Equilibrium and Efficient Land-Use Arrangements under Spatial Externality on a Lattice

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Abstract

Many cases of externalities in agricultural production, such as pesticide drift, cross-pollination, and offensive odors, are attributable to the incompatibility of neighboring land uses and exhibit distance dependence. We characterize equilibrium spatial patterns of externality-generating and -receiving land uses on a two-dimensional lattice with noncooperative, profit-maximizing producers. In equilibrium, generators or recipients form one or more neighborhoods with certain geometric properties, depending on how an externality dissipates with distance and whether there is an externality generated outside the region's boundaries. Efficient land-use arrangements maximize social welfare subject to the implementability constraints stipulating that no farm-level activity, except for land use, can be directly controlled by the social planner. We characterize efficient land-use arrangements when the return to recipient land use decreases linearly with the length of the border shared with incompatible land uses. Under these assumptions, we find circumstances in which an efficient activity arrangement belongs to the set of the Nash equilibrium outcomes. Also, efficient arrangements in a more general case are discussed.

Keywords: graph partitioning, land-use arrangement, spatial externality, supermodular game.

EQUILIBRIUM AND EFFICIENT LAND-USE ARRANGEMENTS UNDER SPATIAL EXTERNALITY ON A LATTICE

1. Introduction

In recent years, agricultural market analysts have paid increasingly more attention to the spatial concentration of production in both animal and crop agriculture. In general, geographic land-use patterns are shaped by a host of factors, including soil qualities, proximity to input markets, vertical integration, farm size, and marketing environment. We will focus on another feature of the grower's decision environment: the presence of spatial externalities due to conflicting land uses.

A number of externalities in agricultural production, such as pesticide drift, cross-pollination, invasion by foreign species and predators, offensive odors, and animal waste pollution, are well documented. For example, Parker (2000) presents anecdotal evidence of spatial negative externalities in crop agriculture, including the damage to cotton crops due to herbicide applied on rice planted in the surrounding area, damage to olive crops produced near cotton, and conflicts among hybrid seed producers due to cross-pollination. Recently, conflicts between growers of non-genetically modified (non-GM) and genetically modified (GM) varieties arising because of the possibility of cross-pollination have become an important issue in crop agriculture (e.g., Jones 2003; Perkins 2003; Belcher, Nolan, and Phillips 2003; Munro 2003; Brasher 2003).[1] For economists, the case of incompatibility between organic and conventional growing practices on neighboring farms presents an almost ideal example of a negative spatial externality because of the identifiable costs associated with it (USDA n.d.).

Parker and Munro (2004) develop econometric tests and find evidence that negative spatial externality is a significant economic factor weighing in on the location decision of certified organic farming operations in California.[2] To be certified as organic, production must be deemed free from potential contamination by prohibited materials that are routinely used in conventional farming. When crops on the surrounding land are grown using conventional farming practices, the quality of the organic crop is impaired because of the drift of prohibited chemicals or possible cross-pollination with GM crops. To avoid a contamination hazard emanating from the neighboring land uses, the organic producer is required to leave a buffer zone between the edge of his certified production site and the neighboring land use.[3] The allocation of crop acreage to buffer zones lowers the profitability of a (organic) farming operation and presents a concrete and directly measurable cost of negative externality (e.g., Parker 2000; Munro 2003).[4]

Palmquist, Roka, and Vukina (1997) and Herriges, Secchi, and Babcock (2003) provide empirical evidence on the presence of negative externalities in animal agriculture. They find that livestock feeding operations have greater negative impacts on the values of residential properties if they are located in the close vicinity. A number of other studies (e.g., Bockstael 1996; Geoghegan et al. 1996; Geoghegan, Wainger, and Bockstael 1997; and Irwin and Bockstael 2002) test for influences of surrounding land uses on property values and the probability of conversion of undeveloped land. They find that land values and conversion probabilities increase with the proportion of surrounding open space and pasture and decrease with the proportion of cropland and the length of incompatible edges. Positive externalities derived through spatial industry concentration in certain geographical areas are also well known (Fujita and Thisse 2002; Parker and Munro 2004).

In all of these cases, the loss or gain from externalities is attributable to the incompatibility or complementarity of neighboring land uses and declines with the distance between the externality generator and recipient. From a policy perspective, this implies that restricting certain land uses to certain areas may control the total externality exposure. There exist a number of policies designed to improve the efficiency of land-use arrangements such as zoning orders, emission regulations, size restrictions, buffer zones, various environmental standards, and other types of legislation. In this light, it is interesting to examine when a desired assignment of certain land uses to certain areas can be sustained by noncooperative behavior without subsequent monitoring and enforcement.

While there is growing empirical literature that analyzes the effect of distance-dependent externalities on equilibrium and optimal spatial land-use patterns, there are only a few theoretical economic models that explicitly address this issue. The question of the choice of a policy instrument to correct for externalities when the damages can be restricted to certain areas of the region is studied in Baumel and Oates (1988), Helfand and Rubin (1994), and Tomasi and Weise (1994) in different settings. The potential for nonconvexities in the aggregate production possibilities frontier and the alleviation of inefficiencies through the spatial separation of conflicting activities is noted in Baumel and Oates (1988). Helfand and Rubin (1994) investigate when it is socially efficient to spread or concentrate environmental damages but not in the context of multiple land sites operated by independent agents.[5] They distinguish between three sources of nonconvexities that may arise in an externality-control problem: technical nonconvexity that occurs when additional units of pollution cause nonincreasing marginal harm to the environment (in our case, to farmers), psychological nonconvexity in social utility function, and production nonconvexity that occurs when the cost function exhibits increasing returns to scale. This paper can be viewed as providing micro-foundations that give rise to technical nonconvexities analyzed in Helfand and Rubin (1994). In their framework, the social planner allocates externality-generating production activity between the two sites, where site-specific environmental quality (externality) is an exogenous function of output. Here, we explicitly model how an "externality" forms and dissipates across multiple production sites (farms). Also, using a spatially continuous one-dimensional model (with atomless farms), Tomasi and Weise (1994) find circumstances when socially optimal intensity of externality-generating farming and the location of a boundary between farm and residential sectors can be achieved through spatial Pigovian taxes. While these papers address the issue of efficiency of land-use arrangements, they do so based on a highly stylized spatial structure that may not adequately capture the decision environment in crop agriculture.

This paper provides two contributions to the literature. First, we formulate a model of land use (crop choice) under spatial externality and noncooperative behavior and examine equilibrium land-use arrangements. Following agent-based models of land use, price-taking producers are located on a two-dimensional regular tessellation (rectangular lattice or grid structure) where each farm is represented by a square cell.[6] Second, we present a general formulation of the socially efficient land-use arrangement problem. It is assumed that the social planner has the ability to set output prices and assign land uses at the farm level but cannot control any other aspects of the environment including farm-level outputs or production intensities on land in the assigned use. We characterize optimal spatial production patterns in environments possibly pertinent to the issue of coexistence of non-GM (organic) and GM (conventional) crops. We also establish conditions when such arrangements are self-enforcing by independent, profit-maximizing producers conditional on the land-use-specific (but not location-specific) payment scheme.

To allow for tractable analysis of land-use arrangements on a two-dimensional lattice with multiple sites, our model is simplified in a number of ways. All farms are identical except for location on a lattice, and the input and output prices are exogenous and invariant across locations. Thus, there is a single source of heterogeneity in equilibrium land use across agents—the difference in externality exposure from other agents. For concreteness, only generators impose an externality on others, and profit-maximizing, farm-level generator output is invariant to externality exposure.

The critical assumption (1) is that the *incremental* return from switching land uses (recipient to generator) increases with the externality. This can be thought of as an (profit-driven) incentive to "conform" to neighboring land uses. Even with just two possible land uses, the framework encompasses a number of real-world situations where the externality can be negative or positive, as in the case of residential housing and open space. In principle, the model can be generalized to multiple activities, each of which generates an externality where the quantity of imposed externality is nondecreasing in the level of activity (e.g., see endnote 26). However, the primary application is to spatial arrangements of agricultural land uses, where the activity choice is dichotomous, in part, because of a minimum efficiency scale of an activity. For example, this includes the production of GM or non-GM crops, organic or conventional farming, and investment in a livestock feeding operation. And so, the terms "generator" and "recipient" are labels associated with "high" and "low" actions given that the agents' payoff functions are supermodular in action and externality, that is, the incremental payoff from increasing the "level" of action is increasing in externality exposure from "higher" actions.[7] This condition is the defining characteristic of "compatibility" among *similar* land uses. And so, for example, if homeowners are entitled to compensation from nearby animal feeding operations, only actions need to be appropriately relabeled. In that case, homeowners are generators and feeding operations are recipients of the externality.

To solve the welfare-maximization problem subject to the aforementioned implementability constraints, we make the following assumptions. (2) The externality is negative and affects only recipients. (3) The total (region-wide) recipient output and cost of production depend only on the *aggregate* loss due to the externality. (4) Individual recipient loss increases *linearly* with the externality. (5) The negative externality only affects neighbors that share a *common* border (and is proportional to the length of the border).[8] Thus, by Assumptions 4 and 5, the recipient's profit falls at a constant rate with the length of the border shared with land in externality-generating use. By Assumption 3, "spatial" efficiency of an arrangement is assessed based on the aggregate recipient (profit) loss conditional on output prices and the number of farms in each use. This objective is assuredly consistent with social welfare-maximization because, keeping prices, the number of generators, and farm-level generator output fixed, the aggregate recipient output is a decreasing function of that statistic. Note that, in general, the problem of cost minimization involves optimization over both locations and levels of production on each farm. However, the only farm-level intervention tool available to the planner is zoning regulation.

In the first part of the paper, we characterize spatial properties of the (strict) Nash equilibrium in pure strategies. Consistent with agent-based simulation models of land use under spatial externality (more to follow), there are multiple land-use patterns that constitute the Nash equilibrium. However, there are certain properties that are invariant to initial conditions and are present in any equilibrium land-use arrangement.[9] The equilibrium properties of land-use patterns depend on how the externality dissipates over distance. We consider three cases of dissipation: (a) across a common border (i.e., Assumption 5 in the efficiency problem), (b) in a small neighborhood surrounding the generating use, and (c) across the entire region. In cases (a) and (b), we find that in the Nash equilibrium, externality-generating or -receiving farmers form one or more rectangular and octagon-shaped neighborhoods depending on whether land surrounding the region is in (permanent) externality-generating use. On the other hand, if the externality from any generator affects all farms in the region and its marginal impact decreases with distance (i.e., it dissipates "slowly" over the region), generators form a single convex-shaped neighborhood. We also obtain a lower bound on the number of generators in the

Nash equilibrium where both crops are produced, which depends on the shape of the externality-dissipation function. A minimum number of generators (maximum number of recipients) assures that there is “enough” mutual externality imposed by generators on each other so that no generator has an incentive to switch to recipient use.

