Buying Ecological Services: Nature's Harmonies, Fragmented Reserves and the Agricultural Extensification Debate

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Abstract

Growing demand for cropland products has placed intense pressure on the ability of land resources to support nature, straining public budgets to purchase environmental goods. Fixing overall agricultural output, two policy options are whether to promote more extensive and nature friendly farming practices or to produce intensively on some land and leave the rest wild. Microeconomic models of the topic have not accommodated widely recognized complementary spatial externalities in providing ecological services. This article does so, identifying also a third policy possibility. This is that environmental services can follow a smoothly varying spatial path characterized by harmonic functions.

Keywords*:* biofuels, environmental policy, spatial externalities, Wirtinger's inequality.
JEL classification: H40, Q28, D62.

1. INTRODUCTION

Rising real incomes, an expanding global population and biofuel policies have increased demand for land under crops over the first decade of the 21st century (Mitchell, 2008). With a marked expansion of cropland and higher land rental rates, governments are posed with the problem of how best to provide a wide variety of environmental services when the price of obtaining these has increased.[1] A widely noted feature of these services is that spatial contiguity matters, and so the spatial arrangement of targeted land matters.

Services at issue include carbon sequestration where output is a global public good and spatial externalities are clearly secondary. They also include green space for outdoor amenities, flora and fauna habitat, as well as riparian buffer zones. For riparian buffer zones to encourage nature, reduce erosion, and filter chemical runoff, spatial effects likely involve local substitution. This is because service provision by neighboring lands may be almost as effective. Substitution effects are also likely when woodland is intended to control rainwater flow and so prevent flooding. In other cases, local complementarity is likely, as with contiguity in scenery or with existence value for an unspoiled ecosystem. Spatial fragmentation is especially harmful for larger plant and animal species, where a reduction in their presence can greatly alter the presence, extent, and behavior of other organisms, including the risk of species invasion; see for example Terborgh *et al.* (2001) or Damschen *et al.* (2006).

In the presence of such spatial spillovers, there has been much debate on how to construct public policy to most efficiently provide ecological public goods. One way of doing so is to buy down farmer profit opportunities through requiring investments that increase environmental outputs or through purchasing restrictions on agricultural practices. In a widely cited paper, Green *et al.* (2005) developed on a suggestion in Waggoner (1995) that it may be optimal to load

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[1] For example, row cropland cash rent increased by about 29% between 2006 and 2008 (Edwards and Smith, 2008).

the buy downs on one set of acres, concentrating agricultural production on the remaining acres. This is the intensification option.

Or it may be optimal to spread the buy downs across all acres so that all acres provide roughly similar bundles of agricultural production and environmental services. This is the extensification option. Which is optimal, they suggest, depends on the relationship between agricultural losses and environmental services. If the marginal reduction in environmental services due to an increase in agricultural outputs declines with an increase in these outputs (is convex) then intensification is the better policy. If the relation is concave, then extensification is the better policy. Their model does not account for spatial spillovers. The intensification policy suggestion is controversial; see Jordan *et al.* (2007), De Fries *et al.* (2007), or Ceotto (2008). See Tichit *et al.* (2007), Scherr and McNeeley (2008), or Fischer *et al.* (2008) for elaboration on how ecosystem and socioeconomic context matters in this debate.

In practice, both policies have been enacted in the past. The Conservation Reserve Program, which buys cropland out of production for periods of a decade or more, has been the primary policy instrument in the United States, with a 2008 budget of about $2 billion. Forestry schemes in the European Union have sought to convert farmland into deciduous and native woodland. Extensification-type agro-environmental policies have been more common in the European Union. Regulation 1257/1997 schemes support such activities as hedgerow restoration, maximum stocking rates, input use reductions, use of native species, production of organic crops, as well as rotation, green manure and fallowing practices, where Donald and Evans (2006) provide a review. These schemes cover about 20 percent of EU farmland at an annual cost of about €3.5 billion (Whitfield, 2006).

The effectiveness of agro-environmental schemes has been brought into question by Kleijn *et al.* (2001) and others, especially with respect to a lack of regard for scientific evaluation (Whitfield, 2006). Many schemes may have had little or even ecologically adverse effects, a belief that may have led to cuts in funding for such EU programs. Regarding evaluation, Wätzold and Schwerdtner (2005) point to a dearth of expertise at the interface of ecology and economics among those designing and implementing schemes.

Even at the level of academic evaluation, we note in particular that spatial interactions seldom enter assessment of such schemes despite widespread agreement that land contiguity and fragmentation are salient in any ecosystem. Recognizing the very decentralized organization of conservation endeavors and with an eye toward strategic public policy, Albers, Ando, and Chen (2008) consider spatial interactions between public land in conservation and private land trusts. In Polasky *et al.* (2008) and papers cited therein, models with land contiguity effects have been developed to answer questions about how to arrange conservation and economic activities. The methods are, however, intended to provide practical assistance to the land manager and to draw lessons from case study applications. The methods are not as well suited to addressing analytic questions on land management.

The intent of this article is to go some way toward including spatial effects in economic analysis of the land use extensification debate. We provide a simple model of a circular ecosystem that allows for spatially complementary spillovers. Under general conditions we show that omission of such spillovers may tilt policy toward extensification when intensification might be optimal. We also show that accommodating spatial spillovers in environmental services can admit a third alternative, not mentioned in the debate to this point. For identical farms located around a disc, this is that an environmental buy down policy may best manage these spatial effects by smoothly varying the provision of environmental services as one moves across space. The provision would follow a linear combination of a sine and cosine function around the circle. A land strip topology is also considered, leading to a markedly different optimal landscape design. We conclude with a brief discussion.

2. MODEL

There are N profit maximizing farms of equal size located on a disc, called the region, with unit radius. Apart from location, these farms are identical. Land quality, resource availabilities, and prices are the same, while all farms have a cheese wedge shape with angle $360/N$ degrees and area π/N. For agricultural output q, each farm has cost function

$$(1)\qquad C(q)=\begin{cases} cq, & q\in[0,\hat{q}]; \\ \infty, & q>\hat{q}. \end{cases}$$

Here, $c>0$ and $\hat{q}>0$ while the market price for output is $P>c$ so that all farms produce at $\hat{q}$ absent a policy intervention. The government seeks to buy down production in order to provide environmental services. It is prepared to pay for a production buy down of R output in total, or an average of $\bar{r}=R/N\in(0,\hat{q})$ per farm. Given (1), the cost of this buy down is $(P-c)R$ and does not depend on which farms participate. Producers are indifferent concerning the size of buy down they choose at auction or are allocated.

For $n\in\{0,1,\dots,N-1\}\equiv\Omega$ as the set of farms enumerated clockwise around the disc, the government seeks understanding on how best to allocate R over these farms where each farm is allocated $r_n\in[0,\hat{q}]$. Environmental benefits are given by $B[\bar{r},v(r),\rho(r)]:\bar{\mathbb{R}}_+^2\times\mathbb{R}\to\bar{\mathbb{R}}_+$ where $v=N^{-1}\sum_{n\in\Omega}(r_n-\bar{r})^2$ is the variance of the coordinates drawn from simplex $S\equiv\{r:r\in[0,\hat{q}]^N$, $\sum_{n\in\Omega}r_n=R\}$ with $[0,\hat{q}]^N\equiv[0,\hat{q}]\times\dots\times[0,\hat{q}]$. Variance is assumed to have domain $[0,v^{\max}]$, $v^{\max}>0$, where v is maximized on S whenever all but one acre is either producing $\hat{q}$ or 0. Benefits depend on $\bar{r}$ with $\partial B(\cdot)/\partial\bar{r}=B_{\bar{r}}(\cdot)>0$. They also depend on v with $B_v(\cdot)$ of unassigned sign.

All other arguments fixed, including the as yet unexplained index ρ, if $B_v(\cdot)<0$ on its domain then benefits would be maximized when $v=0$ and $r_n=\bar{r}\ \forall n\in\Omega$. This would occur under the extensification policy in Green *et al.* (2005). This variance-is-bad situation could arise if there were low-hanging fruit to be had from low intrusion agro-environmental schemes, perhaps requiring slightly wider hedgerows or the use of conservation tillage in the presence of high cultivation costs. All other arguments fixed, if $B_v(\cdot) > 0$ on its domain then benefits would be maximized under $\hat{q}$-or-0, i.e., all-or-nothing, allocations. This is the other policy arrived at in Green *et al.* (2005), intensification. This situation could reflect threshold effects whereby a key top predator, the wolf, tiger or lynx, will not be tolerated in a neighborhood if even minimal livestock farming occurs in an area.

The third statistic entering the benefits function is spatial index ρ. This index accounts for local spatial spillovers in environmental policy and may be viewed as an index of cohesion, i.e., an inverse index of fragmentation. It is obtained by viewing the production reduction r_n on the n^{th} farm as providing environmental benefit $A(r_{[n-1]}, r_n, r_{[n+1]}) = 0.5(r_{[n-1]} - \bar{r})(r_n - \bar{r}) +$ $0.5(r_n - \bar{r})(r_{[n+1]} - \bar{r})$, $n \in \Omega$, where

(2)
$$r_{[n+1]} = \begin{cases} r_{n+1}, & n \in \Omega, \quad n \neq N-1; \\ r_0, & n = N-1; \end{cases}$$
$$r_{[n-1]} = \begin{cases} r_{n-1}, & n \in \Omega, \quad n \neq 0; \\ r_{N-1}, & n = 0. \end{cases}$$

The conditions in (2) are just the modular arithmetic maps required on the disc to ensure that the 0^{th} and $N-1^{\text{th}}$ farms are neighbors. The derivative sign of $\partial^2 A(\cdot)/\partial r_{[n-1]}\partial r_n = \partial^2 A(\cdot)/\partial r_n \partial r_{[n+1]} =$ $0.5 > 0$ is intended to capture farm-level contiguity effects in the provision of environmental services. There is local complementarity so that marginal environmental benefits on a given farm increase with an increase in production buy down on neighboring farms.

