Inspection Intensity and Market Structure

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Abstract

An investigation of financing an inspection policy while allowing the enforcement of a market regulation is described. A simple model shows that the intensity of controls depends on the market structure. Under a given number of firms, the per-firm probability of controls is lower than one, since firms’ incentive to comply with regulation holds under positive profits. In this case, a lump-sum tax is used for limiting distortions coming from financing with a fixed fee. Under free entry, the per-firm probability of controls is equal to one, and only a fixed fee that prevents excess entry is used to finance inspection.

Keywords: inspection policies, market regulation, regulatory funding.
1. INTRODUCTION

In Europe, Canada, and the United States, agencies enforce market regulation by means of a monitoring policy. From counterfeiting to food safety, from nuclear plant safety to stock trading, the lack of money is always the reason put forward to justify some difficulties/inefficiencies in inspection policies.

Funding is particularly important for monitoring firms’ compliance with environmental regulations. Public management to control firms’ efforts at reducing risks and pollution is very costly because each unit needs to be inspected. The European Environmental Agency (2000) has mentioned the lack of resources of public environmental authorities in different member states.

In the United States, the 15,500 sites that are included in the Risk Management Program (RMP) of the Environmental Protection Agency (EPA) because of hazardous chemicals handling are monitored by only 50 inspectors (Kunreuther and Schmeidler 2004). Even if evaluating the optimal number of inspectors with a complete cost-benefit analysis is difficult, this figure of 50 inspectors seems insufficient for guaranteeing safety, and Kunreuther and Schmeidler (2004) mention infrequent inspections coming from the RMP. Minott (2001, p. 1) notes that some experts discussing the RMP said that “it would be preferable if implementing agencies would increase their own enforcement efforts, perhaps with the help of funding generated by fees charged to regulated facilities.”\(^1\) This last point is the topic of this theoretical paper, which is not limited to environmental regulations.

This issue of funding concerns all public audits that check the regulatory compliance of firms or agents. The limited amount of resources available for thorough monitoring raises the

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\(^1\) The U.S. Environmental Protection Agency imposes fees that are directly used to finance environmental cleanup, especially of so-called Superfund sites (see EPA 2003, p. 16).
issue of the available amount of money and the intensity of controls influencing the budget constraints of the regulatory agencies. The common method for financing an inspection program is the general government budget. Nevertheless, financing regulatory programs with general-public taxes can end up limiting programs’ monitoring activities when budgets are tight. As Becker (1999) mentioned, “unfortunately, generous funding for entitlements, farm and urban subsidies, and other programs tend to crowd out desirable forms of government spending” such as the control of environmental or product safety by the government. Because of this, policymakers often turn to alternative ways for funding an agency’s activity, such as fees for particular purposes. The earmarking of the fees is a way to counterbalance the lack of money for safety inspections.

The inspection policy (or the controls intensity) and the way to finance it are often dissociated by the policymaker. However, this paper shows that the probability of controls and financing tools are closely linked and depend on competitive intensity. In other words, a clear examination of the funding and the market context is necessary to define an efficient policy.

A theoretical model seeks a way to finance market regulation as long as it is cost effective to do so, in a context in which the regulatory compliance is costly for (symmetric) firms. We consider various combinations of means of financing commonly used by public agencies around the world: (1) a fixed fee paid by all firms (present on the market) and (2) a public program financed through taxes (as a lump-sum tax incurred by the rest of an economy). The number of controls made by the regulatory agency influences the firms’ incentive to comply with the regulation.

We show that the intensity of controls depends on the optimal choice between a public tax and an industry fee used to finance an inspection policy. Under a given number of firms, the
per-firm probability of controls is lower than one, since firms’ incentive to comply with regulation holds under positive profits. A lump-sum tax is used for limiting distortions coming from financing using a fixed fee that would increase the necessary number of controls. Under free entry, the per-firm probability of controls is equal to one, and only a fixed fee that prevents excess entry is used. Indeed, because of zero profits, the probability of controls is equal to one for impeding a deviation that would avoid the cost of regulatory compliance leading to positive profits.