In the second part of the paper, we formulate the problem of determining socially efficient land-use arrangements that comply with the technological constraints such as the scale of individual farms and the implementability constraints. Under Assumptions 2 and 3, the problem can be decomposed into two stages. First, the number of farms in each use and the farm-level generator output (invariant across all generators) are exogenous, and arrangements that minimize the total externality exposure are determined. Second, given these arrangements, socially optimal outputs of each crop are established. Under Assumption 4, the first-stage problem belongs to the class of quadratic assignment problems (QAP). In a general case, QAP is a notoriously difficult combinatorial problem that has a number of diverse applications in many disciplines (e.g., Cela 1998).[10] Building on the geometric approach developed in Yackel, Meyer, and Christou 1997, we characterize efficient land-use *arrangements* under Assumptions 4 and 5. In efficient arrangements, generators and recipients occupy contiguous areas with the minimum border that separates the generator and recipient sites. The shape of the areas occupied by generator and recipient farms depends on the number of farms in each use.

Based on this characterization, we find circumstances when an efficient arrangement is self-enforcing in the sense that it constitutes one of the strict Nash equilibrium outcomes of the land-use game. Assuming that all externality is generated within the region, this is somewhat more “probable” when the number of externality-generating farms is less than roughly three-fourths of the total number of farms in the region. If the share of generators is (approximately) less than one-fourth, efficiency requires that *generators* be arranged in a (almost) rectangular block located in a corner of the region. If the share of generators is between one-fourth and three-fourths, generators and recipients are arranged in (almost) rectangular blocks. Both of these patterns may coincide with the Nash equilibrium. In contrast, if the share of generators is greater than three-fourths, efficiency requires that *recipients* be arranged in a (almost) rectangular block located in a corner of the (square) region. This has zero probability of occurrence in the Nash equilibrium because it requires that returns to both land uses for some farmers are precisely the same, so that any small change in the parameter values will alter the incentives.[11] On the other hand, if land surrounding the region is in (permanent) externality-generating use, an efficient arrangement belongs to the set of equilibrium arrangements under a considerably wider range of circumstances.

Also, through a series of simple examples we explain why both assumptions (the linearity of the recipient's profit loss and "border" externality) underpinning the analysis in the benchmark case are necessary for our results. If the recipient's returns to farming decrease nonlinearly with the extent of exposure, spatial concentration of generators or recipients may not be optimal since it leads to a skewed distribution of exposures among recipients. Upon dispensing with Assumption 5 and letting the externality extend its impact beyond neighbors with a common border, it is also no longer the case that efficiency involves all generators "agglomerated" in one location in the region. This is because a contiguous area allocated to generator use may necessitate locating some generators "close" to the middle of the region, which may increase exposure for the remaining recipient sites.

The rest of the paper is organized as follows. The remainder of Section 1 offers a brief review of the related literature. A model is formulated in Section 2. In Section 3, equilibrium arrangements are characterized. In Section 4, we determine socially optimal arrangements in an environment reminiscent of non-GM and GM crop production and investigate conditions for compatibility between efficiency and non-cooperative equilibrium. Efficient arrangements in a more general case are also discussed. Conclusions are given in Section 5.

1.1 Literature Review

The modeling of agricultural land-use decision-making frequently employs agent-based models (see Parker et al. 2003 or Janssen 2004 for a review of agent-based models). The main ingredients of agent-based models are the *landscape*, a rectangular array of (equal-sized) cells that cover the land area; *agents*, the landowners who make land-use decisions within their cells, and the specification of the agent's *behavior* and the decision-making process. These complex systems routinely have multiple equilibria and are highly path dependent in the sense that the predicted spatial land-use pattern is very sensitive to subtle differences in the decision-making environment or initial distribution (Brown et al. 2004; Parker 2000; Belcher, Nolan, and Phillips 2003).

Using this kind of a cellular automaton model, Parker (2000) analyzes land-use patterns that may arise as a result of noncooperative profit-maximizing behavior by producers who choose between externality-generating (conventional) and externality-receiving (organic) farming under Assumptions 2 through 5. Parker (2000) gauges the effects of initial conditions and geographic features of the region on the social efficiency of final (equilibrium) arrangements and discusses Pareto-improving rearrangements. However, as the author points out, neither noncooperative nor optimal land-use arrangements have received a conclusive treatment. Also, the question of when optimal arrangements can emerge or be sustained in a noncooperative equilibrium needs more thorough scrutiny.[12]

Some notable exceptions to this methodology of modeling spatial land-use patterns are recent contributions in the urban economics and location theory literatures (for a review, see Kanemoto 1987) such as Page 1999 and Turner 2004. These authors use a game-theoretic approach (as well as computer simulations) to analyze the process of city formation through the choice of residential location in the presence of spatial externalities.[13] In Page 1999, agents decide where to reside on a lattice based on the proximity to other agents and separation (preference for open space) in the environment where multiple residents may reside in the same location. Turner (2004) considers the choice of residential location in a one-dimensional city, which limits the patterns of spatial interactions that are of interest in the present inquiry.[14]

Calvo-Armengol and Zenou (2004) use a model that shares many features with ours to study the role of social networks in promoting criminal activities, and explore the endogenous formation of a criminal network. In their framework, affiliated criminals impose a positive externality on each other by sharing ‘trade secrets’ but compete in criminal activities. They analyze a two-stage game where individuals first decide to work or become criminal (this is analogous to an externality-generating or receiving land-use decision in our model) and then the crime-effort provided if criminals. For simplicity, in our model the second stage, where equilibrium *amounts* of production on each farm are determined, is degenerate because we assume that output prices are exogenous. While

Calvo-Armengol and Zenou (2004) consider arbitrary or endogenous networks (spatial structures positing relations among agents), we analyze geometric equilibrium patterns within a fixed spatial structure (rectangular lattice). Also, the focus of inquiries is different. Calvo-Armengol and Zenou (2004) emphasize the multiplicity of equilibria with different number of active criminals and crime levels that are driven by the geometry of the pattern of links among agents, while we are interested in the compatibility between noncooperative and efficient arrangements.

2. Model

2.1 Spatial Structure

Let $N = \{1,...,n^2\}$ denote the set of farms (convex and non-overlapping plots of land) or agents located on an $n \times n$ lattice (region) with farm indices starting in the left-lower corner and going from the left to the right of each row, $n \geq 3$. The horizontal and vertical coordinates (row and column) of farm (cell) i are given by $y_i = 1 + [(i-1)/n]$ and $x_i = i - n(y_i - 1)$, where $[b]$ denotes the integer part of b. To each lattice point (x_i, y_i) is associated a closed unit square (called farm i) centered at (x_i, y_i). The distance between two farms i and j is measured using Euclidean metrics: $d_{ij} = [(x_i - x_j)^2 + (y_i - y_j)^2]^{1/2}$. For example, in Figure 1, the region consists of nine farms, $n = 3$; farm 7 has coordinates $(x_7, y_7) = (1,3)$, and the distance between farm 7 and farm 2 is equal to $d_{27} = \sqrt{5}$.

2.2 Spatial Externality

Each agent operates one farm, and all farms are identical in all aspects except for location. Each farm i produces one of the two crops: the externality "recipient" crop, r_i,

7	8	9
4	5	6
1	2	3

FIGURE 1. Two-dimensional lattice and farm indices

or the externality "generator" crop, g_i. At the farm level, the externality imposed by farm j on farm i is given by $\gamma(d_{ij})z(g_j, r_j)$, where $\gamma(.) \geq 0$ is the externality dissipation function.[15] For concreteness, we take $z(g_j, r_j) = g_j$, if $g_j > 0$ and $z(g_j, r_j) = 0$ if $r_j > 0$, that is, the externality increases linearly with generator output, and recipients do not generate an externality of any kind. The impact of the externality decreases with distance between the farm plots and may become zero if farms are sufficiently far apart, $\Delta\gamma(d) = \gamma(d+\varepsilon) - \gamma(d) \leq 0$ for any $d \in [1, (n-1)\sqrt{2})$ and $\varepsilon > 0$. The total amount of the externality imposed on farm i is then given by[16]

$$e_i = \sum_{j \neq i} \gamma(d_{ij}) g_j \,. \tag{1}$$

In the analysis to follow, we will make several assumptions regarding the properties of the externality diffusion (dissipation) function, γ. Specifically, we consider production environments where the externality affects recipient neighbors that (a) share a common border with the generator, $\gamma(d) = 1_{d \leq 1}$; (b) belong to a local neighborhood of the generator, $\gamma(d) = 1_{d \leq \sqrt{2}}$; and (c) are located anywhere in the region, $\gamma(d) \geq 0$ for $d \in [1, (n-1)\sqrt{2}]$. Case (c) may pertain when, for example, contamination through cross-pollination by GM material may occur at distances that exceed farm width. Also, in the case of hog confinements and residential properties affected by odor and noise, there may exist additional risks of accidental hog waste pollution in a wider area.

Unless stated otherwise, we assume that the externalities present in the region are generated within the region so that (1) holds for all farms, including those located along the region's boundaries. The case when land surrounding the region is allocated to a "permanent" externality-generating use will be considered in cases (a) and (b). Note that the assumption that there is no externality generated outside the region's boundaries is more natural and emphasizes the endogeneity of "locations" of the externality-generating farms.[17]

2.3 Agents' Payoffs

The maximized profit from growing recipient, r, and generator, g, crops for farmer i is

$$\pi^q(e_i) = \max_q \; p^q q - c^q(q, e_i)\,, \tag{2}$$

where $q = r, g$, output prices p^r and p^g are taken as given by individual producers, and c^r and c^g are increasing and convex cost functions of outputs r and g. For later use, let $r_i^* = r^*(e_i, p^r)$ and $g_i^* = g^*(e_i, p^g)$ denote profit-maximizing outputs of recipient and generator crops conditional on output prices and the externality exposure.

