Strategic Interactions Among Private and Public Efforts when Preventing and Stamping Out an Highly Infectious Animal Disease

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Abstract:

Upon outbreak of a contagious animal disease, a primary motive for restoring disease-free status is often to regain access to international product markets. Efforts applied toward continuing or regaining such access is a public good—all growers benefit regardless of extent of private efforts taken while exclusion is impractical. Private incentives to take preventive measures and stamp-out efforts interact in complex ways. There are intra-farm temporal interactions and also inter-farm contemporaneous interactions. Public effort also takes place and interacts with private efforts. This paper provides a succinct multi-agent model to explore these interactions in social optimum and in Nash equilibrium, and also to explore how socially optimal and Nash behavior differ. Comparative statics under social optimality are more straightforward than under Nash equilibrium. Whether in social optimum or Nash equilibrium, public prevention efforts complement both private prevention and private stamp-out efforts. However, public stamp-out efforts substitute for both private stamp-out and private prevention efforts. Reasonable conditions are identified under which Nash levels of private prevention and stamp-out efforts are both below socially optimal levels. Concerning policy prescriptions, efforts to secure property rights and reduce property transfer costs should promote prevention and eradication efforts. Other things equal, public prevention effort should be more effective in promoting welfare than comparable public stamp-out effort. Subsidies on private efforts should favor prevention efforts because subsidies on eradication effort may discourage prevention effort. Even if produce from diseased animals is safe to consume and acceptable to consumers, it may be optimal to destroy such produce.

Keywords: animal health management, biosecurity; disease prevention; SIS; strategic interactions; trade ban

JEL classification: Q17; D62; I10; H40

The Sanitary and Phytosanitary Agreement of the World Trade Organization (WTO) allows countries to implement trade bans on livestock and produce imports from a country as a precaution in the event of a disease outbreak in the country of origin.[1] In order to provide scientific grounding for such measures while mitigating protectionist opportunism, the World Organization for Animal Health (henceforth, OIE) has put in place a list of animal diseases viewed as specific hazards. If the disease is zoonotic or poses significant health threats to either domesticated or wild animals in the country seeking to impose the ban, a country that is free or almost free of the disease may impose a trade ban on the affected country without violating WTO commitments. Many of the listed diseases have been the subject of significant trade bans in recent years. In many cases, restricted market access has been intermittent where the disease at issue has recurred. This has been the case with Classical Swine Fever, Newcastle Disease and Bovine Spongiform Encephalopathy for various countries, but the most disruptive of these listed diseases has been Foot and Mouth Disease (FMD).

Among Organization for Economic Cooperation and Development (OECD) countries, FMD remains endemic to Turkey. Greece is FMD-free without vaccination, but outbreaks occurred in 1996 and again in 2000 (Junker, Komorowska, and van Tongeren 2009). The United Kingdom had occurrences in 1981, 2001, and again in 2007. Ireland, which had been free since 1941, also suffered an outbreak in 2001. France shared the United Kingdom outbreaks in 1981 and 2001 while the Netherlands had outbreaks in 1984 and again in 2001. Japan had been FMD-free since 1908 until outbreaks in 2000 and again in 2010. South Korea's FMD disease history status is similar—its first outbreak since 1934 occurred in 2000, followed by outbreaks in 2002 and 2010. The virus is readily transmissible by aerosol, wildlife and on-farm equipment, while eradication, or stamp-out, efforts may cause widespread disruption to

[1] Trade bans can be at a sub-country region level. This understood, we will henceforth refer to country bans.

the general economy.

Losses to a country are difficult to assess because market disruptions lead to shifting and volatile prices and require estimates on shifts in spending elsewhere in the economy. In the 1990s, Taiwan's hog sector exported almost one-third of its output to Japan and ranked third among global pork exporters. Its 1997 FMD outbreak decimated the industry (Felt, Gervais, and Larue 2011). Since then, the country has struggled to attain disease-free status, with several subsequent outbreaks, including one in May 2013. Long-term investment in sector development has been impeded due to uncertainty about prospects for international access, as produce from countries that are not disease free, or are free with vaccination, are also subject to import bans. The outbreak in the UK in 2001 involved the slaughter of 6 million animals and is estimated to have cost in the order of $11 billion in 2001 money (Thompson et al. 2002). Pendell et al. (2007) estimated that an outbreak at a medium-sized beef feedlot in southwest Kansas could result in a $200 million reduction in the region's total economic activity.

OIE-listed diseases can presumably be eradicated from a country or region because a disease is not listed whenever they are endemic to all countries. The epidemiology modeling literature refers to diseases where recurrence can occur as SIS diseases.[2] In addition, prevention and stamp-out involve externalities, in that the behavior of one's neighbors matters. One of the pecuniary losses from this class of diseases is reduced market returns due to impeded access to international markets. This loss is shared by all growers regardless of farm disease status. The effort costs are private but the benefits from restored access are shared by all.

It is also important to recognize that prevention and stamp-out interventions are taken in different states. Prevention actions are taken in the susceptible state while stamp-out actions

[2] Other types of disease include susceptible-infected (SI) where the unit of analysis is not cured and susceptible-infected-recovered (SIR) where the unit of analysis is cured and becomes immune.

are taken in the infected state. There is of course reason to believe that these decisions are linked. For example, if it is not costly to prevent recurrence then the incentive to try stamp-out is strong, while if it is not costly to stamp out then the incentive to prevent is weak.

In this paper, we will develop a multi-agent dynamic model of a SIS disease where (*a*) market losses are incident on all growers once a disease is detected, (*b*) prevention efforts, whether public or private, provide pure public goods, and (*c*) stamp-out efforts, public or private, also provide pure public goods. The focus of our paper is on strategic interactions across disease states and agents. For example, we are interested in how one grower's prevention effort when in the non-diseased state affects another grower's stamp-out effort when in the diseased state? Or how does an exogenous shock to stamp-out costs affect prevention actions?

The literature on infectious disease management in human health is vast, but those emphasizing the strategic dimension is, perhaps surprisingly, small. Examples include Kremer (1996), Geoffard and Philipson (1997), and Manfredi and d'Onofris (2013). The literature emphasizing strategic issues in animal health is small. Insights do not generally transfer from human health economics because incentives and ownership structures are not comparable across the two fields while such policies as slaughter and absolute movement control have no parallel among human health policy options.

Horan and Wolf (2005) use optimal control methods to study optimal management of a disease where farmed animals can be cross-infected by a valuable harvested wild animal population, but they do not address private incentives. Gramig, Horan, and Wolf (2009) devise indemnity payments to encourage preventive biosecurity and reporting actions when growers act strategically on private information about disease on their farms. Wang and Hennessy (2014) investigate how market rewards for voluntary release of disease test information can have a rational bandwagon effect, such that successively more herd owners voluntarily test and release information. Hennessy, Roosen, and Jensen (2005) point out that when disease transmission accompanies trade motivated by regional differences in feed availabilities then a socially inefficient low-productivity equilibrium can result such that it is optimal to encumber the live animal trade. Hennessy (2007) characterizes a preventive action when the probability of acquiring a disease depends on what others do and the gains from being disease free are entirely private. Hennessy (2008) demonstrates the complementary nature of private actions to protect against entry of a highly infectious disease into a region, but Reeling and Horan (2013) qualify the finding when grower actions protect against both entry and spread of an infection.

A separate literature considers dynamic issues in animal disease infection, but without accommodating strategic interactions. Mahul and Gohin (1999) note that vaccination in the event of an outbreak precludes such vaccinated animals from certain markets so that trade losses would persist even after stamping out. For this reason there is merit to waiting for further information on the extent of a problem before choosing vaccination as a control method. Olson and Roy (2008) identify conditions under which stamping out a biological invasion is optimal. Private incentives are not incorporated into their analysis. Niemi and Lehtonen (2011) use dynamic programming to study production decisions when a trade ban may materialize.

In this paper, we first explain the sorts of actions of interest, where FMD is used as the contextualizing disease. We then develop a general model of prevention and stamp-out decisions, where the benefits of having collective disease-free status are shared, but some of the costs of becoming and remaining there are private. This paper is distinct in that it integrates prevention and eradication incentives when both efforts provide private externalities. It is also distinct in that the benefit, a trade ban removal, is a pure public good. We use the model to analyze socially efficient and Nash private optimality contexts.

Our analysis finds that an increase in publicly provided stamp-out effort decreases the socially optimal levels of both private prevention and private stamp-out efforts. On the other hand, an increase in public prevention effort increases the socially optimal level of private stamp-out effort and has an ambiguous effect on private prevention effort. There are natural complementarities among prevention efforts as all agents have an incentive not to be the weakest link in letting a disease in. If these complementarities overcome the incentive to free-ride on collective disease prevention efforts then an increase in public prevention effort will increase the socially optimal levels of both prevention and stamp-out efforts. Socially optimal responses to changes in costs are also studied. It is argued that the socially optimal extent of private prevention should decrease in response to an own-cost increase whereas the socially optimal level of private effort to stamp out might plausibly increase, in response to an increase in own cost. The possibility of upward sloping factor demand arises because the diseased state becomes less desirable when cost of exit from the state increases.

In Nash equilibrium, we show that actions to stamp out substitute, whereas prevention actions may be strategic complements. They will be strategic complements whenever free-riding is not a major concern. In general, due to the variety of ways in which responses can occur, little can be inferred about how prevention and stamp-out actions interact and respond to external stimuli in Nash equilibrium unless further assumptions are made. We posit a simple set of technologies that allow for insights on the nature of Nash equilibrium prevention and stamp-out action responses to cost shocks and to public interventions. These lend support to the ideas that private efforts to prevent and stamp-out are too low for the social good and that public effort to prevent are more effective than public efforts to cure. The paper concludes with some brief comments.

A Context

In this section we seek to clarify the nature of actions that are to be modeled. Four types of actions arise in the model, these being privately taken disease prevention and stamp-out actions as well as publicly taken actions of these sorts. In order to better explain what we mean, we note that European Union (EU) Directive 2003/85/EC specifies much detail on required FMD prevention and control efforts. The directive requires that each EU member state maintain or arrange for a reference laboratory focused on ascertaining whether FMD is present. Member states are also required to draw up plans for dealing with an outbreak and to engage in exercises with neighboring states. These are examples of public efforts taken when a country is disease-free to prevent and minimize the extent of an outbreak before it is recognized.

Under this directive the public sector is also required to ensure that leftovers from human food consumption are not fed to cloven-hoofed animals, an endeavor that also calls for private sector preventive efforts. Other private sector prevention activities, not covered in the directive, include on-farm biosecurity protocols concerning who enters a livestock premises, investment in wash-down facilities for people and vehicles entering the premises and maintenance of such records as to support animal tracing in the event of an incident. An aware and attentive stockman might also prevent by isolating an animal with symptoms in a half day period prior to the disease becoming infective (Charleston et al. 2011).

Public stamp-out efforts covered by the directive include maintenance of procedures and resources to initiate an official investigation, to sample and test during a stamp-out campaign, to control animal movement, to conduct an inventory of risky materials on identified farms, to cull animals, and to dispose of animals and other risky materials. Private stamp-out efforts mentioned in the directive include the requirement to promptly notify a suspect case to public authorities, and also costs involved in cleaning and disinfecting the premises. Private costs will also be incurred when providing information to authorities investigating case origin, in maintaining movement-controlled animals, and in disposing of dung other than as manure. Such costs will be incident on movement-controlled farms close to a farm known to be infected, and not just on the farm itself. Although these farms may face legal requirements to incur such costs, the growers will have some discretion on the extent of costs they choose to incur. More broadly, all farms in the country will incur discretionary stamp-out costs when managing livestock movement, feed procurement and labor management, and also when families make lifestyle accommodations during an outbreak.

Model

Setup

This is a continuous time model. There are two possible states, namely the susceptible (*S*) and infected (*I*) states. These states refer to the country's disease status and not to farm or animal disease status. In the susceptible state, each agent earns profit w. There are *N* agents, alternatively referred to as farms or growers, labeled $n \in \{1, 2, \dots, N\} \equiv \Omega_N$. Grower n will take a disease prevention effort of magnitude $a_n \geq 0$ and the continuous flow cost of taking this effort level is $c^a(a_n;\theta)$, a twice continuously differentiable function that is increasing and weakly convex in a_n. Parameter θ represents a cost shift where we assume that cost increases in the shift (i.e., $c^a_\theta(a_n;\theta) \geq 0$). Grower n receives payoff $\pi^{S,n} \equiv w - c^a(a_n;\theta)$ as a continuous flow in the susceptible state.

