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Edward J. Balistreri* and Maxwell Brown†

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Abstract

We propose a method for calibrating an industry-level technology to engineering (bottom-up) estimates with a particular focus on abatement opportunities. As a demonstration, substitution elasticities across inputs are adjusted in the nested cost function for the electricity sector to best fit a target marginal abatement cost (MAC) curve derived from engineering assessments of available technologies. This approach is unique because the elasticities are optimized over an entire relevant range of the bottom-up estimates, whereas other techniques use information local to point estimates under little or no abatement. In the context of fitting to a given MAC we evaluate alternative nesting structures and find that, while complexity in nesting improves the fit, even relatively simple nesting structures can reasonably approximate the target MAC. In our example, focused on the electricity sector, we find standard elasticities adopted in top-down models moderately overstate abatement costs relative to the engineering targets. This conclusion, however, is sensitive to our assumption about output-intensity abatement and consumer price responsiveness, both of which are not delineated in engineering estimates.

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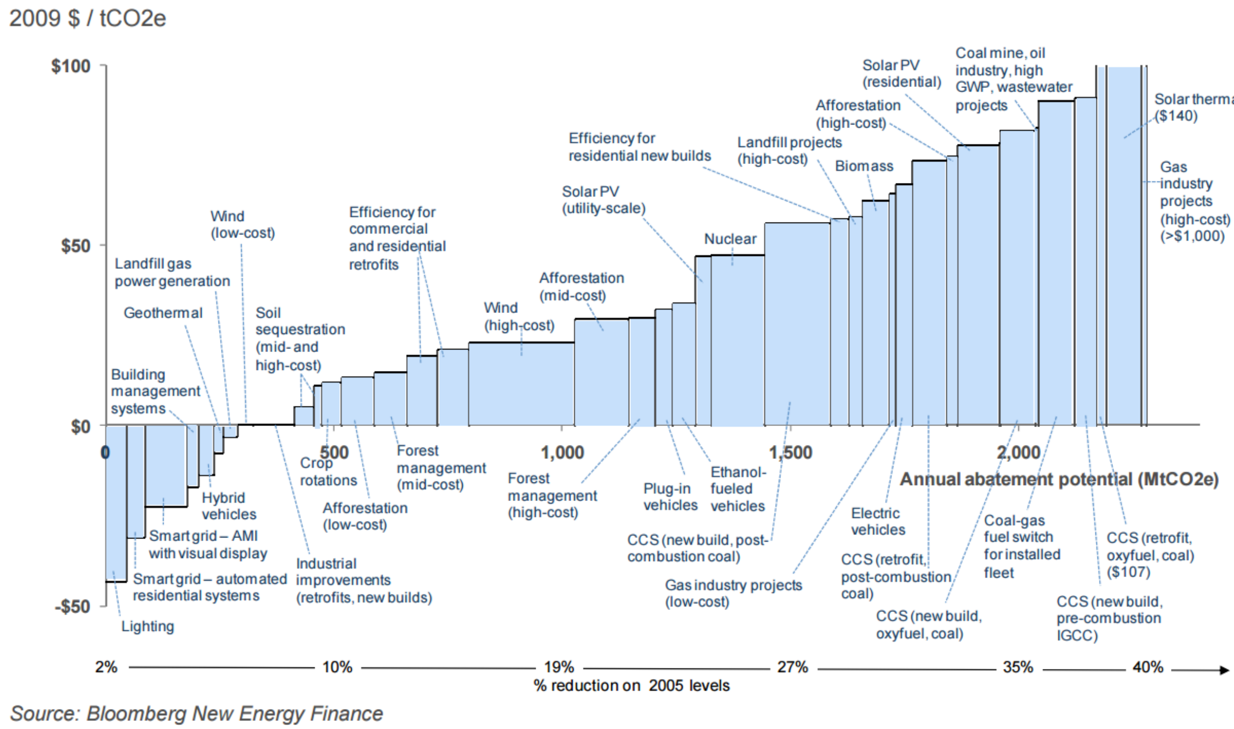

Figure 1: MAC curve as published by Bloomberg (2010)

1 Introduction

In order to translate emissions policy into useful cost estimates we need to have a reasonable estimate of the marginal abatement cost (MAC) curve as it represents available technologies. This is particularly challenging in the context of large-scale models built to analyze carbon policy in a general equilibrium setting. Typically the MAC curve is an implicit outcome of the assumed energy-demand system combined with fuel-specific carbon coefficients. The complexity of fossil energy's role in the economy, as well as the unprecedented nature of a significant carbon-policy shock, precludes a direct empirical estimation and validation of the implied MAC using historical data. As an alternative, researchers often specify their models using a collection of piecemeal evidence from econometric estimates of energy responses. The problem with these approaches is that the resulting energy-demand system will imply a MAC that may be inconsistent with engineering assessments of available abatement technologies. For example, Figure 1 is an engineering assessment of available technologies as published by Bloomberg (2010), and this is likely different from the implicit MAC derived from an economic model calibrated to econometric estimates of fuel price responses. If large policy shocks move us significantly away from the benchmark, the information in engineering assessments is potentially useful in calibration.

In this paper we pursue a general method for systematically calibrating constant elasticity of substitution (CES) technologies typically employed in computable general equilibrium

(CGE) modeling. Although the approach is presented and should be considered broadly, we demonstrate an application focused on the electricity energy-demand system such that the implied MAC for carbon is consistent with bottom-up engineering estimates over a broad range of abatement opportunities. We develop a fitting procedure in which the parameters of the energy-demand system are chosen optimally to target the location and shape of an engineering based MAC. We apply our technique to recalibrate and perform diagnostics on three energy-demand structures adopted in the climate-policy literature. We find that any of these structures can reasonably approximate the engineering MAC when parameterized with that goal in mind.

Many economy-wide (top-down) models include cost functions that are parameterized purely from a local (zero-carbon-abatement) perspective. An example of the traditional approach, which focuses on the fuel demand system is given in Böhringer et al. (2018), who cite the econometric work of Okagawa and Ban (2008) and Steinbuks and Narayanan (2015) to support their elasticity assumptions. In a model with an even closer tie to the underlying econometrics, Jorgenson et al. (2013) estimate translog unit-cost functions directly to calibrate their model. These approaches are valid methods for establishing local price responses and even important efficiency and productivity trends. In general, however, they will not imply a MAC that is consistent with bottom-up studies of available abatement technologies. Our intent is not to suggest that the econometric calibration techniques are inappropriate, nor that the implied MACs are invalid. We simply argue that the implied MACs are different than the bottom-up MACs, and that it is useful to explore these differences as they contribute to a range of views in policy debates. We illustrate a method that accommodates the informational content of the engineering estimates in developing a robust range of potential outcomes.