Index ρ is obtained from averaging $A(r_{[n-1]}, r_n, r_{[n+1]})$ over the set:

(3)
$$\rho(r) = N^{-1} \sum_{n \in \Omega} A(r_{[n-1]}, r_n, r_{[n+1]}) = N^{-1} \sum_{n \in \Omega} (r_{[n-1]} - \bar{r})(r_n - \bar{r}).$$

Note first that $\rho(r)$ is the lag 1 spatial covariance. The index can have a positive or negative value and has upper bound $v(r)$. To observe how it captures cohesion, let $N = 6$ and $R = 3$ while assuming that the r_n must take integer values 0 or 1. Then there are only three distinct arrangements of the reductions. These are I) $\{0,0,0,1,1,1\}$ with $\rho(r) = 1/3$, II) $\{0,0,1,0,1,1\}$ with $\rho(r) = -1/3$, and III) $\{0,1,0,1,0,1\}$ with $\rho(r) = -1$. We hold that $B_\rho(\cdot) > 0$ so as to capture concerns about the loss in environmental services arising from fragmentation.

We seek to understand the nature of

$$\arg\max_{r\in S} B\left[\bar{r}, v(r), \rho(r)\right]. \tag{4}$$

Policy options will be characterized as follows:

DEFINITION 1: *Extensification is said to be optimal if* $\arg\max_{r\in S} B(\cdot) = \bar{r}1_N$ *where* 1_N *is the vector of 1s in* $\mathbb{R}^N$. *Partial intensification is said to be optimal if* $\arg\max_{r\in S} B(\cdot)$ *has one or more ordinates with values 0 or* $\hat{q}$.

Intuitively, our interest in partial intensification arises from the fact that if $B_\rho(\cdot) > 0$ then a landscape arrangement involving $r_n = 0$ and $r_{[n+1]} = \hat{q}$ or involving $r_n = \hat{q}$ and $r_{[n+1]} = 0$ will suffer a benefits penalty. If we do not restrict the size of $B_\rho(\cdot)$, and we do not, the penalty may be large. We will develop on this point in some detail.

3. INDEX PROPERTIES

Note first that $\rho(r)$ is possessed of strong symmetry properties. In particular for $r =$ $(r_0, r_1, r_2, \ldots, r_{N-2}, r_{N-1})^t$, superscript t the transpose operation, write $L^1(r) = (r_1, r_2, r_3, \ldots, r_{N-1}, r_0)^t$ and in general write $L^i(r) = (r_{[i]}, r_{[i+1]}, r_{[i+2]}, \ldots, r_{[i-2]}, r_{[i-1]})^t$ where modular arithmetic has been applied to the subscripts. We say function $f(x)$ is invariant under rotation if $f(x) \equiv f[L^i(x)]$

for all integers i. Also, write $M(r)=(r_0,r_{N-1},r_{N-2},\ldots,r_2,r_1)^t$ as the vector reflection operation and we assert that a function $f(x)$ is invariant under reflection if $f(x)\equiv f[M(x)]$. Finally with $\circ$ as the composition operation, function $f(x)$ is said to be invariant under reflection and rotation composition if $f(x)\equiv f[M\circ L^i(x)]\equiv f[L^i\circ M(x)]$ for all integers i.

PROPERTY 1: *Index* $\rho(r)$ *is invariant under* A) *rotation,* B) *reflection, and consequently* C) *reflection and rotation composition.*

We turn next to one possibility of gleaning inferences from these symmetries. Uniform curvature in conjunction with symmetry provides the potential for exploitable structure on level sets.[2] We will see next that any such opportunities will be qualified. Although qualified, we will show later that such opportunities do exist. The Hessian for the index is given by

$$\begin{pmatrix} 0 & 1 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 1 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & \cdots & 1 & 0 \end{pmatrix}. \tag{5}$$

This is a circulant matrix in that the second row is obtained from applying a rotation operation on the first row, the third row is obtained from applying the same operation on the second row and so on. The eigenvalues of circulant matrices are highly structured, allowing us to obtain:

PROPERTY 2: *The index is neither concave nor convex. The eigenvalues of its Hessian matrix are* $\lambda_n=2\cos[2\pi(n-1)/N]\in[-2,2]$, $n\in\Omega$. *They are negative whenever* $N+4<4n<3N+4$, *zero whenever* $n=(N+4)/4$ *or* $n=(3N+4)/4$, *and positive otherwise.*

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[2] For example, when $f(x):\mathbb{R}\to\mathbb{R}$ is convex then $\sum_{n\in\Omega} f(x_n)$ is permutation symmetric such that any transfer $\varepsilon>0$ to increase x_i by ε and decrease x_j by ε increases the value of the sum

For $N = 6$, then $2\cos[2\pi(n-1)/N] \in \{-2,-1,1,2\}$ where some roots are repeated. For $N = 9$, then $2\cos[2\pi(n-1)/N] \in \{-1.879,-1,0.347,1.532,2\}$. While 2 is always an eigenvalue, -2 is an eigenvalue only when N is even. All other eigenvalues arise twice whenever they arise at all. The eigenvalue symmetries together with Property 1 suggest that symmetric structure will be important to the analysis. Property 2 also suggests that trigonometric, or harmonic, analysis should prove useful in understanding how best to allocate buy downs. Throughout the analysis we will give content to these speculations.

4. OPTIMIZATION PROBLEM

For our first result we fix the values in vector r but allow them to be arranged at will across the farms. Let $T(\hat{r})$ be the set of all $N!$ rearrangements of vector $r = \hat{r} \in \mathbb{R}^N$ where, for convenience in presentation only, we assume the r_n are distinct.

PROPOSITION 1: *Let $r_{(n-1)}$ be the n^th least value of r. When N is*

A) *even then the arrangement $r \in T(\hat{r})$ maximizing $\rho(r)$ is some rotation $L^i(r^{*,ev})$ of*

$$r^{*,ev} = \begin{pmatrix} \hat{r}_{(0)} & \hat{r}_{(1)} & \hat{r}_{(3)} & \cdots & \hat{r}_{(N-3)} & \hat{r}_{(N-1)} & \hat{r}_{(N-2)} & \cdots & \hat{r}_{(4)} & \hat{r}_{(2)} \end{pmatrix}^t, \tag{6}$$

i an integer, or some rotation of reflection $M(r^{*,ev})$ where the largest value $\hat{r}_{(N-1)}$ in $r^{*,ev}$ is at farm $n = N/2$ in Ω.

B) *odd then the maximizing arrangement is some rotation $L^i(r^{*,od})$ of*

$$r^{*,od} = \begin{pmatrix} \hat{r}_{(0)} & \hat{r}_{(1)} & \hat{r}_{(3)} & \cdots & \hat{r}_{(N-2)} & \hat{r}_{(N-1)} & \hat{r}_{(N-3)} & \cdots & \hat{r}_{(4)} & \hat{r}_{(2)} \end{pmatrix}^t, \tag{7}$$

or some rotation of the reflection $M(r^{*,od})$ where the largest value $\hat{r}_{(N-1)}$ in $r^{*,od}$ is at farm n

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whenever $x_j \le x_i$.

$= (N+1)/2$ *in* Ω.

COROLLARY 1.1: *Solutions to* $\arg\max_{r \in S} B(\cdot)$ *have the spatial arrangement laid out in Proposition 1.*

This corollary tells us that even if the policy maker is restricted to rearrangements of some $r = \hat{r} \in S$ then the optimal solution will be highly symmetric. The optimal vector must be at least weakly decreasing over half the disc and weakly increasing over the other half. Notice too that the solutions are entirely ordinal; rank order is all that matters.

We turn now to providing further rationalization of the cohesion index. In the next result, we do not confine attention to the set $r \in T(\hat{r})$ but rather let the values be arbitrary on simplex S.

PROPOSITION 2: *Consider a transfer* $(\tilde{r}_i, \tilde{r}_j) = (r_i + \varepsilon, r_j - \varepsilon)$, $\varepsilon > 0$ *but infinitesimally small, while* $\tilde{r}_n = r_n$ *for all* $n \in \Omega$ *other than i and j. Then* $\rho(\tilde{r}) \geq \rho(r)$ *iff* $r_{[i-1]} + r_{[i+1]} \geq r_{[j-1]} + r_{[j+1]}$ *while* $v(\tilde{r})$ $\geq v(r)$ *iff* $r_i \geq r_j$.

COROLLARY 2.1: *Environmental services increase under the transfer considered in Proposition 2*

A) *whenever i)* $B_v(\cdot) \geq 0$ *and ii) both of* $r_{[i-1]} + r_{[i+1]} \geq r_{[j-1]} + r_{[j+1]}$ *and* $r_i \geq r_j$ *apply, while they decrease whenever both inequalities in ii) are reversed.*

B) *whenever i)* $B_v(\cdot) \leq 0$ *and ii) both of* $r_{[i-1]} + r_{[i+1]} \geq r_{[j-1]} + r_{[j+1]}$ *and* $r_i \leq r_j$ *apply, while they decrease whenever both inequalities in ii) are reversed.*

In particular, suppose $\varepsilon = r_j - r_i$. Then $(\tilde{r}_i, \tilde{r}_j) = (r_j, r_i)$ so that switching the locations of r_i and r_j when both $r_i \leq r_j$ and $r_{[i-1]} + r_{[i+1]} \geq r_{[j-1]} + r_{[j+1]}$ apply increases the value of the cohesion index while holding vector mean and variance fixed, implying an increase in environmental benefits. We also note in passing that $r_{[i-1]} + r_{[i+1]} \geq r_{[j-1]} + r_{[j+1]}$ reduces to one of $r_{[i-1]} \geq r_{[j+1]}$ or $r_{[i+1]} \geq r_{[j-1]}$ when the farms are not adjacent but do have a common neighbor. General condition $r_{[i-1]} + r_{[i+1]} \geq r_{[j-1]} + r_{[j+1]}$ shows how the index captures the idea of cohesion. Regardless of the r_i, the cohesion index increases upon a small buy down transfer from j^{th} farm to i^{th} farm if and only if the farms flanking the latter have an average buy down that is higher than those flanking r_j.

Our findings above point to considerable structure on what increases environmental services. Both propositions 1 and 2 suggest some sort of buy down agglomeration on a segment of the circle might be best. Concentration is an issue elsewhere in economics, as with income inequality and market power. There, such statistics as Gini coefficients and variance have been found to be relevant, where well-known references are Atkinson (1970) and Bergstrom and Varian (1985). But these statistics do not account for spatial effects and so are inappropriate in our context.