This paper is linked to two separate strands of literature. The first strand of literature, initiated by Becker (1968), includes numerous papers on optimal monitoring policies linked to incomplete monitoring activity (Polinsky and Shavell 1979, 1991, and 1992), a dynamic approach (Harford and Harrington 1991 and Harrington 1988) or self revelation (Jones and Scotchmer 1990, Malik 1993, or Livernois and McKenna 1999). This strand of research mainly considers penalties as a credible threat for reaching a regulatory aim, without detailing the regulator’s optimal budgeting choices and the complete choice of funding instruments. These studies abstract from the market context and the firms’ probability depending on the competitive intensity. Our framework differs since we explicitly investigate some agency’s alternative sources of revenue (fee and/or tax) in a context in which the firms’ competition is taken into account.

This paper is also linked to the strand of literature that concerns the optimal way to finance monitoring regulation. For instance, in earlier work in which the number of controls and the number of firms are exogenous, Crespi and Marette (2001) and Marette and Crespi (2005) show that different tools for financing emerge at the equilibrium. Conversely, in the present paper, the number of controls and the number of firms are exogenous, which requires more
studies for capturing the interaction between the funding and the market mechanisms. This paper adds to this literature by showing that the financing and the probability of controls depend on the number of firms, either endogenous or exogenous.

A very simple model is presented in section 2, while the main results are detailed in section 3. Extensions are given in section 4, and section 5 concludes.

2. A SIMPLE MODEL

Our model is a very simple framework (some extensions will be discussed in the fourth section). Trade occurs in a single period, and $n$ producers may choose to produce either low- or high-quality products. This decision is private information for each producer. A “high-quality” good corresponds to a safe product, namely, a good leading to the absence of any damage. The sunk cost for producing only high-quality products is $C$ for each producer and the marginal cost is zero. A “low-quality” good corresponds to a dangerous process of production (or facility) entailing damage $D$ with probability $(1-\lambda)$. Even if the probability $(1-\lambda)$ is the same for the $n$ firms, the realization of damage for a firm is independent from the realization of the other firms’ damages. For simplicity, the marginal and fixed costs of producing only low-quality products are zero. With a probability $(1-\lambda)$, the per-firm damage $D$ is incurred by a third-party. The inverse demand by consumers is $p(Q)=a-Q$, where $p(Q)$ and $Q$ respectively denote the price and the quantity with $a >0$.

Assume that only a single agency is able to ensure a monitoring policy for inducing regulatory compliance that implies only high-quality products. The overall cost of monitoring depends on the number of firms inspected, which we assume will be done randomly. This cost is
denoted $xR$, where $R$ is the fixed cost per inspected firm/plant and $x$ is the number of inspected firms with $x \leq n$. The selection of $x$ determines the number of random inspections and it does not depend on any past experience or voluntary-information signaled by the firm. Thus $x/n$ is the probability that a firm will be inspected. The per-firm cost of monitoring $R$ provides a perfect revelation concerning the quality and the absence of regulatory compliance to the monitoring agency. The instruments for financing this spending are (1) a fixed fee $F$ paid by each firm and/or (2) a public financing by means of a lump-sum tax $T$ paid by taxpayers.\(^2\)

The single round of trading proceeds in four stages. In stage 1, the regulator announces its policy, namely, whether or not to propose a high-quality standard, an inspection policy $x$, and a selection of financing instrument(s) (1) and/or (2). The agency seeks to maximize welfare (defined by the sum of the sellers’ profits, the consumers’ surplus and the third party’s and taxpayers’ losses), while inspecting a minimum number of firms $x$.