Some authors have taken a different approach in an effort to consider economy-wide impacts in general equilibrium while maintaining consistency with detailed bottom-up models of the energy system. Hybrid models have emerged that link top-down and bottom-up models. Early hybrid models, for example ETA-Macro (Manne, 1977), used highly stylized macroeconomic optimization models to represent the general equilibrium. A number of more recent approaches are reviewed by Hourcade et al. (2006) in their introduction to a special issue of *The Energy Journal*. The most promising and flexible approach seems to follow the mixed-complementarity formulations of Böhringer (1998) and Böhringer and Rutherford (2008). These methods accommodate a fully consistent top-down and bottom-up representation of the activity analysis of specific technologies in an equilibrium context (where second-best considerations might be made). Yet, a single unified (top-down) structure remains the preferred tool for timely economy-wide policy studies, because these avoid the complexity of decomposition methods.

We focus on a method for calibrating a smooth top-down representation of the technology that best approximates a target MAC. Our proposed method is most closely related Kiuila and Rutherford (2013). Kiuila and Rutherford (2013) consider fitting an aggregate abatement-cost function to a bottom-up MAC. Their approach relies on the introduction of a specific factor (abatement capacity) to generate an upward sloping abatement supply curve. This is likely the preferred approach when modeling criteria pollutants (e.g., SO_2 or NOx) where abatement is linked to end-of-pipe technologies. In the case of carbon abatement, however, there is a strong physical link between the embodied carbon in fuel use and emissions (at least up to the point that carbon capture and sequestration dominates as the abatement technology). Thus carbon abatement is integral to fuel demand. The representation of fuel demand in most top-down models indicates an *implicit* MAC as opposed to the *explicit* MAC calibrated by Kiuila and Rutherford (2013). Our contribution is to show how the basic proposal of MAC fitting in Kiuila and Rutherford (2013) can be applied in a standard top-down setting that includes a full energy-demand system for a given industry.

We develop a procedure for fitting any well-specified nested CES technology that includes fuel inputs to an arbitrary MAC curve. The input-share parameters of the CES technology are locked down to the observed input-output accounts, but a set of substitution elasticities is chosen to minimize the difference between the implied MAC and the target MAC. To demonstrate the procedure we adopt three different nesting structures from the literature with varying degrees of complexity. We fit the implied MAC for electricity generation in the U.S. to a bottom-up MAC derived from Bloomberg (2010). We find that each of the nesting structures can be parameterized in a way that accurately reflects the target MAC. There appears to be little gain from adding excessive complexity to the nesting structure. Relating specific fitted substitution elasticities to power-system responses is most natural under the nesting suggested by Böhringer et al. (2018) (which has an intermediate level of complexity). Our demonstrations are over a limited set of structures, but we expect the tension between parsimony and complexity in top-down representations to persist across other structures.

In general, the Bloomberg (2010) MAC implies lower abatement costs relative to the implied top-down MACs at the reference elasticities adopted in the cited studies for all but the more extensive nesting structure. That is, with two of the three representations, the fitted elasticities are higher implying more flexibility in the energy demand system than normally assumed in top-down models. This finding is sensitive, however, to our assumption about output-intensity abatement. If we allow some electricity-demand responses to escalate costs, the top-down implied MACs indicate lower abatement costs under some treatments.

The paper is organized as follows. In Section 2 we establish the link between the top-down representation of technology and the implied MAC. We outline our estimation strategy in Section 3. In Section 4 we consider the specific nesting structures and our methods for incorporating the Bloomberg (2010) information as a target MAC. Results of our fitting exercises are presented in Section 5, and concluding remarks are offered in Section 6.

2 Cost functions and the implied MAC

Consider a general linearly-homogeneous nested CES technology for a given industry. We represent this technology by the associated unit cost function. The arguments in the unit cost function are a vector of input prices given by $\boldsymbol{p}$, but the functional outcomes also depend on a vector of parameters. The vector of parameters include a set of substitution elasticities, which we will denote $\boldsymbol{\sigma}$. Finding an appropriate vector $\boldsymbol{\sigma}$ is the goal of our calibration exercise, where we assume that the other parameters of the nested CES technology (share and scale parameters) are measured accurately in a set of consistent input-output accounts. In its general form the unit cost function is given by

$$c(\boldsymbol{p}, \boldsymbol{\sigma}) \equiv \min \left\{ \boldsymbol{p}'\boldsymbol{x} \quad \text{s.t.} \quad y(\boldsymbol{x}, \boldsymbol{\sigma}) = 1 \right\}, \tag{1}$$

where $\boldsymbol{x}$ is the vector of inputs and the function $y(\boldsymbol{x}, \boldsymbol{\sigma})$ is a general CES technology for the industry. Denoting industry output $Y = y(\boldsymbol{x}, \boldsymbol{\sigma})$ the cost function is

$$C(\boldsymbol{p}, \boldsymbol{\sigma}, Y) = c(\boldsymbol{p}, \boldsymbol{\sigma})Y, \tag{2}$$

and in a competitive equilibrium the price of output, p_y, equals marginal cost $[p_y = c(\boldsymbol{p}, \boldsymbol{\sigma})]$. Let us also partition the vector of elasticities into a set to be estimated, $\hat{\boldsymbol{\sigma}}$, and an exogenously-specified set, $\bar{\boldsymbol{\sigma}}$. The reason for partitioning is that some of the elasticities might be informed from information that is independent of the MAC calibration.

A convenient way of introducing carbon emissions is to specify a set of Leontief nests for each fuel, indexed by $f \in \mathcal{F} = \{\text{COL}, \text{GAS}, \text{OIL}\}$, where the fuel input (coal, natural gas, and refined petroleum) is combined with an emissions allowance at a maintained zero substitution rate ($\bar{\sigma}_f = 0 \;\; \forall f \in F$). Denote the allowance price for carbon emissions p_E and the fuel price p_f. The gross fuel-input costs (denoted with an uppercase P_f) is given by the Leontief unit cost function

$$P_f(p_f, p_E) = p_f + \gamma_f p_E, \tag{3}$$

where γ_f is a fuel-specific carbon coefficient. Marginal abatement cost in this setup is given by p_E, which might be zero under no abatement. Emissions from the sector are a function of the parameters, prices, and output: $E(\boldsymbol{p}, \hat{\boldsymbol{\sigma}}, \bar{\boldsymbol{\sigma}}, Y)$. This function is identified by applying Shephard's Lemma:

$$E(\boldsymbol{p}, \hat{\boldsymbol{\sigma}}, \bar{\boldsymbol{\sigma}}, Y) = Y \frac{\partial c(\boldsymbol{p}, \hat{\boldsymbol{\sigma}}, \bar{\boldsymbol{\sigma}})}{\partial p_E} = Y \sum_f \gamma_f \frac{\partial c(\boldsymbol{p}, \hat{\boldsymbol{\sigma}}, \bar{\boldsymbol{\sigma}})}{\partial P_f}. \tag{4}$$

Where the right-hand term applies the chain rule to show that emissions are simply given by the sum of fuel inputs weighted by their respective carbon coefficients.

