (NRI) (USDA, 1981). There are two water restraints for each of the water supply PAs (PAs 48-105), one for groundwater, and one for surface water. These restraints balance the dependable water supply in the region for interbasin transfers, natural flow and runoff, and water use. Water consumed on site by livestock and exogenous crops, by municipal and industrial uses, and water exports is predetermined and is subtracted from the available water supply.
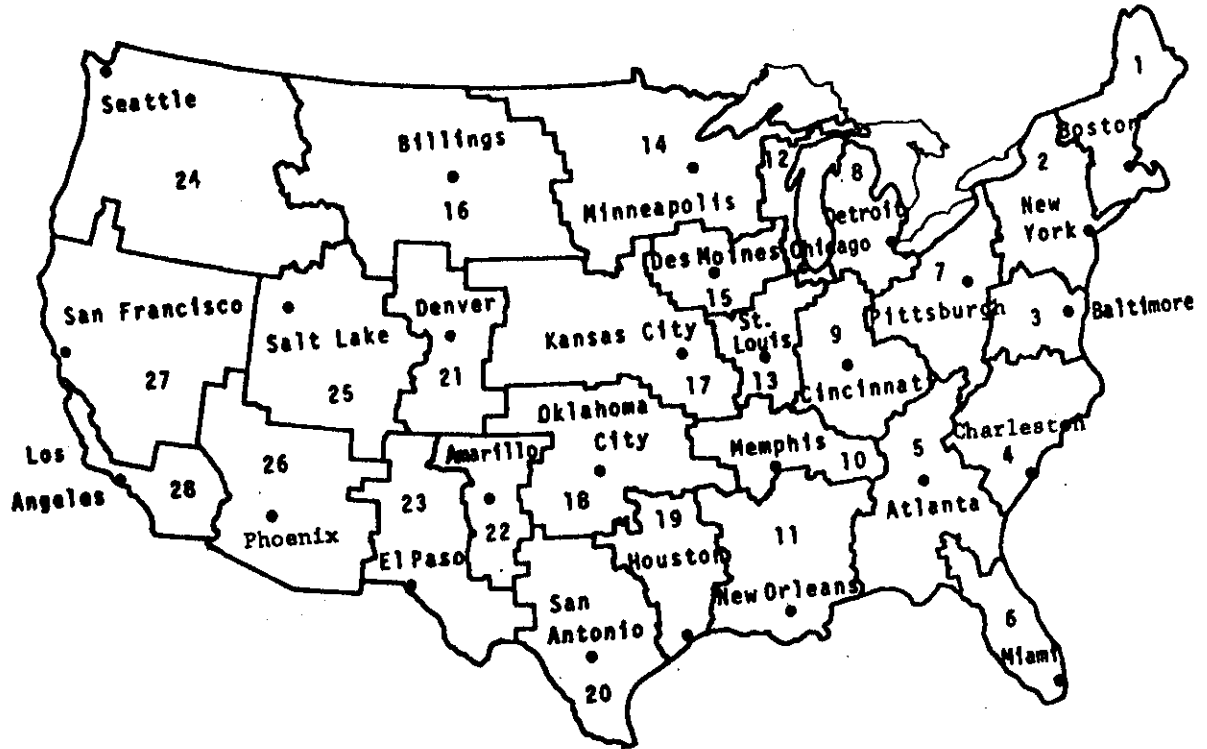
Figure 2. The 28 market regions.

Three classes of activities are defined in the model: crop pro-
duction, commodity transportation, and resource supply. Crop produc-
tion activities are defined to simulate the rotations in use by PA for
barley, corn grain, corn silage, cotton, legume hay, nonlegume hay,
oats, sorghum grain, sorghum silage, soybeans, and wheat. The rota-
tions contain one to four crops and cover from one to five years. Each
rotation may be produced by three tillage methods; conventional tillage
with residue removed, conventional tillage with residue left, or re-
duced tillage. Crop yields are based on functions developed by Stoecker
and modified as documented in Meister and Nicol.