In stage 2, $n$ producers simultaneously choose whether or not to comply with the regulation (with every firm knowing the regulation policy). We distinguish between a situation $(a)$ where the number of firms $n$ is given (due to barriers to entry for instance) and a situation $(b)$ where this number is endogenous, namely, in a context of free entry. Under situation $(a)$, we assume that the agency has no interest in limiting the number of firms, implying instruments (1) to (2) are affordable for the $n$ firms, though under situation $(b)$, the agency sways the number of firms. Under situation $(b)$, the decision of quality compliance is preceded by a simultaneous entry decision in this context of free entry. The entry decision is public information for all firms and the regulatory agency, while the quality choice is private information for each firm.

\(^2\) We abstract from several points. There is no liability for compensating the third party, which is unaware of the damage in the short term. We also abstract from any imperfect detection by the agency during a firm visit.
In stage 3, the public agency makes its inspection if a high-quality standard is selected. The agency imposes the production ban (equivalent to a facility closure) if an infringement of the high-quality standard is found.\footnote{The results are similar if a penalty is imposed on the cheating firm, as the selected penalty would be the largest one} In stage 4, producers simultaneously set quantities (Cournot competition) and earn their profits, which allows them to pay the cost $C$ and/or the fee $F$.

We now turn to the characterization of the subgame perfect Nash equilibrium of this four-stage game (solved by backward induction) and then conduct a welfare analysis allowing the selection among the different rules.

2.1 Firms’ strategy

We successively describe the production choice (stage 4) and the standard compliance (stages 2 and 3) of a firm. Each firm knows the regulation (or its absence), namely, the choice of a high-quality standard, the number of firms inspected $x$, and the values of the fee $F$ imposed on every firm.

If a firm selects high-quality products, no production ban is imposed in the case of control (whatever the selected standard), while the fee $F$ and the fixed cost $C$ are incurred in stage 2. The profit of a firm $i$ with high-quality products is $(a - Q)q_i - (C + F)$.

If a firm selects low-quality products and the regulator imposes a high-quality standard, the production ban occurs in case of inspection. With a probability $x/n$, the firm is inspected and cannot produce because of a product ban. The profit is zero and the firm is not able to pay the fee $F$ (because of the absence of any other assets for the firm). With a probability $(1-x/n)$, no inspection occurs, which leads to a profit $(a - Q)q_i - F$ for the firm $i$, since the firm is able to reimburse the fee $F$. The expected profit of a firm choosing low-quality products is then
In the absence of a high-quality standard, the choice of low-quality products leads to a profit \((a - Q)q - F\) since there is no inspection \((x=0)\).

Let \(I_i\) represent a firm decision regarding the quality selection. A value \(I_i=1\) means that high-quality products are selected, while a value \(I_i=0\) means that low-quality products are selected. By combining the previous expressions, the expected profit function of firm \(i\) is rewritten as \(\pi(I_i) = (1 - (1 - I_i)\frac{x}{n})[(a - Q)q_i - (F + I_iC)]\).

Under Cournot competition in stage 4, the quantity selection leads to the following first-order condition: \(d\pi(I_i)/dq_i = (a - 2q_i - \sum_{k=1}^{n-1} q_k) = 0\). Under a symmetric Cournot-Nash equilibrium, all sellers select the same quantity \((q_i = q_j = q^*)\) equal to \(q^* = a/(n+1)\) with an equilibrium price equal to \(p^* = a - nq^*\). The substitution of those values in \(\pi(I_i)\) leads to the per-firm profit:

\[
\pi(I_i, F) = (1 - (1 - I_i)\frac{x}{n})\left[\frac{a^2}{(n+1)^2} - (F + I_iC)\right]
\]

As the \(n\) firms are equivalent, they adopt the same strategy, leading to \(I_i=I\) for \(i=1,\ldots,n\). A firm selects high-quality products (linked to the investment \(C\)), if \(\pi(1, F) \geq \pi(0, F)\) and low-quality products otherwise. The consumers’ surplus is

\[
CS = \int_0^{q^*} (a - q - p^*)dq = \frac{n^2a^2}{2(n+1)^2}.
\]