Paired combinations of p_E and $E(\boldsymbol{p}, \hat{\boldsymbol{\sigma}}, \bar{\boldsymbol{\sigma}}, Y)$ indicate the implied MAC. This relationship is conditional on a full set of prices, the elasticities, and a given level of output. For exposition, partition the input price vector into $\bar{\boldsymbol{p}}$, which includes all prices except p_E. Thus emissions are given by $E(\bar{\boldsymbol{p}}, p_E, \hat{\boldsymbol{\sigma}}, \bar{\boldsymbol{\sigma}}, Y)$, and abatement is given by the difference $E(\bar{\boldsymbol{p}}, 0, \hat{\boldsymbol{\sigma}}, \bar{\boldsymbol{\sigma}}, Y) - E(\bar{\boldsymbol{p}}, p_E, \hat{\boldsymbol{\sigma}}, \bar{\boldsymbol{\sigma}}, Y)$. This draws a clear link between the calibration of the cost function and the implied MAC. Notice that the implied MAC is conditional on fixed input prices (other than carbon) and a fixed level of output. The implied MAC is a partial equilibrium concept, but this might be desirable to the extent that the construction of the target (bottom-up) MAC ignores general-equilibrium responses.

Representing the technology with the unit-cost function is convenient for exposition but in application the $c(\boldsymbol{p}, \boldsymbol{\sigma})$ that represents a typical nesting structure will be complex. We can add variables to represent the sub-activity level at a node and the unit cost at that node to simplify the formulation. For example, consider an assumed structure that includes a separable energy nest that is a *child* to the top-level nest that combines all other inputs. We can represent the unit cost of the energy composite as a function of the gross fuel costs plus the cost of electricity. Let us denote the composite energy input quantity ENG, which has a price (marginal cost) of P_{ENG}. Adding these two endogenous variables to the system requires the addition of two equilibrium conditions. First, the supply of ENG must equal demand from the *parent* activity:

$$\mathrm{ENG} = Y \frac{\partial c(\boldsymbol{p}, \boldsymbol{\sigma})}{\partial P_{\mathrm{ENG}}}; \tag{5}$$

and second, the unit cost will be the minimum marginal cost of supplying a unit of ENG:

$$P_{\mathrm{ENG}} = C_{\mathrm{ENG}}(P_{\mathrm{coal}}, P_{\mathrm{oil}}, P_{\mathrm{gas}}, p_{\mathrm{ele}}). \tag{6}$$

Emissions are still derived by applying Shephard's Lemma, but now it is applied to the cost function for ENG, which is simply the product of the endogenous variable ENG, given by equation (5) and its unit cost: $\mathrm{ENG} \cdot C_{\mathrm{ENG}}(\bullet)$. Arriving at equation (4) will then include a chain of node-level demands. An arbitrary nesting structure can be represented in this way, by adding two endogenous variables (a quantity and a price) at each node. For exposition, consider collecting all of these endogenous variables in the vector $\boldsymbol{x}$, and collect all of the equilibrium conditions into the vector-valued function

$$F(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{\sigma}) = 0, \tag{7}$$

which implicitly maps from the set of exogenous input prices and elasticities into the endogenous quantities and composite prices, where the set of composite prices includes the fuel prices gross of the emissions charge.

In our application we consider two extensions to the basic partial equilibrium system represented by $F(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{\sigma}) = 0$. First, we add an internally consistent input-price adjustment for the industry's own use of its output. In a typical set of input-output accounts a member of $\boldsymbol{p}$ will be p_y. Abatement escalates the unit cost of producing Y, and this should be reflected in the price that the industry pays for it own inputs. Second, we might also consider abatement that results from output reductions. This allows us to consider that engineering estimates of MACs might implicitly include the output-intensity abatement channel. In order to fit the CES abatement cost curve to estimates that include output-intensity abatement we need to have a measure of output at each point on the MAC. Adding structure to this notion we specify output demand such that Y changes as $p_y = c(\boldsymbol{p}, \boldsymbol{\sigma})$ is affected by changes in p_E. We assume that output is determined by a constant-elasticity demand function;

$$Y = \alpha p_y^{-\eta} \tag{8}$$

where η is the demand elasticity and α is set such that we replicate benchmark output at benchmark marginal cost. For the central analysis in this paper we assume $\eta = 0$, but one could adopt alternative perspectives. For example, in a set of sensitivity runs we set

$\eta = 0.5$ considering that the bottom-up MAC implicitly considers the fact that the quantity demanded falls with higher output prices.[1]

3 Non-linear estimation strategy

With the system $F(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{\sigma}) = 0$ and output levels (Y) well-specified for changes in p_E we can proceed to estimate a set of $\hat{\boldsymbol{\sigma}}$ that fit the implied MAC to a target. Let $\hat{\boldsymbol{E}}$ indicate emissions from the fitted system (the implied MAC), which depends on choices over $\hat{\boldsymbol{\sigma}}$. Now consider a target MAC, usually derived by a rank ordering of engineering estimates of available abatement technologies, which maps allowance prices onto abatement levels over an empirical domain. From this target MAC we can generate a set of paired observations for the vectors $\boldsymbol{E}^0$ and $\boldsymbol{p}_E^0$. The estimation strategy is to minimize the deviations between the $\boldsymbol{E}^0$ and $\hat{\boldsymbol{E}}$ by choosing an appropriate set of $\hat{\boldsymbol{\sigma}}$. Setting up the non-linear least squares problem we have:

$$\begin{array}{cl} \min_{\{\hat{\boldsymbol{\sigma}}\}} & ||\hat{\boldsymbol{E}} - \boldsymbol{E}^0||^2 \\ \text{subject to:} & F(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{\sigma}) = 0 \\ \text{and} & Y = \alpha p_y^{-\eta}, \end{array}$$

where, in general, some subset of elasticities ($\bar{\boldsymbol{\sigma}}$) and input prices ($\bar{\boldsymbol{p}}$) are assumed fixed. The mathematical program, thus, minimizes the sum of squared distances from endogenous emissions to target emissions subject to the nested technology and output demand assumptions.[2]

4 Empirical nesting structures and the target MAC

To illustrate our fitting method we adopt three different nested CES structures previously used in the analysis of climate and energy policy. We apply our method to the electricity generation sector of the U.S. economy. We choose electricity generation because of its prominence in emissions generation and because there are good bottom-up engineering assessments of potential abatement technologies. We consider multiple top-down structures

[1] The system defining the structural model could be generalized to includes additional flexibility in terms of incorporating other potential considerations in construction of the bottom-up MAC. If, for example, the engineering estimates account for the increased cost of capital as capital is substituted for fuel, this is easily accommodated by an added capital supply schedule. We do not include these extensions because we do not think these issues are generally considered in the engineering assessments of the available technologies. Implicitly, by holding their prices fixed, we are assuming a perfectly elastic supply of all inputs other than emissions allowances and the industry's own intermediate inputs.