One way of allowing for spatial effects, and also providing opportunities for econometric study, is to present the chosen buy downs in spectral form. In our case, an additional advantage is the suitability of harmonic analysis for the circular topology. With $\Omega^0 = \{1, 2, \ \ldots \ , N-1\}$, specify the spectral sum

(8) $$r_n - \overline{r} = \sum\nolimits_{k \in \Omega^0} a_k \cos(2\pi nk / N) + \sum\nolimits_{k \in \Omega^0} b_k \sin(2\pi nk / N).$$

Since $2N-2$ orthogonal functions have been used to fit $N \geq 2$ buy down values, this together with some constant a_0 provides an exact representation of the parameters.

PROPOSITION 3: *Let* r *be represented by a spectral sum with parameters* $\{a_0 \cup (a_k, b_k), k \in \Omega^0\}$. *Then* $\rho(r) = 0.5\sum\nolimits_{k \in \Omega^0} (a_k^2 + b_k^2) \cos(2\pi k / N)$ *and* $v(r) = 0.5\sum\nolimits_{k \in \Omega^0} (a_k^2 + b_k^2)$.

Notice next that $\cos(2\pi n / N)$ is reflexively symmetric around π or 180^0 in $n \in \Omega^0$. Thus, $\cos(2\pi / N) \equiv \cos(2\pi(N-1)/N)$, $\cos(4\pi / N) \equiv \cos(2\pi(N-2)/N)$ and so on. For even N there will be a middle entrant in Ω^0, namely $n = N/2$. For it, $\cos(2\pi n/N) = -1$. For N odd, Ω^0 will not have a middle entrant. But break the set in two with $\Omega^{0,low} = \{1, 2, \ \dots \ , (N-1)/2\}$ and $\Omega^{0,high}$ $= \{(N+1)/2, (N+3)/2, \ \dots \ , N-1\}$. The intent of what is to follow is to provide exact conditions under which the mean and variance of production buy downs are held fixed, but the cohesion index increases.

We will first develop what we call summary coefficients. These account for the degrees of freedom that the spectral sum allows in fitting the buy down parameters. Drawing from both sets $\Omega^{0,low}$ and $\Omega^{0,high}$ so that cosine evaluations are the same, write the first summary coefficient as $c_1 = a_1^2 + b_1^2 + a_{N-1}^2 + b_{N-1}^2$ and in general the m^{th} coefficient as $c_m = a_m^2 + b_m^2 + a_{N-m}^2 + b_{N-m}^2 \ \forall m \in$ $\{1, 2, \ \dots \ , (N-1)/2\}$. For even N, place middle entrant $n = N/2$ in $\Omega^{0,low}$ so that $\Omega^{0,low} =$ $\{1, 2, \ \dots \ , N/2\}$ and $\Omega^{0,high} = \{(N+2)/2, (N+4)/2, \ \dots \ , N-1\}$. In this case, and as before, write the m^{th} summary coefficient as $c_m = a_m^2 + b_m^2 + a_{N-m}^2 + b_{N-m}^2 \ \forall m \in \{1, 2, \ \dots \ , (N-2)/2\}$. The only difference when compared with N odd is that $c_{N/2} = a_{N/2}^2 + b_{N/2}^2$.

We are interested in the partial sums of these coefficients. Formally, define

(9)
$$c_m = \begin{cases} a_m^2 + b_m^2 + a_{N-m}^2 + b_{N-m}^2 & \forall m \in \{1, 2, \ \dots \ , (N-1)/2\}, & N \text{ odd}; \\ a_m^2 + b_m^2 + a_{N-m}^2 + b_{N-m}^2 & \forall m \in \{1, 2, \ \dots \ , (N-2)/2\}, & N \text{ even}; \\ a_{N/2}^2 + b_{N/2}^2, & m = N/2, & N \text{ even}; \end{cases}$$
$$\Omega^{0,low} = \begin{cases} \{1, 2, \ \dots \ , (N-1)/2\} & N \text{ odd}; \\ \{1, 2, \ \dots \ , N/2\} & N \text{ even}; \end{cases}$$
$$C_k = \sum_{i=1}^{k} c_i, \quad k \in \Omega^{0,low}.$$

Bearing in mind that a_0 determines the value of $\bar{r}$, we have

PROPOSITION 4: *Let spectral representations of* r' *and* r'' *be given by* $\{a_0' \cup (a_k', b_k'), k \in \Omega^0\}$ *and* $\{a_0'' \cup (a_k'', b_k''), k \in \Omega^0\}$, *respectively. Let the summary representations be* $\{a_0' \cup c_k', k \in \Omega^{0,low}\}$ *and* $\{a_0'' \cup c_k'', k \in \Omega^{0,low}\}$, *respectively. Then*

A) $\bar{r}$ *and* $v(r)$ *do not change whenever*

(10) $$C_k'' \geq C_k' \quad \forall k \in \Omega^{0,low} \quad \text{with} \quad \begin{cases} C_{(N-1)/2}'' = C_{(N-1)/2}' & N \text{ odd;} \\ C_{N/2}'' = C_{N/2}' & N \text{ even.} \end{cases}$$

B) $\rho(r'') \geq \rho(r')$ *under condition set (10).*

C) *Let* $c_m'' = c_m' \ \forall m \in \Omega^{0,low}, m \notin \{j,k\}$ *with* $j,k \in \Omega^{0,low}$ *and* $j < k$. *If* $c_j'' = c_j' + \delta$ *and* $c_k'' = c_k' - \delta$ *with* $\delta \geq 0$ *then* $\rho(r'') \geq \rho(r')$.

COROLLARY 4.1: *Environmental services are larger under* r'' *than under* r' *where these vectors are comparable in the sense of (10) above.*

COROLLARY 4.2: *Among the set of spectral representations given by*

(11) $$\left\{a_0 \cup (a_k, b_k), k \in \Omega^0 : \sum\nolimits_{k \in \Omega^0} \left(a_k^2 + b_k^2\right) = \Gamma\right\}, \qquad \Gamma \text{ a constant,}$$

the value of $\rho(r)$ *is maximized whenever* $a_1^2 + b_1^2 = \Gamma$, *requiring* $(a_k, b_k) = (0,0) \ \forall k \in \Omega^0, k \neq 1$.

Part A) of the proposition assures that only $\rho(r)$ is affected. Part B) then leads to Corollary 4.1 in light of the assumption $B_\rho(\cdot) > 0$. Part C) gives deeper insight into Proposition 3. The loading of variance onto low-frequency harmonics, or $\cos(2\pi k / N)$ where *k* is low, can be seen as ensuring a more coherent or smoother manner of variability and so a larger cohesion index. At the limit we obtain Corollary 4.2.

Corollary 4.2 is very important. It demonstrates part of a discrete version of Wirtinger's inequality, where such demonstration was first provided by different means in Fan, Taussky and Todd (1955). In our setting this inequality asserts that if $r_{[-1]} = r_{N-1}$ and $\sum_{n=0}^{N-1}(r_n - \bar{r}) = 0$ then $\sum_{n=0}^{N-1}(r_n - r_{n-1})^2 \geq 4\sin^2(\pi / N)\sum_{n=0}^{N-1}(r_n - \bar{r})^2$. The inequality is satisfied as an equality if and only if the r_n follow

(12) $$r_n = \overline{r} + K_1\cos(2\pi n / N) + K_2\sin(2\pi n / N),$$

where K_1 and K_2 are free parameters subject to $v = N^{-1}\sum_{n=0}^{N-1}(r_n - \overline{r})^2$. Since we also know from Proposition 3 that $v(r) = 0.5K_1^2 + 0.5K_2^2$, we may write $K_2 = \pm\sqrt{2v - K_1^2}$. Upon re-labeling $K = K_1$, (12) may be written as

(13) $$r_n = \overline{r} + K\cos(2\pi n / N) \pm \left(2v - K^2\right)^{0.5}\sin(2\pi n / N).$$

With $v^+ = 0.5\min[\overline{r}^2, (\hat{q} - \overline{r})^2]$, relation (13) leads us to

PROPOSITION 5: A) *Solution (13) is interior on simplex* S *with variance* $v(r)$ *whenever*

(14) $$v(r) < v^+;$$

B) *In that case,*

(15) $$\rho(r) = \cos(2\pi / N)v(r).$$

In some ways, equation (13) should not be all that surprising given the undulation attribute of the solution to Proposition 1. There of course we confined attention to arrangements of a given vector.[3] Of interest to us is when the path given in (13) emerges as a solution.

DEFINITION 2: *Any solution satisfying (13) is said to be harmonic.*

Now insert (15) into the benefit function to obtain

(16) $$H(\overline{r}, v) \overset{\text{defn}}{=} B\left(\overline{r}, v, \cos(2\pi / N)v\right).$$

This function is sufficient to describe the optimal solution whenever $v(r) < v^+$.

PROPOSITION 6*: If*

[3] Non-spatial models where the idea that identical firms should be treated asymmetrically have

A) $H_v(\bar{r},v)<0\ \forall v\in[0,v^{\max}]$ *then extensification is optimal.*

B) $H_v(\bar{r},v)>0\ \forall v\in[0,v^{+}]$ *then (at least) partial intensification is optimal.*

C) $H(\bar{r},v)$ *is quasiconcave on* $v\in[0,v^{\max}]$ *with an interior maximum on* $v\in[0,v^{+}]$, *then harmonic solution (13) is optimum.*

D) $H(\bar{r},v)$ *is quasiconvex on* $v\in[0,v^{\max}]$ *with* $H(\bar{r},0)<H(\bar{r},v^{+})$, *then (at least) partial intensification is optimal.*

To obtain a better sense of Part C), consider the additively separable form

(17) $$B\left(\bar{r},v,\cos(2\pi/N)v\right)=B^{1}(\bar{r})+B^{2}(v)+B^{3}(\rho),$$

where all of $B_v^2(v)<0$, $B_\rho^3(\rho)>0$, and $B_{vv}^2(v)+\cos^2(2\pi/N)B_{\rho\rho}^3(\rho)<0$ apply on $v\in[0,v^{\max}]$:

COROLLARY 6.1*: For objective function (17), any interior solution* v^* *to*

(18) $$H_v(\bar{r},v)=B_v^2(v)+\cos(2\pi/N)B_\rho^3\left[\cos(2\pi/N)v\right]=0$$

is the unique harmonic solution while if $H_v(\bar{r},v)\big|_{v=0}<0$ *then extensification is optimal and if* $H_v(\bar{r},v)\big|_{v=v^{+}}>0$ *then at least partial intensification is optimal.*

Notice that the solution to (18) would have $v^*=0$ where the spatial spillovers are ignored. As $\cos(2\pi/N)>0$ when $N>4$ and as $B_\rho^3(\rho)>0$, inclusion of spatial effects increases optimal variance. Ignoring spatial complementarities may tilt the identified optimum toward a lower variance, or more extensive, policy choice. This has policy implications in light of *i*) the tendency to ignore these effects in policy assessments, and *ii*) previously mentioned concerns with the effectiveness of implemented agro-environmental schemes.