ASSUMPTION 1. (a) $\partial\pi^r / \partial e < \partial\pi^g / \partial e$ and (b) $\partial^2 c^g / \partial e \partial g = 0$ for all $e, g \geq 0$.

Condition (a) states that the marginal gain (loss) due to the externality for a recipient producer is smaller than that for a generator, or equivalently, the incremental return of switching from the recipient to the generator crop, $\pi^g(e) - \pi^r(e)$, increases with externality. It is consistent with the parable of *conflicting* (or *compatible*) land uses since land uses of the same type typically impose less mutual damage relative to incompatible uses or even provide mutual benefits (positive externality). For example, condition (a) is satisfied if $\partial\pi^r(e)/\partial e < 0$ and $\partial\pi^g(e)/\partial e \geq 0$, which is characteristic of conflicts between organic (recipient) and conventional (generator) farmers. The option to grow the generator crop effectively puts a cap on the losses from the externality exposure accrued when land is in the recipient use. On the other hand, condition (a) is satisfied if $0 \leq \partial\pi^r(e)/\partial e < \partial\pi^g(e)/\partial e$, which is consistent with a positive externality imposed by pasture on the value of nearby houses. Informally, allocating a vacant tract of land to pasture is more profitable relative to housing when there is "too much" open space and pasture surrounding it. Condition (a) is crucial in forming equilibrium land-use patterns, and implies that the recipient's loss (generator's gain) due to the externality exceeds that for the generator (recipient) once the externality exceeds a certain threshold.

Condition (b) assures that the profit-maximizing output of the generator crop is invariant across all generators, $g_i^* = g^*$. It is adopted primarily for ease of exposition. If generator output depends on neighboring land uses, equilibrium production patterns are more difficult to ascertain. In that case, externality exposure, e_i, is no longer a linear-additive function of the number and location of generators (see equation [1]), since each generator output, g_j^*, depends on the other (distance-weighted) generator outputs.

Furthermore, for simplicity, we assume that the region where farms are located is small and that output prices, p^r and p^g, are determined outside the region. Also, for convenience, we normalize the optimal output of the generator crop, g^*, to equal 1.

3. Crop-Choice Game and Equilibrium Arrangements

In this section we study spatial arrangements of production activities under the assumption that farmers act noncooperatively. In other words, there is no coordination among farmers and each farmer chooses what crop to grow taking the choices of others as given (i.e., a simultaneous-move game). Let $\vec{g} = (g_1,..., g_{n^2})$ denote the profile of generator crop farms in the region and $g_{-i} = (g_1,..., g_{i-1}, g_{i+1},..., g_{n^2})$ denote the profile of generator crop farms for all farms but farm i. Each farmer i makes her production decision, $g_i \in \{0,1\}$, (chooses the best response) in accordance with

$$\max_{g_i \in \{0,1\}} \pi(g_i, g_{-i}) = (1 - g_i)\pi^r(e(g_{-i})) + g_i \pi^g(e(g_{-i})) . \qquad (3)$$

Denote this crop choice (production decision) game by $\Gamma(\gamma)$. An important property of the individual farmer payoff (3) is supermodularity in (g_i, g_j) for any $j \neq i$: $\Delta_{g_i}\Delta_{g_j}\pi(g_i, g_{-i}) \geq 0$, which follows by Assumption 1. As a consequence, $\Gamma(\gamma)$ belongs to the class of supermodular games studied in Milgrom and Roberts (1990).[18]

We look for Nash equilibria in pure strategies of the crop choice game.[19] The (strict) pure strategy Nash equilibrium (PSNE) is defined by a profile of crop choices in the region, $\vec{g}^*$, such that for any farmer, $i \in N$, $\pi(g_i^*, g_{-i}^*) \geq (>) \pi(g, g_{-i}^*)$ for $g \neq g_i^*$, $g \in \{0,1\}$. Since $\Gamma(\gamma)$ is a supermodular game, PSNE is known to exist. Observe that either $\pi^r(0) \geq \pi^g(0)$ holds and the crop choice profile $\vec{g}^* = (0,...,0)$, constitutes the PSNE, or $\pi^r(0) < \pi^g(0)$ holds and $\vec{g}^* = (1,...,1)$ is the PSNE (furthermore, this is the only possible Nash equilibrium). We are interested in studying spatial arrangements where both crops are produced, which is equivalent to the existence of a profile $\vec{g}^*$ such that (see equation [3])[20]

$$\pi^r(e_i) \geq \pi^g(e_i) \text{ and } \pi^g(e_j) \geq \pi^r(e_j) \text{ for all } i, j \in N \text{ with } g_i^* = 0 \text{ and } g_j^* = 1. \quad (4)$$

The following result establishes an essential property of equilibrium arrangements. Let $s(\vec{g}) = \sum_i g_i$.

LEMMA 1. In *any* PSNE with $0 < s(\vec{g}^*) < n^2$, we have $e_i \leq \hat{e} \leq e_j$ for all $i, j \in N$ with $g_i^* = 0$ and $g_j^* = 1$, where $\hat{e}$ is a unique solution of equation $\pi^r(e) = \pi^g(e)$.

Proof. Note that (4) implies that $\pi^r(e_i) - \pi^g(e_i) \geq 0 \geq \pi^r(e_j) - \pi^g(e_j)$ for any $i, j \in N$ with $g_i^* = 0$ and $g_j^* = 1$. Because by condition (a) in Assumption 1, $\pi^r(e) - \pi^g(e)$ is (strictly) decreasing in e and $\pi^r(\hat{e}) - \pi^g(\hat{e}) = 0$, it follows that $e_i \leq \hat{e} \leq e_j$.

By Lemma 1, a necessary condition for (4) to hold is that the externality exposure for any recipient is less than that for any generator, that is, for any externality less than $\hat{e}$ the recipient's profit is greater than the generator's profit, while the converse holds for any $e \geq \hat{e}$. For example, if $\partial c^r / \partial e > 0$ and $\partial c^g / \partial e = 0$ so that $\partial \pi^r / \partial e < 0$ and $\partial \pi^g / \partial e = 0$, in any equilibrium where both crops are produced, externality-receiving (organic) farmers earn greater profits than externality-generating (conventional) farmers.

Note that, in general, the total output of the recipient crop depends not only on the number of farms engaged in the recipient crop production but also on the spatial arrangement of externality-generating and -receiving farms in the region. This will not play a role in our analysis because output prices are exogenous ("small" region). In addition, since, by Assumption 1(b), output is invariant across generator farms, the focus is on the characterizations of spatial arrangements of land uses that are undifferentiated by production intensity. In the following sections, we study equilibrium land-use patterns by noncooperative producers under several alternative assumptions about the shape of the externality-dissipation function.

3.1 Local Externality with Four-Neighbor Impact

In this section we consider the case when the externality impacts only the neighbors on the immediate border: $\gamma(d) = 1$ if $d \leq 1$, and $\gamma(d) = 0$ if $d > 1$. The localized impact of the externality suggests that a plural number of neighborhoods that consist of farms with similar (compatible) land uses may exist in equilibrium, because farms that are not in the immediate vicinity of each other are effectively independent in terms of externality exposure.[21] Proofs are provided in the Appendix.

PROPOSITION 1. *Suppose that Assumption 1 holds and* $\gamma(d) = 1_{d \leq 1}$. *Then in any PSNE,*

(i) $s(\vec{g}^*) = 0$ *if* $\hat{e} > 2$*;*

(ii) either $s(\vec{g}^*) = 0$ *or* $s(\vec{g}^*) = n^2$ *or* $4 \leq s(\vec{g}^*) \leq n^2 - n$ *and generators are arranged in rectangular neighborhoods with dimensions* 2×2 *or more if* $1 < \hat{e} < 2$*;*

(iii) either $s(\vec{g}^*) = 0$ *or* $s(\vec{g}^*) = n^2$ *if* $0 \leq \hat{e} < 1$.

If $\pi^r(1) = \pi^g(1)$ (or $\pi^r(2) = \pi^g(2)$) so that producers who have one (or two) generator neighbors are exactly indifferent between either activity, other equilibrium patterns of spatial arrangements exist. However, the probability of any such equilibrium has zero measure since any arbitrarily small perturbation of parameters (e.g., p^g or p^r) will alter producer incentives and result in an equilibrium characterized in cases (i) – (iii) in Proposition 1.

No externality-generating crop is produced in equilibrium if $\pi^g(2) < \pi^r(2)$. This follows by induction because farms along the edge of the region have a smaller number of neighbors with common borders (within the region) which always provides them with an incentive to switch to a recipient use. An example of PSNE where both crops are produced is presented in Figure 2a, where it is assumed that $1 \le \hat{e} \le 2$ so that, by Assumption 1(a), $\pi^r(e) \ge \pi^g(e)$ for $e \in \{0,1\}$ and $\pi^r(e) \le \pi^g(e)$ for $e \in \{2,3,4\}$. In Figure 2a, there are two neighborhoods of externality-generating farms (gray and dotted gray cells) that earn profit $\pi^g(2)$ and $\pi^g(3)$. The profit for recipient farms in light-gray cells is $\pi^r(1)$ because they share a border with a generator. The profit for recipient farms in white cells is $\pi^r(0)$ because none of their "border" neighbors are generators.[22]

In the case when land outside the region is allocated to externality-generating use (as illustrated in Figure 2b where dark gray cells represent "permanent" generators), the analog to Proposition 1 can be easily proved in the same manner (so the proof is omitted). Note that in this case, *recipients* form neighborhoods.

PROPOSITION 2. *Suppose that Assumption 1 holds and* $\gamma(d) = 1_{d \le 1}$, $e_i = 1 + \sum_{j \ne i} 1_{d_{ij} \le 1} g_j$ *for* $i = 2,\ldots, n\text{-}1, n\text{+}1,\ldots, n^2\text{-}2n\text{+}1, 2n,\ldots, n^2 - n, n^2 - n + 2,\ldots, n^2 - 1$, *and* $e_i = 2 + \sum_{j \ne i} 1_{d_{ij} \le 1} g_j$ for $i = 1, n, n^2 - n + 1, n^2$. *Then in any PSNE,*

(i) *either* $s(\vec{g}^*) = 0$ *or* $s(\vec{g}^*) = n^2$ *if* $3 < \hat{e} \le 4$*;*

(ii) *either* $s(\vec{g}^*) = 0$ *or* $s(\vec{g}^*) = n^2$ *or* $n \le s(\vec{g}^*) \le n^2 - 4$ *and recipients are arranged in rectangular neighborhoods with dimensions* 2×2 *or more if* $2 < \hat{e} < 3$*;*[23]

(iii) $s(\vec{g}^*) = n$ *if* $\hat{e} < 2$.