[2] As formulated for computation, the model is solved as a Mathematical Program with Equilibrium Constraints (MPEC). For exposition, and because the solution is an interior point, we present the system $F(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{\sigma})$ as a set of equality conditions. There is a logical extension of our system that binds prices and demand quantities to be weakly positive in a set of complementary slack conditions. For details see Rutherford (1995). The code is publically available: `https://github.com/maxxb77/MAC`.

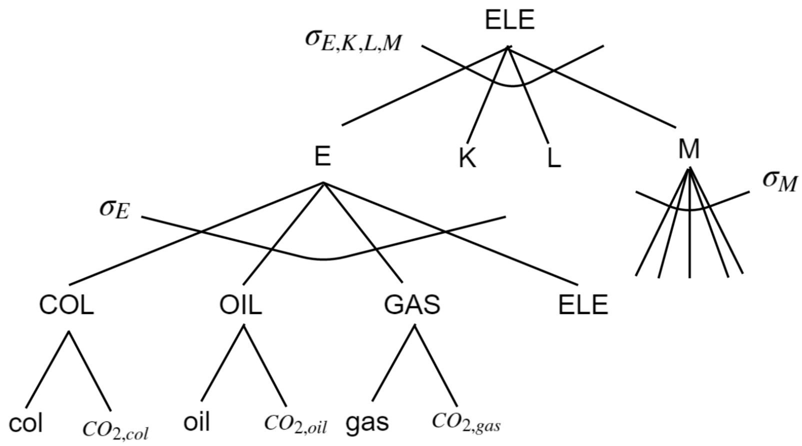

Figure 2: **BASIC** nesting structure (McKibbin and Wilcoxen, 1999)

that are increasing in complexity to explore their respective flexibility in accurately reflecting the target MAC.[3] The three structures and the corresponding models are as follows

- **BASIC** from an early version of the G-Cubed Model (McKibbin and Wilcoxen, 1999).
- **STANDARD** from a GTAPinGAMS climate-policy application (Böhringer et al., 2018).
- **EXTENDED** from the WorldScan model (Lejour et al., 2006).

Starting with the least complex **BASIC** nesting, illustrated in Figure 2, we have an output nest at the top that combines Capital, Labor, Energy, and Materials (often denoted as KLEM) at a constant elasticity of substitution equal to $\sigma_{K,L,E,M}$, where a comma in the subscript indicates separable inputs in the nest. The inputs of energy and materials, however, are CES composites. The energy composite is a CES aggregation of the fuels plus electricity, and the materials composite is a CES aggregation of all other intermediate inputs. The elasticities in these subnests are σ_E and σ_M, respectively.

Under our **STANDARD** nesting structure, illustrated in Figure 3, additional complexity is added. A value-added composite of capital and labor trades off with the energy composite directly at a CES of $\sigma_{KL,E}$. This composite then combines with materials at the top level at a CES of $\sigma_{KLE,M}$. Additional complexity is again added under the **EXTENDED** nesting adopted in the WorldScan model (Figure 4). Under the **EXTENDED** structure a Coal and Gas-Oil nest is added to the energy nest, which allows for a different elasticity of substitution between the various fuels. The code for the partial equilibrium models and calibration exercises for each nesting structure is available[4] but requires the GTAP9 database.

[3]We only consider separable nested CES functions, which dominate the climate and energy policy literature. For a nested CES cost function to be a true *flexible* functional form non-separabilities would be added (Perroni and Rutherford, 1995).

[4]See: `https://github.com/maxxb77/MAC`

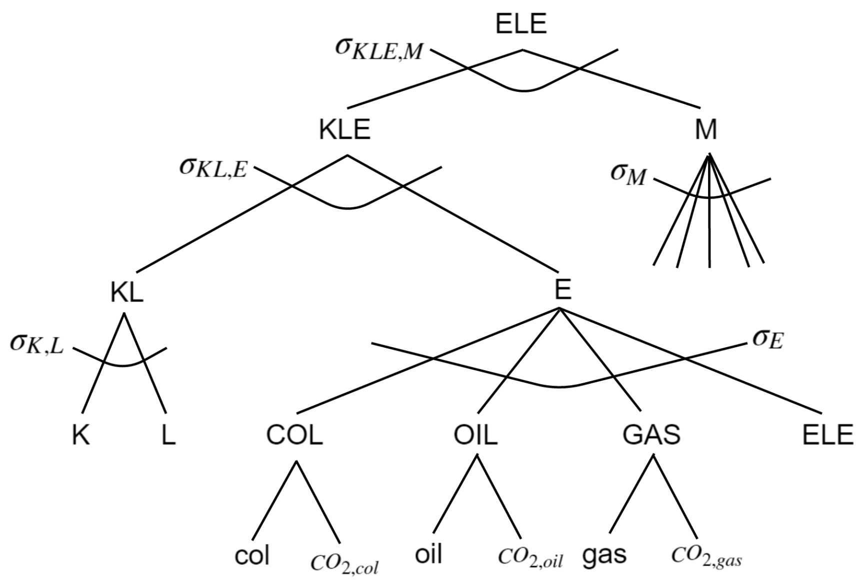

Figure 3: **STANDARD** nesting structure (Böhringer et al., 2018)