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arisen include works by Salant and Shaffer (1999) and Long and Soubeyran (2001).

5. BETWEEN CITY AND WILDERNESS

Finally we ask how the production buy downs should be arranged under an alternative topographical setting. So as to be explicit, suppose that the optimization problem is to choose $r \in S$ to maximize

$$\sum_{n\in\{1,2,\,...\,,N-2\}} \alpha_n r_n - 0.5\chi \sum_{n\in\Omega} r_n^2 + \tau \sum_{n\in\Omega} r_n r_{n+1}, \qquad \chi > 0, \qquad \tau > 0, \tag{19}$$
$$\text{subject to} \quad r_0 = \hat{r}_0, \quad r_{N-1} = \hat{r}_{N-1};$$

where $\chi - 2\tau > 0$ is assumed to ensure concavity. Here the boundary values are to capture external effects arising from, say, *i*) a city located on a circle so that no environmental services are provided and $\hat{r}_0 = \hat{r}_{N-1} = 0$, or *ii*) a city at one end of a line, $\hat{r}_0 = 0$, and a National Park at the other end so that $\hat{r}_{N-1}$ is large. Parameters α_n are intended to capture farm-specific benefit heterogeneities due to locational idiosyncracies, perhaps arising from geographic features such as rivers, wetlands, or geological formations. These parameters are net of the shadow cost of raising taxes so no explicit constraint on the sum of buy downs has been included.

The assumption is made that (19) is concave in r. So were $\alpha_n = \alpha\ \forall n \in \Omega$ on the disc topology studied to this point then extensification would be optimal. Consequently, there should be a tendency for the buy downs to be similar across farms in this setting too. This we call the cohesion force, and the resulting levels of r_n will depend on the opportunity cost of tax dollars. A simple calculation shows that setting $r_n = \alpha/(\chi - 2\tau)\ \forall n \in \{1,2,\,...\,,N-2\}$ is optimal whenever $\alpha_n = \alpha$ for all *n* and $\hat{r}_0 = \hat{r}_{N-1} = \alpha/(\chi - 2\tau)$. However, there is a second force at play. Low $\hat{r}_0$ and $\hat{r}_{N-1}$ values should tend to depress the values for r_n near to the boundaries when compared with farms near the center of the line. This we call the boundary action force or the edge effect.

The first-order optimality conditions are:[4]

[4] We have used terminology that may bring physical problems to mind. System (20) is an example of a Sturm-Liouville difference equation. Sturm-Liouville systems are widely

$$
\begin{aligned}
&r_1: && \alpha_1 - \chi r_1 + \tau \hat{r}_0 + \tau r_2 = 0; \\
(20) \qquad &r_n: && \alpha_n - \chi r_n + \tau r_{n-1} + \tau r_{n+1} = 0, \qquad n \in \{2,3, \ldots, N-4, N-3\}; \\
&r_{N-2}: && \alpha_{N-2} - \chi r_{N-2} + \tau r_{N-3} + \tau \hat{r}_{N-1} = 0.
\end{aligned}
$$

Thus, asymmetries arise due to imposed boundary values. The system may be written as

$$
(21) \qquad \begin{pmatrix} 1+2\xi & -\xi & 0 & \cdots & 0 & 0 \\ -\xi & 1+2\xi & -\xi & \cdots & 0 & 0 \\ 0 & -\xi & 1+2\xi & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1+2\xi & -\xi \\ 0 & 0 & 0 & \cdots & -\xi & 1+2\xi \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_{N-3} \\ r_{N-2} \end{pmatrix} = \frac{1}{\kappa} \begin{pmatrix} \alpha_1 + \tau \hat{r}_0 \\ \alpha_2 \\ \alpha_3 \\ \vdots \\ \alpha_{N-3} \\ \alpha_{N-2} + \tau \hat{r}_{N-1} \end{pmatrix};
$$

where $\kappa = \chi - 2\tau > 0$ and $\xi = \tau / \kappa > 0$. Notice that the roots ϕ_l and ϕ_h of the characteristic equation for system (21), $\xi x^2 - (1+2\xi)x + \xi = 0$, are

$$
(22) \quad \phi_l = 1 + \frac{1-\Delta}{2\xi} \in (0,1); \qquad \phi_h \equiv \phi_l^{-1} = 1 + \frac{1+\Delta}{2\xi} = \phi_l + \frac{\Delta}{\xi} > 1; \qquad \Delta = \sqrt{1+4\xi} \in (1, 1+2\xi);
$$

and are both positive. We have

PROPOSITION 7: A) *With* $\Psi = 1/[\Delta(\phi_h^{N-1} - \phi_l^{N-1})]$, *the inverse matrix* P *of the* $(N-2)\times(N-2)$ *matrix in (21) has row i, column j entries*

$$
(23) \qquad p_{i,j} = \begin{cases} \left(\phi_h^i - \phi_l^i\right)\left(\phi_h^{N-1}\phi_l^j - \phi_h^j \phi_l^{N-1}\right)\Psi, & i \le j; \\ \left(\phi_h^j - \phi_l^j\right)\left(\phi_h^{N-1}\phi_l^i - \phi_h^i \phi_l^{N-1}\right)\Psi, & i > j. \end{cases}
$$

B) *The entries are all strictly positive.*

C) *The entries are all symmetric in that* $p_{i,j} = p_{j,i}$ $\forall i, j \in \{1, 2, \ldots, N-2\}$, *and furthermore, centro-symmetric in that* $p_{i,j} = p_{N-1-i, N-1-j} = p_{N-1-j, N-1-i}$ $\forall i, j \in \{1, 2, \ldots, N-2\}$.

D) *The entries satisfy the unimodality properties* $p_{i-1,j} \le p_{i,j}$ $\forall i \le j$, $p_{i,j} \ge p_{i+1,j}$ $\forall i \ge j$, $p_{i,j-1} \ge p_{i,j}$ $\forall i \le j$, *and* $p_{i,j+1} \ge p_{i,j}$ $\forall i \ge j$. *This means that* $p_{1,j}$ *is decreasing in* $j, \forall j \in \{1, \ldots, N-2\}$,

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encountered in physics when modeling a variety of phenomena from sound waves to quantum *while* $p_{i,N-2}$ *is increasing in* $i, \forall i \in \{1, \dots, N-2\}$.

E) *If* N *is odd then* $p_{(N-1)/2,k+(N-1)/2} = p_{(N-1)/2,-k+(N-1)/2} = p_{k+(N-1)/2,(N-1)/2} = p_{-k+(N-1)/2,(N-1)/2}$ $\forall k \in \{1, \dots, (N-3)/2\}$.

F) *If* N *is odd then* $p_{i,i}$ *increases on* $i \in \{1, 2, \dots, (N-1)/2\}$ *to attain maximum value at* $i = (N-1)/2$ *and decreases on* $i \in \{(N-1)/2, (N+1)/2, \dots, N-2\}$. *If* N *is even then* $p_{i,i}$ *increases on* $i \in \{1, 2, \dots, (N-2)/2\}$ *to attain maximum value when at* $i \in \{(N-2)/2, N/2\}$ *and decreases on* $i \in \{N/2, (N+2)/2, \dots, N-2\}$.

G) *For* N *odd then* $p_{i,1} + p_{i,N-2} < p_{i-1,1} + p_{i-1,N-2}$ *on* $i \in \{2, 3, \dots, (N-1)/2\}$ *and* $p_{i,1} + p_{i,N-2} > p_{i-1,1} + p_{i-1,N-2}$ *on* $i \in \{(N+1)/2, (N+3)/2, \dots, (N-2)\}$. *For* N *even then* $p_{i,1} + p_{i,N-2} < p_{i-1,1} + p_{i-1,N-2}$ *on* $i \in \{2, 3, \dots, (N-2)/2\}$ *and* $p_{i,1} + p_{i,N-2} > p_{i-1,1} + p_{i-1,N-2}$ *on* $i \in \{(N+2)/2, \dots, (N-2)\}$, *with* $p_{i',1} + p_{i',N-2} = p_{i'-1,1} + p_{i'-1,N-2}$ *at* $i' = N/2$.

H) *With* $P_i = \sum_{j=1}^{N-2} p_{i,j}$, *then* $P_i = P_{N-2-i}$. *If* N *is odd then* P_i *increases on* $i \in \{1, 2, \dots, (N-1)/2\}$ *to attain maximum value at* $i = (N-1)/2$ *and decreases on* $i \in \{(N-1)/2, (N+1)/2, \dots, N-2\}$. *If* N *is even then* P_i *increases on* $i \in \{1, 2, \dots, (N-2)/2\}$ *to attain maximum value when at* $i \in \{(N-2)/2, N/2\}$ *and decreases on* $i \in \{N/2, (N+2)/2, \dots, N-2\}$.

As an example, when $\xi = 1$ and $N = 7$ then the inverted matrix is

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mechanics.

$$
(24) \qquad \begin{pmatrix} 0.382 & 0.146 & 0.056 & 0.021 & 0.007 \\ 0.146 & 0.437 & 0.167 & 0.062 & 0.021 \\ 0.056 & 0.167 & 0.444 & 0.167 & 0.056 \\ 0.021 & 0.062 & 0.167 & 0.437 & 0.146 \\ 0.007 & 0.021 & 0.056 & 0.146 & 0.382 \end{pmatrix}.
$$

Positivity and symmetry are clearly satisfied while centro-symmetry can be seen from observing reflections in the off diagonal. Unimodality is confirmed by observing that the largest entry in any row is on the main diagonal while the entry values decline monotonically as one moves away from the diagonal along a row. Symmetry ensures that unimodality is true for columns as well as rows. As $N = 5$, odd, we can observe a cross pattern around the central entry $p_{3,3} =$ 0.444. So $p_{2,3} = p_{4,3} = p_{3,2} = p_{3,4} = 0.167$ while $p_{1,3} = p_{5,3} = p_{3,1} = p_{3,5} = 0.056$. As for row sums, they are $P_1 = P_5 = 0.612 \leq P_2 = P_4 = 0.833 \leq P_3 = 0.89$. That F) applies in this case can be seen from unimodality along the main diagonal.