Suppose that the externality's impact on recipients is negative, $\partial \pi^r / \partial e < 0$. Propositions 1 and 2 provide a sense in which in equilibrium recipients "stay away" from generators when the edges of the region provide "protection" from the externality, while

(a) Region with "protected" borders, $1 \le \hat{e} \le 2$ (b) "Unprotected" borders, $2 \le \hat{e} \le 3$

FIGURE 2. Equilibrium arrangements: $\gamma(d) = 1_{d \le 1}$, $n = 7$

they "get closer together" when there are "permanent" generators outside the region (see Figure 2). If the externality is positive, $\partial \pi^r / \partial e \geq 0$, the interpretation is that generators "get closer together" when there are no "permanent" generators outside the region, and they "stay away" from relatively less "beneficial" recipients when the region is surrounded by generators.

3.2 Local Externality with Eight-Neighbor Impact

In this section, we consider the case when the externality impacts neighbors located within an eight-farm neighborhood surrounding a generator, $\gamma(d) = 1$ if $d \leq \sqrt{2}$, and $\gamma(d) = 0$ if $d > \sqrt{2}$. We will refer to an (irregular) octagon-shaped neighborhood determined by the intersection of parallel vertical, horizontal, and diagonal lines (passing through the centers of cells) as an octagon neighborhood, given that the length of each side of the smallest octagon containing all farms (cells) in the neighborhood is at least $\sqrt{2}$ (see Figure 3).

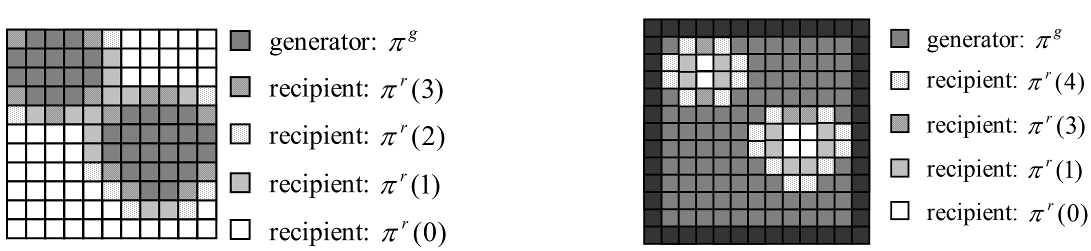
PROPOSITION 3. *Suppose that Assumption 1 holds and* $\gamma(d) = 1_{d \leq \sqrt{2}}$. *Then in any PSNE,*

(i) $s(\vec{g}^*) = 0$ *if* $4 < \hat{e}$;

(ii) *either* $s(\vec{g}^*) = 0$ *or* $s(\vec{g}^*) = n^2$ *or* $12 \leq s(\vec{g}^*) \leq n^2 - 4$ *and generators are arranged in octagon neighborhoods, if* $3 < \hat{e} < 4$;

(iii) *either* $s(\vec{g}^*) = 0$ *or* $s(\vec{g}^*) = n^2$ *or* $s(\vec{g}^*) = 4k$, *where* $1 \leq k \leq round(n/3)^2$, *and generators are arranged in* 2×2 *squares if* $2 \leq \hat{e} < 3$;[24]

(iv) *either* $s(\vec{g}^*) = 0$ *or* $s(\vec{g}^*) = n^2$ *if* $0 \leq \hat{e} < 2$.



(a) Region with "protected" borders, $3 \leq \hat{e} \leq 4$

(b) "Unprotected" borders, $4 \leq \hat{e} \leq 5$

FIGURE 3. Octagon-shaped neighborhoods with $\gamma(d) = 1_{d \leq \sqrt{2}}$, $n = 11$

To prove Proposition 3, we take into account that farms located at the edge of the region have fewer neighbors than farms in the middle of the region. The limited number of possible local configurations implied by the rectangular grid structure has an immediate consequence for the shape of equilibrium generator neighborhoods. However, if $\pi^r(4) = \pi^g(4)$ or $\pi^r(3) = \pi^g(3)$, other (unstable and unlikely) equilibrium patterns of land uses exist. An example of PSNE where both crops are produced is presented in Figure 3a, where it is assumed that $3 \le \hat{e} \le 4$ so that, by Assumption 1(a), $\pi^r(e) \ge \pi^g(e)$ for $e \in \{0,...,3\}$ and $\pi^r(e) \le \pi^g(e)$ for $e \in \{4,...,8\}$. To simplify the diagrams in Figure 3, generators are represented by gray color independent of the profit (the number of generator neighbors). In Figure 3a, there are two octagon-shaped neighborhoods of generators (gray cells) that earn $\pi^g(e)$, where $e \ge 4$ (therefore, they have no incentive to switch). Recipient farms in dotted gray (light gray, dotted white, and white) cells earn $\pi^r(3)$ ($\pi^r(2)$, $\pi^r(1)$, and $\pi^r(0)$) because they have three (two, one, and zero) neighbors within a $\sqrt{2}$ radius that are generators.

The following result demonstrates that when the region is surrounded by land in externality-generating use, the equilibrium land-use patterns are "inverted" so that *recipients* form neighborhoods in PSNE where both crops are produced (see Figure 3b). This is a counterpart to Proposition 3, and so the proof is omitted.

PROPOSITION 4. *Suppose that Assumption 1 holds and* $\gamma(d) = 1_{d \le \sqrt{2}}$, $e_i = 3 + \sum_{j \ne i} 1_{d_{ij} \le 1} g_j$ *for* $i = 2,..., n\text{-}1, n\text{+}1,..., n^2\text{-}2n\text{+}1, 2n,..., n^2 - n$, $n^2 - n + 2,..., n^2 - 1$, *and* $e_i = 5 + \sum_{j \ne i} 1_{d_{ij} \le 1} g_j$ for $i = 1, n, n^2 - n + 1$, n^2. *Then in any PSNE,*

(i) $s(\vec{g}^*) = 0$ *if* $6 < \hat{e} \le 8$;

(ii) *either* $s(\vec{g}^*) = 0$ *or* $s(\vec{g}^*) = n^2$ *or* $s(\vec{g}^*) = n^2 - 4k$ *where* $1 \le k \le round(n/3)^2$, *and recipients are arranged in* 2×2 *squares if* $5 < \hat{e} \le 6$;

(iii) *either* $s(\vec{g}^*) = 0$ *or* $s(\vec{g}^*) = n^2$ *or* $4 \le s(\vec{g}^*) \le n^2 - 12$, *and recipients are arranged in octagon neighborhoods if* $4 < \hat{e} < 5$;

(iv) $s(\vec{g}^*) = n^2$ *if* $\hat{e} < 4$.

The intuition behind Propositions 3 and 4 is analogous to that for Propositions 1 and 2. Together, these results demonstrate that equilibrium land-use patterns depend on the nature of externality dissipation in the spatial dimension. In some agricultural contexts, externality does not dissipate completely beyond the immediate neighbors but extends some of its impact across a larger area or possibly the entire (small) region. The equilibrium land-use arrangements in these cases are investigated next.

3.3 Global Externality

In this section we examine some properties of noncooperative equilibrium spatial arrangements when the externality is not local. First, we derive a lower bound on the number of generators in any PSNE, which turns out to depend on the curvature of the externality dissipation function. Then we provide a geometric characterization of equilibrium land-use patterns when the externality's impact declines by more as the distance between a source and receiver increases (i.e., the dissipation function is globally concave). For concreteness, we hold that the externality is only generated by farms within the region.

Lower Bounds on Equilibrium Number of Generators. The following result formalizes the intuition that in equilibrium where both crops are produced, the number of generators is sufficient to assure that there is "enough" mutual externality exposure among generators, so that no generator has a *unilateral* incentive to switch to recipient land use (which is more profitable in the absence of other generators).

PROPOSITION 5. *Let Assumption 1 hold. In any PSNE either* $s(\vec{g}^*) = 0$ *or* $s(\vec{g}^*) \geq 1 + \gamma(1)/[\gamma(1) - \gamma(2)]$ *if* $\Delta^2\gamma(d) \geq 0$ *or* $s(\vec{g}^*) \geq 1 + \gamma(1)/[\gamma((n-1)\sqrt{2} - 1) - \gamma((n-1)\sqrt{2})]$ *if* $\Delta^2\gamma(d) \leq 0$ *for all* $d \in [1, (n-1)\sqrt{2})$.

The tightness of the lower bound on the equilibrium share of generator farms depends on the attributes of the externality dissipation function such as concavity or convexity. The following example illustrates how the lower bound is used to restrict the set of candidate equilibrium arrangements when the externality-dissipation function is linear.

EXAMPLE 1. Consider a linear case of externality diffusion-decay: $\gamma(d) = (n-1)\sqrt{2} - d$ so that $\gamma(d) > 0$, $\Delta\gamma(d) < 0$, and $\Delta^2\gamma(d) = 0$ for all $d \in [1, (n-1)\sqrt{2})$. From Proposition 5, we obtain a lower bound on the equilibrium number of generators: $s(\vec{g}) \geq (n-1)\sqrt{2}$. Using this bound, we can immediately rule out spatial arrangement in Figure 4a, where $s(\vec{g}) = 4$ and $n = 4$, as a candidate for PSNE because $4 < 3\sqrt{2}$. However, it is easy to check that the arrangement in Figure 4b constitutes PSNE with $s(\vec{g}^*) = 9$ given that $18.6 \leq \hat{e} \leq 19.2$ where $\pi^r(\hat{e}) = \pi^g(\hat{e})$.

Concavity of Externality-Dissipation Function and Convexity of Generator Neighborhood. To further characterize possible equilibrium arrangements, we assume that the externality exposure decreases by more as the distance between farms increases: $\Delta^2\gamma(d) \leq 0$ for all $d \in [1, (n-1)\sqrt{2}]$. Note that this assumption implies that $\gamma(d) > 0$ for any $d \in [1, (n-1)\sqrt{2})$, and if $\gamma(d) = 0$ then $d = (n-1)\sqrt{2}$. In other words, the externality exhibits a "slow" diffusion-decay in the spatial dimension so that each generating unit is "felt" throughout the entire region by each farm. We will need the following definition.