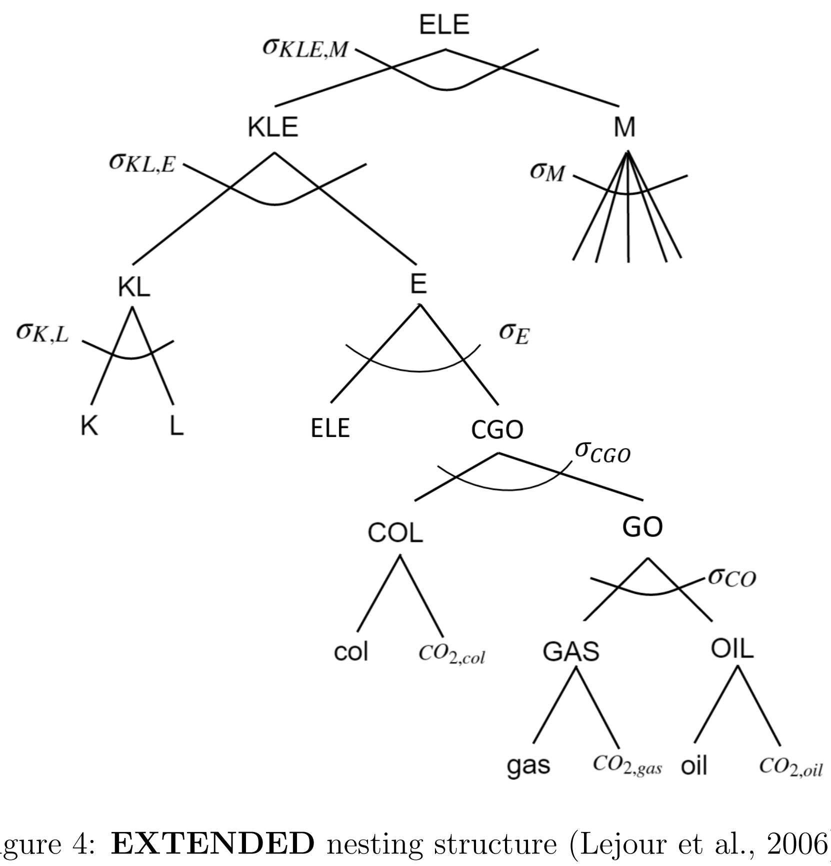

Figure 4: **EXTENDED** nesting structure (Lejour et al., 2006)

The outlined nesting structures are calibrated to the U.S. electricity-generation factor and intermediate input demands in the GTAPinGAMS database used in the paper by Balistreri et al. (2018). These data are aggregated from the original GTAP9 data (Narayanan and Walmsley, 2008). The GTAP data establish the CES scale and distribution parameters. GTAP also provides a set of default response (elasticity) parameters, but we do not use these. Rather we adopt those elasticities assumed by the authors associated with each nesting structure.

The target MAC for the U.S. electricity sector is derived from Bloomberg (2010). We use Bloomberg's compiled assessment of the 2030 U.S. MAC curve with current policies. We filter out those technologies that are specific to electricity generation to generate the target MAC. The technologies that we consider are presented in the derived MAC for electricity generation, Figure 5. Typical of a bottom-up assessment of available technologies notice that there are some technologies that are measured to have negative or zero cost. This is inconsistent with optimization in the benchmark. To reconcile the target with the structural model we employ alternative scaling methods following Kiuila and Rutherford (2013). The scaling methods are as follows:

- **A** shifts the MAC upward by the absolute value of the minimum price,
- **B** treats negative costs as zero but retains their abatement quantities,
- **C** removes all negative costs and their associated abatement quantities.

The bottom-up target MAC represents a step function as each additional abatement technology, at a higher cost, is employed. We select the midpoint of the step as the cost associated with abatement as each new technology is adopted. The constructed target MACs are presented in Figure 6. Specific measures of abatement costs are taken as digital measurements off of the Bloomberg (2010) graphics.

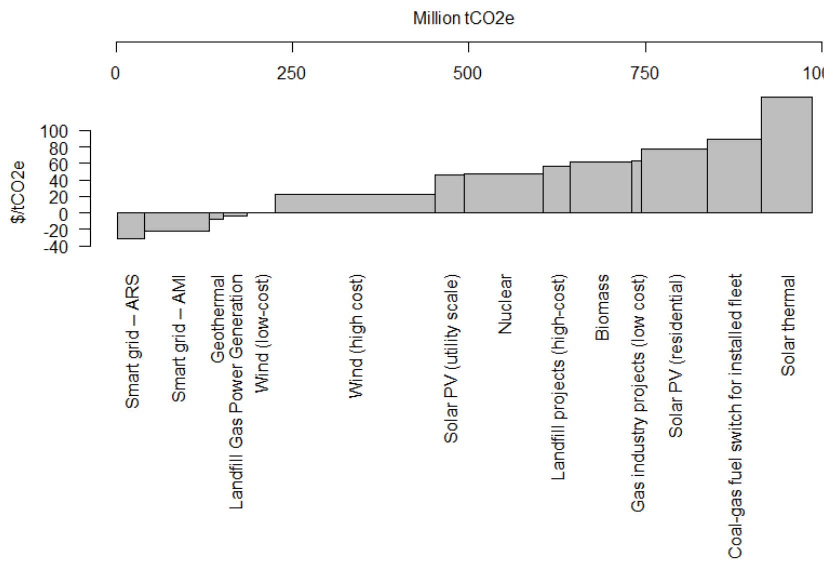

Figure 5: Filtered abatement cost curve, original data from Bloomberg (2010)

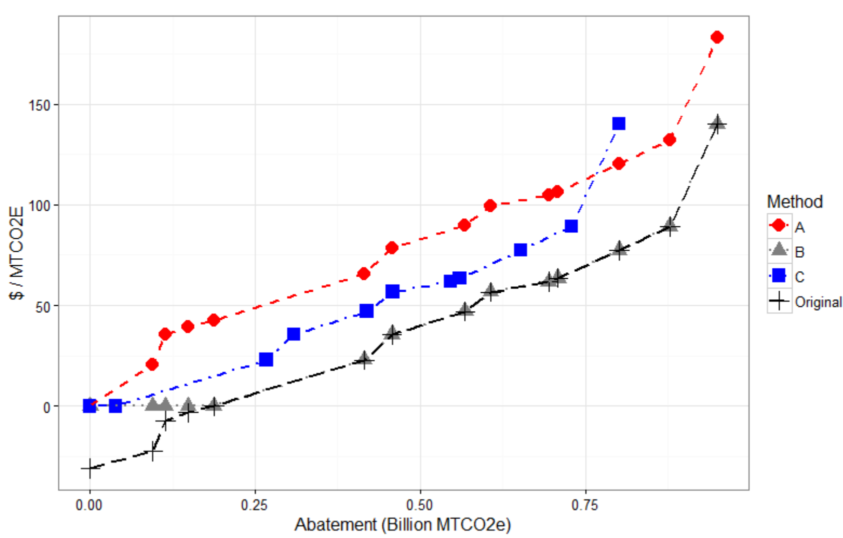

Figure 6: Target abatement cost curves

5 Results

We solve the non-linear least squares problem presented in Section 3 where the system $F(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{\sigma}) = 0$ is specified to represent the proposed nesting structures. Observations on the fitted and target levels of abatement are made by applying alternative CO_2 prices (given by the price points in Figure 6). Initially we assume no electricity-output demand response ($\eta = 0$). Table 1 shows the reference and optimized elasticities across the different nesting structures and scaling methods for the Bloomberg MAC. The reference elasticities are taken from literature applications of the corresponding nesting structure. In general we find that the reference elasticities are somewhat lower than the optimized elasticities with the **BASIC** and **STANDARD** nesting structures. The opposite is true, however, in the **EXTENDED** nesting structure. This indicates that the top-down applications assume higher implied abatement costs relative to the bottom-up target, a point generally illustrated in Figure 7 where the dotted curves are the implied CES MACs at the reference elasticities ('Reference') and the dashed curves are best fit ('Fitted') curves. Figure 8 focuses on scaling method **B** to illustrate the reference and fitted curves.