Ceteris paribus, an increase in prevention effort by any one grower lowers the hazard rate, $\phi(h_0, a_1, \dots, a_N)$, for disease presence in a country where the government's level of prevention efficiency or effort is represented by $h_0 > 0$ and $\phi(\cdot)$ is decreasing in each action. To be more specific, we assume that $\phi(h_0, a_1, \dots, a_N) = H(h_0 + h_1 A)$, $A = \sum_{n \in \Omega_N} a_n$, where $H(\cdot)$ is twice continuously differentiable, decreasing and weakly convex, while $h_1 > 0$. The hazard rate function is decreasing to reflect positive marginal benefit of actions to prevent infection and the function is convex to reflect diminishing returns to the action. The linear aggregation, $A = \sum_{n \in \Omega_N} a_n$, reflects our view that prevention efforts are 'public goods' provided to other growers. That is, the benefit from avoiding a trade ban is non-excludable and non-rival. It is a country public good where the public in this case is the set of all growers in the country (Kaul, Grunberg, and Stern 1999). Linear summation requires that actions be technical substitutes in prevention)i.e., $H_{a_j a_k}(\cdot) = h_1^2 H''(\cdot) \geq 0$) or an increase in a_j renders a_k less effective in reducing the hazard of contracting the disease. Parameter h_0, public prevention effort, could reflect expenditure on a public health campaign or on border measures to reduce disease prevalence within a region. It also substitutes with private effort as $H_{h_0 a_j}(\cdot) = h_1 H''(\cdot) \geq 0$. We refer to $h_0 + h_1 A$ as total prevention effort.

In the infected state, each grower earns amount $w - L$ where L is the loss from being locked out from international markets. Thus, we assume that a trade ban is put in place immediately when the diseases occur. We think that this is a reasonable approximation for the sorts of animal diseases at issue. We also assume that trade will not resume until the disease is eradicated in the country. This is the intent of the WTO when overseeing country-to-country trade bans, although in reality it may be difficult to ascertain when the disease has truly been eradicated and some countries may seek to continue the ban long after objective evidence indicates little remaining cause for concern.

Denote the stamp-out effort taken by grower n as magnitude $b_n \geq 0$ where the continuous flow of taking effort level b_n is $c^b(b_n;\tau)$, again increasing and convex in choice argument b_n and increasing in cost parameter τ. Therefore, in the infected state grower n receives a payoff of $\pi^{I,n} \equiv w - L - c^b(b_n;\tau)$ as a continuous flow. The probability rate for recovery is $\eta(g_0, b_1, \ldots, b_N)$ where public sector effort is given as $g_0 > 0$. Public sector effort might be viewed as expenditure on a public animal health campaign, or effort to reduce the extent of background infection through channels other than inter-personal interactions. Of course each effort has some effect (i.e., $\eta(\cdot)$ is increasing in each of the effort levels g_0 and $b_n, n \in \Omega_N$).

We will also require the form of $\eta(\cdot)$ to be such that the b_n are technical substitutes because success involves a joint effort to recover. Stamp-out efforts are public goods because it would be practically impossible to exclude a grower from the benefits while grower benefits are non-rival. To provide a specific and tractable form we let $\eta = G(g_0 + g_1 B)$, $B = \sum_{n \in \Omega_N} b_n$, where $G(\cdot)$ is twice continuously differentiable, increasing and (at least weakly) concave while $g_1 > 0$. Here $g_0 + g_1 B$, the argument in $G(\cdot)$, is referred to as total stamp-out effort.

Discounted Expected Present Value

We define $V^{S,n}$ and $V^{I,n}$ as grower *n*'s discounted expected present value (DEPV) in the susceptible and infected states, respectively. Let *r* be the continuous time discount rate. Let μ be the intensity parameter for a Poisson process, independent of other variables in the model, such that valuation collapses to 0 if the shock materializes (Taylor and Karlin 1984). This process is intended to reflect exogenous risks to each of the farms. As is well-known, such a process modifies the discount rate from r to $r + \mu$. The fundamental valuation equations are as given by

$$(r+\mu)V^{S,n} = \pi^{S,n} + \phi \times (V^{I,n} - V^{S,n}); \qquad (r+\mu)V^{I,n} = \pi^{I,n} + \eta \times (V^{S,n} - V^{I,n}); \qquad (1)$$

see Shapiro and Stiglitz (1984) for a similar application. Each equation takes the form that DEPV times the sum of interest rate and valuation collapse shock parameter equals the instantaneous payoff plus the expected gains or losses from a state transition. That is, benefit flow from being in the given state equals instantaneous benefit given the present state plus the valuation implication of an instantaneous state transition.

The solution to (1) is

$$V^{S,n} = \frac{(r+\mu+\eta)\pi^{S,n} + \phi\pi^{I,n}}{(r+\mu)\psi(r,\mu,g_0,h_0,A,B)}; \qquad V^{I,n} = \frac{\eta\pi^{S,n} + (r+\mu+\phi)\pi^{I,n}}{(r+\mu)\psi(r,\mu,g_0,h_0,A,B)};$$
$$\psi(r,\mu,g_0,h_0,A,B) \equiv r+\mu+\eta+\phi \equiv r+\mu+G(g_0+g_1B)+H(h_0+h_1A). \tag{2}$$

Equation set (2) may be written as

$$V^{S,n} = \frac{w-c^a(\cdot)}{r+\mu} - \frac{H(h_0+h_1A)[L+c^b(\cdot)-c^a(\cdot)]}{(r+\mu)\psi(\cdot)};$$
$$V^{I,n} = \frac{w-L-c^b(\cdot)}{r+\mu} + \frac{G(g_0+g_1B)[L+c^b(\cdot)-c^a(\cdot)]}{(r+\mu)\psi(\cdot)}. \tag{3}$$

The difference in the DEPV between the two states is:

$$V^{S,n} - V^{I,n} = \frac{L+c^b(\cdot)-c^a(\cdot)}{\psi(\cdot)}. \tag{4}$$

The surplus value in the susceptible state can be viewed as a bond paying benefit flow $L+c^b(\cdot)-c^a(\cdot)$ at discount rate $\psi(\cdot)$ (Hennessy 2007).

First Best

Under the susceptible state, first-best choices are given by the choice vector $(a_1, \ldots, a_N)$ that maximizes $\mathfrak{L}^S \equiv \sum_{n\in\Omega_N} V^{S,n}$, in other words,

$$\max_{(a_1,\ldots,a_N)} \frac{Nw - \sum_{n\in\Omega_N} c^a(\cdot)}{r+\mu} - \frac{H(\cdot)\left[NL + \sum_{n\in\Omega_N} c^b(\cdot) - \sum_{n\in\Omega_N} c^a(\cdot)\right]}{(r+\mu)\psi(\cdot)}. \tag{5}$$

Notice that the objective function is both symmetric and concave in the a_n. Thus, for any value of A and any *N*-tuple $(b_1, \ldots, b_N)$, the equal values *N*-tuple $(a_1, \ldots, a_N) = (A/N, \ldots, A/N)$ maximizes benefit. By symmetry it should also be clear that the infected state objective function is also concave in the b_n. Therefore, we may write the objective function as[3]

$$\max_a \frac{[w - c^a(a;\theta)]N}{r+\mu} - \frac{H(h_0 + h_1 Na)M^{sc}(a,b;\theta,\tau)}{(r+\mu)\psi(r,\mu,g_0,h_0,Na,Nb)};$$
$$M^{sc}(a,b;\theta,\tau) \equiv [L + c^b(b;\tau) - c^a(a;\theta)]N; \tag{6}$$

where $M^{sc}(\cdot)$ is the difference across states in social cost.

In optimality it will necessarily be the case that $M^{sc}(\cdot) > 0$. This is because it would never be optimal for infected state loss $L + c^b(\cdot)$, including stamp-out effort costs, to exceed what one spends seeking to avoid the loss. The optimality condition is

$$\mathfrak{L}_a^S = -\frac{[r+\mu+G(\cdot)]N}{(r+\mu)\psi(\cdot)}\left[c_a^a(\cdot) + \frac{h_1 H'(\cdot)M^{sc}(\cdot)}{\psi(\cdot)}\right] = 0 \tag{7}$$

We write the solution as $a^{so}(b)$ where we will shortly explore the nature of the response in $a^{so}(b)$.

Condition (7) can be viewed as a marginal cost-benefit calculation. The certain marginal cost of the prevention action a in state S is $c_a^a(\cdot)$ per unit time. The expected benefit from this action arises from net social savings per unit time amounting to $M^{sc}(\cdot)$. These savings need to be multiplied by the marginal effect of action a on state transition probabilities as reflected in $h_1 H'(\cdot)/\psi(\cdot)$. At any optimum the second-order condition satisfies $\mathfrak{L}_{aa}^S < 0$.[4]

Similarly, under the infected state first-best choices are given by the choice vector $(b_1, \ldots, b_N)$ that maximizes $\mathfrak{L}^I \equiv \sum_{n\in\Omega_N} V^{I,n}$ (i.e., upon recognizing concavity in $(b_1, \ldots, b_N)$) and what that means for socially optimal choices,

$$\max_b \frac{[w - L - c^b(b_n;\tau)]N}{r+\mu} + \frac{G(g_0 + g_1 Nb)M^{sc}(a,b;\theta,\tau)}{(r+\mu)\psi(r,\mu,g_0,h_0,Na,Nb)}. \tag{8}$$

[3] Technically, we only need Schur-concavity. See Marshall, Olkin, and Arnold (2010, 21).
[4] See supplemental materials for calculations.

The optimality condition for (8) is

$$\mathfrak{L}_b^I = -\frac{[r+\mu+H(\cdot)]N}{(r+\mu)\psi(\cdot)}\left[c_b^b(\cdot)-\frac{g_1G'(\cdot)M^{sc}(\cdot)}{\psi(\cdot)}\right]=0, \tag{9}$$

where we write the solution as $b^{so}(a)$ and the marginal interpretation of condition (9) is as with (7). The second-order optimality condition satisfies $\mathfrak{L}_{bb}^I < 0$.

Finally, it is readily calculated that $\mathfrak{L}_{ab}^S = \mathfrak{L}_{ba}^I = 0$, where details are provided in supplemental materials. These vanishing cross derivatives mean that, at the optimum, social welfare maximizing choices are locally independent across states (i.e., the b is not relevant in $a^{so}(b)$ and the a is not relevant in $b^{so}(a)$). This may be surprising, but remember that choices are made in different states of nature so that the other decision may be viewed as given.

In summary, we have the pair of optimality conditions

$$\begin{aligned} -\frac{h_1H'(h_0+Nh_1a^{so})M^{sc}(a^{so},b^{so};\theta,\tau)}{\psi(r,\mu,g_0,h_0,A^{so},B^{so})} &= c_a^a(a^{so};\theta); \\ \frac{g_1G'(g_0+Ng_1b^{so})M^{sc}(a^{so},b^{so};\theta,\tau)}{\psi(r,\mu,g_0,h_0,Na^{so},Nb^{so})} &= c_b^b(b^{so};\tau). \end{aligned} \tag{10}$$

We consider now how the socially optimum levels respond to exogenous parameter changes including exogenous changes in the levels of prevention and stamp-out efforts (i.e., public efforts). It is readily established, and should not be surprising to learn, that $da^{so}/dL \geq 0$ and $db^{so}/dL \geq 0$. More interesting are the effects of an increase in public effort in one disease state on the socially optimal level of private effort in the other state. We find

$$\frac{da^{so}}{dg_0} \leq 0; \qquad \frac{db^{so}}{dh_0} \geq 0; \tag{11}$$

and so we have

Result 1: An increase in exogenous stamp-out effort decreases the socially optimal level of privately chosen prevention effort. An increase in exogenous prevention effort increases the socially optimal level of privately chosen stamp-out effort.

The divergent responses are as one might expect. The socially optimal response to an exogenous increase in the provision of stamp-out effort is to reduce private prevention efforts. This is because the likely sojourn time in the infected state declines as exogenous stamp-out effort increases. The response does not involve moral hazard as we are referring to socially optimal effort levels. The response is one of crowding out, but across states of nature. On the other hand, the socially optimal response to an exogenous increase in prevention effort is to increase private levels of stamp-out effort. This is because the likely sojourn time in the susceptible state increases, raising the benefits arising from eradication. The response reflects complementarity between government prevention effort and private stamp-out effort.

We turn now to socially optimal within state responses to exogenous (or government-supported) increases in effort. The responses are shown to have qualitative signs (i.e., $\overset{\text{sign}}{=}$) as follows:

$$\frac{da^{so}}{dh_0} \overset{\text{sign}}{=} [H'(\cdot)]^2 - \psi(\cdot)H''(\cdot); \qquad \frac{db^{so}}{dg_0} \overset{\text{sign}}{=} \psi(\cdot)G''(\cdot) - [G'(\cdot)]^2 < 0. \tag{12}$$

Within the infected state, the exogenous increase in stamp-out effort increases the transition probability and so reduces private incentives to act. This response is captured by the term $-[G'(\cdot)]^2$. There is also a term, as reflected in $\psi(\cdot)G''(\cdot)$, that captures decreasing returns to stamp-out effort where the transition and decreasing returns effects buttress each other. On the other hand, the response to an exogenous increase in prevention effort is ambiguous without further information. The transition probability effect, $[H'(\cdot)]^2$, is positive but is counteracted by a crowding out effect due to decreasing returns on total effort applied to prevention.