DEFINITION 1. An eight-connected neighborhood $L \subset \{(x_i, y_i) \mid i \in N\}$ is said to be discretely (digitally) convex if no point (farm center) outside of L lies in the convex hull of L.[25]

It can be shown that this attribute is equivalent to the following properties (Chauhuri and Rosenfeld 1998). First, there exists no triple of collinear (real) points such that the first and last ones lie in L and the middle one lies outside of L. Second, the line segment joining is

FIGURE 4. Arrangements under global externality, $n = 4$

any two farms in L lies everywhere "near" L, in the sense that every (real) point of it strictly within a unit distance of some farm in L. Third, for any two farms in L there exists a digital line segment between those farms ("the shortest path") that belongs to L.

PROPOSITION 6. *Suppose that Assumption 1 holds and the externality dissipation function is globally concave,* $\Delta^2\gamma(d) \leq 0$ *for all* $d \in [1,(n-1)\sqrt{2}]$. *Then the set of generator locations* $L^* = \{(x_i, y_i): g_i^* = 1, \forall i \in N\}$ *is discretely convex in any PSNE.*

Because externality dissipates "slowly" with distance, the externality exposure for any recipient located "among" generators is greater than the average (distance-weighted) externality imposed by generators on each other. This implies that in equilibrium generators form a *single* neighborhood with "no holes."[26] The following example illustrates how the concavity of the externality-dissipation function leads to the discrete convexity of the equilibrium set of locations of generator farms.

EXAMPLE 2. Assume a quadratic externality diffusion-decay function: $\gamma(d) = 2(n-1)^2 - d^2$. By Lemma 1, PSNE where both crops are produced exists if $2(n-1)^2 + \sum_t(d_{ij}^2 - d_{jt}^2)\, g_t^* \leq 0$ for all $i, j \in N$ with $g_i^* = 1$, $g_j^* = 0$. Substituting for the Euclidean distances yields $2(n-1)^2/s + x_i^2 + y_i^2 - x_j^2 - y_j^2 + 2(x_i - x_j)\bar{x} + 2(y_i - y_j)\bar{y} \leq 0$ for all $i, j \in N$ with $g_i^* = 1$, $g_j^* = 0$, where $\bar{x} = (1/s)\sum_t x_t g_t^*$, $\bar{y} = (1/s)\sum_t y_t g_t^*$, and $s = s(\vec{g}^*)$. For convenience, relabel the coordinates in order to place the origin in the center of the region: let $x_i^o = x_i - (n+1)/2$, $y_i^o = y_i - (n+1)/2$ so that farm $o = (n^2+1)/2$ has coordinates (0,0) if n is odd. Consider a ball-shaped (symmetric) neighborhood of generators that is located in the center of the region as in Figure 4c. Then we have $\bar{x} = 0$ and $\bar{y} = 0$. And the condition for the existence of PSNE where both crops are produced reduces to $(x_j^o)^2 + (y_j^o)^2 \geq 2(n-1)^2/s + (x_i^o)^2 + (y_i^o)^2$ for all i, j with $g_i^* = 1$, $g_j^* = 0$, that is, in equilibrium recipient farms are located "farther away" from the center of the region than generator farms. An arrangement with $s(g^*) = 12$ that satisfies the last condition and constitutes PSNE given that $140 \le \hat{e} \le 146$ where $\pi^r(\hat{e}) = \pi^g(\hat{e})$ is presented in Figure 4c. Note that the set of generator locations, L^*, is discretely convex.

4. Efficient Land-Use Arrangements and Compatibility with Nash Equilibrium

In this section we take a policy perspective and look for spatial production patterns that maximize social welfare:

$$\max_{Q^r, Q^g} U(Q^r, Q^g) - C(Q^r, Q^g), \tag{5}$$

where Q^r, Q^g are the total outputs of recipient and generator crops. Here, U is the social utility of consumption of both crops, $\partial U / \partial Q^r > 0$ and $\partial U / \partial Q^g > 0$, and C is the minimum total cost of production given the size of the region and the technological and implementability-through-zoning-orders constraints:

$$C(Q^r, Q^g) = \min_{\{r_i\}_{i\in N}, \{g_i\}_{i\in N}, p^r, p^g} \sum_i c^r(r_i, e_i) + \sum_i c^g(g_i, e_i) \text{ subject to} \tag{6}$$

(i) $\sum_{i\in N} r_i \ge Q^r$, $\sum_{i\in N} g_i \ge Q^g$, (“output quantity constraints”)

(ii) $g_i r_i = 0$, $g_i + r_i > 0 \;\; \forall \; i \in N$, (“zoning orders assign generator or recipient use”)

(iii) $r_i \in \{0, r^*(e_i, p^r)\}$, $g_i \in \{0, g^*(e_i, p^g)\}$, (“farmers are noncooperative profit-maximizers”)

where e_i is the externality exposure given by (1), $r^*(e_i, p^r)$ and $g^*(e_i, p^g)$ are the profit-maximizing output levels on farm i - optimal solutions to (2). We assume that the participation constraints are met at the optimum: $\pi^r(e_i, p^r) \ge 0$ and $\pi^g(e_i, p^g) \ge 0$, and that it is socially optimal to produce both crops (e.g., $\partial U(0,.) / \partial Q^r = \partial U(.,0) / \partial Q^g = \infty$).

The formulation of equation (6) implies that the social planner can assign land use to each farm but is able to control farm-level production activities only through output prices, p^r and p^g (common to all producers). In particular, constraints (ii) state that one (and only one) of the two crops is produced on each farm. In other words, because of a small scale of farming operations or other technological reasons, no farmer can (or, rather, has an incentive to) produce a mix of the two crops.[27] Also implicit is the assumption that there exist no other (profitable) farm-level activities that neither generate nor bear externality. Constraint (iii) may arise as a result of high costs of monitoring and enforcing location-specific farm-level production and input-use intensities. Depending on the context, these may or may not be plausible assumptions. An unconstrained welfare-maximization problem, where outputs of both crops are endogenous and vary across farms in each use, is not studied in this paper (however, see endnote 15).[28]

ASSUMPTION 2. (i) $\partial c^r / \partial e > 0$ (negative externality); (ii) $\partial c^g / \partial e = 0$ (generators are externality-neutral).

Condition (i) implies that $\partial \pi^r(e) / \partial e < 0$, so that the profit from growing recipient crop decreases with externality. Condition (ii) assures that generator profit, π^g, as well as profit-maximizing output, g^*, is invariant to externality and, hence, to location. Under Assumption 2, upon incorporating constraints (i) - (iii), the joint maximization problems (5) and (6) can be restated more concisely as

$$\max_{p^r, p^g, \{g_i\}_{i \in N}, s} U(\sum_i r^*(e_i, p^r) 1_{g_i=0}, s g^*(p^g)) - \sum_i c^r(r^*(e_i, p^r), e_i) 1_{g_i=0} - s c^g(g^*(p^g)), \quad (7)$$

where $g_i \in \{0,1\}$, $\sum_i g_i = s$, and $e_i = g^*(p^g) \sum_{j \neq i} \gamma(d_{ij}) g_j$. Note that externalities e_i enter both the benefit and cost components of the objective in (7). To simplify problem (7) we make the following assumption.

ASSUMPTION 3. For any $\{g_i\}_{i \in N}$, $g_i \in \{0, g^*\}$, the *total* farm-level profit-maximizing recipient output, R, and cost, C^r, are both functions of the *total* externality damage, E:

$\sum_i r^*(e_i, p^r)1_{g_i=0} = R(E, p^r)$ and $\sum_i c^r(r^*(e_i, p^r), e_i)1_{g_i=0} = C^r(E, p^r)$, where $\partial R / \partial E \le 0$, $\partial R / \partial p^r > 0$, $\partial C^r / \partial E \ge 0$, $\partial C^r / \partial p^r > 0$, $E = \sum_i f(e_i)1_{g_i=0}$, $f(.) > 0$, and $\partial f / \partial e > 0$.

Assumption 3 adheres if (a) $c^r(r,e) = c^r(r + f(e))$, that is, externality damage amounts to the leftward shift of the farm-level cost function so that $R(E, p^r) = \varphi(p^r) - E$ and $C^r(E, p^r) = \varphi(p^r)$; or (b) $c^r(r,e) = c^r(r) + f(e)$, that is, the optimal output level is invariant to the extent of externality exposure ($\partial^2 c / \partial r \partial e = 0$ and $\partial R / \partial E = 0$) so that $R(E, p^r) = \varphi(p^r)$ and $C^r(E, p^r) = \varphi(p^r) + E$, where $\varphi(p^r) = \sum_i 1_{g_i=0}[\partial c^r / \partial r]^{-1}(p^r)$.

Hence, problem (7) can be decomposed into two steps. First, we minimize the *aggregate* damages (profit losses), E (or $p^r E$), given the number of farms in generating use, s, and output prices, p^r and p^g:

$$E^*(s) = \min_{\{g_i\}_{i \in N}} E \text{ subject to } g_i \in \{0,1\}, \sum_i g_i \ge s. \tag{8}$$

Then, based on the optimal arrangements of land uses in (8), output prices and the number of farms in generating use are optimally chosen. Note that generator and recipients outputs can be controlled by two instruments. Output prices affect farm-level production intensities, while zoning orders control the total number (and location) of farms in each use. We do not pursue the analysis of optimality of these two control measures any further but instead focus on problem (8) and characterize cost-minimizing arrangements.

Observe that, under Assumptions 2 and 3, the objective in (8) can be equivalently posed as $\Pi^r(s, p^r, p^g) = \max_{\{g_i\}_{i \in N}} \sum_i \pi^r(e_i) = p^r R(E^*(s), p^r) - C^r(E^*(s), p^r)$, so that minimizing the region-wide externality exposure, E, is equivalent to maximizing the total recipient profits, Π^r, conditional on s, p^r, and p^g. The next two assumptions suffice to permit an analytical solution to (8).

ASSUMPTION 4. The recipient's profit decreases linearly with externality $\partial^2 \pi^r / \partial e^2 = 0$.

