ANALYSIS OF DEMAND AND SUPPLY FOR SOME U.S. CROPS THROUGH TATONNEMENT MODELING*

by

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During the last decade, two concerns have been raised about the future of U.S. crop production. These concerns are: a) Will the United States have enough land available in the future that is suitable for crop production? and b) What will future crop yields be? Future U.S. crop production is highly dependent upon both these issues.

The real problem may not be the quantity of future crop production, but rather the price of the quantity that is available. The development of a model to answer this question is one of the objectives of this study. The other objective is the projection of prices and quantities for barley, corn, oats, sorghum, soybeans, and wheat for the year 2000 under alternative yield and land availability assumptions.

Methodology

Samuelson (1952) established desired formal equivalence between the equilibrium of interregional trade and a maximum problem. The concept was further developed by Smith (1963) who showed that a dual of the equivalence exists and therefore a competitive spatial equilibrium can be identified by the minimization of economic rent. Since then, spatial programming models have been used to examine how the agricultural sector works and to analyze the implications of a range of policy actions. Spatial programming models have been formulated in several ways. However, linear models have enjoyed widespread use because of the powerful algorithm available to obtain their solutions.

The use of linear programming models has one serious drawback in analyzing aggregate equilibrium conditions. That drawback is that the prices or the quantities must be assumed fixed. They both cannot be solved by the linear programming model. Linear programming can determine the optimal pattern of production including resource use, production location, transportation flows, and supply prices given fixed quantities of demand. Or, given a fixed level of prices, the supply quantities can be determined along with the resource use, production location, and transportation flows. The Center for Agricultural and Rural Development (CARD), Iowa State University has developed a continuing sequence of multi-product models for U.S. agriculture with many spatially separated markets and producing regions [Meister and Nicol (1975); Dvoskin, Heady, and English (1978); English, Alt, and Heady (1982); and Turhollow, Short, and Heady (1982)].

The assumption of fixed demands in linear programming models is restrictive, limiting the usefulness of the results. Some early linear programming studies used an iterative solution process with changing quantities of demand to obtain the equilibrium price and quantity relationships. The iterative process was proposed by Fox (1953) and further explored by Judge and Wallace (1958) and Schrader and King (1962). Their results were consistent with the competitive equilibrium solution. However, the rationale for the method was not firmly based in mathematics or in economic theory. In addition, the iterative procedure was both expensive and time consuming. In 1964, Takayama and Judge developed an extension of the Samuelson maximization approach which solved the equilibrium problem by means of concave programming. Plessner and Heady (1965) and Stoecker (1974) applied a quadratic programming model, a form of concave programming, to the U.S. agricultural economy. Quadratic programming also has a major limitation because the solution algorithms are much more expensive than the simplex algorithm used for linear programming when equivalent-sized problems are examined. Quadratic programming models are therefore usually solved using a much smaller set of production activities than the linear programming models contain.

Because of the high cost of quadratic programming, separable programming was developed and refined using linear approximations of the nonlinear functions to solve the nonlinear model. Separable programming has been used by Yaron and Heady (1961), Duloy and Norton (1975), and Huang and Hogg (1976) to solve nonlinear programming models. Separable programming models have the disadvantage that the results are sensitive to changes in the segments used to linearize the nonlinear function. Also, the optimality conditions for a competitive equilibrium are only approximately satisfied because separable programming uses linearized functions to approximate the nonlinear functions.

In the study reported here, an iterative technique is used to solve a spatial linear programming model for equilibrium prices and quantities. The iterative process is based on the economic theory of tatonnement. While in the past iterative processes were avoided because of the computer expense and time required to adjust the demand levels, recent advances in computer software make this technique attractive from both a cost and flexibility point of view. The technique can be applied to the spatial linear programming model with no modification to the coefficient matrix. The adjustments to the demand levels and the determination of the approximation to the equilibrium point can be done completely internally to the computer by using the appropriate computer programming software.