Now suppose that the susceptible state hazard rate is of constant relative curvature form

$H(h_0 + h_1 A) = \lambda_0 e^{-\lambda_1 (h_0 + h_1 A)}, \lambda_0 > 0, \lambda_1 > 0$, a form commonly used in empirical analysis (Greene 2003, 792–797). Then $[H'(\cdot)]^2 - \psi(\cdot) H''(\cdot) = -[r + \mu + G(\cdot)]\lambda_0 \lambda_1^2 h_1^2 e^{-\lambda_1 (h_0 + h_1 A)} < 0$ and the decreasing returns effect would dominate the transition probability effect. Indeed, the comparative strength of the transition probability effect is determined by discount rate r, exogenous value collapse intensity parameter μ, and infected state hazard rate $G(\cdot)$. If any of these is large then one's sense of concern about future adverse states is small and the transition probability effect is comparatively weaker.

We turn now to consider total responses to endowment shocks, and there we have

$$\frac{d(h_0 + N h_1 a^{so})}{dh_0} > 0; \qquad \frac{d(g_0 + N g_1 b^{so})}{dg_0} = \frac{c_{bb}^b(\cdot)[\psi(\cdot)]^2 - N g_1^2 [G'(\cdot)]^2 M^{sc}(\cdot)}{\psi(\cdot)\left[c_{bb}^b(\cdot)\psi(\cdot) - N g_1^2 G''(\cdot) M^{sc}(\cdot)\right]}. \tag{13}$$

Total prevention effort increases in response to h_0 and we can therefore infer that the expected time that the population spends in the diseased state declines. However we cannot sign the socially optimal response of total stamp-out effort to public stamp-out effort without further information. Upon comparing the first and second expressions in (12) with the corresponding expressions in (13), it can be seen that the total effort response in social optimum can be signed only if the private effort response in social optimum cannot be signed.

Upon inserting optimality condition (10) into (13) we have

$$\frac{d(g_0 + N g_1 b^{so})}{dg_0} = \frac{c_{bb}^b(\cdot)[\psi(\cdot)]^2 - N g_1^2 [G'(\cdot)]^2 \dfrac{\psi(\cdot) c_b^b(\cdot)}{g_1 G'(\cdot)}}{\psi(\cdot)\left[c_{bb}^b(\cdot)\psi(\cdot) - N g_1^2 G''(\cdot) M^{sc}(\cdot)\right]} \overset{\text{sign}}{=} \frac{b^{so} c_{bb}^b(\cdot)}{c_b^b(\cdot)} - \frac{N g_1 b^{so} G'(\cdot)}{\psi(\cdot)}. \tag{14}$$

If stamp-out marginal cost is elastic to effort when compared with stamp-out hazard rate sensitivity to effort then overall stamp-out effort in social optimum will increase in response to public stamp-out effort. This is because crowding out of the stamp-out action will be small.

As for the impact of cost shocks, comparative statics yield

$$\frac{da^{so}}{d\tau} \overset{\text{sign}}{=} -h_1 H'(\cdot) N c_\tau^b(\cdot) \geq 0; \qquad \frac{db^{so}}{d\theta} \overset{\text{sign}}{=} -g_1 G'(\cdot) N c_\theta^a(\cdot) \leq 0;$$
$$\frac{da^{so}}{d\theta} \overset{\text{sign}}{=} h_1 H'(\cdot) c_\theta^a(\cdot) N - \psi(\cdot) c_{a\theta}^a(\cdot); \qquad \frac{db^{so}}{d\tau} \overset{\text{sign}}{=} g_1 G'(\cdot) N c_\tau^b(\cdot) - \psi(\cdot) c_{b\tau}^b(\cdot); \tag{15}$$

which may be summarized as:

Result 2: An increase in the cost of stamp-out effort increases the socially optimal level of prevention effort while an increase in the cost of prevention effort decreases the socially optimal level of stamp-out effort. If an increase in the cost of prevention effort also increases the marginal cost of prevention effort (i.e., $c_{a\theta}^a(\cdot) \geq 0$), then such a cost increase decreases the socially optimal level of prevention effort.

It seems quite intuitive that as the cost of stamping out increases then the benefits of prevention actions increase to avoid entering the more costly infection state. Similarly, if the cost of prevention increases then the benefits of stamp-out actions decrease because success in stamping out has become less rewarding. It is reasonable to assume that $c_{a\theta}^a(\cdot) \geq 0$ so the socially optimal level of prevention likely decreases in response to an increase in the cost of prevention effort.

The remaining comparative static, $db^{so} / d\tau$ is least readily signed. As it is reasonable to assume that $c_{b\tau}^b(\cdot) \geq 0$, it follows that there are two opposing effects.

Result 3: If the stamp-out marginal cost shock is sufficiently inelastic with respect to stamp-out effort, or $g_1 b^{so} G'(\cdot) N / \psi(\cdot) > b^{so} c_{b\tau}^b(\cdot) / c_\tau^b(\cdot)$, then the socially optimal level of stamp-out effort will increase with an increase in the cost of stamp-out effort.

This somewhat contrary optimal response arises because the cost shock changes the desirability of being in the disease-free state. If the change in cost of stamp-out effort in the infected state, $c_\tau^b(\cdot)$, is sufficiently large in comparison with the change in marginal cost of the action, $c^b_{b\tau}(\cdot)$, then it would be best to bite the bullet and put in more effort in order to hasten escape from the infected state.

Private Optimality Conditions

Given (3), the private optimization problems for the *n*th agent in the susceptible and infected states are to $\max_{a_n} V^{S,n}$ and $\max_{b_n} V^{I,n}$, respectively, in other words,

$$\begin{aligned}
&\max_{a_n} \frac{w - c^a(\cdot)}{r+\mu} - \frac{H(\cdot)M^{pc,n}(\cdot)}{(r+\mu)\psi(\cdot)};\\
&\max_{b_n} \frac{w - L - c^b(\cdot)}{r+\mu} + \frac{G(\cdot)M^{pc,n}(\cdot)}{(r+\mu)\psi(\cdot)};\\
&M^{pc,n}(a_n, b_n; \theta, \tau) = L + c^b(b_n;\tau) - c^a(a_n;\theta).
\end{aligned} \tag{16}$$

The Nash private optimality conditions are

$$\begin{aligned}
&-\frac{[r+\mu+G(\cdot)]}{(r+\mu)\psi(\cdot)}\left[c^a_a(a^{po}_n;\theta) + \frac{h_1 H'(\cdot)M^{pc,n}(\cdot)}{\psi(\cdot)}\right] = 0;\\
&-\frac{[r+\mu+H(\cdot)]}{(r+\mu)\psi(\cdot)}\left[c^b_b(b^{po}_n;\tau) - \frac{g_1 G'(\cdot)M^{pc,n}(\cdot)}{\psi(\cdot)}\right] = 0;
\end{aligned} \tag{17}$$

with private optimality choices a^{po}_n and b^{po}_n. It bears emphasis that $M^{pc,n}(\cdot)$ is similar to $M^{sc}(\cdot)$, as given in (6), in that a loss difference across states $L + c^b(\cdot) - c^a(\cdot)$ is involved. However the difference is not multiplied by *N* as only private losses enter the decision calculations.

In order to establish uniqueness of any pure strategy solution to this game, we need to confirm that the cumulative reaction function is decreasing with slope larger, or less negative, than -1 (Vives 1999, theorem 2.8).[5] That is, we require that

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[5] The conditions for the theorem to apply are that the best reply of one action to the sum of other actions be a single-valued function on a compact set. We have no reason to suspect that

$$-\frac{V^{S,n}_{a_n A_{\backslash n}}}{V^{S,n}_{a_n a_n}} \in (-1,0); \qquad A_{\backslash n} = \sum_{j\in\Omega_{N,j\neq n}} a_j. \tag{18}$$

where calculations are provided in supplemental materials. Condition $-V^{S,n}_{a_n A_{\backslash n}} / V^{S,n}_{a_n a_n} < 0$ is true if and only if $[H'(\cdot)]^2 < \psi(\cdot)H''(\cdot)$. The latter condition is certainly true whenever $H(\cdot)$ is log-convex, or $d^2 \ln[H(\cdot)]/dh_0^2 \geq 0$ so that it would apply were $H(h_0 + h_1 A) = \lambda_0 e^{-\lambda_1 (h_0 + h_1 A)}$, $\lambda_0 > 0, \lambda_1 > 0$. Indeed, from (12) we have that log-convexity of $\psi(\cdot)$ in h_0 is both necessary and sufficient to ensure the intuitive comparative static $da^{so}/dh_0 < 0$. As we show in the supplemental materials, $-V^{S,n}_{a_n A_{\backslash n}} / V^{S,n}_{a_n a_n} > -1$ is true so we can be quite confident about the uniqueness of any pure-strategy equilibrium.

Similarly, for uniqueness in pure strategy solutions we require that

$$-\frac{V^{I,n}_{b_n B_{\backslash n}}}{V^{I,n}_{b_n b_n}} \in (-1,0); \qquad B_{\backslash n} = \sum_{j\in\Omega_{N,j\neq n}} b_j. \tag{19}$$

Now $-V^{I,n}_{b_n B_{\backslash n}} / V^{I,n}_{b_n b_n} < 0$ if and only if $\psi(\cdot)G''(\cdot) < [G'(\cdot)]^2$, which is true whenever $G(\cdot)$ is concave. As we also show in supplemental materials, $-V^{I,n}_{b_n B_{\backslash n}} / V^{I,n}_{b_n b_n} > -1$ can be boiled down to the elasticity condition $g_1 b_n^{po} G'(\cdot)/\psi(\cdot) < b_n^{po} c^b_{bb}(\cdot)/c^b_b(\cdot)$. If the elasticity of stamp-out marginal cost-to-own action is large when compared with the relative sensitivity of stamp-out 'hazard' to stamp-out action, then this sufficient condition for uniqueness in pure strategies applies. Notice that the condition is weaker (without *N* multiplying g_1) than the condition identified in (14) above for the socially optimal value of total stamp-out effort to increase with an exogenous government increase in stamp-out effort, or $d(g_0 + Ng_1 b^{so})/dg_0 > 0$. So the condition appears to be quite reasonable.

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these conditions are onerous in this application.

The own-effect cross-derivatives are $V^{S,n}_{a_n b_n} = V^{I,n}_{a_n b_n} = 0$. As in social optimality, this independence between own actions across states is because choices are made in different states and may be taken as given. Furthermore, the in-state, cross-agent cross-derivatives are

$$V^{S,n}_{a_n a_j}\,|_{j\neq n} = \frac{[r+\mu+G(\cdot)]h_1^2 M^{pc,n}(\cdot)\left\{[H'(\cdot)]^2 - \psi(\cdot)H''(\cdot)\right\}}{(r+\mu)[\psi(\cdot)]^3};$$
$$V^{I,n}_{b_n b_j}\,|_{j\neq n} = \frac{[r+\mu+H(\cdot)]g_1^2 M^{pc,n}(\cdot)\left\{\psi(\cdot)G''(\cdot) - [G'(\cdot)]^2\right\}}{(r+\mu)[\psi(\cdot)]^3} < 0. \tag{20}$$

Result 4: Stamp-out actions are strategic substitutes. Prevention actions are complements if and only if $da^{so}/dh_0 > 0$ (i.e., if and only if an increase in exogenous prevention effort elicits an increase in the socially optimal level of private prevention effort).

As discussed under (12) above, if prevention actions are to complement then technical decreasing returns to prevention effort, as captured by $-\psi(\cdot)H''(\cdot)$, should not be large when compared with the transition probability effect, as reflected in $[H'(\cdot)]^2$. To be clear here, even though we have stacked the tables in favor of substitution between efforts, through technical substitutability between efforts via $H(h_0 + h_1 A)$ and $G(g_0 + g_1 B)$, it is plausible that prevention efforts are strategic complements. However, we have already found $da^{so}/dh_0 < 0$ to be a sufficient condition for a unique solution in Nash equilibrium. So if prevention efforts are strategic complements then there may be multiple solutions, a situation that is well-known to students of games with strategic complementarities (Milgrom and Shannon 1994).

The relations in (20) allow us to comment on system-wide responses to shocks under Nash equilibrium. The approach taken is to establish whether the game we study is supermodular (Milgrom and Roberts 1990) (i.e., whether the agents' strategies interact in a qualitatively unidirectional manner). If so, then an exogenous adjustment to the system could impact all actions in a beneficial, reinforcing way. For example, a subsidy on any one action would induce an increase in all equilibrium choices under optimal behavior. Is this likely to happen? We need to consider all grower responses in order to provide a theoretically grounded opinion.