If Assumption 2 holds (and hence, $\partial f^2 / \partial e^2 = 0$[29]), problem (8) is a well-known combinatorial problem that can be formulated as a graph partitioning problem, a specific instance of the quadratic assignment problem, or a quadratic optimization problem using graph-theoretic or matrix notation (Cela 1998; Burkard et al. 1998). This problem arises in a number of settings, including facility layout, manufacturing, circuit board and microchip design, parallel computing, and other numerous areas of engineering, physics, and management. The number of different arrangements with s generator farms on a square $n \times n$ grid is $n^2!/s!(n^2 - s)!$, so even for a small region, complete enumeration of all arrangements is infeasible. For example, there are more than 10^{29} possible arrangements for $n^2 = 100$ farms and $s = 50$ generators. In general, the graph partitioning problem belongs to the class of problems for which the time required to check the optimality of a solution grows polynomially with the size of the problem. The variety of the suggested solution algorithms based on different approaches can be grouped into four categories: spectral and geometric methods, multilevel algorithms, discrete optimization-based methods, and continuous optimization-based methods (e.g., see Hager and Krylyuk 1999 and references therein).

So, to characterize optimal arrangements in (8), we will also need the following assumption.

ASSUMPTION 5. Externality impacts only neighbors with a common border: $\gamma(d) = 1_{d \le 1}$.

We follow the approach suggested by Yackel, Meyer, and Christou (1997), which is based on the equivalence between the (constrained) optimal assignment problem (8) under Assumptions 4 and 5 and tiling the region so as to minimize the total tile perimeter, where each tile corresponds to the collection of farms in each use.[30] Some consequences of relaxing Assumptions 4 and 5 are discussed in Sections 4.3 and 4.4.

4.1. Efficient Arrangements under a Negative Linear "Border" Externality

In this section, we determine efficient arrangements (EA) under Assumptions 2 through 5. For ease of reference, we restate problem (8) as follows:

$$E^*(s) = \min_{\{g_i\}_{n\in N}} \sum_i (1-g_i) \sum_j 1_{d_{ij}\le 1} g_j \quad \text{subject to } g_i \in \{0,1\} \text{ and } \sum_i g_i = s. \qquad (9)$$

We will need the following result reported in Yackel, Meyer, and Christou 1997 and Rosenberg 1979. Let $\lceil x \rceil = \min\{z : z \ge x, z \text{ is integer}\}$ and $\lfloor x \rfloor = \max\{z : z \le x,$ $z \text{ is integer}\}$.

THEOREM A (Yackel, Meyer, and Christou 1997; Rosenberg 1979). The minimum perimeter of all configurations of s cells, $P^*(s)$, is given by

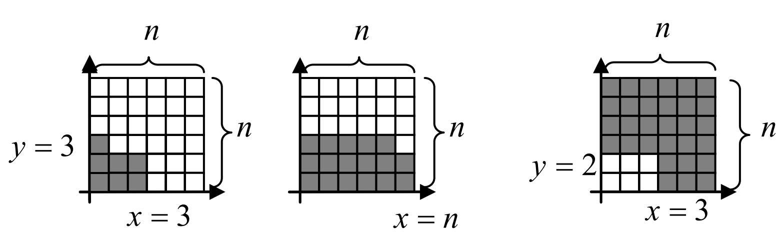
$$P^*(s) = \begin{cases} 2(\lceil s^{1/2} \rceil + \lfloor s^{1/2} \rfloor), \text{if } \lceil s^{1/2} \rceil \lfloor s^{1/2} \rfloor \ge s \\ 4\lceil s^{1/2} \rceil, \text{ oherwise} \end{cases}. \qquad (10)$$

Using Theorem A and formula (10), we state the following result (see Figure 5 for an illustration).

PROPOSITION 7. *Suppose that Assumptions* 2 - 5 *hold. For a given number of generators,* s, *EA consists of*

(a) *a minimum-perimeter neighborhood of generators that is tightly fitted in a corner of the region if* $s \le n^2/2$ *and* $P^*(s)/2 \le n + 1_{s/n > [s/n]}$;

(b) *(almost) rectangular neighborhoods of generators and recipients that share a border that is "almost" a row or a column if* $P^*(\min[s, n^2 - s])/2 \ge n + 1_{s/n > [s/n]}$;



Case (a), s=7 Case (b), s=17 Case (c), s=30

FIGURE 5. Optimal arrangements

(c) *a minimum-perimeter neighborhood of recipients that is tightly fitted in a corner of the region if* $s \geq n^2/2$ *and* $P^*(n^2 - s)/2 \leq n + 1_{s/n>[s/n]}$.

The optimal amount of externality is given by $E^*(s) = \min[P^*(\min[s, n^2 - s])/2,$
$n + 1_{s/n>[s/n]}]$.

Proposition 7 is based on the observation (see Yackel, Meyer, and Christou 1997) that the quadratic assignment problem (9) can be reformulated as

$$E^*(s) = \min_{G,R} \mathrm{P}(G) + \mathrm{P}(R) - 4n \text{ such that } |G| = s,\ |R| = n^2 - s,\ G \cup R = N, \quad (11)$$

where $\mathrm{P}(G)$ is the perimeter of a configuration of farms in set G (only the borders that are shared with farms not in set G are counted). Because the perimeter of the region's boundary is fixed ($\mathrm{P}(N) = 4n$), minimizing the length of the border separating generators and recipients is equivalent to minimizing the total perimeter of both neighborhoods, or minimizing the length of the outer border that does not coincide with the region's boundary for recipient (or generator) farms. In the proof of Proposition 7 it is shown that any EA must consist of recipient and generator neighborhoods with "filled" rows and columns. And so, in a square and symmetric region, there can be only three distinct forms of solution as illustrated in Figure 5.[31]

A useful property of the square grid is that the length of the border separating recipients and generators in Cases (a) and (c) in Figure 5 is equal to $x + y$ when there are "no gaps" in row and columns in the two neighborhoods.[32] Here, $x + y$ (both are positive integers) is the number of distinct rows and columns that intersect the neighborhood of generators (recipients) and contribute 1 to $E^*(s)$. Arrangements in Figure 5 solve the following optimization problem:

$$E^*(s) = \min_{x,y} \begin{cases} x + y, & \text{if } \max[x, y] < n \\ n + 1_{s/n>[s/n]}, & \text{if } \max[x, y] = n \end{cases} \text{ subject to } \min[s, n^2 - s] \leq xy. \quad (12)$$

If $s = 7$ as in Case (a), it is optimal to minimize the perimeter of the generator neighborhood. If $s = 17$ as in Case (b), it is optimal to set $x = n$, or $y = n$ in (12). Then, clearly, the length of the border cannot be less than n if s can be expressed as kn for some integer k, or $n+1$, if otherwise. If $s=30$ as in Case (c), it is optimal to minimize the perimeter of the recipient neighborhood. All EAs can be obtained by rotation and/or reflection of EAs in Cases (a) through (c). (There can be several EAs in Cases (a) and (c).)

Note that when land surrounding the region is allocated to externality-generating use (as in Figure 2b), the search for EA is very simple. EA consists of any perimeter-optimal neighborhood of recipients located anywhere in the region, with the total amount of the externality born by recipients equal to $P^*(n^2-s)$. Next we ascertain when EAs may arise as a result of (or be "cheaply" implemented through) noncooperative behavior.

4.2. When Are Efficient Arrangements Compatible with Nash Equilibrium?

In this section, we investigate when EAs are self-enforcing, in the sense that (farm-level) zoning restrictions, output and input prices (common to all producers), and per farm tax/subsidies conditional on land use constitute the Nash equilibrium.

DEFINITION 2. The assignment of land uses $\vec{g}$ is self-enforcing in a noncooperative setting if there exists a crop-specific payment scheme (per farm tax or subsidy), t^r and t^g, such that $\pi^r(p^r,e_i)>\pi^g(p^g)+t>\pi^r(p^r,e_j)$ for each $g_i=0$ and $g_j>0$, where $t=t^g-t^r$ is a net positive or negative farm-level monetary transfer that depends on land use but not on farm location.

Note that an arrangement can be self-enforcing if and only if the externality damage for any recipient is strictly less than the (avoided) damage for any generator, which is the necessary condition for the strict Nash equilibrium (see Lemma 1).

Under Assumption 5, by Proposition 1, we can narrow our search for PSNE-compatible EAs to those that are comprised of the rectangles of generators. Yackel, Meyer, and Christou (1997) provide the following theorem characterizing the rectangular blocks that have minimum perimeter among all arrangements with a given number of cells (farms).

THEOREM B (Yackel, Meyer, and Christou 1997). A rectangle $x \times y$ or $y \times x$ is perimeter-optimal if and only if

$$x - y \leq \max[2round(y^{1/2}), 2\lfloor y^{0.5} - 1 \rfloor + 1] \text{ where } y \leq x . \qquad (13)$$

Using this characterization of all perimeter-optimal rectangles (Theorem B), we state the following result.

PROPOSITION 8. *Let Assumptions 2–5 hold,* $4 \leq s \leq n^2 - n$, *and* $1 < \hat{e}(p^r, p^g, t) < 2$, *where* $\pi^r(p^r, \hat{e}) = \pi^g(p^g) + t$. *Then an EA is self-enforcing (compatible with the strict PSNE) if and only if one of the following holds:*

(i) $s < n^2/2$, $P^*(s)/2 \leq n + 1_{s/n > [s/n]}$, *and there exist integers* x *and* $y \leq x$ *such that* $s = xy$, $y \geq 2$, $x < n$, *and condition (13) is satisfied.*

(ii) $s/n = [s/n]$ *and* $P^*(\min[s, n^2 - s])/2 \geq n$.

Proposition 8 provides precise conditions when it is possible that an efficient arrangement will emerge as a result of noncooperative profit-maximizing behavior. If condition (i) holds then $g_i^* = 1_{(x_i, y_i) \leq (x, y)}$ for all $i \in N$ is a self-enforcing EA (strict PSNE). If condition (ii) holds then $g_i^* = 1_{i \leq s}$ for all $i \in N$ is a self-enforcing EA (strict PSNE). Also, this result provides a sense in which it is "more likely" that EA is compatible with PSNE if the number of recipients exceeds the number of generators. In that case, EAs consist of (almost) rectangular blocks of generators, which, by Proposition 1, coincide with one of the PSNE outcomes when condition (13) is satisfied. However, if there are sufficiently more generators than recipients and $P^*(n^2 - s)/2 < n$, efficiency requires that recipients be arranged in a (almost) rectangular block, which cannot occur in the strict Nash equilibrium (Case (c) in Figure 5).