Iterative process

The iterative process used in this study is based on the tatonnement process of market adjustments. Negishi (p. 191, 1972) defines tatonnement as a trial and error process representing the market mechanism under free competition. The tatonnement process can be described by the following sequence of events.

a) An auctioneer sets a price for each good.

b) The consumers specify the quantity of each good they want to buy at the given price.

c) The producers specify the quantity of each good they want to sell at the given price.

d) If the aggregated quantity demanded equals the aggregated quantity supplied for each good, the markets are cleared and the equilibrium prices and quantities have been found.

e) If the quantities are not equal, the auctioneer adjusts the prices by raising the prices of the goods in excess demand and by lowering the prices of the goods in excess supply.

f) These new prices become the prices offered, and the sequence of events, b through e, is repeated until equilibrium prices and quantities are reached.

By stimulating the above process of adjustment, the problems caused by using fixed demands in the spatial linear programming model can be solved. The iterative process above does not fit the cost minimization spatial programming model, so the process is modified slightly. A modification was first defined by English, Short, and Heady (1981) and can be described by the following sequence of events where a spatial linear programming model is used to stimulate the producers' actions and estimated demand equations are used to simulate the consumers' actions.

a) A set of national production quantities is determined and distributed to the regions as done in previous spatial linear programming studies.

b) The spatial linear programming model with these regional quantities of demand is solved by cost minimization.

c) The national supply prices for each commodity is determined from the linear programming model's shadow prices.

d) These national supply prices are then used in the estimated demand functions to determine the quantities demanded.

e) The quantity demanded is compared to the quantity produced for each good. If the two quantities are equal for all the commodities, the equilibrium prices and quantities have been determined.

f) If the two quantities are not equal for one or more of the commodities, a new set of national production levels is determined by increasing the level of those commodities in excess demand and decreasing those in excess supply.

g) The new set of production levels are distributed to the regions and the sequence of events b through f is repeated until equilibrium prices and quantities are reached.

In actuality, English, Short, and Heady stopped the sequence of events whenever the difference between the production quantity and demand quantity for each of the commodities was less than 1 percent of the demand quantities. The same criteria is used in this study.

The iterative process used can be described mathematically in the following way, assuming only one good, Q. Begin with an arbitrary supply price P_0. This price is used to determine the arbitrary starting level of Demand D_0 using the demand equation as specified by Equation 1.

$$D_0 = g(P_0) \qquad (1)$$

The starting quantity of demand becomes the quantity of production, S_1, used in the linear programming model to determine the supply price. The linear programming model has a theoretical supply curve, Equation 2,

$$P_t = f(S_t), \qquad (2)$$

from which the new price P_1 can be determined. The new price can be used in Equation 1 to determine the new demand level D_1. If the difference between S_1 and D_1 is sufficiently small, it is assumed the approximate equilibrium prices and quantities have been found. If not, a new level of supply to be used in Equation 2 is determined. For example, if:

$$\left| \frac{D_t - S_t}{D_t} \right| > Y \qquad (3)$$

where: Y is an arbitrarily small number based on the desired accuracy level,

then: $S_{t+1} = wD_t + (1-w)S_t$, (4)

where: w is a weight between 0 and 1.

The process is continued until Equation 3 is found to be false.

The stability of the process for a single good can easily be shown for the current problem if a restraint is put on w and the demand is assumed to be negatively sloping and the supply positively sloping. The excess demand, X, is defined as:

$$X_t = D_t - S_t. \tag{5}$$

The conditions for stability can be illustrated by substituting for D_t and then P_t in Equation 5 and taking the derivative of the excess demand with respect to a change in supply resulting in Equations 6 and 7.

$$X_t = g(f(S_t)) - S_t \tag{6}$$

$$X_t' = \frac{dX_t}{dS_t} = \frac{dg}{dP_t}\frac{dP_t}{dS_t} - 1 = g'f' - 1, \tag{7}$$

where: $X'_t = \frac{dX_t}{dS_t}$,

$g' = \frac{dg}{dP_t}$, and

$f' = \frac{dP_t}{dS_t}$.