The expected third-party loss is \(V = n(1-I)(1-\lambda)D\) and it depends on the \(n\) firms’ choice \(I\).
2.2 The financing of the agency

Regarding the financing of the agency, the overall cost of the policy $Rx$ has to be covered by the different instruments. Thus, the budget constraint for the agency is

\begin{equation}
B(F,T)=nF+T-Rx=0
\end{equation}

The welfare given by the sum of the profits, the consumers’ surplus and the third party’s and tax payers’ losses depends on the number of firms selecting high- or low-quality products. As such, the overall welfare is

\begin{equation}
W(I,F,T) = n\pi(I,F) - n(1-I)(1-\lambda)D + CS - T.
\end{equation}

Note that if $I=1$ (respectively $I=0$), no damage (respectively no cost $C$) is incurred by society (respectively by the firms). The welfare corresponding to the different types of financing instruments is detailed in the appendix (in the proofs of the different propositions).

3. THE POLICY

The public agency maximizes the welfare given by (4) subject to (3) and the firms’ quality choice represented by the comparison between $\pi(1,F)$ and $\pi(0,F)$. In this section, we distinguish between a situation $(a)$ where the number of firms $n$ is given and a situation $(b)$ where this number is endogenous.

3.1 A given number of firms

First, under situation $(a)$, the agency does not limit the number of firms $n$, implying instruments (1) to (4) are affordable for the $n$ firms. This means that the profits according to quality choices are $\pi(1,F)\geq0$ and $\pi(0,F)\geq0$. We also assume that the welfare with low-quality
products and without any control is \( W(0,0,0) \geq 0 \), which corresponds to a case where the per-firm damage \( D \) is not too high.\(^4\)

Comparing welfare under the agency financing constraint allows us to derive proposition 1. Below, we present the propositions and provide an intuitive interpretation (see mathematical details and the proofs of propositions in the appendix).

**Proposition 1.** When the number of firms \( n \) is given, the socially optimal policy is

(i) a high-quality standard, with a lump-sum tax \( T_1 > 0 \) and a probability of control lower than one, leading to high-quality products if \( R \leq R_1 \).

(ii) the absence of standard with no financing instruments \( (F=T=0) \) and no control \( (x(0)/n=0) \), leading to low-quality products, if \( R > R_1 \).

When the monitoring cost \( R \) is relatively low, the regulation is optimal since there is no low-quality product entailing a risk of damage. The probability of being control \( x(F)/n \) is lower than one as soon as firms have positive profits with high-quality products, namely, for \( a^2/(n+1)^2-C-F > 0 \), which is the case when \( C \) and \( n \) are relatively low. This possibility of profits limits the firms’ incentive to deviate and the number of necessary controls. The per-firm probability of control \( x(F)/n \) increases with the fixed fee \( F \).\(^5\) Indeed, a large fee results in negative firms’ incentives to select high-quality products, which increases the number of controls.

\(^4\) The alternative assumption could be considered with the product ban scenario linked to the case where \( W(0,0,0) < 0 \).

\(^5\) This result comes from the absence of initial assets, which leads to zero profits in case of a production ban because of low-quality products (see section 2.1). The expected cost is lower with low-quality products than with high-
and lowers overall welfare. To thwart this distortion, the regulatory agency chooses to replace this fixed fee $F$ with public financing $T$.

If, on the other hand, the monitoring cost $R$ is relatively large, the absence of regulation (and control) is optimal and low-quality products are offered. The costs of regulation exceed the benefits for $R > R_f$. Under the absence of regulatory controls and sanctions, firms have no incentive to invest in high quality by incurring the fixed-cost $C$.

We now turn to the impact of the variation on the number of firms $n$ and study the issue of the optimal number of firms and controls.