The interactions among efforts for different states and agents comprise part of a system-wide response, and these interactions are:

$$V^{S,n}_{a_n b_j}\,|_{j\neq n} \leq 0; \qquad V^{I,n}_{b_n a_j}\,|_{j\neq n} \geq 0. \tag{21}$$

An increase in stamp-out actions on the part of someone else decreases marginal product of prevention effort, whereas an increase in prevention action on the part of someone else increases marginal product of stamp-out actions. The rationales for these responses are essentially as given in Result 1 above. In Nash equilibrium, someone else's choice can be viewed as an exogenous public choice (i.e., b_j and a_j in (21) replace g_0 and h_0 in (11), respectively). Furthermore,

$$\begin{array}{lll} V^{S,n}_{a_n\theta} \leq 0 \text{ if } c^a_{a\theta}(\cdot) \geq 0; & V^{S,n}_{a_n L} \geq 0; & V^{S,n}_{a_n\tau} \geq 0; \\ V^{I,n}_{b_n\theta} \leq 0; & V^{I,n}_{b_n L} \geq 0; & V^{I,n}_{b_n\tau} = -\dfrac{[r+H(\cdot)]}{(r+\mu)\psi(\cdot)}\left\{c^b_{b\tau}(\cdot) - \dfrac{g_1 G'(\cdot)c^b_\tau}{\psi(\cdot)}\right\}. \end{array} \tag{22}$$

Interactions (20)–(22) allow for contemplation of Nash equilibrium responses to external shocks. Consider what would happen in the system were the value of loss L to increase. Marginal products of both actions would increase as $V^{S,n}_{a_n L} \geq 0$ and $V^{I,n}_{b_n L} \geq 0$. Under Result 4 we have conditions such that any incentive to increase a_n would reinforce incentives to increase the level of $a_j, j \neq n$. Then, from $V^{I,n}_{b_n a_j}\,|_{j\neq n} \geq 0$ in (21), this would reinforce private incentives to increase b_n in Nash equilibrium. However, another path of incentive spillovers that one could follow is that L increases the incentive to increase b_j on the part of some agent *j*. Then, from Result 4, this increase in b_j reduces the marginal product of b_n and so the incentive to take that stamp-out action decreases along this path. Whether the first, second, or another pathway is more important in determining the ultimate response of b_n to a change in L is not clear. Such dissonance pervades the system so that the game is not supermodular and, in general, little can be established about how (a_n^{po}, b_n^{po}) respond to $(L, \theta, \tau, h_0, g_0)$ in Nash equilibrium. An implication is that it is, in general, difficult to compare the levels of first-best actions with privately optimal actions.

Specific Technology

The above has not enabled us to compare first-best with Nash equilibrium solutions. In the following example we seek to shed some light on the comparison. Let

$$\begin{aligned}
&c^a(a_n) = \kappa_0^a e^{\kappa_1^a a_n}, \qquad \kappa_0^a > 0, \qquad \kappa_1^a > 0; \\
&c^b(b_n) = \kappa_0^b e^{\kappa_1^b b_n}, \qquad \kappa_0^b > 0, \qquad \kappa_1^b > 0; \\
&H(h_0 + h_1 A) = \hat{h} - h_0 - h_1 A \quad \forall A \in [0, (\hat{h} - h_0)/h_1], \qquad \hat{h} > h_0 > 0, \qquad h_1 > 0; \\
&G(g_0 + g_1 B) = g_0 + g_1 B \quad \forall B \geq 0, \qquad g_0 > 0, \qquad g_1 > 0;
\end{aligned} \tag{23}$$

where the cost functions are convex. Transition rate function $G(\cdot)$ is concave, while $H(\cdot)$ is convex as previously assumed, although neither are strictly so. A point to note here is that, due to linearity, decreasing marginal benefits to actions do not play a role in hazard rate functions from here on out.

In supplemental materials we demonstrate that symmetric Nash equilibrium solutions for prevention actions solve

$$\begin{aligned}
&\rho + \frac{N g_1 \ln\left(\dfrac{g_1 \kappa_0^a \kappa_1^a}{h_1 \kappa_0^b \kappa_1^b}\right)}{\kappa_1^b} - \left(\frac{g_1}{\kappa_1^b} - \frac{h_1}{\kappa_1^a}\right) + \left(\frac{g_1}{\kappa_1^b} - \frac{h_1}{\kappa_1^a}\right) N \kappa_1^a a^{po} = \frac{h_1 L}{\kappa_0^a \kappa_1^a} e^{-\kappa_1^a a^{po}}, \\
&\rho \equiv r + \mu + g_0 + \hat{h} - h_0,
\end{aligned} \tag{24}$$

which we also write as $\theta_0 + \theta_1 a^{po} = \theta_2 e^{-\kappa_1^a a^{po}}$ with the obvious values for θ_0, θ_1 and θ_2. Now

$e^{-\kappa_1^a a}$ is positive and decreasing in the value of a. If $\kappa_1^a / h_1 > \kappa_1^b / g_1$ then we may say that a is 'less responsive' than b. We use that term because the condition has the prevention marginal cost increase quickly and the prevention hazard rate decrease slowly when compared with stamp-out marginal cost and hazard rate.

When $\theta_0 < \theta_2$ and $\theta_1 \geq 0$ then it is clear that a unique and strictly positive solution exists, as depicted in Figure 1a. When $\theta_0 > \theta_2$ and $\theta_1 > 0$ then no positive solution exists. The parameters summing to ρ are too large to support a unilateral deviation from $a^{po} = 0$. This would be the case when the discount rate and/or probability of collapse are large, public contribution to stamp-out is large, or public contribution to prevention is small. When $\theta_0 < \theta_2$ and $\theta_1 < 0$ then there may be zero or two solutions, ignoring the case of tangency. Figure 1b depicts the case where there are two solutions, with the smaller of the two being the only stable solution.[6] Figure 1c depicts one of the two cases where there are no strictly positive stable solutions. The other is when $\theta_0 \geq \theta_2$, then either the line does not intersect the exponentially decaying curve in the strictly positive domain or the line intersects the curve just once and from above so that the solution is not stable.

Some calculation also provides

$$\rho + \frac{Nh_1 \ln\left(\dfrac{g_1\kappa_0^a\kappa_1^a}{h_1\kappa_0^b\kappa_1^b}\right)}{\kappa_1^a} - \left(\frac{g_1}{\kappa_1^b} - \frac{h_1}{\kappa_1^a}\right) + \left(\frac{g_1}{\kappa_1^b} - \frac{h_1}{\kappa_1^a}\right) N\kappa_1^b b^{po} = \frac{g_1 L}{\kappa_0^b\kappa_1^b} e^{-\kappa_1^b b^{po}}. \tag{25}$$

[6] To see this, write (24) as $\kappa_0^a\kappa_1^a e^{\kappa_1^a a^{po}} = h_1 L / (\theta_0 + \theta_1 a^{po})$ where the left-hand side is marginal cost and the right-hand side can be viewed as marginal private benefit. The first intersection, at $a^{po,l}$ in Figure 1b, involves marginal cost rising from below to meet marginal private benefit so that profit would fall upon a small unilateral deviation. The second intersection, at $a^{po,h}$ in Figure 1b, involves marginal benefit rising to meet marginal cost so that profit would rise upon a small unilateral deviation. The first intersection is locally stable to small perturbations whereas the second is not.

Similar to (24), a unique, strictly positive solution is assured whenever $g_1/\kappa_1^b \geq h_1/\kappa_1^a$ and $g_1L/(\kappa_0^b\kappa_1^b) > \rho + Nh_1 \ln[g_1\kappa_0^a\kappa_1^a/(h_1\kappa_0^b\kappa_1^b)]/\kappa_1^a - (g_1/\kappa_1^b - h_1/\kappa_1^a)$. Notice that the left-hand sides of (24) and (25) are both upward sloping in their respective actions whenever prevention is less responsive than stamp-out activities. Whether or not $g_1/\kappa_1^b \geq h_1/\kappa_1^a$, we can state from (24) and (25):

Result 5: If a stable solution exists in pure strategy then an increase in trade ban loss L increases the privately optimal levels of both prevention and stamp-out actions.

Given $V_{a_nL}^{S,n} \geq 0$ and $V_{b_nL}^{I,n} \geq 0$ in (22), Result 5 may not appear surprising. However, follow through $V_{b_nL}^{I,n} \geq 0$ to $V_{a_nb_j}^{S,n}|_{j\neq n} \leq 0$ in (21) to obtain an indirect contrary effect of L on prevention efforts that might conceivably have overwhelmed the direct effect. However, not so for this technology.

A further point to note about (24) and (25) is that in (24) both prevention cost parameters κ_0^a and κ_1^a have reinforcing impacts in that an increase in their magnitude decreases the value of the left-hand side and increase the value on the right-hand side. Thus, in Nash equilibrium an increase in prevention costs decreases private prevention activities. By contrast, in (25) eradication cost parameters κ_0^b and κ_1^b have contrary impacts in that an increase in their magnitude decreases the value of both sides of the equation. Private stamp-out efforts may increase with an increase in private costs incurred. This Nash equilibrium comparative static corresponds to the social first-best comparative static provided in Result 3 above, and the rationale is the same. An increase in any cost of being in the infected state, even the cost of trying to exit the state, may motivate growers to try all the harder to exit.

Next we turn to first-best solutions. In light of optimality condition set (10) and the functional forms given in (23) above, we arrive at

$$\rho+\frac{Ng_1\ln\left(\frac{g_1\kappa_0^a\kappa_1^a}{h_1\kappa_0^b\kappa_1^b}\right)}{\kappa_1^b}-\left(\frac{g_1}{\kappa_1^b}-\frac{h_1}{\kappa_1^a}\right)N+\left(\frac{g_1}{\kappa_1^b}-\frac{h_1}{\kappa_1^a}\right)N\kappa_1^a a^{so}=N\frac{h_1 L}{\kappa_0^a\kappa_1^a}e^{-\kappa_1^a a^{so}};$$
$$\rho+\frac{Nh_1\ln\left(\frac{g_1\kappa_0^a\kappa_1^a}{h_1\kappa_0^b\kappa_1^b}\right)}{\kappa_1^a}-\left(\frac{g_1}{\kappa_1^b}-\frac{h_1}{\kappa_1^a}\right)N+\left(\frac{g_1}{\kappa_1^b}-\frac{h_1}{\kappa_1^a}\right)N\kappa_1^b b^{so}=N\frac{g_1 L}{\kappa_0^b\kappa_1^b}e^{-\kappa_1^b b^{so}}. \tag{26}$$

It is apparent from (24)–(26) that when $g_1/\kappa_1^b \geq h_1/\kappa_1^a$, then both $a^{so} \geq a^{po}$ and $b^{so} \geq b^{po}$. Figure 2 illustrates a comparison between social optimum and Nash private optimum. In panel A, $\theta_1 \geq 0$ so prevention action is less responsive than stamp-out action. The line for the socially optimal level of prevention is lower than that for the Nash equilibrium level of prevention. Nash equilibrium does not emphasize prevention in part because it is comparatively ineffective. The exponentially decaying curve is higher due to the inclusion of *N* in (26) to acknowledge the public good effect of stamping out. Thus, the socially optimal level of stamp-out action must be larger than the privately optimal choice.

In panel b), $\theta_1 < 0$ ensures that the line is higher under social optimum. The exponentially decaying curve is also higher, as before. By itself, the upward line shift would decrease the optimal level of stamp-out action but the action's public good effect counters so it is conceivable that the socially optimal level of private stamp-out effort is smaller than the Nash optimal level of private stamp-out effort.

Result 6: Under the technologies provided in (23), whenever a is less responsive than b (i.e., $\kappa_1^a/h_1 > \kappa_1^b/g_1$) then the Nash optimal levels of private prevention and stamp-out efforts are both lower than the socially optimal levels.

What is most interesting here is the asymmetry between prevention and stamp-out activities. If prevention is comparatively less responsive then more of both actions should be taken than are taken. This is because the disease is managed primarily by stamp-out efforts.

From $V^{I,n}_{b_n b_j}|_{j\neq n} < 0$ in (20) and $V^{S,n}_{a_n b_j}|_{j\neq n} \leq 0$ in (21), we can conclude that comparatively high levels of stamp-out effort reduce the incentives of others to support stamp-out efforts and to prevent. If prevention is comparatively more effective than efforts to stamp-out then a comparison between socially optimal choices and privately optimal choices is less clear cut, as reflected in panel B of Figure 2.

Notice, too, in (24)–(26) that an increase in $\rho = r + \mu + g_0 + \hat{h} - h_0$ through any of an increase in interest rate, exogenous increase in farm value collapse, increase in public stamp-out effort, or decrease in public prevention effort will shift the value of θ_0 up. The equations together with the figures reveal the consequences.