BASIC		Scaling Method		
	Reference	A	B	C
σ_E	0.20	0.51	0.71	0.55
$\sigma_{K,L,E,M}$	0.76	4.30	4.96	4.63
STANDARD				
	Reference	A	B	C
σ_E	0.50	0.51	0.71	0.55
$\sigma_{KL,E}$	0.26	4.48	5.26	4.92
$\sigma_{KLE,M}$	0.10	0.00	0.04	0.02
EXTENDED				
	Reference	A	B	C
σ_{GO}	0.50	4.08	4.65	4.49
$\sigma_{C,GO}$	0.70	0.32	0.48	0.35
σ_E	0.25	0.00	0.00	0.00
$\sigma_{KL,E}$	0.50	0.00	0.09	0.00
$\sigma_{KLE,M}$	0.00	2.79	3.15	0.05

Table 1: Reference (as assumed in the respective study) and fitted elasticities across structures and scaling methods

The different nesting structures indicate elasticity escalation at different nodes. For example, in the **BASIC** nesting structure elasticity expansion is at the KLEM level, but in the **STANDARD** nesting structure the elasticities are expanded at the substitution between value added (KL) and the energy nest. In the case of the **BASIC** structure, however, the only

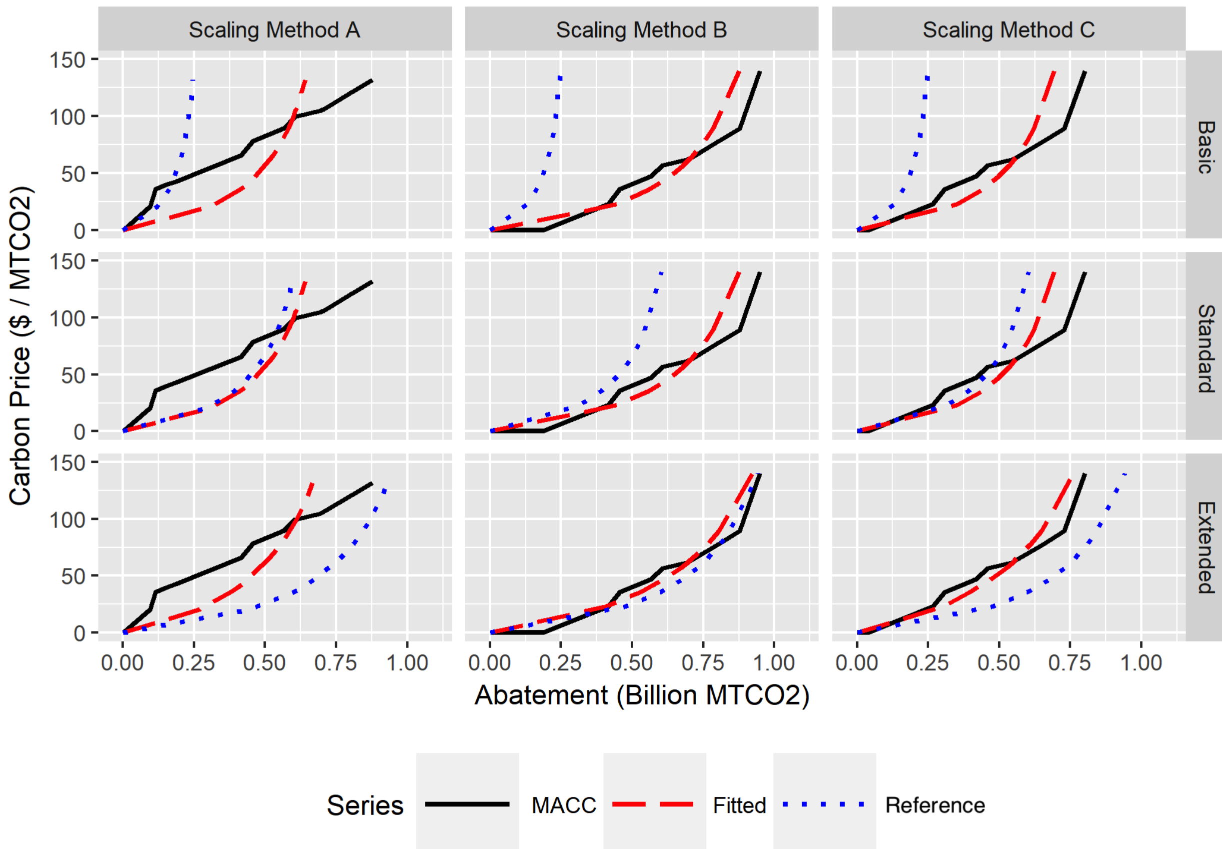

Figure 7: Target (solid black), Reference (dotted blue), and fitted (dashed red) MACs across structures and scaling methods

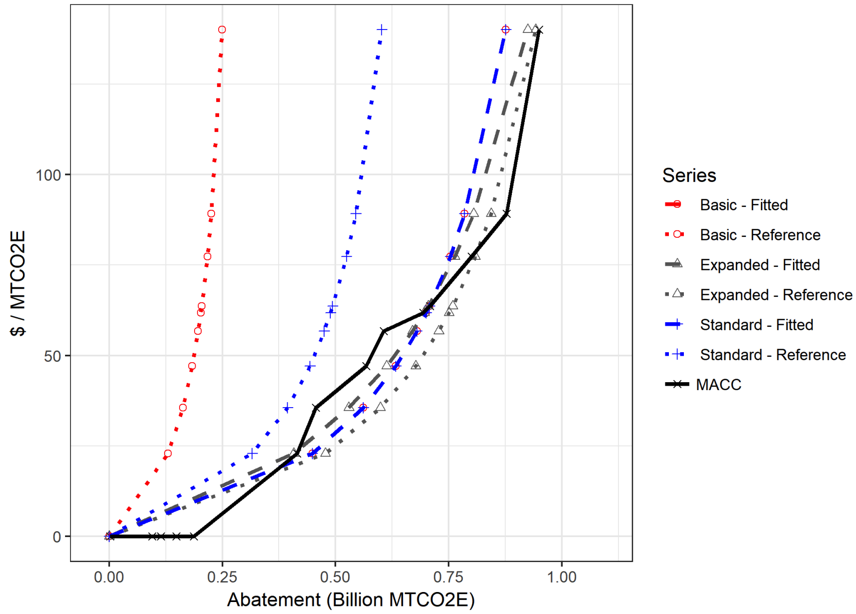

Figure 8: Similarity in fitted MACs across structures (scaling method B)

parameter available to increase is $\sigma_{K,L,E,M}$, and when this increases the rate of substitution with the materials composite automatically increases. To the degree that we think energy substitution is primarily into capital, as opposed to materials, the **BASIC** structure does not have enough flexibility.