The costs for the rotations are derived from the Federal Enter-
prise Data System (FEDS) (USDA, 1977). The rotation costs represent
the per acre non-land-variable cost, excluding nitrogen costs. These
costs are adjusted to reflect the given conservation-tillage practice
that the rotation represents. The adjustment is primarily based on
timing factors that indicate the time variance for each practice.

Commodity transportation activities define the shipment of a commodity
from one market region to another -- one activity for shipment in each
direction. Transportation activities are defined for barley, corn grain,
oats, sorghum grain, soybeans, and wheat. All transportation is assumed
to be done by railroads since the majority of long hauls are by rail-
roads.

Resource supply activities are defined for water, nitrogen, and
land conversion. Water activities allow for the movement of the water

from the water supply rows to the water demand row. Other water activities

allow for movement of water from one region to another through down-

stream flows or interbasin canal flows. Nitrogen activities allow for

the purchase of commercially produced nitrogen once a specific amount

of nitrogen derived from livestock wastes is exhausted. Nitrogen derived

from livestock wastes is determined exogenously as explained in Short

and Dvoskin.

The demands for four crops not represented by the demand side of

the model are determined exogenously. These crops are silage, cotton,

legume hay, and nonlegume hay. Cotton demand is on a national basis,

while silage and hay demands are on a regional basis. Silage and hay

demands are distributed to market regions based on livestock feed demands

(Boggess). National cotton demand is set at 17.8 million bales (USDA,

1979) with 108.9, 82.2, and 65.5 million tons assumed for silage, legume

hay and nonlegume hay, respectively. Nonlegume hay demand can be lowered

by irrigating pasture in the PAs which allow irrigation.

Linkage between the demand and
supply sides of the model

The demand equations provide the commodity demand restraints for

the linear programming model. The linear programming model provides

the demand equations with commodity supply prices. The linear program-

ming model can be solved using MPSX (IBM, 1972) and the demand equations

can be solved using a computer program written in FORTRAN (Cress,

Dirksen, and Graham). The linkage between the two is accomplished using

the READCOMM (IBM, 1971) feature of MPSX which allows a FORTRAN sub-routine to be called by MPSX.

The estimated demand equations determine the quantity demanded on a national basis. The linear programming model is driven by market region demands. To distribute the national demands to each of the market regions linear programming activities are developed. These activities distribute net exports based on port weights, food demand based on population weights, and feed demand based on livestock feed weights by crop. The linear programming model then provides national average shadow prices for each crop by food, feed, and net export demand. The demand equations use a single national average price for each crop, so a weighted average shadow price for each crop is determined using the food, feed and net export components as weights.

Since the model works in a circular process, starting values for the prices are needed to solve for the starting quantities to be used in the linear programming model. The average value of the 1950 to 1979 prices is used in the demand equations to determine the starting quantities. The average prices and starting quantities along with the actual 1979 values for each of the commodities are presented in Tables 1 and 2 respectively.

The linear programming model provides the supply prices which are then used to determine the quantity demanded. If the difference between the quantity demanded and the quantity supplied is more than plus or minus 1 percent of the quantity demanded, then new quantities are deter-

Table 1. Average crop prices for 1950 to 1979 used in demand equations
to determine starting quantities and the actual 1979 prices
(values in 1975 dollars)

Crop	1979 price	Starting price
	------------dollars per bushel----------	
Barley	1.42	1.85
Corn	1.67	2.21
Oats	0.89	1.18
Sorghum	1.50	1.93
Wheat	2.21	3.10

Table 2.   Starting quantities used in the linear programming model as

computed from the demand equations and the actual 1979 quantities

Variable	1979 quantity	Starting quantity
	---------------million bushels-----------	
Barley net exports	15	28.5
Barley feed	207	376.3
Barley food	157	176.1
Corn net exports	2,132	3,295.5
Corn feed	4,198	5,134.3
Corn food	557	57.1
Oats net exports	12	11.2
Oats feed	530	575.3
Oats food	42	57.1
Sorghum feed	566	1,283.0
Sorghun food	5	10.3
Soybean meal net exports	1,034	1,885.0
Soybean meal feed	743	1,191.0
Wheat net exports	1,193	1,129.1
Wheat food	178	159.5
Wheat feed	592	520.2