Result 7: Under the technologies provided in (23), an increase in interest rate, Poisson value collapse intensity parameter or public stamp-out effort, or decrease in public prevention effort will decrease all of a^{po}, b^{po}, a^{so} and b^{so} (i.e., all of privately and socially optimal prevention and stamp-out efforts).

The reason is quite straightforward and we illustrate with reference to business collapse intensity. All efforts regard seeking to either continue in the susceptible state or attain the susceptible state. Costs of effort are immediate whereas benefits are deferred. When the probability of a business collapse not related to disease increases, then the decision calculus shifts toward cutting back on effort because the prospective benefit is less likely to materialize. A similar phenomenon arises in human capital formation. Oster, Shoulson, and Dorsey (2013) compare education decisions among individuals that were ex ante equally likely to be subject to a genetic mutation, giving rise to the fatal Huntington's disease. Their evidence shows that those who learn through a genetic test that they do have the causal mutation subsequently apply less effort to job training and education when compared with those who learn the converse.

Finally we ask what would be the impact of scaling up costs on Nash equilibrium solutions.

We consider (24)–(25) as a system. When only prevention costs are scaled up, as captured by an increase in κ_0^a, then neither $g_1 L/(\kappa_0^b \kappa_1^b)$ nor $g_1/\kappa_1^b - h_1/\kappa_1^a$ are affected. However, the value of $\ln[g_1\kappa_0^a\kappa_1^a/(h_1\kappa_0^b\kappa_1^b)]$ increases so that the value of the intercept on the linear term in (25) increases. We can conclude that the Nash equilibrium choice of private efforts to stamp out decreases in response to an increase in prevention cost. Similarly, if only κ_0^b increases then neither $h_1 L/(\kappa_0^a \kappa_1^a)\ \theta_2$ nor $g_1/\kappa_1^b - h_1/\kappa_1^a$ are affected while $\ln[g_1\kappa_0^a\kappa_1^a/(h_1\kappa_0^b\kappa_1^b)]$ decreases. The value of the intercept in (24) shifts down, leading to an increase in Nash equilibrium private prevention effort in response to an increase in the cost of stamping out.

Result 8: Let the technologies be as provided in (23). A subsidy on overall prevention effort cost, or parameter κ_0^a, increases the Nash equilibrium level of private stamp-out effort. A subsidy on overall stamp-out effort cost, or parameter κ_0^b, decreases the Nash equilibrium level of private prevention effort.

This result is consistent with socially optimal responses as identified in Result 2. It is also consistent with findings in Hennessy (2008), who considered prevention and cure efforts for a human disease where strategic dimensions to infection that arise from person-to-person infection were not accounted for. He found that prevention and cure efforts complement in the sense that an increase in prevention cost decreases cure effort. However, prevention and cure also substitute in the sense that an increase in cure cost increases prevention effort. The key to understanding these seemingly conflicting responses is to recognize that the actions are not taken in the same states of nature (i.e., one is taken when non-diseased while the other is taken when diseased). Here we show that the apparently inconsistent responses occur even when the private actions of other agents are recognized and accommodated.

Alternatively, suppose that both κ_0^a and κ_0^b decrease proportionately because policymakers seek to ensure that subsidies are balanced. Then nothing changes on the left-hand sides of (24) and (25). This is because $\ln(g_1(\lambda\kappa_0^a)\kappa_1^a / [h_1(\lambda\kappa_0^b)\kappa_1^b]) \equiv \ln(g_1\kappa_0^a\kappa_1^a / [h_1\kappa_0^b\kappa_1^b])$. However, the right-hand sides of both equations increase.

Result 9: Under the technologies given in (23), equal proportional subsidies on private prevention and stamp-out efforts increase Nash equilibrium levels of both sorts of effort.

Policy Implications

Notwithstanding the complexity of the disease management context, we are prepared to identify four policy implications from the analysis. These are:

Point 1: *A decision environment conducive to forward-planning matters when seeking to remain free of an infectious animal disease.*

We have seen from Result 7 that an exogenous threat to a business decreases incentives to prevent a disease and to stamp out any disease that enters. There is a strong correlation between the countries where infectious animal diseases are endemic and the quality of the country's governance structure. The OECD seeks to promote good governance among its comparatively wealth member countries. Among its members countries, only South Korea, Israel, and Turkey were not listed by the OIE as being FMD free in October 2013 whereas few other countries in the world were free of the disease (without vaccination). The correlation is unsurprising because wealthy countries can support strong public animal disease management infrastructure. In addition, sound public sector governance with objective performance-based criteria for promotion and curtailed opportunities for corruption is likely to support management choices that emphasize longer-run goals over political expediency or favoritism.

However, the correlation could also be due in part to concerns about other risks, such as the risk of a war, government land appropriation, unpredictable trade policies that might affect access to world markets, or monetary instability that could threaten firm valuations. Finally, the collapse intensity parameter can be taken as a stochastic asset transaction cost as would arise due to asset market frictions upon a sudden need to sell a business or an estate tax. The presence of such costs would adversely affect disease management incentives.

Point 2: *Public stamp-out efforts countenance moral hazard concerns that public prevention efforts do not.*

A reading of Result 8 that views public stamp-out efforts as something to be avoided would take the model too seriously as a depiction of the complex environment in which animal disease management occurs. The model treats public inputs as exogenous, but governments also respond to incentives including costs of exchequer funds and state-dependent political demands. It is likely that an analysis of endogenized public input choices would reveal some subtleties about how public state-contingent efforts interact when in the presence of private efforts that they might crowd out. These considerations are beyond the present work's scope.

Our stance is that a public commitment to be parsimonious in public stamp-out efforts would be viewed as incredible, or subgame imperfect, in light of political risks that a government might face once the state is realized. Consequently such an ex-ante commitment would be ineffective in encouraging ex-ante private prevention efforts. See, e.g., Innes' (2003) study of a similar problem that arises upon the event of a widespread crop disaster. But our model does point to the superiority of public prevention efforts over stamp-out efforts. Public prevention provides a three-fold return in that it directly reduces the probability of an event, indirectly encourages more private sector preventive effort, and indirectly increases private sector stamp-out efforts. Public stamp-out effort managers should pay acute regard to implications for private efforts. Independent policy instruments not considered in our model, such as legal prosecution, may allow a government to mitigate moral hazard concerns of public interventions in the diseased state.

Point 3: *An increase in the loss due to a disease outbreak will increase private incentives to prevent and to stamp-out while insurance indemnities will reduce these incentives.*

The possibility of a country trade ban should increase private incentives to prevent and to stamp-out. Although we have not shown that the finding applies beyond the specific technology studied, the technology's simplicity leaves us with a qualified assurance that the threat of a trade ban is likely to improve biosecurity incentives all round.[7] Other restrictions on sale of produce from a country afflicted with the disease, such as restrictions on sales within the country, are also likely to incentivize private prevention and stamp-out actions. The collective punishment for a disease break-down will encourage private prevention actions and the knowledge that private prevention activities will be high should stiffen resolves on the part of actors in the private sector to return to the uninfected state. The same response also implies that the presence of an indemnity program in the event of an outbreak will discourage such actions, a point addressed in Hennessy (2007), but in the narrower context of preventive actions only. If punishment for breakdown is low then the incentives to prevent are low so why try to stamp-out a disease that is quite likely to re-emerge?

As qualification to the above, it is recognized that issues not addressed in this model also matter. One is business continuity. Indemnities serve the purpose of providing a cash flow lifeline to growers at a critical time. These farmers may well be severely credit constrained because they would have had no time to prepare for the event and because extra feed and other costs may arise due movement controls during the disease outbreak. In addition, banks may be cautious to provide liquidity when the length of the crisis is unknown. A further consideration

[7] Acemoglu and Jensen (2013) have developed a systemic approach to aggregation games with strategic substitutes similar in spirit to the theory of games with strategic complementarities, as in Milgrom and Roberts (1990) and Milgrom and Shannon (1994). Our model fits loosely into the Acemoglu and Jensen framework and their approach may allow for a generalization of the technology set for which definite monotone responses can be obtained.

is the nature of the livestock industry's business support network. Cattle marts, feed mills and slaughter plants in a region may have large fixed operating costs and sufficient scale can be important to ensuring profitability. If some farms in the region do not survive then these input providing and produce using businesses may increase animal production input prices or decrease livestock prices, which may drive more growers out of the business.

Point 4: *Although caution is warranted when devising biosecurity action subsidy schemes, it is generally better to subsidize prevention efforts than subsidize stamp-out efforts.*

Within the confines of the context we have modeled, and referring to Results 8 and 9 in particular, a subsidy on private prevention efforts would likely better secure a country from a disease. Of course, other aspects of an input may also be relevant. Some inputs, such as better fences, better record keeping, and vaccination, are dual purpose in the sense that they serve to help in prevention and extermination. In addition, many animal agriculture sectors are small in scale and geographically dispersed, especially in less developed countries and also the beef sector in many developed countries. It may be very difficult to monitor the extent of some activities that a government might seek to subsidize.

There may also be some private inputs considered to be so critical to stamp-out endeavors that general point 4 should be ignored. A case that comes to mind is subsidies to facilitate the development of a livestock premises registry in a region confirmed to have a disease. However the general point is that, having sought to account for such operational concerns as monitoring and effectiveness, there is likely merit in tilting subsidies toward preventive efforts.

Concluding Comment

The management of defenses against a possible infectious animal disease incursion is an involved partnership between the public and private sectors. Inevitably uncertainty, dynamics, and strategic behavior inconsistent with the public good confront management. By providing a model that incorporates these complications in a tractable way, we see our work as a first effort at developing a more coherent perspective on appropriate roles for government in securing a country from animal disease entry. The approach is general and does not seek to address a particular intervention, such as subsidies on vaccination.

Adaption to the salient details of some particular context would shed light on the framework's potential. So too would an analysis of how disease transmission parameters affect incentives, a development that might be accomplished through parameterizing the relation between effort and state transition in the two state transition probability functions. Adaptation in that way would not involve much adjustment to the present framework. Possible uses are an exploration of how openness in farmed animal industries and how specific disease management practices such as vaccination affect equilibrium incentives and outcomes. In our view, perhaps the most interesting extension would be to step back and endogenize government efforts with the end of introducing political incentives. Endogenous government effort would allow for a better understanding of best public management of a virulent disease when mindful of own credibility and of grower responses to public choices.

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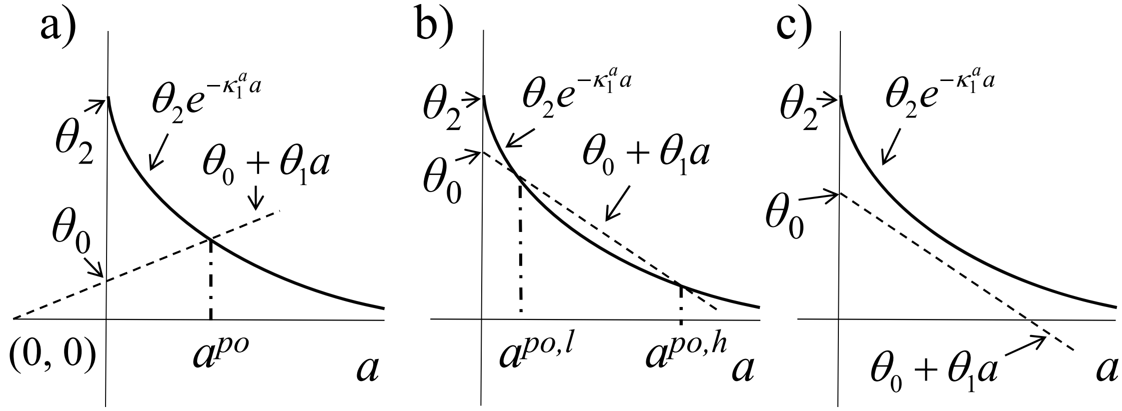

Figure 1. Nash equilibrium for prevention actions: a) $\theta_1 \geq 0$ and $0 < \theta_0 < \theta_2$; b) $\theta_1 < 0$ and $0 < \theta_0 < \theta_2$ such that there are two solutions; c) $\theta_1 < 0$ and $0 < \theta_0 \leq \theta_2$ such that there are no solutions.