In the situations when land outside the region is permanently allocated to externality-generating use (as illustrated in Figure 2b), it is easy to show that EA is compatible with PSNE if there exist integers x and $2 \leq y \leq x$ such that $s = n^2 - xy$ and condition (13) is satisfied. Thus, the range of circumstances when EAs are self-enforcing in the region with "unprotected edges" is broader compared with that in the region with "protected edges."

While a characterization in the manner of Proposition 7 of EA when externality has a local eight-farm neighborhood impact ($\gamma(d) = 1_{d \le \sqrt{2}}$) is not attempted in this paper, it is easy to show that, in such cases, EA may or may not be compatible with PSNE as well. For example, a square neighborhood of four generators fitted in a corner of the region (see Part (iii) in Proposition 3 with $2 \le \hat{e} < 3$ and $s = 4$), and an octagon neighborhood of $n^2 - 4$ generators (see Part (ii) in Proposition 3 with $3 < \hat{e} < 4$ and $s = n^2 - 4$), where $n \ge 4$, are EA and the strict PSNE for some output prices and per farm payments t^r and t^g .

More generally, by Proposition 5, EA may be self-enforcing only if the desired number of generators, s , exceeds a lower bound. However, Proposition 8 demonstrates that without knowing the properties of the externality-dissipation function no "negative" universal statements regarding the compatibility of EA and PSNE are available. Next we examine some consequences of relaxing (one at a time) Assumptions 4 and 5.

4.3. Efficient Arrangements under Non-Linear Profit Loss Due to Externality

The property that in EA generators are located in a contiguous area with the minimum border length depends on Assumption 4—that returns to externality-receiving land use fall linearly with externality, e . The spatial concentration of generators may result in smaller on average but (possibly) more dispersed externality damages among the recipients. This is because a decrease in the total number of "border" neighbors with incompatible uses may necessitate locating the "bordering" recipients next to a greater number of generators.

Observe that input adjustment at the farm level may assure that the marginal profit loss due to the externality decreases with a greater exposure. For example, suppose that the amount of productive acreage is reduced because of the buffer zone requirement. This is likely to result in a smaller-than-proportionate decline in farm income due to the optimal adjustment of the other farm inputs such as labor as the available acreage shrinks. However, it is not inconceivable that the reverse relationship holds when the externality enters the production function differently. For example, it may be increasingly difficult for a non-GM crop grower to coordinate planting activities to reduce the probability of cross-pollination when there are more GM crop farmers in the surrounding area and buffer zones are not effective.[33]

Let $c^r(r,e) = c^r(r + f(e))$ so that $\pi^r(e) = p^r(\varphi(p^r) - f(e)) - c^r(\varphi(p^r))$, or $c^r(r,e) = c^r(r) + f(e)$ so that $\pi^r(e) = p^r\varphi(p^r) - c^r(\varphi(p^r)) - f(e)$, where $\varphi(p^r) = [\partial c^r / \partial r]^{-1}(p^r)$. Suppose that $\partial^2 f / \partial e^2 \le 0$, which implies that the marginal recipient loss increases with the amount of exposure, $\partial^2 \pi^r / \partial e^2 \ge 0$. Then it may be optimal to let a smaller number of recipients bear most of the externality damage while lowering the damage for the remaining recipients. In contrast, suppose that $\partial^2 f / \partial e^2 \ge 0$, and hence, $\partial^2 \pi^r / \partial e^2 \le 0$. Then it may not be socially optimal to expose some recipients to the damage from multiple externality generators. In some cases, a spatial spreading of generators across the region is liable to yield a more even distribution of the externality damages among recipients and improve efficiency. We will need the following measure of dispersion.

DEFINITION 3. A vector $x = (x_1,..., x_n)$ is sub-majorized by the vector $y = (y_1,..., y_n)$ (denoted by $\prec_w$) if $\sum_{i=1}^{k} x_{(i)} \le \sum_{i=1}^{k} y_{(i)}$ for $k = 1,2,..., n$, where $x_{(1)} \ge x_{(2)} \ge ... \ge x_{(n)}$ and $y_{(1)} \ge y_{(2)} \ge ... \ge y_{(n)}$ are their components in non-increasing order.

Note that $x \prec_w y$ can be equivalently stated as $\sum_{i=1}^{n} f(x_i) \le \sum_{i=1}^{n} f(y_i)$ for all increasing convex functions f (see Marshall and Olkin 1979 for details). The following result shows that if EA under Assumptions 2–5 is self-enforcing then it is also efficient (and of course, self-enforcing) if $\partial^2 \pi^r / \partial e^2 \le 0$. The reason is that the self-enforcement condition requires that each recipient be exposed to at most one unit of externality. Therefore, a self-enforcing EA under Assumptions 2–5 is characterized not only by the least number of exposed recipients but also by the least dispersion of exposures among them.

COROLLARY 1. EA under Assumptions 2–5 is self-enforcing if and only if it is a self-enforcing EA under Assumptions 2, 3, and 5, and $\partial^2 \pi^r / \partial e^2 \le 0$.

However, if $\partial^2 \pi^r / \partial e^2 \geq 0$ or EA under Assumptions 2–5 is not self-enforcing, EAs are affected by the curvature of the recipient's profit function. The following result presents circumstances ("sufficient" convexity of $\pi^r(e)$) when an arrangement where generators (recipients) form a triangular neighborhood (illustrated in Figure 6a), dominates EA under Assumptions 2–5. We say that an arrangement is triangular if recipient farms that have one or two generator neighbors form a diagonal (discrete) line. Let $k(s,n) = 0$ if $s > n^2/2$ and $E^*(s) = n$, $k(s,n) = 1$ if $s < n^2/2$ and the corresponding EA (Proposition 7) is self-enforcing, or $s > n^2/2$ and $E^*(s) = n+1$, or $s > n^2/2$ and $E^*(s) < n$ and condition (13) is met for some integer x and $2 \leq y \leq x$ with $n^2 - s = xy$, and $k(s,n) = 2$ if $s < n^2/2$ and EA (Proposition 7) is not self-enforcing, or $s > n^2/2$ and $E^*(s) < n$ and condition (13) is not met for any integer x and $2 \leq y \leq x$ with $n^2 - s = xy$, where $E^*(s)$ is given by (9). Function $k(s,n)$ counts the number of recipients with two generator neighbors in EA under Assumptions 2–5 plus one if there are recipients with one generator neighbor in the corresponding triangular arrangement (i.e., $s < n^2/2$).

COROLLARY 2. Suppose that $s = a(a+1)/2$ or $s = n^2 - a(a+1)/2$ for some integer $2 < a < n$. Then a triangular arrangement dominates EA under Assumptions 2–5 in the sense of social welfare if $-[\partial^2 \pi^r(0)/\partial e^2]/[\partial \pi^r(0)/\partial e] \geq (2a - E^*(s))/(a - k(s,n))$.

Note that $E^*(s) < 2a$ when $s = a(a+1)/2$ or $s = n^2 - a(a+1)/2$, $2 < a < n$, so that a triangular arrangement may improve social welfare only if $\Delta^2 \pi^r(0) > 0$, and the index of

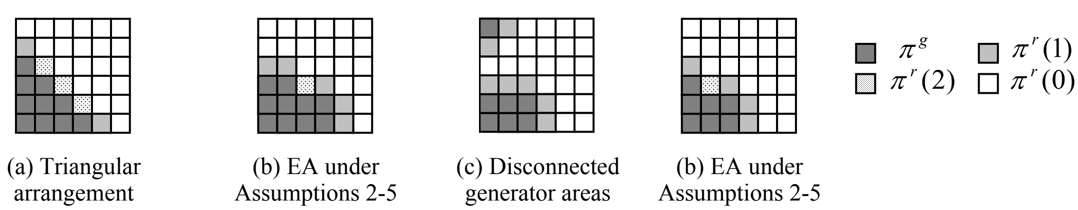


FIGURE 6. Welfare-improving arrangements under non-linear recipient profit

"convexness" of the recipient's profit in externality, $-[\partial^2 \pi^r(0)/\partial e^2]/[\partial \pi^r(0)/\partial e]$, is sufficiently large. The following example illustrates.

EXAMPLE 3. Suppose that Assumptions 2, 3, and 5 hold, $s = 10$, and $n = 6$. Consider arrangements in Figure 6a and 6b. Arrangement in Figure 6b is an EA under Assumptions 2–5. However, arrangement in Figure 6a improves the social welfare compared with that in Figure 6b if $-[\partial^2 \pi^r(0)/\partial e^2]/[\partial \pi^r(0)/\partial e] \geq 1/2$. In arrangement (a), there are five exposed recipients, at the expense of a "double" exposure for three recipients, while in arrangement (b) the exposures among recipients are more "evened out," at the expense of increasing the number of the exposed ones to six.

It is clear that, under Assumptions 2, 3, and 5, and $\partial^2 \pi^r / \partial e^2 \geq 0$, any EA consists of at most one neighborhood of recipients and one neighborhood of generators. If an EA consisted of more than one neighborhood of farms with the same land use, then, by gluing the neighborhoods (see Yackel, Meyer, and Christou 1997), one could decrease the number of the exposed recipients without decreasing the measure of "unevenness" in the distribution of exposures among the remaining recipients (see Definition 3). On the other hand, an EA may consist of more than one neighborhood of generators if $\partial^2 \pi^r / \partial e^2 \leq 0$ and none of the EAs under Assumptions 2–5 is self-enforcing. The following example illustrates.

EXAMPLE 4. Suppose that Assumptions 2, 3, and 5 hold, and $s = 7$, and $n = 6$. Consider arrangements in Figures 6c and 6d. Arrangement in Figure 6d is EA under Assumption 4 ($\partial^2 \pi^r / \partial e^2 = 0$). However, arrangement in Figure 6c improves the social welfare compared with that in Figure 6d, if $\partial^2 \pi^r(0)/\partial e^2 \leq \partial \pi^r(0)/\partial e < 0$. In the arrangement in Figure 6(c), there are six recipients that are each exposed to one generator, while in the arrangement in Figure 6(d), there are five exposed recipients but one of them has two generator neighbors. When the recipient profit function is "sufficiently" concave, that is, $[\partial^2 \pi^r(0)/\partial e^2]/[\partial \pi^r(0)/\partial e] \geq 1$, the benefits of a more "evened out" distribution of damages outweigh the additional costs of exposing a greater number of recipients as in the case of arrangements in Figures 6(c) and 6(d).