mined to be used as demand restraints in the linear programming model. The new quantities are determined in one of two ways. If it is the first iteration or the excess demand has the same sign as the previous iteration, one-half of the excess demand is added to the supply quantity. If the excess demand has the opposite sign of the previous iteration, the equation for a line drawn through two points is computed. The current and previous excess demand quantities are used as one of the two coordinates for each point, while the current and previous supply quantities are used as the other coordinates. The excess demand is then set to zero and the equation is solved for the new supply quantity. The iterations continue until the constraints on excess demand are met for each of the disaggregated demands for each commodity.

A limit of 15 iterations is placed on the model to allow the results to be checked manually for oscillations about a step in the supply function of one or more crops. Since the linear programming model produces a stepped supply function, there is the possibility that the model will converge to the point of a vertical step. If that occurs, the difference between the crop's demand at the lower price and at the higher price may be larger than 1 percent of the demand at either price. If the difference is greater than 1 percent and the model is trying to converge to a price somewhere on the vertical step, the model will never converge. The model can then be assumed to have converged. If the model is not oscillating about a step in the supply function, then it can be restarted and 15 more iterations allowed.

Results

Once the Tatonnement model is designed, the model needs testing. To test the model, two future levels of crop yields are used in the linear programming segment. The first level of crop yields are the expected crop yields for the year 2000 based on the yield equations developed by Meister and Nicol. The second level is more optimistic. The yields are developed by taking the projected yields for 2015 and using then for 2000. The two yield alternatives are labelled LOW for the expected yields and HIGH for the more optimistic yields.

The LOW scenario tatonnement model converged in 9 iterations while the HIGH scenario took 11 iterations. For the LOW scenario, the largest convergence check value was .95 percent with the rest less than .1 percent. For the HIGH scenario, the largest was .87 percent, the next .23 percent and the rest less than .11 percent with 13 out of the 15 less than .01 percent. Both scenarios are very close to the equilibrium points.

In addition, the tatonnement model solution is compared to the linear programming solution for the starting demand quantities for each yield level. The linear programming solution for the starting demand quantities is the solution that would be used if only a linear programming model is used to analyze the two yield levels. A comparison of the linear programming solution to the tatonnement solution provides insights into how close the linear programming model comes to a price equilibrium solution and when the linear programming model may be a good approximation for the equilibrium solution.

The model provides regional results that are too numerous to analyze in this paper. Therefore, only national results for each of the yield levels for both the tatonnement and linear programming solutions are presented. Results for prices, quantities, acres and yield by crop are given in Tables 3 through 6.

Average U.S. prices for each of the crops are presented in Table 3. If a linear programming model is used for the analysis, corn price for 2000 is projected to be $3.69 per bushel with LOW yields and $1.58 with HIGH yields, a difference of $2.11 per bushel. If the quantity demanded is allowed to adjust to the price instead of being held constant, the tatonnement model projects corn price to be $2.50 per bushel with LOW yields and $1.58 per bushel with the HIGH yields, a difference of $0.92 per bushel. The change in price between the two yield scenarios is quite different, depending upon the model chosen. Similar results can be seen for the rest of the crops. Even the supply prices for silage, legume hay, other hay, and cotton, whose quantities are exogenous are influenced as the tatonnement model adjusts the quantities of the endogenous crops.

If instead of comparing the LOW yield price with the HIGH yield price, the LP price is compared to the TATONNEMENT price, one sees a large difference under LOW yields and only a small difference under HIGH yields. The linear programming solution under the HIGH yield scenario is very close to the equilibrium solution. However, with LOW

Table 3. Estimated prices[a] in 1975 dollars for crops in 2000 for

each solution

Crop	Linear Programming Solutions		Tatonnement Solutions	
	LOW Yields	HIGH Yields	LOW Yields	HIGH Yields
Barley	4.15	1.42	2.59	1.45
Corn	3.69	1.58	2.50	1.58
Oats	3.78	1.37	2.02	1.32
Sorghum	4.45	1.88	2.29	1.86
Wheat	6.43	2.29	3.98	2.29
Soybeans	9.06	3.35	5.96	3.37
Silage	28.44	13.08	19.50	13.11
Legume Hay	93.87	40.95	64.22	41.13
Other Hay	120.58	42.62	74.84	42.60
Cotton	297.34	185.81	230.55	185.47