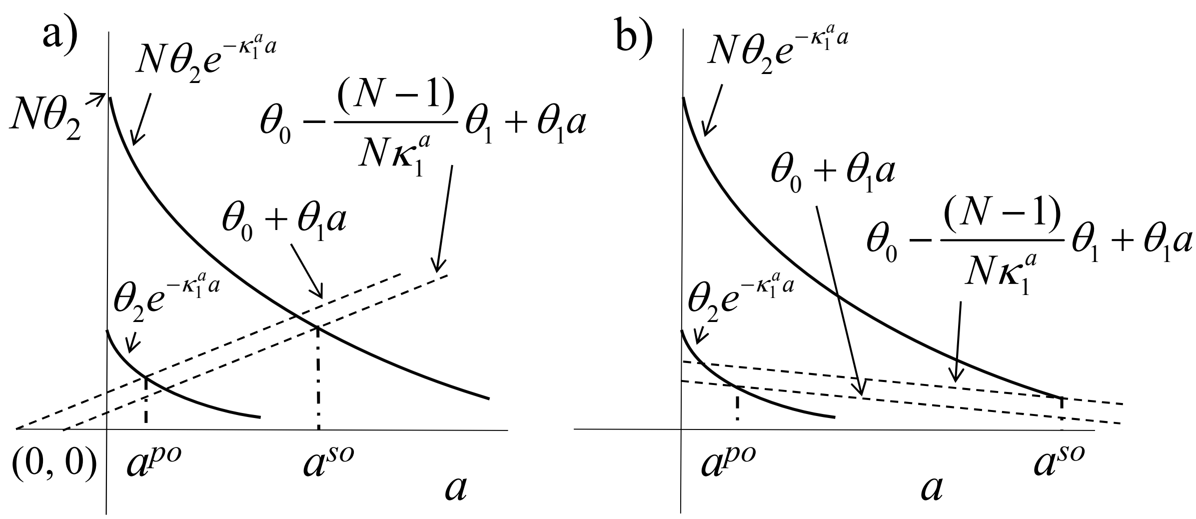

Figure 2. Comparison between social optimum and Nash private optimum level of prevention: a) when $\theta_1 \geq 0$; b) when $\theta_1 < 0$.

Supplemental Materials

Second-order condition for social optimum in susceptible and infected states:

Define $\Phi_a^S \equiv c_a^a(\cdot) + h_1 H'(\cdot) M^{sc}(\cdot)/\psi(\cdot)$ in (7) and $\Phi_b^I \equiv c_b^b(\cdot) - g_1 G'(\cdot) M^{sc}(\cdot)/\psi(\cdot)$ in (9) as the key terms of interest from the optimality conditions upon which to apply the implicit function theorem. Term $-[r+\mu+G(\cdot)]N/[r\psi(\cdot)]$ in (7) and $-[r+\mu+H(\cdot)]N/[(r+\mu)\psi(\cdot)]$ in (9) are both strictly negative so that $\Phi_a^S = \Phi_b^I = 0$. From (7) we have

$$\mathfrak{L}_{aa}^S = -\Phi_a^S \frac{d}{da}\left(\frac{[r+\mu+G(\cdot)]N}{(r+\mu)\psi(\cdot)}\right) - \frac{[r+\mu+G(\cdot)]N}{(r+\mu)\psi(\cdot)}\Phi_{aa}^S. \qquad \text{(A1)}$$

Upon applying the first-order condition, the first right-hand expression falls out so that

$$\mathfrak{L}_{aa}^S = -\frac{[r+\mu+G(\cdot)]N}{(r+\mu)\psi(\cdot)}\left\{c_{aa}^a(\cdot) + Nh_1^2 M^{sc}(\cdot)\frac{\psi(\cdot)H''(\cdot)-[H'(\cdot)]^2}{[\psi(\cdot)]^2} - \frac{Nh_1 H'(\cdot)c_a^a(\cdot)}{\psi(\cdot)}\right\}. \qquad \text{(A2)}$$

Insert first-order condition (7) for $c_a^a(\cdot)$ to obtain:

$$\begin{aligned}\mathfrak{L}_{aa}^S &= -\frac{[r+\mu+G(\cdot)]N}{(r+\mu)\psi(\cdot)}\left\{c_{aa}^a(\cdot) + Nh_1^2 M^{sc}(\cdot)\frac{\psi(\cdot)H''(\cdot)-[H'(\cdot)]^2}{[\psi(\cdot)]^2} + \frac{Nh_1 H'(\cdot)}{\psi(\cdot)}\frac{h_1 H'(\cdot)M^{sc}(\cdot)}{\psi(\cdot)}\right\} \\ &= -\frac{[r+\mu+G(\cdot)]N}{(r+\mu)\psi(\cdot)}\left\{c_{aa}^a(\cdot) + \frac{Nh_1^2 H''(\cdot)M^{sc}(\cdot)}{\psi(\cdot)}\right\} < 0.\end{aligned} \qquad \text{(A3)}$$

The last inequality follows because $c_{aa}^a(\cdot) > 0$, $H''(\cdot) \geq 0$ and $M^{sc}(\cdot) > 0$.

From (9) we have

$$\mathfrak{L}_{bb}^I = -\Phi_b^I \frac{d}{db}\left(\frac{[r+\mu+H(\cdot)]N}{(r+\mu)\psi(\cdot)}\right) - \frac{[r+\mu+H(\cdot)]N}{(r+\mu)\psi(\cdot)}\Phi_{bb}^I. \qquad \text{(A4)}$$

Upon applying first-order condition (9), the first right-hand expression falls out so that

$$\mathfrak{L}_{bb}^I = -\frac{[r+\mu+H(\cdot)]N}{(r+\mu)\psi(\cdot)}\left\{c_{bb}^b(\cdot) - Ng_1^2 M^{sc}(\cdot)\frac{\psi(\cdot)G''(\cdot)-[G'(\cdot)]^2}{[\psi(\cdot)]^2} - \frac{Ng_1 G'(\cdot)c_b^b(\cdot)}{\psi(\cdot)}\right\}. \qquad \text{(A5)}$$

Insert the first-order condition for $c_b^b(\cdot)$ to obtain:

$$\mathfrak{L}^I_{bb} = -\frac{[r+\mu+H(\cdot)]N}{(r+\mu)\psi(\cdot)}\left\{c^b_{bb}(\cdot) - Ng_1^2 M^{sc}(\cdot)\frac{\psi(\cdot)G''(\cdot)-[G'(\cdot)]^2}{[\psi(\cdot)]^2} - \frac{Ng_1G'(\cdot)}{\psi(\cdot)}\frac{g_1G'(\cdot)M^{sc}(\cdot)}{\psi(\cdot)}\right\}$$

$$= -\frac{[r+\mu+H(\cdot)]N}{(r+\mu)\psi(\cdot)}\left\{c^b_{bb}(\cdot) - \frac{Ng_1^2G''(\cdot)M^{sc}(\cdot)}{\psi(\cdot)}\right\} < 0. \tag{A6}$$

The last inequality follows because $c^b_{bb}(\cdot) > 0$, $G''(\cdot) \le 0$ and $M^{sc}(\cdot) > 0$.

Next, from (7) and (9) we have

$$\mathfrak{L}^S_{ab} = -\Phi^S_a \frac{N}{(r+\mu)}\frac{d}{db}\left(\frac{[r+\mu+G(\cdot)]}{\psi(\cdot)}\right) - \frac{[r+\mu+G(\cdot)]N}{(r+\mu)\psi(\cdot)}\Phi^S_{ab}$$

$$= -\frac{[r+\mu+G(\cdot)]N}{(r+\mu)\psi(\cdot)}\Phi^S_{ab} = -\frac{[r+\mu+G(\cdot)]N}{(r+\mu)\psi(\cdot)}\left[\frac{Nh_1H'(\cdot)c^b_b(\cdot)}{\psi(\cdot)} - \frac{Ng_1h_1G'(\cdot)H'(\cdot)M^{sc}(\cdot)}{[\psi(\cdot)]^2}\right] = 0; \tag{A7}$$

$$\mathfrak{L}^I_{ba} = -\Phi^I_b \frac{N}{(r+\mu)}\frac{d}{da}\left(\frac{[r+\mu+H(\cdot)]}{\psi(\cdot)}\right) - \frac{[r+\mu+H(\cdot)]N}{(r+\mu)\psi(\cdot)}\Phi^I_{ba}$$

$$= -\frac{[r+\mu+H(\cdot)]N}{(r+\mu)\psi(\cdot)}\Phi^I_{ba} = -\frac{[r+\mu+H(\cdot)]N}{(r+\mu)\psi(\cdot)}\left[\frac{G'(\cdot)H'(\cdot)g_1h_1NM^{sc}(\cdot)}{[\psi(\cdot)]^2} + \frac{Ng_1G'(\cdot)c^a_a(\cdot)}{\psi(\cdot)}\right] = 0. \tag{A8}$$

Socially optimal responses to endowments:

From the social optimality conditions (10) together with some calculations, we have

$$\Phi^S_{aa}da^{so} = -\frac{Nh_1H'(\cdot)}{\psi(\cdot)}dL \Rightarrow \frac{da^{so}}{dL} = -\frac{Nh_1H'(\cdot)}{\psi(\cdot)\Phi^S_{aa}} > 0. \tag{A9}$$

$$\Phi^I_{bb}db^{so} = \frac{Ng_1G'(\cdot)}{\psi(\cdot)}dL \Rightarrow \frac{db^{so}}{dL} = \frac{Ng_1G'(\cdot)}{\psi(\cdot)\Phi^I_{bb}} > 0. \tag{A10}$$

$$\Phi^S_{aa}da^{so} = \frac{h_1G'(\cdot)H'(\cdot)M^{sc}(\cdot)}{[\psi(\cdot)]^2}dg_0 \Rightarrow \frac{da^{so}}{dg_0} = \frac{h_1G'(\cdot)H'(\cdot)M^{sc}(\cdot)}{[\psi(\cdot)]^2\Phi^S_{aa}} < 0. \tag{A11}$$

$$\Phi^I_{bb}db^{so} = -\frac{g_1G'(\cdot)H'(\cdot)M^{sc}(\cdot)}{[\psi(\cdot)]^2}dh_0 \Rightarrow \frac{db^{so}}{dh_0} = -\frac{g_1G'(\cdot)H'(\cdot)M^{sc}(\cdot)}{[\psi(\cdot)]^2\Phi^I_{bb}} > 0. \tag{A12}$$

$$\Phi^S_{aa} da^{so} = -h_1 M^{sc}(\cdot)\frac{\psi(\cdot)H''(\cdot) - [H'(\cdot)]^2}{[\psi(\cdot)]^2} dh_0$$

$$\Rightarrow \frac{da^{so}}{dh_0} = -h_1 M^{sc}(\cdot)\frac{\psi(\cdot)H''(\cdot) - [H'(\cdot)]^2}{[\psi(\cdot)]^2 \Phi^S_{aa}}$$

$$\Rightarrow \frac{d(h_0 + Nh_1 a^{so})}{dh_0} = 1 - \frac{Nh_1^2 M^{sc}(\cdot)\left\{\psi(\cdot)H''(\cdot) - [H'(\cdot)]^2\right\}}{[\psi(\cdot)]^2 \Phi^S_{aa}} \tag{A13}$$

$$= 1 - \frac{Nh_1^2 M^{sc}(\cdot)\left\{\psi(\cdot)H''(\cdot) - [H'(\cdot)]^2\right\}}{[\psi(\cdot)]^2 \left[c^a_{aa}(\cdot) + \frac{Nh_1^2 H''(\cdot) M^{sc}(\cdot)}{\psi(\cdot)}\right]}$$

$$= \frac{c^a_{aa}(\cdot)[\psi(\cdot)]^2 + Nh_1^2 M^{sc}(\cdot)[H'(\cdot)]^2}{[\psi(\cdot)]^2 \Phi^S_{aa}} > 0.$$

$$\Phi^I_{bb} db^{so} = \frac{\left\{\psi(\cdot)G''(\cdot) - [G'(\cdot)]^2\right\} g_1 M^{sc}(\cdot)}{[\psi(\cdot)]^2} dg_0$$

$$\Rightarrow \frac{db^{so}}{dg_0} = \frac{\left\{\psi(\cdot)G''(\cdot) - [G'(\cdot)]^2\right\} g_1 M^{sc}(\cdot)}{[\psi(\cdot)]^2 \Phi^I_{bb}} < 0$$

$$\Rightarrow \frac{d(g_0 + Ng_1 b^{so})}{dg_0} = 1 + \frac{\left\{\psi(\cdot)G''(\cdot) - [G'(\cdot)]^2\right\} Ng_1^2 M^{sc}(\cdot)}{[\psi(\cdot)]^2 \Phi^I_{bb}} \tag{A14}$$

$$= \frac{\psi(\cdot)\left[c^b_{bb}(\cdot)\psi(\cdot) - Ng_1^2 G''(\cdot) M^{sc}(\cdot)\right] + Ng_1^2 M^{sc}(\cdot)\left\{\psi(\cdot)G''(\cdot) - [G'(\cdot)]^2\right\}}{\psi(\cdot)\left[c^b_{bb}(\cdot)\psi(\cdot) - Ng_1^2 G''(\cdot) M^{sc}(\cdot)\right]}$$

$$= \frac{c^b_{bb}(\cdot)[\psi(\cdot)]^2 - Ng_1^2 [G'(\cdot)]^2 M^{sc}(\cdot)}{\psi(\cdot)\left[c^b_{bb}(\cdot)\psi(\cdot) - Ng_1^2 G''(\cdot) M^{sc}(\cdot)\right]}.$$

$$\Phi^S_{aa} da^{so} = \left[\frac{h_1 H'(\cdot) N c^a_\theta(\cdot)}{\psi(\cdot)} - c^a_{a\theta}(\cdot)\right] d\theta \Rightarrow \frac{da^{so}}{d\theta} = \frac{h_1 H'(\cdot) c^a_\theta(\cdot) N - \psi(\cdot) c^a_{a\theta}(\cdot)}{\psi(\cdot)\Phi^S_{aa}} \leq 0 \tag{A15}$$

whenever both $c^a_\theta(\cdot) \geq 0$ and $c^a_{a\theta}(\cdot) \geq 0$.