We have also from the proposition that sensitivity to edge values decreases with displacement and that, all else equal, edge values can create a variety of optimal buy down arrangements.

COROLLARY 7.1: $0 < dr_{N-2} / d\hat{r}_0 < ... < dr_n / d\hat{r}_0 < ... < dr_1 / d\hat{r}_0$ *and* $dr_{N-2} / d\hat{r}_{N-1} > ... > dr_n / d\hat{r}_0 > ...$ $> dr_1 / d\hat{r}_0 > 0$.

COROLLARY 7.2: *Suppose that* $\alpha_i = \alpha \ \forall i \in \{1, 2, ... , N-2\}$.

A) *For* $\hat{r}_0 = \hat{r}_{N-1} > \alpha / \kappa$ *then* r_n *forms a U shape. That is,* r_n *decreases in* n *for small n, reaches a minimum and increases in n for large n.*

B) *For* $\hat{r}_0 = \hat{r}_{N-1} < \alpha / \kappa$ *then* r_n *forms an inverted U shape. That is,* r_n *increases in* n *for small n, reaches a maximum and decreases in n for large n.*

C) *For* $\hat{r}_0 < \alpha / \kappa$ *and* $\hat{r}_{N-1} > \alpha / \kappa$ *then* r_n *is monotone increasing.*

D) *For* $\hat{r}_0 > \alpha / \kappa$ *and* $\hat{r}_{N-1} < \alpha / \kappa$ *then* r_n *is monotone decreasing.*

The proposition's economic content is best illustrated through the corollaries. Corollary 7.1 shows how the complementary spillovers diminish over space. All else equal, a nature reserve at one end of a strip should encourage relatively large buy downs over neighboring farms when compared with more distant farms. Parts A) and B) of Corollary 7.2 identify two opposing forces. For low values of $\hat{r}_0$ and $\hat{r}_{N-1}$, as when there are cities at both ends of a strip, then buy downs should rise when moving from the edges toward the center. This is just as viscous fluids dip toward the sides of a glass. The reverse is true for high values of $\hat{r}_0$ and $\hat{r}_{N-1}$. When the edges are wilderness, then buy downs should taper off so that the most intensively farmed land lies at the center of the strip.

6. DISCUSSION

Working with a simple spatial model on a disc, this paper has sought to clarify some issues in agri-environmental policy. For farms identical in all ways, we find it may be optimal to treat them asymmetrically in order to avoid loss in ecosystem services due to fragmentation. In doing so, a trade-off can arise if eco-service benefits on any given farm are concave. This trade-off leads to the possibility of a third policy option, one not considered in the formal literature to this point. A smoothly varying buy down policy around the disc may be best, where we find a closed-form trigonometric solution for the optimal policy.

We also allow for farm-level heterogeneities in the provision of eco-services. In order to better understand the implications of topological structure, we do so for a strip of land rather than for a closed disc. While the setting is very different, spatial spillovers lead to a preference for smooth variation in this situation too. If the boundaries are wilderness and the land between is homogeneous then the buy downs should largely be near the bounds and will decrease steeply toward the center whenever the opportunity cost of tax funds is high. If the bounds are urban and the opportunity cost of tax funds is low then the buy downs will be at the center of the land strip.

Beyond the problem at hand, we see other uses for our approach. One is to better understand spatial effects within city residency patterns where positive neighborhood spillovers can be seen in equilibria that involve family wealth gradients. Somewhat more abstractly, parents worry about the friends children keep, perhaps in part arising from beliefs about behavioral norms, peer effects, and mutual re-enforcement. Public health studies lend credence to these concerns. For example, Christakis and Fowler (2007) have identified social effects in the propensity to become obese where these effects are not entirely explained by the endogenous formation of social ties by people of different body mass indices. If a group of individuals are arranged in a circle and each person is viewed as being friends of just the contiguous neighbors on either side, then our model may be able to say something about equilibrium behavior concerning diet, social deviancy, and personal discipline.

APPENDIX: OMITTED PROOFS

PROOF OF PROPERTY 1:

Since $\rho(r) = N^{-1}\sum_{n\in\Omega} r_{[n-1]}r_n - \bar{r}^2$ and $\bar{r}$ is invariant to reflection, rotation or their composition, we need only consider the effect on $\theta(r) = \sum_{n\in\Omega} r_{[n-1]}r_n$. For part A), note that $\sum_{n\in\Omega} r_{[n-1]}r_n \equiv \sum_{n\in\Omega} r_{[n-1+i]}r_{[n+i]}$ $\forall i \in \Omega$. Parts B) and C) follow from

(A1)
$$\begin{aligned} & r_0 r_1 + r_1 r_2 + \ldots + r_{n-1} r_n + \ldots + r_{N-2} r_{N-1} + r_{N-1} r_0 \\ & \equiv r_{N-1} r_0 + r_{N-2} r_{N-1} + \ldots + r_{N-n} r_{N-n+1} + \ldots + r_1 r_2 + r_0 r_1 \\ & \equiv r_{[N-1+i]} r_{[0+i]} + r_{[N-2+i]} r_{[N-1+i]} + \ldots + r_{[N-n+i]} r_{[N-n+i+1]} + \ldots + r_{[1+i]} r_{[2+i]} + r_{[0+i]} r_{[1+i]}; \end{aligned}$$

i.e., impose reflection and composition permutations on the arguments of $\theta(r)$. □

PROOF OF PROPERTY 2:

Eqn. (3.1.5') on p. 68 and eqn. (3.2.6) on p. 73 in Davis (1994) provide the eigenvalue formula for circulant matrices

(A2) $$\lambda_j = \sum_{n\in\Omega} c_n \times \left(\cos[2\pi n(j-1)/N] + \left(\sqrt{-1}\right)\sin[2\pi n(j-1)/N]\right),$$

where $(c_0, c_1, \ \ldots \ , c_{N-1})$ is the first row of the matrix. In our case, for eqn. (5), $c_1 = c_{N-1} = 1$ and all other entries are zero so that

(A3) $$\begin{aligned}\lambda_j &= \cos[2\pi(j-1)/N] + \left(\sqrt{-1}\right)\sin[2\pi(j-1)/N] \\ &+ \cos[2\pi(N-1)(j-1)/N] + \left(\sqrt{-1}\right)\sin[2\pi(N-1)(j-1)/N] \\ &= 2\cos[2\pi(j-1)/N],\end{aligned}$$

as $\cos[2\pi(j-1)/N] \equiv \cos[2\pi(N-1)(j-1)/N]$ and $\sin[2\pi(j-1)/N] \equiv$ $-\sin[2\pi(N-1)(j-1)/N]$. The other statements follow from cosine function properties. □

PROOF OF PROPOSITION 1:

Theorem 10 in Chao and Liang (1992) shows that $r^{*,ev} = \min_{r\in T(\hat{r})} \sum_{n\in\Omega} f\left(\left|r_{[n+1]} - r_n\right|\right)$ when N is even and $r^{*,od} = \min_{r\in T(\hat{r})} \sum_{n\in\Omega} f\left(\left|r_{[n+1]} - r_n\right|\right)$ for $f(\cdot): \mathbb{R} \to \mathbb{R}$ a convex function and $|\cdot|$ the absolute value function. Let $f(x) = x^2$ and note that $\sum_{n\in\Omega}\left(\left|r_{[n+1]} - r_n\right|\right)^2 = 2\sum_{n\in\Omega} r_n^2$ $-2\sum_{n\in\Omega} r_n r_{[n+1]}$. Since $\sum_{n\in\Omega} r_n^2$ is invariant for $r \in T(\hat{r})$, it follows that these vectors must maximize $\rho(r)$ over domain $r \in T(\hat{r})$. □

PROOF OF COROLLARY 1.1:

Rearrangements of any r do not affect $\bar{r}$ or $v(r)$, while $B_\rho(\cdot) > 0$. So the optimal r in some set $T(\hat{r})$ will be the one maximizing $\rho(r)$. As it is true of any vector, it is true of

$T\left(\arg\max_{r\in S} B(\cdot)\right)$. □

PROOF OF PROPOSITION 2:

Again, we need only consider the effect on $\theta(r)=\sum_{n\in\Omega} r_{[n-1]}r_n$. There are two cases: either the i^{th} and j^{th} farms are adjacent or they are not. Suppose they are adjacent, or $j=[i+1]$ where symmetry ensures that consideration of $j=[i-1]$ is identical. Then

(A4)
$$\begin{aligned}\theta(\tilde{r}) &= r_0 r_1 + \ldots + r_{[i-1]}(r_i+\varepsilon)+(r_i+\varepsilon)(r_{[i+1]}-\varepsilon)+(r_{[i+1]}-\varepsilon)r_{[i+2]}+\ldots+r_{N-1}r_0\\ &=\theta(r)+r_{[i-1]}\varepsilon - r_i\varepsilon + r_{[i+1]}\varepsilon-\varepsilon^2-r_{[i+2]}\varepsilon,\end{aligned}$$

with ε derivative $\partial\theta(\tilde{r})/\partial\varepsilon\big|_{\varepsilon=0}=r_{[i-1]}-r_i+r_{[i+1]}-r_{[i+2]}$ where $i=[j-1]$ and $[i+2]=[j+1]$.

Suppose instead the farms are not adjacent. Then

(A5)
$$\theta(\tilde{r})=\theta(r)+r_{[i-1]}\varepsilon+r_{[i+1]}\varepsilon-r_{[j-1]}\varepsilon-r_{[j+1]}\varepsilon,$$

with ε derivative $\partial\theta(\tilde{r})/\partial\varepsilon\big|_{\varepsilon=0}=r_{[i-1]}+r_{[i+1]}-r_{[j-1]}-r_{[j+1]}$.