[a]Prices for barley, corn, oats, sorghum, wheat, and soybeans in $/bushel; for silage and hay in $/ton; and for cotton in $/bale. All linear programming prices are supply prices while tatonnement prices for barley, corn, oats, sorghum, wheat, and soybeans are market equilibrium prices.

Table 4. Estimated quantities for crops in 2000 for each solution

Crop	Linear Programming Solutions		Tatonnement Solutions	
	LOW Yields	HIGH Yields	LOW Yields	HIGH Yields
	----------------------millions of bushels------------------			
Barley	603.4	598.2	597.5	643.1
Corn	9,133.4	9,132.3	9,153.2	9,220.6
Oats	668.0	673.5	322.8	456.1
Sorghum	1,503.7	1,503.9	1,406.6	1,409.8
Wheat	1,869.1	1,867.2	1,796.9	1,917.3
Soybeans	3,125.8	3,116.7	3,103.6	3,177.2

Table 5. Estimated planted acres for each crop, acres of fallow and idle cropland available for each solution

Crop	Linear Programming Solutions		Tatonnement Solutions	
	LOW Yields	HIGH Yields	LOW Yields	HIGH Yields
	----------------------millions of acres----------------------			
Barley	13.0	9.98	11.26	10.48
Corn	78.06	74.36	80.00	75.84
Oats	9.44	11.51	4.32	7.44
Sorghum	23.18	24.51	20.02	22.87
Wheat	49.16	47.58	48.00	48.93
Soybeans	96.66	78.83	94.11	79.94
Silage	7.37	7.24	7.61	7.24
Legume Hay	22.76	21.15	22.29	21.20
Other Hay	25.74	25.97	28.56	26.00
Cotton	11.85	13.04	15.36	13.02
Total cropped	337.26	314.18	331.52	312.96
Fallowed	15.64	31.52	21.27	32.42
Idled	0.0	7.52	0.11	7.53
Total Available	352.90	352.90	352.90	352.90

Table 6. Average yield[a] per acre for each crop in 2000 for each solution

Crop	Linear Programming Solutions		Tatonnement Solutions	
	LOW Yields	HIGH Yields	LOW Yields	HIGH Yields
Barley	46.23	59.93	53.04	60.52
Corn	117.01	122.81	114.42	121.58
Oats	70.78	58.49	74.66	61.33
Sorghum	64.88	61.36	70.27	61.63
Wheat	38.02	39.24	37.43	39.19
Soybeans	32.34	39.54	32.98	39.74
Silage	14.78	15.05	14.32	15.04
Legume Hay	3.61	3.89	3.69	3.88
Other Hay	2.40	2.49	2.19	2.49
Cotton	1.50	1.36	1.16	1.36

[a]Yield for barley, corn, oats, sorghum, wheat, and soybeans in bushels; for silage and hay in tons; and for cotton in bales.

yields the linear programming solution is a long ways from the equilibrium point. The results are just the opposite for the quantity of production solution values presented in Table 4.

The change between the linear programming solution and the tatonnement solution is smaller under the LOW yield scenario than under the HIGH yield scenario. The LOW and HIGH yield linear programming results for the quantity of production are about equal. The results would be the same except the quantity of production required for seed use is higher under LOW yields since a larger number of acres are needed to get the same quantity of output. The changes between the linear programming solution quantities and the tatonnement solution is much less than the changes in prices on a percentage basis. The price adjustment is larger than the quantity adjustment because the linear programming prices are shadow prices or supply prices. The shadow price is the value of the last unit produced and includes an imputed cost for resources that are constrained. If the results are in short supply, decreasing the quantity of production a little will lower the imputed cost of the resource since less of it will be required. The imputed cost will also change as the production of one crop changes relative to another. As the imputed cost of resources fall the shadow prices fall.