With the **EXTENDED** nesting structure we also see significant differences in where the technology becomes more responsive. Unlike the **BASIC** and **STANDARD** structures, the location of elasticity expansion under the **EXTENDED** structure is sensitive to the particular MAC scaling method. With scaling methods **A** and **B** we see that the bottom and top nests of the **EXTENDED** nesting structure (σ_{GO} and $\sigma_{KLE,M}$) increase dramatically whereas all the nests in-between become more inelastic. With scaling method **C**, however, we see that only the bottom nest elasticity increases whereas all others decrease.

We see trade offs between the level of parsimony in the nesting structure and the actual technology in the power sector. To the extent that we focus on top-down representations this tension is not resolved by our fitting procedure. Regardless of how well we may fit the overall MAC it is not clear that we pick the correct margins for adjusting the CES technology. That said, accurately representing the overall MAC might be quite useful in communicating how different structures generate different outcomes. It is useful in this context to measure the gains from complexity. Table 2 reports the level of the least-squares objectives across the estimations. In general, there seems to be small gains, in terms of fit, to added complexity. In these estimations we are optimizing over a set of elasticities. As a lesson, freeing the full set of elasticities (in the structures that we consider) results in some elasticities being adjusted such that the target MAC is reasonably approximated. It remains a judgment call as to whether the correct elasticities are adjusted.

	A	B	C
BASIC	0.4841	0.1169	0.0659
STANDARD	0.4838	0.1168	0.0657
EXTENDED	0.4685	0.1078	0.0632

Table 2: Minimized objective (sum of squared errors) across structures and scaling methods

	Elasticity			ObjVal	Ratio to Optimal
Reference:	$\bar{\sigma}_E$=0.50	$\bar{\sigma}_{KL,E}$=0.26	$\bar{\sigma}_{KLE,M}$=0.10	0.52	4.464
	$\hat{\sigma}_E$=**0.74**	$\bar{\sigma}_{KL,E}$=0.26	$\bar{\sigma}_{KLE,M}$=0.10	0.12	1.058
	$\bar{\sigma}_E$=0.50	$\hat{\sigma}_{KL,E}$ > **500**	$\bar{\sigma}_{KLE,M}$=0.10	0.42	3.576
	$\bar{\sigma}_E$=0.50	$\bar{\sigma}_{KL,E}$=0.26	$\hat{\sigma}_{KLE,M}$=**40.03**	0.52	4.464
	$\hat{\sigma}_E$=**0.71**	$\hat{\sigma}_{KL,E}$=**5.25**	$\bar{\sigma}_{KLE,M}$=0.10	0.12	1.002
	$\hat{\sigma}_E$=**0.74**	$\bar{\sigma}_{KL,E}$=0.26	$\hat{\sigma}_{KLE,M}$=**12.13**	0.12	1.058
	$\bar{\sigma}_E$=0.50	$\hat{\sigma}_{KL,E}$ > **500**	$\hat{\sigma}_{KLE,M}$=**0.00**	0.42	3.576
Optimal:	$\hat{\sigma}_E$=**0.71**	$\hat{\sigma}_{KL,E}$=**5.26**	$\hat{\sigma}_{KLE,M}$=**0.04**	0.12	1.000

Table 3: Component contributions of elasticities to the overall fit (**STANDARD** nesting structure with scaling method **B**)

Looking at Table 2 we see that the non-convexities implied by scaling method **A** make it significantly more difficult for the CES technology to represent the target MAC. Recall that scaling method A shifts the target MAC up by an amount equal to the most negative-cost technology. This is apparent in the first column of graphs in Figure 7. Under scaling methods **B** and **C**, where the negative-cost technologies are either set to zero or ignored, the MAC is more convex and the overall CES fit is better. Figure 8 focuses on scaling method **B** to illustrate the reference and fitted curves. Again the key take away is that the optimized curves are almost identical, although in Table 2 we do see that more complexity does offer a slightly better fit (in terms of a smaller sum of squared errors).

In Table 3 we report a set of diagnostics on the component contributions of each estimated parameter. We again focus on scaling method **B** and consider sequentially freeing up different combinations of elasticities while other elasticities are held at their reference values. In a given row of the table an elasticity embellished with a 'bar' is held at its reference value, while an elasticity embellished with a 'hat' is optimized and the optimized value of the elasticity is in **boldface** type.[5] Notice that significant improvement in fit is available by simply freeing up the energy elasticity (σ_E) which allows for fuel substitution as well as some electricity substitution. This is not very appealing, however, relative to cases where both σ_E and $\sigma_{KL,E}$ are optimized because this includes the opportunity for energy to substitute more freely with value added inputs.

We now turn to a set of fitting exercises where we consider elastic electricity demand. In particular we set $\eta = 0.5$. This has two main impacts. First, our ability to fit the

[5]Note that the lower bound for estimated elasticities is 0 and the upper bound is 500.

BASIC		Scaling Method		
	Reference	A	B	C
σ_E	0.20	0.06	0.39	0.19
$\sigma_{K,L,E,M}$	0.76	0.00	0.00	0.00

STANDARD				
	Reference	A	B	C
σ_E	0.50	0.08	0.40	0.20
$\sigma_{KL,E}$	0.26	0.00	0.00	0.00
$\sigma_{KLE,M}$	0.10	0.10	0.11	0.10

EXTENDED				
	Reference	A	B	C
σ_{GO}	0.50	2.82	5.12	4.99
$\sigma_{C,GO}$	0.70	0.00	0.22	0.04
σ_E	0.25	0.00	0.00	0.00
$\sigma_{KL,E}$	0.50	0.00	0.36	0.00
$\sigma_{KLE,M}$	0.00	0.01	0.01	0.01

Table 4: Reference and fitted elasticity values by nesting structure and MAC scaling method with elastic demand ($\eta = 0.5$)

target MAC is improved because output-intensity based abatement is available. Second, relative to the reference elasticities the fitted elasticities imply very little input flexibility in the production structure. The fitted optimal elasticities across nesting structures and scaling methods are presented in Table 4. With elastic demand we find that under the reference substitution elasticities the implied top-down MAC greatly understates abatement costs relative to the target with the **BASIC** and **STANDARD** nesting structures. This is opposite of the case under inelastic demand (and this is at a modest demand elasticity of 0.5). While fit is improved, the fitted elasticities are unstable due to the reliance on output-intensity abatement. In each of the nesting structures we see that the optimal fit is with very low input substitution elasticities. This drives the technology to escalated abatement costs that are dominated by output reductions.