### 3.2 An endogenous number of firms

The ability of firms to enter and/or leave an industry changes the optimal policy and introduces some interesting features of the financing instruments selection. When the number of firms is endogenous (scenario $b$), the choice of financing instruments may influence the number of firms able to enter the market. Recall that in stage 2, the decision of quality compliance is preceded by a simultaneous entry decision by firms under scenario $b$. This entry decision is public information, while quality needs to be inspected. The choice of an optimal policy leads to the following proposition (see mathematical details in the appendix).

**Proposition 2.** Under free entry, the socially optimal policy is

(i) a high-quality standard with a fixed fee $F_a > 0$, a transfer $T_a < 0$, and $x(F_a) / n^*_f = 1$,

leading to the entry of $n^*_f$ firms, if $R \leq R_2$ and $R \leq R_3$. 

quality products (see section 1).
(ii) the absence of an inspection policy \((x/n_0=0)\) with a fixed fee \(F_\beta>0\) and a transfer \(T_\beta<0\), leading then to entry of \(n_0\) firms with low-quality products, if \(D\leq D_1\) and \(R\geq R_3\).

(iii) a fixed fee \(F_a>0\) that leads to the absence of entry and production, if \(R\geq R_2\) and \(D>D_1\).

The selection of a fixed fee allows the prevention of excess entry when numerous firms are able to enter the market. The comparison between the per-firm expected damage \((1-\lambda)D\) coming from low-quality products and the per-firm full cost of high-quality products \(C+R\) (including the per-firm cost of inspection) determines the policy.

If \(R\leq R_2\), having high-quality products leads to a higher welfare than the welfare with low-quality products. As firms have zero profits under free entry with high-quality products, the probability of control has to be equal to one in order to avoid any deviation by a firm with low-quality products that are less costly. The fixed fee is the only tool financing the public inspection. As the fixed fee is larger than the cost of inspection, the agency transfers the “profit” to the rest of society through \(T_a<0\), allowing a balanced budget for the agency. The agency keeps some firms out of a market in the context of fixed costs \(C\) and \(R\). The regulator increases the fixed fee \(F\) for reducing the number of firms and reducing the number of inspected firms.\(^6\) The issue associated with entry showed that under free-entry equilibrium, the number of firms is greater than the socially optimal number with positive fixed costs (see Mankiw and Whinston 1986 and Perry 1984). Thus we see that it is the financing of the quality regulation itself that is implicated in the concentration.

\(^6\) The optimal number of firms in the industry declines with \(R\) and \(C\), an interesting result because each firm has the
If $R > R_2$, having low-quality products without inspection policy leads to a higher welfare than the welfare with high-quality products. Without a fixed fee $F_\beta > 0$ limiting the entry, the number of firms $n \rightarrow +\infty$ would lead to a welfare $W(0,0,0,0) \rightarrow -\infty$ for $D > 0$. This fixed fee $F_\beta > 0$ allows limitation of the number of firms and the overall damage. Eventually, when the expected damage $(1-\lambda)D = 0$, the fixed fee is $F_\beta = 0$, leading to a competitive situation with $n_0 \rightarrow +\infty$.

A lucid analysis of the market context matters for defining a policy. While a complex combination of instruments and a probability of being inspected lower than one is socially optimal with a given number of firms, a fixed fee reflecting the fixed cost of inspection and a probability of being inspected equal to one are selected when the number of firms is endogenous.

Note that the optimal number of firms may be lower or larger than the number of firms $n$ considered in proposition 1. If the optimal number of firms is lower than $n$, the fixed fee can be selected for reducing the number of firms. Conversely, a number of firms $n$ lower than the optimal number of firms should lead the regulator to select the policy presented in proposition 1. In this case, the public agency may even choose more than the necessary amount of regulation, depending on the incumbent’s influences on the agency. Kim (1997) underscores how regulation is sub-optimal when an incumbent behaves strategically against the government (the regulator, as a follower, deters entry by newcomers (with some fees), protecting the incumbents’ oligopoly situation), an aspect we do not consider here. Further, the argument for restricting the number of firms needs to be mitigated with respect to the government’s ability to collect information regarding parameters such as firms’ fixed costs and the optimal number of firms.