Since $g' < 0$ and $f' > 0$ by assumption, x'_t will be less than minus one. Since $X'_t < -1$, a change in supply, dS_t, will result in a change in excess demand, dX_t, of a larger magnitude in the opposite direction. By rearranging Equation 4 into Equation 8,

$$S_{t+1} = S_t + w(D_t - S_t) = S_t + wX_t \qquad (8)$$

it can be seen that the change in supply is a function of the excess demand. The value of the weight needed to cause the change in supply to move the model to a solution in one iteration can be determined as shown in Equations 9 through 13.

$$dS_t = wX_{t-1}, \tag{9}$$

$$dX_t = (g'f'-1)dS_t = (g'f'-1)wX_{t-1} \tag{10}$$

But for a solution:

$$dX_t = -X_{t-1}, \tag{11}$$

therefore:

$$-X_{t-1} = (g'f'-1)wX_{t-1}, \tag{12}$$

and therefore:

$$w = -(g'f'-1)^{-1} = (1-g'f')^{-1}. \tag{13}$$

For the change in supply to cause a movement to the equilibrium point, the change in excess supply must equal the minus value of the current excess demand, Equation 11. The change in supply is a function of the current excess demand, Equation 9. By manipulating Equation 7 and substituting for dX_t and dS_t as done in Equations 10 and 12, it is shown that the weight, w, must equal $(1-g'f')^{-1}$ for convergence in one iteration. If $w < (1-g'f')^{-1}$, the change in supply will result in a movement towards the equilibrium point. If $w > (1-g'f')^{-1}$, the change in supply will result in a movement past the equilibrium point, resulting in oscillations about the equilibrium point. Whether the oscillations move the model closer to the equilibrium point depends again upon the value of the weight, w. For the oscillations to converge, the change in excess supply must be of a magnitude less than twice the current value of the excess demand. The value of w that allows for oscillations to converge can be found as shown in Equations 14 through 16.

$$\text{Set:}\quad dX_t < -2X_{t-1}, \tag{14}$$

and Equation 12 becomes:

$$-2X_{t-1} < (g'f'-1)wX_{t-1}, \tag{15}$$

and therefore:

$$w < 2(1-g'f')^{-1}. \tag{16}$$

A value of w between $(1-g'f')^{-1}$ and $2(1-g'f')^{-1}$ will therefore result in convergence to the equilibrium point through oscillations. If $W = 2(1-g'f')^{-1}$, the model will bounce back and forth from one side to the other of the point of convergence with only the sign of the value of excess demand changing. If $w > 2(1-g'f')^{-1}$, the value of the excess demand will get larger and larger, resulting in an unstable market.

There will be a system of demand and supply equations with interaction terms when there is more than one commodity or region. Metzler (1945) has shown that the sufficient conditions for stability in such a linear system is all the commodities must be gross substitutes for all positive adjustment factors.

The tatonnement programming model

The linear programming model is based upon the models previously developed at the Center for Agricultural and Rural Development (CARD). The model is a regionalized one land group model representing the continental United States. [Turhollow, Short, and Heady (1982) and Dvoskin, Heady and English (1978)]. The objective of the linear programming model is to minimize the total cost of crop production and transportation. The costs are in 1975 dollars and the restraints are set up based on the expected year 2000 situation. The linear programming model also must minimize the cost of the production of silage, hay, and cotton, in addition to the crops of interest in this study.

The demand equations are estimated econometrically based on time series data for 1950 to 1979. Demand is disaggregated into domestic-feed demand, domestic human and industrial demand, and foreign demand for U.S. crops. The equations are estimated using seemingly unrelated regression. The equations are described in Schatzer and Heady (1982). The demand eauations are linked to the linear programming model using a FORTRAN subroutine that is linked to the MPSX linear programming package using the READCOMM feature of MPSX.

The linear programming model provides the supply prices which are then used to determine the quantity demanded. If the difference between the quantity demanded and the quantity supplied is more than plus or minus 1 percent of the quantity demanded, then new quantities are determined to be used as demand constraints in the linear programming model. The new quantities are determined in one of two ways. If it is the first iteration or the excess demand has the same sign as the previous iteration, one-half of the excess demand is added to the supply quantity. If the excess demand has the opposite sign of the previous iteration, the equation for a line drawn through two points is computed. The current and previous excess demand quantities are used as one of the two coordinates for each point, while the current and previous supply quantities are used as the other coordinates. The excess demand is then set to zero and the equation is solved for the new supply quantity. The iterations continue until the constraints on excess demand are met for each of the disaggregated demands for each commodity or until 15 iterations are completed. A limit of 15 iterations is placed on the model to allow the results to be checked manually for oscillations about a step in the supply function of one or more crops.