Figure 9 shows the reference and fitted MACs when we have elastic electricity demand. In terms of overall fit we do not see a great deal of variation across structures, again because of the heavy reliance on output reductions for abatement. The accuracy of the fitted curves to the MAC data does not vary greatly by nesting structure, regardless of MAC scaling method. Table 5 presents the values of the objective functions (sum of squared errors) under the different treatments. Comparing these values to those presented in Table 2 we see significant improvement in the overall fit. Our read, however, is that there is less useful information in the estimates of substitution elasticities. First, most abatement is through the output channel. Second, it is not clear to us that the engineering assessments of abatement

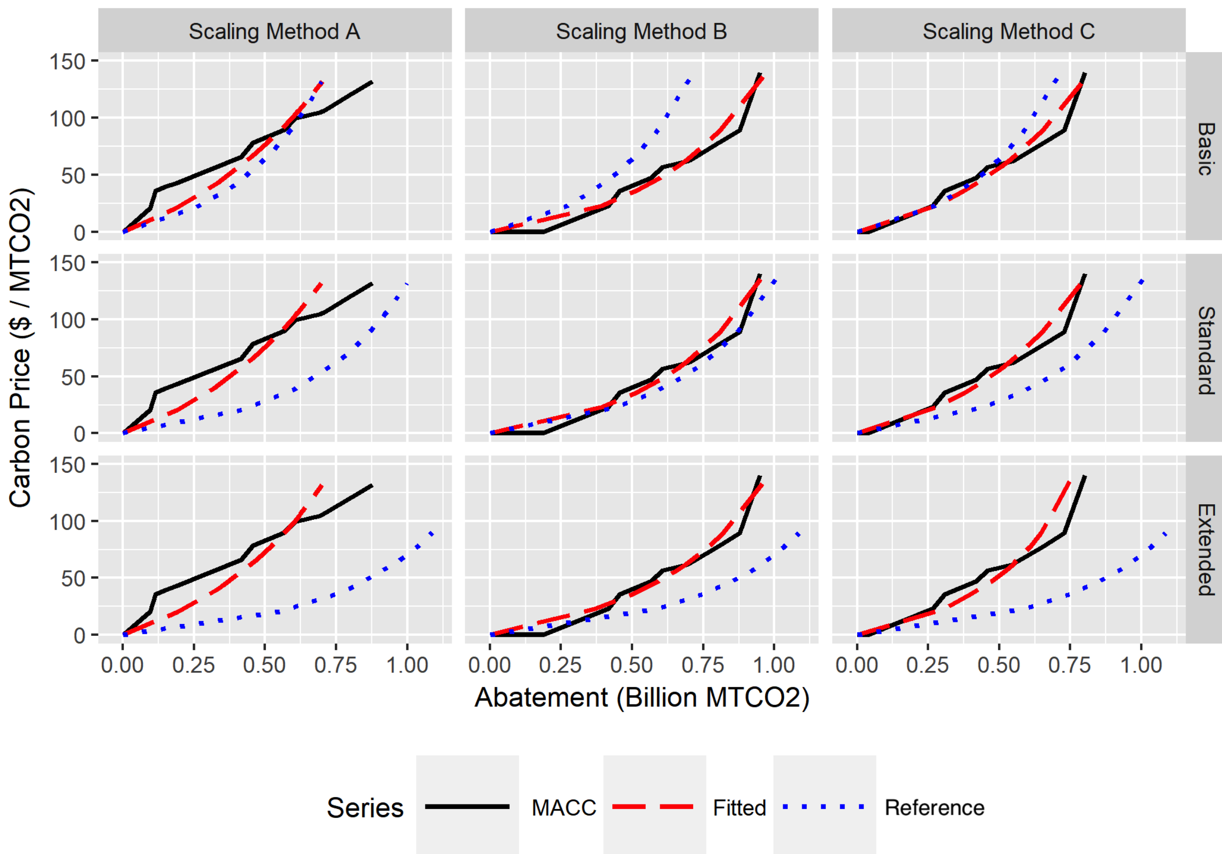

Figure 9: Target (solid black), Reference (dotted blue), and Fitted (dashed red) MACs across structures and scaling methods with elastic demand ($\eta = 0.5$)

technologies actually considers output changes. So, although output reductions are a real economic response, it may not be appropriate to consider these when targeting a MAC derived from engineering assessments.

6 Conclusion

This paper demonstrates a method by which bottom-up engineering assessments of abatement technologies can be incorporated into a standard CES representation. This method is useful for reconciling the MACs implied in a standard top-down model with beliefs about future technologies. Contemporary methods for calibrating nested CES technologies might be quite appropriate in their examination of fuel demand responses to variation in prices. The danger, however, is that significant carbon abatement might pull us considerably away from historical price changes. Furthermore, price variability as observed in the data is not the same as a simultaneous escalation of fuel prices based on their carbon coefficients. The tools developed here have significant potential as an alternative calibration method.

	A	B	C
BASIC	0.2791	0.0988	0.0300
STANDARD	0.2085	0.0947	0.0219
EXTENDED	0.1940	0.0942	0.0172

Table 5: Minimized objective (sum of squared errors) across structures and scaling methods with elastic demand ($\eta = 0.5$)

While we find the method sound it is only useful to the extent that we have an accurate target MAC that is well understood. We find it challenging to understand and adopt an 'off-the-shelf' MAC as published. Negative abatement costs are generally inconsistent with the behavioral assumptions built into any equilibrium model. We dealt with negative abatement costs with various scaling methods as proposed by Kiuila and Rutherford (2013). This is not entirely satisfying, however, because it would be better to know how these negative costs were estimated and how they might be reconciled with economic behavior. We also find ourselves questioning the specific input-pricing and output demand conditions under which a given target MAC is appropriate. Our suspicion is that these issues were simply not considered in the development of the target MAC. This lead to our central assumption that demand is perfectly inelastic and input prices are fixed. Given their application in policy debates it is troubling that bottom-up MAC assessments do not clearly indicate any stance on consumer responds to the indicated prices. Our application indicates the importance of these implicit assumptions that change our interpretation of the information content of the MAC. We show that the CES calibration is critically sensitive to our assumption about what was assumed about output demand responses in the development of the engineering estimates of abatement technologies.

Under our central assumptions we find that the best fit elasticities were generally larger than those adopted in general-equilibrium models. In particular, the evidence suggests that the bottom-up technologies imply more substitution between energy (fuel inputs) and value added. This is not surprising given that bottom-up MACs are specifically focused on new (out of sample) technologies, while the typical CES calibration relies on local (in sample) responses to changes in fuel prices. While there is nothing wrong with using evidence on local price changes to inform the CES calibration, there is potentially other information available that could yield more accurate responses to large policy changes; or, at a minimum, accommodate a range of general-equilibrium results that include the positions held by the engineering community. Our contribution is to develop and illustrate a method by which a CES technology can be calibrated to any target MAC over an entire relevant range of potential abatement opportunities.

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