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same cost structure (symmetric firms); thus, the result does not occur because higher-cost (presumably smaller) firms are driven out of the market.
For restricting entry or encouraging exit in a context of numerous potential producers, a fixed fee larger than the per-firm cost of inspection is favored. Implementing such a fee may be thwarted by the lobbying of an industry that threatens to lay off workers or to locate abroad. Even if an “optimal number of firms” may be politically charged, the point we are emphasizing is simply that the choice by the regulatory authority to maximize societal welfare subject to the constraints of the financing mechanisms necessarily affects the number of firms in an industry. In particular, strengthening regulations without hurting the industry is impossible in such a context.

4. EXTENSIONS

In defining the analytical framework, very restrictive assumptions were made for simplicity. The link between the market structure and the choice of instruments is robust under alternative assumptions. In order to fit different problems coming from various contexts, some extensions could be easily integrated into the model presented here.

(1) We abstracted from heterogeneity among firms. The consideration of size/cost differences would lead to heterogeneous profits, implying a possible fee/tax/penalties discrimination. For instance, the firms with the largest profit could pay a larger fixed fee than the other firms. Different levels of damages, risks (probability of accident) or costs of inspection also appeal for different fees or penalties among firms. As profits would be different, the incentive to comply with regulation could differ at the equilibrium with some firms preferring not to respect regulation. In this case, penalties that would be incurred by inspecting cheating firms could finance the budget of the agency.
(2) We focus only on public inspection. However, the private auditing made by the third party may partially replace the public auditing when there is a lack of public money. In this case, the public agency needs to monitor the third-party auditors and this task must be financed according to mechanisms similar to the ones previously presented. For instance, the Public Company Accounting Oversight Board (PCAOB) oversees the auditors making the financial audit of public companies on the stock market (SEC 2003). The PCAOB shall establish a fee for enforcing inspection and disciplinary programs, where the aggregate annual accounting support fee may not exceed the PCAOB’s aggregate “recoverable budget expenses,” as we shown with our equation (3).

(3) A possible explanation for the reluctance to earmark fees for inspection policies is the collusion between the auditor and the audited firms. The public agency may choose more or less than the necessary amount of controls, depending on any political influences upon the agency. Obviously, the risk of “collusion” between the agency and the inspected firms would obfuscate the agency’s regulatory obligations. However, as the Economist (2002, p. 1) noted for financial auditing, “a firm’s relationship with its auditors is, after all, a curious one: it pays the fees of the institution with the prime responsibility, in the first instance, for spotting any irregularities. That is not an insuperable problem: that taxpayers pay for the police does not lead them to expect to be allowed to get away with daylight robbery.” The sanctions for firms and members of agencies for abusive collusion should deter fraudulent behaviors.
(4) We assume perfect monitoring when a firm was inspected by the agency. One extension could be to allow for imperfect monitoring during the inspection, which may be introduced with an additional probability parameter. We expect that imperfect monitoring would reduce the attractiveness of the monitoring policy. The agency could allocate money to improve the monitoring process (through inspector training or/and new technologies), which would reduce the number of inspections (defined by $x$ in our framework). However, the bureaucracy may stifle the agency’s improvements of the monitoring process.

(5) Eventually, a very simple per-firm expected damage $(1-\lambda)D$ was assumed. Several extensions could be introduced. First, the parameter $\lambda$ corresponding to the absence of risk could depend on a costly firm’s effort that should be taken into account in the number of controls defined by (A1) in the appendix. Second, the per-firm damage $D$ could depend on the firm’s output $q$ with $D'(q)>0$ and $D''(q)>0$. In this case, a per-unit tax (that depends on quantity) could complete the other tools presented in sections 1 and 2, if the per-firm damage is not fully eliminated (as with low-quality products).