The two major resources in the linear programming model are land and water. The acres of land required for the production of each crop are given in Table 5 for each scenario. The total number of land available for crop production is 352.9 million acres. Under the LOW yield

scenario the linear programming solution used all of the land avail-

able and even had a relatively small amount of land in summer fallow.

(Some land in the plains require summer fallow rotations to achieve a

reasonable yield.) Since all the land is used, a high imputed cost of

land is obtained,an average of $195.93 per acre for the U.S. Under the

HIGH yield scenario, the linear programming solution leaves 7.21 million

acres of the crop land available idle and has 31.51 million acres in

summer fallow. The imputed cost of land is only an average of $32.14

per acre for the U.S., mush smaller than with LOW yields. As the quan-

tity of production is allowed to adjust to the prices, the imputed cost

of land changes. Under the tatonnement solutions, the amount of idle

and summer fallow land increases. The imputed cost of land is an average

of $93.38 per acre for the U.S. under the LOW yield scenario and $32.19

under the HIGH yield scenario. The $93.38 is still quite high but it is

almost one half the $195.93 obtained with the linear programming solution.

The acres of each crop also change across solutions. These changes

are a function of the quantity produced and the average yield per acre

of the crop. Table 6 presents the average yield for each crop for each

solution. Some idea of the difference in yields between the LOW and

HIGH yield scenario can be obtained by comparing the two linear pro-

gramming solutions. The exact difference cannot be obtained because it

varies by state. Also average oats, sorghum, and cotton yields decline

in the solutions between the LOW and HIGH scenarios. These yields for

a given PA increase when moving from the LOW to HIGH scenario, however,

these crops get pushed onto less productive land as the comparative advantages of each PA for a given crop change.

Conclusions

The tatonnement model provided results very close to the equilibrium values when export, feed and food components of demand for each crop meet the convergence criteria of excess demand being less than 1 percent of the quantity demanded. The methodology worked quite well although improvements in the adjustment mechanisms should improve the speed of convergences. The results from the two scenarios analyzed suggest that the tatonnement procedure is only needed when the resources available in the linear programming model are in short supply and have very high imputed costs. The opposite, all resources in large excess supply resulting in very low imputed costs, may also result in the tatonnement model changing the results. In this case, however, quantities would probably change much more relative to prices than the results present here.

One limitation of the linear programming model used here is that it contained an exogenous livestock sector. Meat demands were held constant therefore feed demands do not vary as much as perhaps they should as feed price changes. Addition of an endogenous livestock sector to the model would therefore be an improvement.

Also more land classes in the linear programming model would help provide a smoother step supply function. They would also provide for better representation of each PA's potential cropping patterns.

In conclusion, the iterative model based on the tatonnement process outlined in this study has the potential for improving the results of interregional programming models. There would be little increase in the cost of constructing or solving the models. The tatonnement model would make linear programming a better normative tool for analyzing changes in agricultural policy or changes in input prices or availability which cause shifts in the supply functions.

Finally, the projections of crop prices made in this study must be viewed with caution and taken with an understanding of the assumptions made about land and water availability, future yields and future relative input prices. The results are only as good as the data from which they are derived and the assumptions made. Many things can influence future crop yields, land availability, and crop demands. Therefore, any projection of the future is at best an educated guess.

REFERENCES

Boggess, William G. "The Development of an Exogenous Livestock Sector for Use in CARD Linear Programming Models." Working Paper, Center for Agricultural and Rural Development, Iowa State University, Ames, 1977.

Cress, Paul, Paul Dirksen, and J. Wesley Graham. FORTRAN IV with WATFOR and WATFIV. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1970.

Duloy, J. M., and R. D. Norton. "Prices and Incomes in Linear Programming Models." American Journal of Agricultural Economics 57(1975): 591-600.

English, Burton D., Klaus Alt, and Earl O. Heady. "A Documentation of the Resource Conservation Act's Assessment Model of Regional Agricultural Production, Land and Water Use, and Soil Loss." CARD Report 107T. Center for Agricultural and Rural Development, Iowa State University, Ames, 1982.