Results

The iterative model is used to estimate approximate equilibrium prices and quantities for barley, corn, oats, sorghum, soybeans, and wheat and supply prices for corn silage, sorghum silage, legume hay, other hay, and cotton for the year 2000. Seven scenarios consisting of three alternative yield levels and three alternative levels of land constraints are run and analyzed. The three levels of yields are determined using three different time trend values for the year 2000 in the yield functions developed by Stoecker (1974) and updated by Meister and Nicol (1975). The three alternative cropland constraints are developed based on the amount of land the Soil Conservation Service has classified as having a potential for conversion to cropland in the future. (A detailed description of the scenarios and the results is provided in Schatzer and Heady, 1982.)

The seven scenarios used in this study are low yields with the possibility of converting the high potential land (LYML); low yields with the possibility of converting the high and medium potential land (LYHL); medium yields with no land conversion (MYLL); medium yields with the possibility of converting the high potential land (MYML); medium yields with the possibility of converting the high and medium potential land (MYHL); high yields with no land conversion (HYLL); and high yields with the possibility of converting the high potential land (HYML).

The results from the seven scenarios suggest that the future equilibrium prices and quantities are highly dependent upon the assumptions made about future crop yields and future cropland availability (see Tables 1 and 2). The highest equilibrium price for barley, corn, oats, sorghum, wheat, and soybeans is 182.1, 158.8, 145.3, 165.1, 182.4, and 208.9 percent higher than the lowest equilibrium price, respectively.

The highest supply price for corn silage, sorghum silage, legume hay, other hay, and cotton is 116.7, 158.0, 140.7, 234.0, and 67.4 percent higher than the lowest price, respectively. Future quantities also vary across scenarios. The largest total equilibrium quantity for barley, corn, oats, sorghum, wheat, and soybeans is 7.0, 2.4, 135.4 24.9, 15.7, and 6.6 percent larger than the smallest total equilibrium quantity, respectively. The total quantities do not vary as much as the prices.

The amount of land required for production also varies greatly depending upon the assumptions made about yields. If the land area is held constant and only yields are varied, then for the medium land scenarios the total land area used for the production of the crops examined in this study is 344.04, 379.90, and 388.89 million acres for the high, medium, and low yield scenarios, respectively.

The future average crop yields are also influenced by the assumptions made about future land availability. If the yields at the activity level in the linear programming model are held constant and only the size of the land area is varied, then for the medium yield scenarios, the average corn yield is 114.42, 109.05, and 108.43 bushels per acre for the low, medium, and high land scenarios, respectively.

Conclusions

The iterative model based on the tatonnement process outlined in this study has the potential for improving the results of interregional programming models. There would be little increase in the cost of constructing or solving the models. The iterative model would make linear programming models. There would be little increase in the cost of constructing or solving the models. The iterative model would make linear programming a better normative tool for analyzing changes in agricultural policy, changes in activity coefficients, or changes in input prices or availability which cause shifts in the supply function. The iterative model would provide better estimates of price changes from one scenario to another than would linear programming. Results for this study suggest that the difference may have a significant impact on the solution.

However, the livestock sector is exogenous for this study. Exogenous livestock production limits the ability of the feed demands to adjust to changes in feed prices. Livestock production should be allowed to change as feed prices change, which would result in larger shifts in feed demand. An improvement would be the addition of an endogenous livestock sector to the linear programming model and the additional of meat demand equations to the demand sector. This addition would allow the tatonnement model to also solve for equilibrium prices and quantities of meat.

Results from the alternatives analyzed in this study show that future crop prices will depend upon what happens to crop yields and to the amount of land available for crop production. The results show that unless crop yields continue to increase, future demand for crops may place a large strain upon our current cropland base. This strain would cause the conversion of some of the United States' current rural lands which are in pasture, range, or forest to cropland uses. Conversely, large increases in crop yields may put the United States in another surplus cropland situation as it was in the 1960s.

Finally, the projections of crop prices made in this study must be viewed with caution. The results are only as good as the data from which they are derived and the assumptions made. Many things can influence future crop yields, land availability, and crop demands. Therefore, any projection of the future is at best an educated guess.