As for variance, $\bar{r}$ does not change so we need only consider the effect on the sum of squares. The expression

(A6)
$$r_0^2+\ldots+(r_i+\varepsilon)^2+\ldots+(r_j-\varepsilon)^2+\ldots+r_{N-1}^2$$

has ε derivative $2r_i-2r_j$ when $\varepsilon=0$. □

PROOF OF COROLLARY 2.1:

This follows from the assumed sign on $B_v(\cdot)$ in addition to $B_\rho(\cdot)>0$. □

PROOF OF PROPOSITION 3:

Insert (8) into (3) and expand to obtain

(A7)
$$\begin{aligned}\rho(r) &= N^{-1}\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} a_k\cos(2\pi nk/N)\sum_{k\in\Omega^0} a_k\cos(2\pi(n+1)k/N)\Big]\\ &+N^{-1}\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} a_k\cos(2\pi nk/N)\sum_{k\in\Omega^0} b_k\sin(2\pi(n+1)k/N)\Big]\\ &+N^{-1}\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} b_k\sin(2\pi nk/N)\sum_{k\in\Omega^0} a_k\cos(2\pi(n+1)k/N)\Big]\\ &+N^{-1}\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} b_k\sin(2\pi nk/N)\sum_{k\in\Omega^0} b_k\sin(2\pi(n+1)k/N)\Big].\end{aligned}$$

Consider each of these four right-hand terms in turn. For the first, use

(A8) $$\cos(2\pi(n+1)k/N) = \cos(2\pi nk/N)\cos(2\pi k/N) - \sin(2\pi nk/N)\sin(2\pi k/N),$$

to write

(A9)
$$\begin{aligned}&\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} a_k\cos(2\pi nk/N)\sum_{l\in\Omega^0} a_l\cos(2\pi(n+1)l/N)\Big]\\ &=\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} a_k\cos(2\pi nk/N)\sum_{l\in\Omega^0} a_l\cos(2\pi nl/N)\cos(2\pi l/N)\Big]\\ &-\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} a_k\cos(2\pi nk/N)\sum_{l\in\Omega^0} a_l\sin(2\pi nl/N)\sin(2\pi l/N)\Big].\end{aligned}$$

But multiplication of the first inner product of sums and then use of the orthogonality property

(A10) $$\sum_{n\in\Omega}\cos(2nk\pi/N)\cos(2nl\pi/N) = 0 \text{ for } k \text{ and } l \neq k \text{ integers},$$

on the sum over $n \in \Omega$ leads to

(A11)
$$\begin{aligned}&\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} a_k\cos(2\pi nk/N)\sum_{l\in\Omega^0} a_l\cos(2\pi nl/N)\cos(2\pi l/N)\Big]\\ &=\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} a_k^2\cos^2(2\pi nk/N)\cos(2\pi k/N)\Big]\\ &=\sum_{k\in\Omega^0} a_k^2\cos(2\pi k/N)\sum_{n\in\Omega_N}\cos^2(2\pi nk/N).\end{aligned}$$

Multiply out the remaining inner product of sums in (A9) and apply orthogonality property

(A12) $$\sum_{n\in\Omega}\cos(2nk\pi/N)\sin(2nl\pi/N) = 0 \text{ for } k \text{ and } l \text{ integers},$$

to confirm

(A13) $$\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} a_k\cos(2\pi nk/N)\sum_{l\in\Omega^0} a_l\sin(2\pi nl/N)\sin(2\pi l/N)\Big] = 0$$

so that

(A14)
$$\begin{aligned}&\sum_{n\in\Omega}\Big[\sum_{k\in\Omega^0} a_k\cos(2\pi nk/N)\sum_{l\in\Omega^0} a_l\cos(2\pi(n+1)l/N)\Big]\\ &=\sum_{k\in\Omega^0} a_k^2\cos(2\pi k/N)\sum_{n\in\Omega_N}\cos^2(2\pi nk/N) = 0.5N\sum_{k\in\Omega^0} a_k^2\cos(2\pi k/N),\end{aligned}$$

where $\sum_{n\in\Omega_N}\cos^2(2\pi nk/N)=0.5N\ \forall N\geq 3$ simplifies. With the additional use of

$$\text{(A15)}\qquad \begin{aligned}&\sin(2\pi(n+1)k/N)=\sin(2\pi nk/N)\cos(2\pi k/N)+\sin(2\pi k/N)\cos(2\pi nk/N);\\ &\sum_{n\in\Omega}\sin(2nk\pi/N)\sin(2nl\pi/N)=0\ \text{ for } k \text{ and } l\neq k \text{ integers};\\ &\sum_{n\in\Omega_N}\sin^2(2\pi nk/N)=0.5N\ \forall N\geq 3,\end{aligned}$$

it can be shown that

$$\text{(A16)}\qquad \begin{aligned}&\sum_{n\in\Omega}\left[\sum_{k\in\Omega^0}a_k\cos(2\pi nk/N)\sum_{k\in\Omega^0}b_k\sin(2\pi(n+1)k/N)\right]\\ &=0.5N\sum_{k\in\Omega^0}a_kb_k\sin(2\pi k/N);\\ &\sum_{n\in\Omega}\left[\sum_{k\in\Omega^0}b_k\sin(2\pi nk/N)\sum_{k\in\Omega^0}a_k\cos(2\pi(n+1)k/N)\right]\\ &=-0.5N\sum_{k\in\Omega^0}a_kb_k\sin(2\pi k/N);\\ &\sum_{n\in\Omega_N}\left[\sum_{k\in\Omega^0}b_k\sin(2\pi nk/N)\sum_{k\in\Omega^0}b_k\sin(2\pi(n+1)k/N)\right]\\ &=0.5N\sum_{k\in\Omega^0}b_k^2\cos(2\pi k/N).\end{aligned}$$

The sum in (A7) therefore resolves to

$$\text{(A17)}\qquad \rho(r)=0.5\sum_{k\in\Omega^0}\left(a_k^2+b_k^2\right)\cos(2\pi k/N).$$

As for variance, insert (8) into the expression for variance to obtain

$$\text{(A18)}\qquad \begin{aligned}v(r)&=N^{-1}\sum_{n\in\Omega}\left[\sum_{k\in\Omega^0}a_k\cos(2\pi nk/N)\sum_{k\in\Omega^0}a_k\cos(2\pi nk/N)\right]\\ &+N^{-1}\sum_{n\in\Omega}\left[\sum_{k\in\Omega^0}a_k\cos(2\pi nk/N)\sum_{k\in\Omega^0}b_k\sin(2\pi nk/N)\right]\\ &+N^{-1}\sum_{n\in\Omega}\left[\sum_{k\in\Omega^0}b_k\sin(2\pi nk/N)\sum_{k\in\Omega^0}a_k\cos(2\pi nk/N)\right]\\ &+N^{-1}\sum_{n\in\Omega}\left[\sum_{k\in\Omega^0}b_k\sin(2\pi nk/N)\sum_{k\in\Omega^0}b_k\sin(2\pi nk/N)\right].\end{aligned}$$

The orthogonality conditions laid out above, i.e., where the sum of a product equals 0, readily leads to $v(r)=0.5\sum_{k\in\Omega^0}(a_k^2+b_k^2)$. □

PROOF OF PROPOSITION 4:

Part A): For mean, it is readily shown that $\sum_{n\in\Omega}\cos(2\pi n/N)=\sum_{n\in\Omega}\sin(2\pi n/N)=0$ so that $\sum_{n\in\Omega}r_n=N\overline{r}$ regardless of how values of the spectral sum coefficients are rearranged. On variance, $v(r) = 0.5\sum_{k\in\Omega^0}(a_k^2 + b_k^2)$ in Proposition 3 may be viewed in terms of partial sums. The condition that $C''_{(N-1)/2} = C'_{(N-1)/2}$ when N is odd and $C''_{N/2} = C'_{N/2}$ when N is even ensures that variance is held fixed.

Part B): Use Proposition 3 and summation by parts to write

$$
\begin{aligned}
\rho(r) &= 0.5\sum_{k\in\Omega^{0,low}} c_k \cos(2\pi k/N) \\
&= 0.5c_1\cos(2\pi/N) + 0.5c_2\cos(4\pi/N) + \ \ldots\ + 0.5c_{(N-1)/2}\cos(\pi(N-1)/N) \\
&= 0.5C_1\left[\cos(2\pi/N) - \cos(4\pi/N)\right] + 0.5C_2\left[\cos(4\pi/N) - \cos(6\pi/N)\right] \\
&\quad +0.5C_3\left[\cos(6\pi/N) - \cos(8\pi/N)\right] + \ \ldots\ + 0.5C_{(N-1)/2}\cos(\pi(N-1)/N),
\end{aligned} \tag{A19}
$$

when N is odd and replace the last term by $0.5C_{N/2}\cos(\pi N/N) = -0.5C_{N/2}$ when N is even. If buy down vectors r' and r'' are represented by vectors c' and c'', respectively, then

$$
\begin{aligned}
\rho(r'') - \rho(r') &= 0.5\overbrace{\left(C''_1 - C'_1\right)}^{\geq 0}\overbrace{\left[\cos(2\pi/N) - \cos(4\pi/N)\right]}^{\geq 0} \\
&\quad +0.5\overbrace{\left(C''_2 - C'_2\right)}^{\geq 0}\overbrace{\left[\cos(4\pi/N) - \cos(6\pi/N)\right]}^{\geq 0} + 0.5\overbrace{\left(C''_3 - C'_3\right)}^{\geq 0}\overbrace{\left[\cos(6\pi/N) - \cos(8\pi/N)\right]}^{\geq 0} \\
&\quad + \ \ldots\ + 0.5\overbrace{\left(C''_{(N-1)/2} - C'_{(N-1)/2}\right)}^{=0}\overbrace{\cos(\pi(N-1)/N)}^{<0}
\end{aligned} \tag{A20}
$$

when N is odd and replace the last term by $0.5(C''_{N/2} - C'_{N/2})\cos(\pi) = -0.5(C''_{N/2} - C'_{N/2})$ when N is even. Clearly, condition set (10) together with $\cos(x)$ decreasing on $x \in (0,\pi)$ ensure that $\rho(r'') \geq \rho(r')$.