Fox, K. A. "A Spatial Equilibirum Model of the Livestock Feed Economy in the United States." Econometrica 21(1953):547-566.

Ginsburgh, Victor A. and Jean L. Waelbroeck. Activity Analysis and General Equilibrium Modelling. Amsterdam: North-Holland Publishing Company, 1981.

Huang, Wen-Yuan, and H. C. Hogg. "Estimating Land Use Patterns: A Separable Programming Approach." Agricultural Economics Research 28(1976):22-33.

IBM. Mathematical Programming System Extended (MPSX), Read Communications

Format (READCOMM) Program Description Manual. White Plains, New York:

IBM Corporation, 1971.

IBM. Mathematical Porgramming System -- Extended (MPSX), and Generalized

Upper Bounding (GUB) Program Description. White Plains, New York:

IBM Corporation, 1972.

Judge, G. C., and T. D. Wallace. "Estimation of Spatial Price Equilibrium

Models." Journal of Farm Economics 40(1958):801-820.

Meister, Anton D., and Kenneth J. Nicol. "A Documentation of the National

Water Assessment Model of Regional Agricultural Production, Land and

Water Use, and Environmental Interaction." Miscellaneous Report.

Center for Agricultural and Rural Development, Iowa State University,

Ames, 1975.

Netishi, Takashi. General Equilibrium Theory and International Trade.

Amsterdam: North-Holland Publishing Company, 1972.

Plessner, Y., and Earl O. Heady. "Competitive Equilibrium Solutions

with Quadratic Programming." Metroeconomica 17(1965):117-130.

Samuelson, P. A. "Spatial Price Equilibrium and Linear Programming."

American Economic Review 42(1952):282-303.

Schatzer, Raymond Joe. "Tatonnement Modeling with Linear Porgramming:

Demand and Supply for Some United States Crops in 2000." Ph.D.

Dissertation, Department of Economics, Iowa State University, Ames,

1982.

Schatzer, Raymond Joe and Earl O. Heady. "National Results for Demand
and Supply Equilibrium for Some United States Crops in 2000: Theory
and Application of Tatonnement Modeling." CARD Report 106. Center
for Agricultural and Rural Development, Iowa State University, Ames,
1982.

Schrader, L. F., and G. A. King. "Regional Location of Cattle Feeding --
A Spatial Equilibrium Analysis." Hilgardia 34(1963):331-616.

Short, Cameron, and Dan Dvoskin. "Recycling of Livestock Wastes as a
Source of Nitrogen." Miscellaneous Report. Center for Agricultural
and Rural Development, Iowa State University, Ames, 1977.

Stoecker, A. L. "A Quadratic Programming Model of United States Agriculture
in 1980: Theory and Application." Ph.D. Dissertation, Department
of Economics, Iowa State University, Ames, 1974.

Takayama, T., and George C. Judge. "An Interregional Activity Analysis
Model for the Agricultural Sector." Journal of Farm Economics 46(1964):
349-365.

Turhollow, Anthony F., Jr., Cameron Short, and Earl O. Heady. "Potential
Impacts of Future Energy Price Increases on U.S. Agricultural Production."
CARD Report 116. Center for Agricultural and Rural Development,
Iowa State University, Ames, 1983.

U.S. Department of Agriculture. "Firm Enterprise Data System, 1975
Crop Budgets." Economic Research Service, U.S. Department of
Agriculture, Oklahoma State University, Stillwater, 1977.

U.S. Department of Agriculture. "Unpublished Data from the National Inter-
regional Projection System." Economics Statistics Service, U.S.
Department of Agriculture, Washington, D.C., 1979.

U.S. Department of Agriculture. "Basic Statistics 1977 National Resources
Inventory." (Mineographed) Soil Conservation Service, U.S. Department
of Agriculture, Washington, D.C., 1980.

Yaron, Dan, and Earl O. Heady. "Approximate and Exact Solutions to
Nonlinear Programming Problem with Separable Objective Function."
Journal of Farm Economics 43(1961):57-69.