5. CONCLUSION

Using a simple single-period model based on asymmetric information that also takes into account the link between the competitive structure and financing of the regulatory program, we showed how fees and public financing may be optimally used. The previous sections
demonstrated the benefits of a policy that links the probability of controls, the market structure, and the choice of financing instrument.

Key factors in the choice of the optimal regulation are the market context, the number of firms that are likely to comply with the regulation, and, especially, firms’ incentives. The simple model presented here underscores the importance in choosing appropriate financing structures. Although the type of budgetary financing may seem mundane, what this simple model shows is that the choice of financing may have important implications for industry structure and firm compliance and the intensity of controls. Thus, this paper suggests that it is especially imperative for governments not only to examine the types of regulations imposed upon an industry but also to scrutinize the type of financing used by the agencies charged with enforcing those regulations.

This analysis needs to be extended with complete cost-benefit analysis and more empirical details about agents and specific markets concerned with regulation. However, all of the questions and results of this paper are crucial for developing a debate regarding the improvement of public inspection. At this juncture in the debate over public inspection and efficiency, it is important for economists to bring their knowledge to the fore. Our hope is that this paper will serve as a reference base for policymakers and governments (in charge of public funding) on whom the decisions of inspection policies ultimately rest.
REFERENCES

APPENDIX

Proof of proposition 1.

When a high-quality standard is selected, the agency minimizes the number of inspections and the financing instruments that influence profits and/or tax-payer losses. By using equation (1), the equality \( \pi(1, F) = \pi(0, F) \) leads to a number of inspected firms

\[
x(F) = \frac{nC}{\sqrt{\left[a^2 + (n+1)^2 - F\right]}}.
\]

(A1)

The minimization of the number of inspections leads to \( F=0 \) since \( dx(F)/dF > 0 \). The value of \( x(0) \) is substituted in condition (3). Thus, a lump-sum tax \( T_i \) satisfying (3) with \( F=0 \) is selected with

\[
T_i = \frac{-RnC}{\sqrt{\left[a^2 + (n+1)^2\right]}}.
\]

(A2)

with a number of controls \( x(0) \). It is easy to check the probability of control \( x(0)/n<1 \) for positive profits with high-quality products, namely, for \( a^2 + (n+1)^2 - F - C > 0 \).

The welfare is

\[
W(1,0,T_i) = \frac{a^2n(1+n/2)}{(n+1)^2} - \frac{RnC}{\sqrt{\left[a^2 + (n+1)^2\right]}} - nC.
\]

(A3)

Under the absence of standard, low-quality products are selected (\( I=0 \)) since no seller incurs the cost \( C \). No cost of inspection is incurred by the regulatory agency, since inspections are useless (\( F=T=0 \)). The welfare is

\[
W(0,0,0) = \frac{a^2n(1+n/2)}{(n+1)^2} - n(1-\lambda)D.
\]

(A4)
The frontier between regions 3 and 4 is given by $W(1,0,T) = W(0,0,0)$, which is equivalent to

\[(A5) \quad R_i = \alpha^2 [(1-\lambda)D-1]/(n+1)^2 C.\]

The welfare of high-quality producers is larger (respectively lower) than the welfare with $n_0$ low-quality producers if $R \leq R_1$ (respectively, $R > R_1$). \(\square\)

**Proof of proposition 2.**

We mainly detail point (i), as the method for point (ii) is similar. Recall that the entry decisions are public information.