Part C): The transfers in question ensure satisfaction of condition set (10). □

PROOF OF PROPOSITION 5:

Part A): We seek the extremes on (13) to establish when a K exists such that the r_n are in the interior of S given variance value v. Differentiate (12) and set equal zero as the necessary condition for a maximum or minimum:

$$-K_1\sin(2\pi n/N) + K_2\cos(2\pi n/N) = 0; \tag{A21}$$

with solution set $n^* \in [0, N)$ where the solutions need not be in Ω. Re-write as

(A22) $$n^* = \frac{N}{2\pi} \tan^{-1}\left(K_2 / K_1\right).$$

Consideration of the shape of the inverse tangent function shows that there are two solutions on $n^* \in [0, N)$. Furthermore, trigonometric periodicity ensures that $-K_1 \sin(2\pi n / N) + K_2 \cos(2\pi n / N) = K_1 \sin\left[(2n + N)\pi / N\right] - K_2 \cos\left[(2n + N)\pi / N\right]$, implying that the two solutions are π radians apart. Since (12) is continuous in n, one of the solutions must be the unique maximum and the other must be the unique minimum. So any candidate optima satisfy

(A23) $$r_n - \bar{r} = K_1 \cos\left[\tan^{-1}\left(K_2 / K_1\right)\right] + K_2 \sin\left[\tan^{-1}\left(K_2 / K_1\right)\right].$$

Now

(A24) $$\cos\left[\tan^{-1}\left(K_2 / K_1\right)\right] = \pm \frac{K_1}{\sqrt{K_1^2 + K_2^2}}; \qquad \sin\left[\tan^{-1}\left(K_2 / K_1\right)\right] = \pm \frac{K_2}{\sqrt{K_1^2 + K_2^2}};$$

where the signs must match. Consequently the knowledge $v(r) = 0.5K_1^2 + 0.5K_2^2$ provides

(A25) $$r_n - \bar{r} = \pm\sqrt{K_1^2 + K_2^2} = \pm\sqrt{2v(r)}$$

as maximum and minimum. Thus for interior solutions on S we need $0 < \bar{r} - \sqrt{2v(r)}$ and $\bar{r} + \sqrt{2v(r)} < \hat{q}$, or

(A26) $$v(r) < 0.5 \min\left[\bar{r}^2, (\hat{q} - \bar{r})^2\right] \equiv v^+;$$

Part B): Use Wirtinger's inequality and specification (12) to re-write $\sum_{n=0}^{N-1} (r_n - r_{n-1})^2 = 4\sin^2(\pi / N) \sum_{n=0}^{N-1} (r_n - \bar{r})^2$ as $2\sum_{n=0}^{N-1} r_n^2 - 2\sum_{n=0}^{N-1} r_n r_{[n-1]} = 4\sin^2(\pi / N) Nv$ or $\left(\sum_{n=0}^{N-1} r_n^2 - \bar{r}^2\right) - \left(\sum_{n=0}^{N-1} r_n r_{[n-1]} - \bar{r}^2\right) = 2\sin^2(\pi / N) Nv$ or $Nv - N\rho = 2\sin^2(\pi / N) Nv$ or

(A27) $$\rho = v - 2\sin^2(\pi / N) v = \cos(2\pi / N) v,$$

as $\cos(2\pi / N) \equiv 1 - 2\sin^2(\pi / N)$. □

PROOF OF PROPOSITION 6:

Part A): Clearly, setting $v = 0$ maximizes the value of $H(\overline{r}, v)$ on $v \in [0, v^{\max}]$.

Part B): Setting $v = v^{+}$ maximizes the value of $H(\overline{r}, v)$ on $v \in [0, v^{+}]$. The only solutions that have not been ruled out involve at least partial intensification.

Part C): Quasiconcavity rules out a solution on $v \in (v^{+}, v^{\max}]$ given an interior maximum on $v \in [0, v^{+}]$.

Part D) Quasiconvexity with $H(\overline{r}, 0) < H(\overline{r}, v^{+})$ ensures that the solution is in $[v^{+}, v^{\max}]$ while all solutions on $v \in [v^{+}, v^{\max}]$ must involve at least partial intensification. □

PROOF OF PROPOSITION 7:

Part A): This is an adaptation of Remark 2, p. 110, in Yamamoto and Ikebe (1979). Note, from (22) that $\phi_h \phi_l = 1$, so that (23) can be rewritten as:

(A28) $$p_{i,j} = \begin{cases} \left(\phi_h^i - \phi_l^i\right)\left(\phi_h^{N-1-j} - \phi_l^{N-1-j}\right)\Psi, & i \le j; \\ \left(\phi_h^j - \phi_l^j\right)\left(\phi_h^{N-1-i} - \phi_l^{N-1-i}\right)\Psi, & i > j. \end{cases}$$

Part B): Since $\Delta > 0$ and $\phi_h^{N-1} > \phi_l^{N-1}$, therefore $\Psi > 0$. Since $\phi_h > \phi_l, i > 0,\ j > 0$, and $N - 1 - i > 0$, $N - 1 - j > 0$ for all $i, j \in \{1, \ldots, N-2\}$, it immediately follows that all terms in parentheses in (A28) are strictly positive, proving the assertion.

Part C): For symmetry, arbitrarily assume that $i \le j$. Using (A28), $p_{i,j}$ and $p_{j,i}$ are:

(A29) $$\begin{aligned} p_{a,b} &= \left(\phi_h^a - \phi_l^a\right)\left(\phi_h^{N-1-b} - \phi_l^{N-1-b}\right)\Psi, && a \le b; \\ p_{b,a} &= \left(\phi_h^a - \phi_l^a\right)\left(\phi_h^{N-1-b} - \phi_l^{N-1-b}\right)\Psi, && b \ge a. \end{aligned}$$

Centro-symmetry follows from inspection of (A28); e.g., $i \le j$ implies $N - 1 - i \ge N - 1 - j$ so that $p_{i,j} = (\phi_h^i - \phi_l^i)(\phi_h^{N-1-j} - \phi_l^{N-1-j})$ and

(A30)
$$\begin{aligned} p_{N-1-i,N-1-j} &= \left(\phi_h^{N-1-j} - \phi_l^{N-1-j}\right)\left(\phi_h^{N-1-(N-1-i)} - \phi_l^{N-1-(N-1-i)}\right) \\ &= \left(\phi_h^{N-1-j} - \phi_l^{N-1-j}\right)\left(\phi_h^{i} - \phi_l^{i}\right) = p_{i,j}. \end{aligned}$$

Part D): This is also immediate from inspection of (A28). For $i < j$, and $\phi_h > 1 > \phi_l = \phi_h^{-1}$ then the top line of (A28) is increasing in *i*, whereas the bottom line is decreasing in *i*, implying that $p_{i,j}$ is increasing in *i* for $i < j$ and $p_{i,j}$ is decreasing in *i* for $i > j$, thus proving the conjecture. The second part of the statement, how $p_{i,j}$ varies with *j*, follows from the symmetry $p_{i,j} = p_{j,i}$ in part C).

Part E): Note from centro-symmetry in part C) that $p_{(N-2)/2,j} = p_{(N-2)/2,N-1-j}$. Then set $j =$ $-k + (N-1)/2$ so that $p_{(N-2)/2,-k+(N-1)/2} = p_{(N-2)/2,N-1-(N-1)/2+k} = p_{(N-2)/2,(N-1)/2+k}$.

Part F): From (A28),

(A31)
$$p_{i,i} = \left(\phi_h^{i} - \phi_l^{i}\right)\left(\phi_h^{N-1-i} - \phi_l^{N-1-i}\right)\Psi = \left(\phi_h^{i} - \phi_h^{-i}\right)\left(\phi_h^{N-1-i} - \phi_h^{-N+1+i}\right)\Psi.$$

Specifying $\Delta_i \equiv p_{i+1,i+1} - p_{i,i}$, it follows that

(A32)
$$\begin{aligned} \Delta_i &= \left\{-\left(\phi_h^{i} - \phi_h^{-i}\right)\left(\phi_h^{N-1-i} - \phi_h^{-N+1+i}\right) + \left(\phi_h^{i+1} - \phi_h^{-i-1}\right)\left(\phi_h^{N-2-i} - \phi_h^{-N+2+i}\right)\right\}\Psi \\ &= \left\{-\phi_h^{N-1} + \phi_h^{-N+1+2i} + \phi_h^{N-1-2i} - \phi_h^{-N+1} + \phi_h^{N-1} - \phi_h^{-N+3+2i} - \phi_h^{N-3-2i} + \phi_h^{-N+1}\right\}\Psi \\ &= \left\{-\phi_h^{-N+1+2i}\left(\phi_h^{2} - 1\right) + \phi_h^{N-3-2i}\left(\phi_h^{2} - 1\right)\right\}\Psi = \left\{\phi_h^{N-3-2i}\left(\phi_h^{2} - 1\right)\left(1 - \phi_h^{4i+4-2N}\right)\right\}\Psi. \end{aligned}$$

Since $\phi_h > 1$, Δ_i has the sign of $2N - 4i - 4$, i.e., $\Delta_i \geq 0$ if and only if $i \leq (N-2)/2$. For *i* even and $i' \equiv (N/2) - 1$ then $p_{i,i}$ is increasing in *i* when $i < i'$, $p_{i',i'} = p_{i'+1,i'+1}$, and $p_{i,i}$ is decreasing in *i* when $i > i'$. For *i* odd and $i'' \equiv (N-1)/2$, $p_{i,i}$ increases when $i < i''$, reaches a maximum at $i = i''$ and decreases when $i \geq i''$.