During the use of a model, limitations of the model make themselves known and possible improvements of the model are seen. One problem that appears for the present model is the possibility of cycling about steps in the supply functions. Cycling occurs for two of the scenarios in this study. In one case, the cycling occurs around a step of less than a cent. A smoother step supply function would help decrease the changes of cycling. A smoother function can be created by adding land quality differentials to the model. Also an improvement in the quantity adjustment procedure should lead to faster convergence of the model and help alleviate the cycling problem.

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Table 1. Estimated prices[a] in 1975 dollars for crops in 2000 under seven scenarios.

Crop	LYML	LYHL	MYLL	MYML	MYHL	HYLL	HYML
Barley	$ 4.09	$ 2.29	$ 2.59	$ 1.85	$ 1.56	$ 1.45	$ 1.45
Corn	3.83	2.36	2.50	1.95	1.74	1.58	1.48
Oats	2.87	1.84	2.02	1.53	1.34	1.32	1.17
Sorghum	4.56	2.70	2.89	2.27	1.99	1.86	1.72
Wheat	6.10	3.45	3.98	2.88	2.45	2.29	2.16
Soybeans	9.70	5.60	5.96	4.37	3.76	3.37	3.14
Corn Silage	27.04	17.31	19.59	15.46	13.94	13.23	12.48
Sorghum Silage	28.61	16.15	18.08	13.81	12.05	11.22	11.09
Legume hay	94.77	57.25	64.22	49.16	43.84	41.13	39.38
Other hay	133.21	70.27	74.84	53.93	48.69	42.60	39.88
Cotton	298.42	217.54	230.55	200.57	188.05	185.47	178.22

[a]Barley, corn, oats, sorghum, wheat, and soybeans in $/bushel; silages and hays in $/ton and cotton in $/bale.

tities produced for crops in 2000 u

YHL	MYLL	MYML	MYH
- - - - - -	- - million bushels -		
7.20	597.47	616.59	631.
1.89	9,153.13	9,197.68	9,211.
5.59	322.82	432.34	479.
4.58	1,406.62	1,141.12	1,411.
0.84	1,796.88	1,882.08	1,914.
1.07	3,103.58	3,147.06	3,163.6

Table 1. Estimated prices[a] in 1975 dollars for crops in 2000 under seven scenarios.

Crop	LYML	LYHL	MYLL	MYML	MYHL	HYLL	HYML
Barley	$ 4.09	$ 2.29	$ 2.59	$ 1.85	$ 1.56	$ 1.45	$ 1.45
Corn	3.83	2.36	2.50	1.95	1.74	1.58	1.48
Oats	2.87	1.84	2.02	1.53	1.34	1.32	1.17
Sorghum	4.56	2.70	2.89	2.27	1.99	1.86	1.72
Wheat	6.10	3.45	3.98	2.88	2.45	2.29	2.16
Soybeans	9.70	5.60	5.96	4.37	3.76	3.37	3.14
Corn Silage	27.04	17.31	19.59	15.46	13.94	13.23	12.48
Sorghum Silage	28.61	16.15	18.08	13.81	12.05	11.22	11.09
Legume hay	94.77	57.25	64.22	49.16	43.84	41.13	39.38
Other hay	133.21	70.27	74.84	53.93	48.69	42.60	39.88
Cotton	298.42	217.54	230.55	200.57	188.05	185.47	178.22

[a]Barley, corn, oats, sorghum, wheat, and soybeans in $/bushel; silages and hays in $/ton and cotton in $/bale.

Table 2. Estimated total quantities produced for crops in 2000 under seven scenarios.

Crop	LYML	LYHL	MYLL	MYML	MYHL	HYLL	HYML
	- - - - - - - - - - - - - - - - -	- - million bushels - - - -	- - - - - - - - - - -				
Barley	592.61	617.20	597.47	616.59	631.27	634.09	596.21
Corn	9,013.96	9,171.89	9,153.13	9,197.68	9,211.24	9,220.56	9,231.74
Oats	216.79	375.59	322.82	432.34	479.58	456.11	510.31
Sorghum	1,364.94	1,404.58	1,406.62	1,141.12	1,411.86	1,409.79	1,425.21
Wheat	1,662.76	1,850.84	1,796.88	1,882.08	1,914.36	1,917.33	1,923.49
Soybeans	2,980.28	3,111.07	3,103.58	3,147.06	3,163.64	3,177.15	3,177.16