$$\Phi^S_{aa} da^{so} = -\frac{h_1 H'(\cdot) N c^b_\tau(\cdot)}{\psi(\cdot)} d\tau \Rightarrow \frac{da^{so}}{d\tau} = -\frac{h_1 H'(\cdot) N c^b_\tau(\cdot)}{\psi(\cdot)\Phi^S_{aa}} \geq 0 \tag{A16}$$

whenever $c^b_\tau(\cdot) \geq 0$.

$$\Phi_{bb}^{I} db^{so} = \left[\frac{g_1 G'(\cdot) N c_{\tau}^{b}(\cdot)}{\psi(\cdot)} - c_{b\tau}^{b}(\cdot) \right] d\tau \Rightarrow \frac{db^{so}}{d\tau} = \frac{g_1 G'(\cdot) N c_{\tau}^{b}(\cdot) - \psi(\cdot) c_{b\tau}^{b}(\cdot)}{\psi(\cdot)\Phi_{bb}^{I}}. \qquad \text{(A17)}$$

$$\Phi_{bb}^{I} db^{so} = -\frac{g_1 G'(\cdot) N c_{\theta}^{a}(\cdot)}{\psi(\cdot)} d\theta \Rightarrow \frac{db^{so}}{d\theta} = -\frac{g_1 G'(\cdot) N c_{\theta}^{a}(\cdot)}{\psi(\cdot)\Phi_{bb}^{I}} \leq 0 \qquad \text{(A18)}$$

whenever $c_{\theta}^{a}(\cdot) \geq 0$.

Uniqueness of any pure-strategy equilibrium

The two derivatives are

$$V_{a_n A_{\setminus n}}^{S,n} = -\frac{[r+\mu+G(\cdot)] h_1^2 M^{pc,n}(\cdot)}{(r+\mu)\psi(\cdot)} \left\{ \frac{\psi(\cdot) H''(\cdot) - [H'(\cdot)]^2}{[\psi(\cdot)]^2} \right\}; \qquad \text{(A19)}$$

$$\begin{aligned} V_{a_n a_n}^{S,n} &= -\frac{[r+\mu+G(\cdot)]}{(r+\mu)\psi(\cdot)} \left\{ c_{aa}^{a}(\cdot) + \frac{h_1^2 H''(\cdot) M^{pc,n}(\cdot) - c_a^a(\cdot) h_1 H'(\cdot)}{\psi(\cdot)} - \frac{h_1^2 [H'(\cdot)]^2 M^{pc,n}(\cdot)}{[\psi(\cdot)]^2} \right\} \\ &= -\frac{[r+\mu+G(\cdot)]}{(r+\mu)\psi(\cdot)} \left\{ c_{aa}^{a}(\cdot) + h_1^2 M^{pc,n}(\cdot) \frac{\psi(\cdot)\left\{ H''(\cdot) + \frac{[H'(\cdot)]^2}{\psi(\cdot)} \right\} - [H'(\cdot)]^2}{[\psi(\cdot)]^2} \right\} \qquad \text{(A20)} \\ &= -\frac{[r+\mu+G(\cdot)]}{(r+\mu)\psi(\cdot)} \left\{ c_{aa}^{a}(\cdot) + \frac{h_1^2 H''(\cdot) M^{pc,n}(\cdot)}{\psi(\cdot)} \right\} < 0; \end{aligned}$$

so that the ratio is

$$\begin{aligned} -\frac{V_{a_n A_{\setminus n}}^{S,n}}{V_{a_n a_n}^{S,n}} &= -\frac{-\frac{[r+\mu+G(\cdot)] h_1^2 M^{pc,n}(\cdot)}{(r+\mu)\psi(\cdot)} \left\{ \frac{\psi(\cdot) H''(\cdot) - [H'(\cdot)]^2}{[\psi(\cdot)]^2} \right\}}{-\frac{[r+\mu+G(\cdot)]}{(r+\mu)\psi(\cdot)} \left[c_{aa}^{a}(\cdot) + \frac{h_1^2 H''(\cdot) M^{pc,n}(\cdot)}{\psi(\cdot)} \right]} \qquad \text{(A21)} \\ &= -h_1^2 M^{pc,n}(\cdot) \frac{\psi(\cdot) H''(\cdot) - [H'(\cdot)]^2}{\psi(\cdot)\left[c_{aa}^{a}(\cdot)\psi(\cdot) + h_1^2 H''(\cdot) M^{pc,n}(\cdot) \right]}. \end{aligned}$$

Now

$$-\frac{V^{S,n}_{a_n A_{\setminus n}}}{V^{S,n}_{a_n a_n}} < 0 \Leftrightarrow [H'(\cdot)]^2 < \psi(\cdot)H''(\cdot) \tag{A22}$$

which is certainly true if $H(\cdot)$ is log-convex, or $d^2 \ln[H(\cdot)]/dh_0^2 \geq 0$ so that it would apply were $H(h_0 + h_1 A) = \lambda_0 e^{-\lambda_1(h_0+h_1 A)}, \lambda_0 > 0, \lambda_1 > 0$. Furthermore,

$$\begin{aligned} -\frac{V^{S,n}_{a_n A_{\setminus n}}}{V^{S,n}_{a_n a_n}} > -1 &\Rightarrow -h_1^2 M^{pc,n}(\cdot)\frac{\psi(\cdot)H''(\cdot) - [H'(\cdot)]^2}{\psi(\cdot)\left[c^a_{aa}(\cdot)\psi(\cdot) + h_1^2 H''(\cdot)M^{pc,n}(\cdot)\right]} > -1 \\ &\Rightarrow -h_1^2 M^{pc,n}(\cdot)[H'(\cdot)]^2 < [\psi(\cdot)]^2 c^a_{aa}(\cdot), \end{aligned} \tag{A23}$$

which is certainly true.

Also,

$$V^{I,n}_{b_n B_{\setminus n}} = \frac{[r+\mu+H(\cdot)]g_1^2 M^{pc,n}(\cdot)}{(r+\mu)\psi(\cdot)}\left\{\frac{\psi(\cdot)G''(\cdot) - [G'(\cdot)]^2}{[\psi(\cdot)]^2}\right\}; \tag{A24}$$

while

$$\begin{aligned} V^{I,n}_{b_n b_n} &= -\frac{[r+\mu+H(\cdot)]}{(r+\mu)\psi(\cdot)}\left\{c^b_{bb}(\cdot) - \frac{g_1 G'(\cdot)c^b_b(\cdot)}{\psi(\cdot)} - \frac{g_1^2 M^{pc,n}(\cdot)\left\{\psi(\cdot)G''(\cdot) - [G'(\cdot)]^2\right\}}{[\psi(\cdot)]^2}\right\} \\ &= -\frac{[r+\mu+H(\cdot)]}{(r+\mu)\psi(\cdot)}\left\{c^b_{bb}(\cdot) - \frac{\dfrac{g_1^2[G'(\cdot)]^2 M^{pc,n}(\cdot)}{\psi(\cdot)}}{\psi(\cdot)} - \frac{g_1^2 M^{pc,n}(\cdot)\left\{\psi(\cdot)G''(\cdot) - [G'(\cdot)]^2\right\}}{[\psi(\cdot)]^2}\right\} \\ &= -\frac{[r+\mu+H(\cdot)]}{(r+\mu)\psi(\cdot)}\left[c^b_{bb}(\cdot) - \frac{g_1^2 G''(\cdot)M^{pc,n}(\cdot)}{\psi(\cdot)}\right] < 0; \end{aligned} \tag{A25}$$

and so

$$-\frac{V^{I,n}_{b_nB_{\backslash n}}}{V^{I,n}_{b_nb_n}} = -\frac{\dfrac{[r+\mu+H(\cdot)]g_1^2M^{pc,n}(\cdot)}{(r+\mu)\psi(\cdot)}\left\{\dfrac{\psi(\cdot)G''(\cdot)-[G'(\cdot)]^2}{[\psi(\cdot)]^2}\right\}}{-\dfrac{[r+\mu+H(\cdot)]}{(r+\mu)\psi(\cdot)}\left[c^b_{bb}(\cdot)-\dfrac{g_1^2G''(\cdot)M^{pc,n}(\cdot)}{\psi(\cdot)}\right]} \tag{A26}$$

$$= \frac{g_1^2M^{pc,n}(\cdot)\left\{\psi(\cdot)G''(\cdot)-[G'(\cdot)]^2\right\}}{\psi(\cdot)\left[c^b_{bb}(\cdot)\psi(\cdot)-g_1^2G''(\cdot)M^{pc,n}(\cdot)\right]}.$$

Now

$$-\frac{V^{I,n}_{b_nB_{\backslash n}}}{V^{I,n}_{b_nb_n}} < 0 \Leftrightarrow \psi(\cdot)G''(\cdot) < [G'(\cdot)]^2, \tag{A27}$$

which is certainly true if $G(\cdot)$ is log-concave, or $d^2\ln[G(\cdot)]/dg_0^2 \le 0$ so that it would apply were $G(g_0+g_1B)=\hat{\lambda}-\lambda_0e^{-\lambda_1(g_0+g_1B)}, \lambda_0>0, \lambda_1>0$. Furthermore,

$$-\frac{V^{I,n}_{b_nB_{\backslash n}}}{V^{I,n}_{b_nb_n}} > -1 \Rightarrow \frac{g_1^2M^{pc,n}(\cdot)\left\{\psi(\cdot)G''(\cdot)-[G'(\cdot)]^2\right\}}{\psi(\cdot)\left[c^b_{bb}(\cdot)\psi(\cdot)-g_1^2G''(\cdot)M^{pc,n}(\cdot)\right]} > -1$$

$$\Rightarrow -g_1^2[G'(\cdot)]^2M^{pc,n}(\cdot) > -c^b_{bb}(\cdot)[\psi(\cdot)]^2$$

$$\Rightarrow \left(\frac{g_1G'(\cdot)}{\psi(\cdot)}\right)^2 M^{pc,n}(\cdot) < c^b_{bb}(\cdot) \tag{A28}$$

$$\Rightarrow \left(\frac{g_1G'(\cdot)}{\psi(\cdot)}\right)^2 \frac{c^b_b(\cdot)\psi(\cdot)}{g_1G'(\cdot)} < c^b_{bb}(\cdot) \quad \text{(from Nash optimality)}$$

$$\Rightarrow \frac{g_1b^{po}_nG'(\cdot)}{\psi(\cdot)} < \frac{b^{po}_nc^b_{bb}(\cdot)}{c^b_b(\cdot)}.$$