**Point (i).** Under free entry, profits are equal to zero. When the high-quality standard is imposed, the inequality $\pi(1,F) \geq \pi(0,F)$ is only satisfied for $x=n$ (or a probability of being inspected $x/n=1$) and $F > 0$. As every firm is inspected ($x=n$), the choice of low-quality products leads to a product ban. In this case, firms only choose high-quality products ($I=1$) with a profit $\pi(1,F) \geq 0$. The free entry leads to a profit equal to $\pi(1,F) = 0$. In this context with $x=n$, the budget of the agency given by (3) may be rewritten as $\tilde{F}(T) = T/n + R$. The substitution of $\tilde{F}(T)$ in the welfare defined by (4) allows computation of the optimal number of firms. Note that $T$ is the transfer, which does not influence welfare $W(1,\tilde{F}(T),T)$. It is straightforward to show that overall welfare is at a maximum \((dW(1,\tilde{F}(T),T)/dn=0)\) when

\[(A6) \quad n_1^* = INT \left[ \frac{a^{2/3}}{(R + C)^{1/3}} - 1 \right],\]
by letting \( \text{INT}[.] \) be a function that returns the maximum integer satisfying a constraint. The profit equal to zero, namely, \( \pi(1,F)=0 \), determines the number of firms entering the market

\[
(A7) \quad n_1^* = \text{INT} \left[ \frac{a}{(F + C)^{1/2}} - 1 \right].
\]

The choice of \( F \) allows the regulator to cap the number of firms to a level equal to \( n_1^* \). Thus, the equality \( n_1 = n_1^* \) leads to

\[
(A8) \quad F_a = \left[ a(R + C) \right]^{2/3} - C.
\]

As \( F_a > R \), then the budget constraint (3) leads to

\[
(A9) \quad T_a = n(R + C) - n\left[ a(R + C) \right]^{2/3}.
\]

This value \( T_a \) is negative, which corresponds to a transfer to the rest of the economy. It is easy to check that \( n_1 \geq 1 \) for \( R \leq R_2 \) with

\[
(A10) \quad R_2 = a^2/8 - C.
\]

**Point (ii).** If the selection of low-quality products \( (I=0) \) by all sellers leads to the highest welfare, inspections are useless with \( x=0 \). In this case, the budget constraint (3) for the agency is equal to \( T = -nF \). It is straightforward to show that overall welfare is at a maximum \( (dW(0,F,-nF)/dn=0) \) when

\[
(A11) \quad n_0^* = \text{INT} \left[ \frac{a^{2/3}}{((1 - \lambda)D)^{1/3}} - 1 \right].
\]

The profit equal to zero, namely, \( \pi(0,F)=0 \), determines the number of firms entering the market
(A12) \[ n^*_0 = \text{INT} \left[ \frac{a}{F^{1/2}} - 1 \right]. \]

The choice of \( F \) allows the regulator to cap the number of firms to a level equal to \( n_0 \). Thus, the equality \( n_0^* = n_0^* \) leads to the selection of a fixed fee equal to

\[ (A13) \quad F_\beta = \left[ a(1 - \lambda)D \right]^{2/3}. \]

and to a lump-sum tax

\[ (A14) \quad T_\beta = -n_0^* F_\beta. \]

It is easy to check that \( n_0^* \geq 1 \) for \( D \leq D_1 \) with

\[ (A15) \quad D_1 = \frac{a^2}{\sqrt[3]{8(1-\lambda)}}. \]

The choice between the policies described in point (i) and (ii) depends on the welfare comparison. The welfare \( W(1,F_\alpha,T_\alpha) \) with \( n_1^* \) high-quality producers is larger (respectively lower) than the welfare \( W(0,F_\beta,T_\beta) \) with \( n_0^* \) low-quality producers if \( R \leq R_3 \) (respectively \( R > R_3 \)), with

\[ (A16) \quad R_3 = D(1-\lambda) - C. \]

Point (iii). For values \( R > R_2 \) and \( D > D_1 \), a fixed fee \( F_\alpha \geq \text{Max}[F_\alpha, F_\beta] + \epsilon \) with \( \epsilon \) positive leads to the absence of entry/production since the firm’s profit would be negative with firm number \( n=1 \). A welfare equal to zero (linked to the absence of production) is a maximum if \( R > R_2 \) and \( D > D_1 \), since a production with low- or high-quality products and \( n=1 \) entails a negative welfare. \( \square \)