Part G): This requires writing out the relevant terms,

(A33)
$$\begin{aligned} p_{i,1} &= \Psi(\phi_h - \phi_l)\left(\phi_h^{N-1-i} - \phi_l^{N-1-i}\right); \\ p_{i,N-2} &= \Psi\left(\phi_h^{i} - \phi_l^{i}\right)\left(\phi_h^{N-1-(N-2)} - \phi_l^{N-1-(N-2)}\right) = \Psi\left(\phi_h - \phi_l\right)\left(\phi_h^{i} - \phi_l^{i}\right). \end{aligned}$$

Thus we may define

(A34) $$A \equiv p_{i,1} + p_{i,N-2} = \Psi\left(\phi_h - \phi_l\right)\left[\left(\phi_h^{N-1-i} - \phi_l^{N-1-i}\right) + \left(\phi_h^{i} - \phi_l^{i}\right)\right].$$

Similarly, with

(A35) $$\begin{aligned} &p_{i-1,1} = \Psi\left(\phi_h - \phi_l\right)\left(\phi_h^{N-i} - \phi_l^{N-i}\right); \qquad p_{i-1,N-2} = \Psi\left(\phi_h^{i-1} - \phi_l^{i-1}\right)\left(\phi_h - \phi_l\right); \\ &B \equiv p_{i-1,1} + p_{i-1,N-2} = \Psi\left(\phi_h - \phi_l\right)\left[\left(\phi_h^{N-i} - \phi_l^{N-i}\right) + \left(\phi_h^{i-1} - \phi_l^{i-1}\right)\right]; \end{aligned}$$

then

(A36) $$\begin{aligned} \Delta_A(i) \equiv A - B &= \left\{\left[p_{i,1} + p_{i,N-2}\right] - \left[p_{i-1,1} + p_{i-1,N-2}\right]\right\} \\ &= \Psi\left(\phi_h - \phi_l\right)\left[\left(\phi_h^{N-1-i} - \phi_l^{N-1-i}\right) + \left(\phi_h^{i} - \phi_l^{i}\right) - \left(\phi_h^{N-i} - \phi_l^{N-i}\right) - \left(\phi_h^{i-1} - \phi_l^{i-1}\right)\right] \\ &= \Psi\left(\phi_h - \phi_l\right)\left[\left(\phi_h^{N-1-i}\left(1 - \phi_h\right) - \phi_l^{N-1-i}\left(1 - \phi_l\right)\right) + \phi_h^{i-1}\left(\phi_h - 1\right) + \phi_l^{i-1}\left(1 - \phi_l\right)\right] \\ &= \Psi\left(\phi_h - \phi_l\right)\left(\phi_h - 1\right)\left[\left(-\phi_h^{N-1-i} - \phi_l^{N-1-i}\phi_h^{-1}\right) + \phi_h^{i-1} + \phi_l^{i-1}\phi_h^{-1}\right], \end{aligned}$$

where $(1-\phi_l) = \phi_h^{-1}(\phi_h - 1)$ has been used. Factorize further to obtain

(A37) $$\begin{aligned} \Delta_A(i) &= \Psi\left(\phi_h - \phi_l\right)(\phi_h - 1)\left[\left(-\phi_h^{N-1-i} - \phi_h^{-N+i}\right) + \phi_h^{i-1} + \phi_h^{-i}\right] \\ &= \Psi\left(\phi_h - \phi_l\right)(\phi_h - 1)\left[\phi_h^{i-1}\left(1 - \phi_h^{-N+1}\right) - \phi_h^{-i}\left(\phi_h^{N-1} - 1\right)\right] \\ &= \Psi\left(\phi_h - \phi_l\right)(\phi_h - 1)\left(\phi_h^{N-1} - 1\right)\phi_h^{-i}\left(\phi_h^{2i-N} - 1\right). \end{aligned}$$

For *N* even, let $i' \equiv N/2$. Then $\Delta_A(i)$ is negative on $i < i'$ and positive on $i > i'$, implying $p_{i,1} + p_{i,N-2} < p_{i-1,1} + p_{i-1,N-2}$ on $i \in \{2, 3, \ldots, (N/2) - 1\}$, $p_{i,1} + p_{i,N-2} = p_{i-1,1} + p_{i-1,N-2}$ when $i = N/2$ and $p_{i,1} + p_{i,N-2} > p_{i-1,1} + p_{i-1,N-2}$ on $i \in \{(N/2) + 1, \ldots, N-2\}$. For *N* odd, the same reasoning implies $p_{i,1} + p_{i,N-2} < p_{i-1,1} + p_{i-1,N-2}$ on $i \in \{2, 3, \ldots, (N-1)/2\}$, and $p_{i,1} + p_{i,N-2} > p_{i-1,1} + p_{i-1,N-2}$ on $i \in \{(N+1)/2, \ldots, N-2\}$.

Part H): From part C), $p_{i,j} = p_{N-2-i,N-2-j}$. Sum each over *j* to conclude that $P_i = P_{N-2-i}$. Rewrite (21) in the text as:

$$\mathbf{\Gamma r} = \mathbf{b}; \qquad \mathbf{\Gamma} \equiv \begin{pmatrix} 1+2\xi & -\xi & 0 & \cdots & 0 & 0 \\ -\xi & 1+2\xi & -\xi & \cdots & 0 & 0 \\ 0 & -\xi & 1+2\xi & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1+2\xi & -\xi \\ 0 & 0 & 0 & \cdots & -\xi & 1+2\xi \end{pmatrix};$$

(A38)

$$\mathbf{r} \equiv \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_{N-3} \\ r_{N-2} \end{pmatrix}; \qquad \mathbf{b} \equiv \frac{1}{\kappa} \begin{pmatrix} \alpha_1 + \tau \hat{r}_0 \\ \alpha_2 \\ \alpha_3 \\ \vdots \\ \alpha_{N-3} \\ \alpha_{N-2} + \tau \hat{r}_{N-1} \end{pmatrix}.$$

By definition, $p_{i,j}$ is the element in the i^{th} row and j^{th} column of $\mathbf{\Gamma}^{-1}$. By definition:

(A39)
$$\begin{pmatrix} P_1 \\ P_2 \\ \vdots \\ P_{N-3} \\ P_{N-2} \end{pmatrix} = \mathbf{\Gamma}^{-1}\mathbf{U} \qquad \text{where} \qquad \mathbf{U} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{pmatrix}.$$

Also,

(A40)
$$\mathbf{\Gamma U} = \begin{pmatrix} 1+\xi \\ 1 \\ \vdots \\ 1 \\ 1+\xi \end{pmatrix} = \mathbf{U} + \mathbf{E} \qquad \text{where} \qquad \mathbf{E} = \begin{pmatrix} \xi \\ 0 \\ \vdots \\ 0 \\ \xi \end{pmatrix}.$$

Thus:

(A41)
$$\mathbf{U} = \mathbf{\Gamma}^{-1}\mathbf{\Gamma U} = \mathbf{\Gamma}^{-1}\left(\mathbf{U} + \mathbf{E}\right) = \mathbf{P} + \mathbf{\Gamma}^{-1}\mathbf{E}.$$

Now pick off the i^{th} row in $\mathbf{U}$ and write it as U_i. From (A41) it is $U_i = 1 = P_i +$ $(p_{i,1} + p_{i,N-2})\xi$. Since part B) of the proposition has $p_{i,j} > 0$, therefore

(A42)
$$P_i = 1 - \left(p_{i,1} + p_{i,N-2}\right)\xi < 1.$$

Hence:

(A43) $$P_i - P_{i-1} = \xi\left\{\left(p_{i-1,1} + p_{i-1,N-2}\right) - \left(p_{i,1} + p_{i,N-2}\right)\right\} = \frac{\tau}{\kappa}\left\{p_{i-1,1} + p_{i-1,N-2} - p_{i,1} - p_{i,N-2}\right\}.$$

From part G) of the proposition, if N is odd, the expression on the right-hand side of (A43) is positive for $i \in \{2,3, \ldots ,(N-1)/2\}$ and negative for $i \in \{(N+1)/2, \ldots ,N-2\}$, implying P_i increases for $i \le (N-1)/2$, attains its maximum at $i = (N-1)/2$ and decreases thereafter. Similarly, for N even, the expression on the right-hand side of (A43) is positive for $i \in$ $\{2,3, \ldots ,(N/2)-1\}$, zero when $i = N/2$ and negative for $i \in \{(N/2)+1, \ldots ,N-2\}$, implying P_i increases for $i \le (N-2)/2$, reaches a maximum at $P_{(N-2)/2} = P_{N/2}$, and decreases thereafter. □

PROOF OF COROLLARY 7.1: From (21) and (23), $dr_n / d\hat{r}_0 = p_{n,1}\tau / \kappa$. Parts D) and C) of the proposition have that $p_{n,1} \equiv p_{1,n}$ is decreasing in the value of n . Similarly, $dr_n / d\hat{r}_{N-1} =$ $p_{n,N-2}\tau / \kappa$ while parts D) and C) have that $p_{n,N-2} \equiv p_{N-2,n}$ is increasing in the value of n . □

PROOF OF COROLLARY 7.2: From inverting (21) in the text we can write

(A44) $$r_i = P_i\alpha + \tau\left(p_{i,1}\hat{r}_0 + p_{i,N-2}\hat{r}_{N-1}\right) = \alpha + \tau\left[p_{i,1}\left(\hat{r}_0 - \frac{\alpha}{\kappa}\right) + p_{i,N-2}\left(\hat{r}_{N-1} - \frac{\alpha}{\kappa}\right)\right],$$

where, in (A44), we use the result from the proof of part H), Proposition 7, that:

(A45) $$P_i = 1 - \left(p_{i,1} + p_{i,N-2}\right)\xi; \qquad \xi \equiv \tau / \kappa.$$

Part A): For $\hat{r}_0 = \hat{r}_{N-1} > \alpha / \kappa$, (A44) becomes:

(A46) $$r_i = \alpha + \tau\left(\hat{r}_0 - \frac{\alpha}{\kappa}\right)\left(p_{i,1} + p_{i,N-2}\right).$$

Since part G) of Proposition 7 establishes that the sequence $p_{i,1} + p_{i,N-2}$ is U-shaped, with minimum in the middle of the interval $\{1, \ldots ,N-2\}$, and since $\hat{r}_0 = \hat{r}_{N-1} > \alpha / \kappa$, it immediately follows from (A46) that the sequence r_i is also U-shaped with $r_{\min} > \alpha$.

Part B): The logic is the same as part A), except that since $\hat{r}_0 - \alpha / \kappa < 0$, the sequence r_i forms an inverted U, with maximum $r_{\max} < \alpha$ in the middle of the interval, and relative minima at the end points.

Part C): Note that, from Proposition 7, part D), $p_{i,1}$ is monotonically decreasing in *i* and that $p_{i,N-2}$ is monotonically increasing in *i*. Thus, for $\hat{r}_0 - < 0 < \hat{r}_{N-1} - \alpha / \kappa$, it follows that $p_{i,1}\left(\hat{r}_0 - \alpha / \kappa\right) + p_{i,N-2}\left(\hat{r}_{N-1} - \alpha / \kappa\right)$ is monotonically increasing in *i*, proving the assertion.

Part D): Proof of D) is identical to C), except that since $\hat{r}_0 - \alpha / \kappa > 0 > \hat{r}_{N-1} - \alpha / \kappa$, $p_{i,1}\left(\hat{r}_0 - \alpha / \kappa\right) + p_{i,N-2}\left(\hat{r}_{N-1} - \alpha / \kappa\right)$ - and hence r_i - is monotone decreasing in *i*. □

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