Cross-action derivatives

$$V^{S,n}_{a_nb_n} = -\frac{[r+\mu+G(\cdot)]}{(r+\mu)\psi(\cdot)}\frac{d}{db_n}\left[c^a_a(\cdot)+\frac{h_1H'(\cdot)M^{pc,n}(\cdot)}{\psi(\cdot)}\right]$$

$$= -\frac{[r+\mu+G(\cdot)]h_1}{(r+\mu)\psi(\cdot)}\left[\frac{H'(\cdot)}{\psi(\cdot)}c^b_b(\cdot)+M^{pc,n}(\cdot)\frac{d}{db_n}\frac{H'(\cdot)}{\psi(\cdot)}\right] \tag{A29}$$

$$= -\frac{[r+\mu+G(\cdot)]h_1}{(r+\mu)\psi(\cdot)}\left[\frac{H'(\cdot)}{\psi(\cdot)}c^b_b(\cdot)-\frac{g_1G'(\cdot)H'(\cdot)M^{pc,n}(\cdot)}{[\psi(\cdot)]^2}\right]=0;$$

$$V^{I,n}_{a_n b_n} = -\frac{[r+\mu+H(\cdot)]}{(r+\mu)\psi(\cdot)}\frac{d}{da_n}\left[c^b_b(\cdot) - \frac{g_1 G'(\cdot)M^{pc,n}(\cdot)}{\psi(\cdot)}\right]$$

$$= \frac{[r+\mu+H(\cdot)]g_1}{(r+\mu)\psi(\cdot)}\left[\frac{G'(\cdot)}{\psi(\cdot)}c^a_a(\cdot) - M^{pc,n}(\cdot)\frac{d}{da_n}\frac{G'(\cdot)}{\psi(\cdot)}\right] \quad \text{(A30)}$$

$$= \frac{[r+\mu+H(\cdot)]g_1}{(r+\mu)\psi(\cdot)}\left[\frac{G'(\cdot)}{\psi(\cdot)}c^a_a(\cdot) + \frac{h_1 G'(\cdot)H'(\cdot)M^{pc,n}(\cdot)}{[\psi(\cdot)]^2}\right] = 0;$$

$$V^{S,n}_{a_n a_j}|_{j\neq n} = -\frac{[r+\mu+G(\cdot)]}{(r+\mu)\psi(\cdot)}\frac{d}{da_j}\left[c^a_a(\cdot) + \frac{h_1 H'(\cdot)M^{pc,n}(\cdot)}{\psi(\cdot)}\right]$$

$$= -\frac{[r+\mu+G(\cdot)]h_1 M^{pc,n}(\cdot)}{(r+\mu)\psi(\cdot)}\frac{d}{da_j}\frac{H'(\cdot)}{\psi(\cdot)} \quad \text{(A31)}$$

$$= -\frac{[r+\mu+G(\cdot)]h_1^2 M^{pc,n}(\cdot)}{(r+\mu)\psi(\cdot)}\left\{\frac{\psi(\cdot)H''(\cdot) - [H'(\cdot)]^2}{[\psi(\cdot)]^2}\right\};$$

$$V^{I,n}_{b_n b_j}|_{j\neq n} = -\frac{[r+\mu+H(\cdot)]}{(r+\mu)\psi(\cdot)}\frac{d}{db_j}\left[c^b_b(\cdot) - \frac{g_1 G'(\cdot)M^{pc,n}(\cdot)}{\psi(\cdot)}\right]$$

$$= \frac{[r+\mu+H(\cdot)]g_1 M^{pc,n}(\cdot)}{(r+\mu)\psi(\cdot)}\frac{d}{db_j}\frac{G'(\cdot)}{\psi(\cdot)} \quad \text{(A32)}$$

$$= \frac{[r+\mu+H(\cdot)]g_1^2 M^{pc,n}(\cdot)}{(r+\mu)\psi(\cdot)}\left\{\frac{\psi(\cdot)G''(\cdot) - [G'(\cdot)]^2}{[\psi(\cdot)]^2}\right\} < 0.$$

$$V^{S,n}_{a_n b_j}|_{j\neq n} = \frac{[r+\mu+G(\cdot)]g_1 h_1 G'(\cdot)H'(\cdot)M^{pc,n}(\cdot)}{(r+\mu)[\psi(\cdot)]^3} \leq 0;$$

$$V^{I,n}_{b_n a_j}|_{j\neq n} = -\frac{[r+\mu+H(\cdot)]g_1 h_1 G'(\cdot)H'(\cdot)M^{pc,n}(\cdot)}{(r+\mu)[\psi(\cdot)]^3} \geq 0. \quad \text{(A33)}$$

$$V^{S,n}_{a_n \theta} = -\frac{[r+\mu+G(\cdot)]}{(r+\mu)\psi(\cdot)}\left[c^a_{a\theta}(\cdot) - \frac{h_1 H'(\cdot)c^a_\theta(\cdot)}{\psi(\cdot)}\right] \leq 0;$$

$$V^{S,n}_{a_n L} = -\frac{[r+\mu+G(\cdot)]h_1 H'(\cdot)}{(r+\mu)[\psi(\cdot)]^2} \geq 0; \quad \text{(A34)}$$

$$V^{S,n}_{a_n \tau} = -\frac{[r+\mu+G(\cdot)]h_1 H'(\cdot)c^b_\tau(\cdot)}{(r+\mu)[\psi(\cdot)]^2} \geq 0.$$

$$V^{I,n}_{b_n\theta} = -\frac{[r+\mu+H(\cdot)]g_1 G'(\cdot)c^a_\theta(\cdot)}{(r+\mu)[\psi(\cdot)]^2} \le 0;$$

$$V^{I,n}_{b_n L} = \frac{[r+\mu+H(\cdot)]g_1 G'(\cdot)}{(r+\mu)[\psi(\cdot)]^2} \ge 0; \qquad \text{(A35)}$$

$$V^{I,n}_{b_n\tau} = -\frac{[r+\mu+H(\cdot)]}{(r+\mu)\psi(\cdot)}\left[c^b_{b\tau}(\cdot) - \frac{g_1 G'(\cdot)c^b_\tau}{\psi(\cdot)}\right].$$

Algebra in example in the case of Nash equilibrium:

Under the symmetric solution we drop the agent identification subscript. We let $\rho =$ $r+\mu+g_0+\hat{h}-h_0$ and we can conclude that, in Nash equilibrium, (17) becomes

$$\kappa^a_0\kappa^a_1 e^{\kappa^a_1 a^{po}} = \frac{h_1(L+\kappa^b_0 e^{\kappa^b_1 b^{po}} - \kappa^a_0 e^{\kappa^a_1 a^{po}})}{\rho + g_1 N b^{po} - h_1 N a^{po}};$$

$$\kappa^b_0\kappa^b_1 e^{\kappa^b_1 b^{po}} = \frac{g_1(L+\kappa^b_0 e^{\kappa^b_1 b^{po}} - \kappa^a_0 e^{\kappa^a_1 a^{po}})}{\rho + g_1 N b^{po} - h_1 N a^{po}}. \qquad \text{(A36)}$$

Taking ratios and then logging, we arrive at $b^{po} = [\kappa^a_1 a^{po} + \ln(\delta)]/\kappa^b_1$ where $\delta =$ $g_1\kappa^a_0\kappa^a_1/(h_1\kappa^b_0\kappa^b_1)$. Also, (A36) allows us to assert that $e^{\kappa^b_1 b^{po}} = \delta e^{\kappa^a_1 a^{po}}$ and so

$$\kappa^a_0\kappa^a_1 e^{\kappa^a_1 a^{po}} = \frac{h_1}{\rho + g_1 B^{po} - h_1 A^{po}}\left[L + \frac{(g_1\kappa^a_1 - h_1\kappa^b_1)\kappa^a_0}{h_1\kappa^b_1} e^{\kappa^a_1 a^{po}}\right], \qquad \text{(A37)}$$

so that

$$e^{\kappa^a_1 a^{po}} = \frac{h_1/(\kappa^a_0\kappa^a_1)}{\rho + N g_1 \frac{\kappa^a_1 a^{po} + \ln(\delta)}{\kappa^b_1} - h_1 N a^{po}}\left[L + \frac{(g_1\kappa^a_1 - h_1\kappa^b_1)\kappa^a_0}{h_1\kappa^b_1} e^{\kappa^a_1 a^{po}}\right]; \qquad \text{(A38)}$$

Now write $\theta_1 = (g_1\kappa^a_1 - h_1\kappa^b_1)N/\kappa^b_1$ and $\theta_2 = h_1 L/(\kappa^a_0\kappa^a_1)$ so that (A38) becomes

$$\left[1 - \frac{(g_1\kappa^a_1 - h_1\kappa^b_1)/(\kappa^a_1\kappa^b_1)}{\rho + \theta_1 a^{po} + \frac{N g_1 \ln(\delta)}{\kappa^b_1}}\right] e^{\kappa^a_1 a^{po}} = \frac{\theta_2}{\rho + \theta_1 a^{po} + \frac{N g_1 \ln(\delta)}{\kappa^b_1}}; \qquad \text{(A39)}$$

It follows that

$$\left[\rho+\theta_1 a^{po}+\frac{Ng_1\ln(\delta)}{\kappa_1^b}-\frac{(g_1\kappa_1^a-h_1\kappa_1^b)}{\kappa_1^a\kappa_1^b}\right]e^{\kappa_1^a a^{po}}=\theta_2. \tag{A40}$$

Therefore we may write

$$\rho+\frac{Ng_1\ln(\delta)}{\kappa_1^b}-\left(\frac{g_1}{\kappa_1^b}-\frac{h_1}{\kappa_1^a}\right)+\left(\frac{g_1}{\kappa_1^b}-\frac{h_1}{\kappa_1^a}\right)N\kappa_1^a a^{po}=\frac{h_1 L}{\kappa_0^a\kappa_1^a}e^{-\kappa_1^a a^{po}}. \tag{A41}$$

A strictly positive and unique solution is ensured whenever $g_1/\kappa_1^b \geq h_1/\kappa_1^a$ and $h_1 L/(\kappa_0^a\kappa_1^a) > \rho + Ng_1\ln(\delta)/\kappa_1^b - (g_1/\kappa_1^b - h_1/\kappa_1^a)$.

Insert $a^{po}=[\kappa_1^b b^{po}-\ln(\delta)]/\kappa_1^a$ into (A41) to obtain

$$\rho+\frac{Nh_1\ln(\delta)}{\kappa_1^a}-\left(\frac{g_1}{\kappa_1^b}-\frac{h_1}{\kappa_1^a}\right)+\left(\frac{g_1}{\kappa_1^b}-\frac{h_1}{\kappa_1^a}\right)N\kappa_1^b b^{po}=\frac{g_1 L}{\kappa_0^b\kappa_1^b}e^{-\kappa_1^b b^{po}}. \tag{A42}$$

Here too a strictly positive and unique solution is assured whenever $g_1/\kappa_1^b \geq h_1/\kappa_1^a$ and $g_1 L/(\kappa_0^b\kappa_1^b) > \rho + Nh_1\ln(\delta)/\kappa_1^a - (g_1/\kappa_1^b - h_1/\kappa_1^a)$.

Algebra in example in the case of Social optimum:

Next we turn to first-best solutions. In light of optimality conditions (10) and the functional forms given in (23) above, we have social optimality conditions

$$\begin{aligned}\kappa_0^a\kappa_1^a e^{\kappa_1^a a^{so}} &= \frac{Nh_1(L+\kappa_0^b e^{\kappa_1^b b^{so}}-\kappa_0^a e^{\kappa_1^a a^{so}})}{\rho+g_1 Nb^{so}-h_1 Na^{so}};\\ \kappa_0^b\kappa_1^b e^{\kappa_1^b b^{so}} &= \frac{Ng_1(L+\kappa_0^b e^{\kappa_1^b b^{so}}-\kappa_0^a e^{\kappa_1^a a^{so}})}{\rho+g_1 Nb^{so}-h_1 Na^{so}}.\end{aligned} \tag{A43}$$

Therefore $e^{\kappa_1^b b^{so}}=\delta e^{\kappa_1^a a^{so}}$ as before, and

$$\kappa_0^a\kappa_1^a e^{\kappa_1^a a^{so}}=\frac{Nh_1}{\rho+g_1 Nb^{so}-h_1 Na^{so}}\left[L+(\kappa_0^b\delta-\kappa_0^a)e^{\kappa_1^a a^{so}}\right]; \tag{A44}$$

and

$$e^{\kappa_1^a a^{so}} = \frac{Nh_1 / (\kappa_0^a \kappa_1^a)}{\rho + \dfrac{Ng_1 \kappa_1^a a^{so}}{\kappa_1^b} - h_1 N a^{so} + \dfrac{Ng_1 \ln(\delta)}{\kappa_1^b}} \left[L + (\kappa_0^b \delta - \kappa_0^a) e^{\kappa_1^a a^{so}} \right]. \tag{A45}$$

This means that

$$\left[1 - \frac{N(g_1 \kappa_1^a - h_1 \kappa_1^b) / (\kappa_1^a \kappa_1^b)}{\rho + \theta_1 a^{so} + \dfrac{Ng_1 \ln(\delta)}{\kappa_1^b}} \right] e^{\kappa_1^a a^{so}} = \frac{N\theta_2}{\rho + \theta_1 a^{so} + \dfrac{Ng_1 \ln(\delta)}{\kappa_1^b}}. \tag{A46}$$

Therefore,

$$\left[\rho + \theta_1 a^{so} + \frac{Ng_1 \ln(\delta)}{\kappa_1^b} - \frac{N(g_1 \kappa_1^a - h_1 \kappa_1^b)}{\kappa_1^a \kappa_1^b} \right] e^{\kappa_1^a a^{so}} = N\theta_2, \tag{A47}$$

and we may write

$$\theta_0 - \frac{(N-1)\theta_1}{N\kappa_1^a} + \theta_1 a^{so} = N\theta_2 e^{-\kappa_1^a a^{so}}. \tag{A48}$$

Similarly, insert $a^{so} = (\kappa_1^b / \kappa_1^a) b^{so} - \ln(\delta) / \kappa_1^a$ into (A48) to obtain

$$\theta_0 - \frac{\theta_1 \ln(\delta)}{\kappa_1^a} - \frac{(N-1)\theta_1}{N\kappa_1^a} + \frac{\kappa_1^b \theta_1}{\kappa_1^a} b^{so} = N\delta\theta_2 e^{-\kappa_1^b b^{so}}. \tag{A49}$$