4.4. Efficient Arrangement under Global Spatial Externality

The property, characterized in Proposition 7, that in EA generators are concentrated in a contiguous area also hinges on the restriction that the externality dissipates only across borders (Assumption 5). This is because the contiguity of the areas allocated to generator and recipient uses is a necessary condition for minimizing the total number of immediate "border" neighbors with incompatible uses. However, the externality may affect recipient farms located at greater distances (see Section 3.3). In such cases, optimal patterns may no longer involve the "agglomeration" of generators in one location even when Assumption 4 is satisfied. The reason is that concentrating generators in a contiguous area may entail locating some generators "too close" to the center of the region.

Suppose that Assumptions 2, 3, and 4 hold, and consider a linear externality-decay case: $\gamma(d) = \bar{d} - d$, $\bar{d} \geq (n-1)\sqrt{2}$. As illustrated in Figure 7, the concentration of generators may increase the number of exposed recipients and/or intensify the rate at which the externality reaches recipients by more than it minimizes the number of "maximally" exposed recipients. Note that in the EA in Figure 7b, generators are not located in a contiguous area (do not form a neighborhood). In EA (b), disconnected farms 1 and 3 are generators because, compared with a suboptimal arrangement in Figure 7(a), even though the exposure for recipient farm 2 in arrangement (b), $e_2 = 2(\bar{d} - 1)$, is greater than that for recipient farm 3 in Figure 7a, $e_3 = 2(\bar{d} - 1) - 1$, the other recipients "gain" by distancing themselves (on average) from the sources of externality. In other words, generator farms 1 and 3 are located "on average" farther away from the other (recipient) farms in arrangement in Figure 7b compared with the "average" distance between generator farms 1 and 2 and recipients in arrangement in Figure 7a.

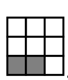
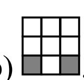
(a) , $E(2) = 7\bar{d} - 25.02$ (b) , $E^*(2) = 7\bar{d} - 25.43$

FIGURE 7. Inefficient (a) and efficient (b) arrangements: $\gamma(d) = \bar{d} - d$, $n = 3$, $\bar{d} \geq 2\sqrt{2}$

5. Conclusions

This paper takes a close look at the equilibrium and efficient arrangements of activities (conflicting and non-conflicting land uses) that generate and receive a distance-dependent negative externality. Similar land uses are *non-conflicting* (*compatible*) in the sense that the incremental return from switching land uses increases with the externality given by the distance-weighted output of neighbors with the same land use. Then the pure strategy Nash equilibrium is characterized by a simple condition that the amount of externality exposure for each farm in the externality-receiving land use cannot be greater than that for each farm in the externality-generating use. To improve the efficiency of an arrangement, the social planner can assign land uses through zoning restrictions and manipulate production intensities through output prices. The compatibility of the (constrained) efficient arrangements with Nash equilibrium depends on how an externality dissipates with distance, the desired number of generators, and the curvature of the recipient's profit function. A main policy implication is that there exist circumstances when an (constrained) efficient arrangement can be implemented through self-enforcing zoning orders, which obviates the need for the investment in subsequent monitoring or enforcement. However, in other situations such zoning orders may conflict with profit-maximizing objectives of individual (noncooperative) land owners. And so, a policymaker may face a potential trade-off between the gain in efficiency and the cost of implementing a particular land-use arrangement.

There are a number of restrictive assumptions implicit in the formulation of the model and the "constrained" efficiency problem, such as perfect information among the agents, the observability of the production activities in the entire region, and the lack of countermeasures to combat the effects of the externality (e.g., pollution abatement) on the part of generators. Also, the limitation of (profitable) production activities to be either externality generating or externality receiving may be an implausible assumption. For example, agricultural land enrolled in the Conservation Reserve Program and retired from production may also serve as a buffer (or barrier) zone to prevent externality exposure (Munro 2003). From the producers' perspective, an externality-neutral activity amounts to "convexifying" the profit from recipient land use as it effectively puts a cap on the incurred damage from the externality.

The temporal dimension of the activity choice and, as a consequence, the fixed costs that are frequently associated with land-use change, as well as the uncertainty of the future income flow contingent on the surrounding land uses, are completely left out of the model. Dispensing with these and other assumptions regarding the participants' behavior is likely to glean valuable insights into the problem of improving the efficiency of the arrangement of land uses under spatial externality. It is also worth noting that the issue of the spatial arrangement of conflicting or benefiting activities is not confined to crop agriculture and is pertinent in other areas of economics such as urban economics and social sciences (e.g., Brock and Durlauf 2001).

Endnotes

1. In 2001, a conference in Minneapolis titled “Strategies for the Coexistence of GMO, Non-GMO, and Organic Crop Production” was attended by GMO, non-GMO, and organic producers; specialists from the USDA; academic researchers; and representatives from biotechnology companies. One of the primary issues on the agenda was the development of strategies for the coexistence of potentially incompatible farming practices, including “neighbor relations” (the meeting’s agenda and summary are available at http://www.biotech.iastate.edu/publictions/IFAFS/coexistence.html). Public interest in this issue is also exemplified by the grants awarded by the National Science Foundation to research the unintended spread of engineered plant genes (*Feedstuffs* 2004a, p. 15).

2. In the data sample of California growers used in their study, the certification as an organic producer is a substantial economic decision. To use the “organic” label for crops produced on land that was not previously certified, a farm is required to undergo a three-year transition period with annual inspections, during which organic production practices are followed.

3. See USDA n.d. for more information on organic farming and Riddle 2004 for a recommendation of strategies for organic and conventional growers to minimize genetic drift, commingling, and other GMO contamination. An account of the latest research results on corn pollen drift and the effectiveness of buffer strips planted to corn is reported in *Feedstuffs* 2004b, p. 15.

4. Also, there may be increases in the cost of production or yield losses attributed to the conflicting pest management approaches used by organic and conventional growers, and the severity of such problems is greatest along borders between organic and conventional farms (Parker and Munro 2004).

5. Also, the presence of externalities that extend across borders between land uses plays a role in the analysis of optimal management decisions in the case of tropical forests in Albers 1996.

6. Of course, nothing changes if there is a large pool of homogenous farm operators with a fixed reservation wage who competitively bid for land rent.

7. We use the terms “generator” and “recipient” following the agent-based simulation literature (Parker and Meretsky 2004). A function is called “supermodular” if increasing one variable increases the incremental return to another variable. This concept is immediately related to the important economic notion of complementarity (e.g., Topkis 1998).

8. We also report certain properties of equilibrium land-use arrangements and provide a discussion of efficient arrangements when Assumptions 4 and 5 are relaxed.

9. The timing of moves (simultaneous or sequential), including incentive-based asynchronous updating in agent-based models, is known to qualitatively alter both dynamics and end states (e.g., Huberman and Glance 1993; Page 1997). In addition, the set of Nash equilibrium outcomes may not coincide with agent-based learning simulation models such as the local adjustment and experience-based models used in Laine and Busemeyer 2004.

10. In fact, it was first introduced by economists Koopmans and Beckman (1957) in the context of a plant location problem.

11. It is impossible in the strict Nash equilibrium where no agent is indifferent between her equilibrium action and some other action, given the other agents' actions.

12. Also, Belcher, Nolan, and Phillips (2003) use a cellular automata simulation program known as the game of life to study a dynamic process of contamination and decontamination in the case of GMO and non-GMO crops. They find that under certain initial distributions of crops there exist stable equilibria where both crops are produced. Munro (2003) assigns a fixed number of externality generating uses in a random manner and determines an upper and lower bound on the total externality damage imposed on the recipients. Spatial externality considered in both studies corresponds to Case (b), analyzed in Section 3.2, where the externality imposed by a GMO grower affects at most eight non-GMO growers located in the adjacent cells. In addition, an extension of Parker 2000 is presented in Parker and Meretsky 2004, where a simulation model of parcel managers under conflicts between urban and agricultural uses is developed.

13. There is a rich literature on city formation where externalities drive spatial agglomeration and specialization (e.g., Fujita and Thisse 2002; and Berliant, Peng, and Wang 2002).

14. Also, Lucas and Rossi-Hansberg (2002) develop a very rich framework to analyze an endogenous formation of the city structure under spatial externality. They consider atomless firms that become more productive as the amount of production activity in their proximity increases with the impact of a unit increase in nearby production activity restricted to decay exponentially with its distance from the target firm. While in Lucas and Rossi-Hansberg firms balance the benefits from locating in high-employment density areas against the costs of longer commutes for workers, in our set-up there is only one type of agent and one force behind the heterogeneity in equilibrium land-use across agents. Our analysis is much simpler because equilibrium is characterized by a single condition that the exposure to the externality by any farm in the externality-receiving use be smaller than that for any farm in the externality-generating use. This condition is the sole determinant of the geometric properties of equilibrium arrangements analyzed in the paper.

15. Heterogeneity in farm size, land quality, or other production characteristics can be accommodated by allowing for asymmetry in the externality impacts in multiplicative or additive form as follows: $\gamma(d_{ij}) + h_{ij}$ or $\gamma(d_{ij})h_{ij}$. Furthermore, suppose that $h_{ij} = a_i + b_j$ or $h_{ij} = a_i b_j$. In that case, a_i can be interpreted as farm *i*'s susceptibility to externality and b_j as farm *j*'s generating capacity. Equilibrium and efficient arrangements under the assumption of heterogeneous farms (but in the absence of spatial structure, i.e., $\gamma(d) = k$ for all d) are easy to obtain and are not reported in this paper because of space constraints.

16. Equivalently, in terms of spatial weight matrices used in spatial econometrics (e.g., Anselin, Florax, and Rey 2004), externality exposures are given by $\vec{e} = W\vec{g}$, where $\vec{e} = \{e_1, ..., e_{n^2}\}$ is a vector of externality exposures, $W = \{w_{ij}\}$ is the distance-weighting matrix with $w_{ii} = 0$ and $w_{ij} = \gamma(d_{ij})$ for each $i \neq j$, $i, j \in N$, and $\vec{g} = (g_1, ..., g_{n^2})$ is a vector of generator crop outputs on each farm. Also, note the analogy with the expected form of the utility function over lotteries. Expression (1) for the externality damage can be derived from "preferences" over the distribution of land uses in the region if the "preferences" satisfy continuity and independence axioms. The independence axiom amounts to requiring that the impact of any generator farm on the recipient farm is independent of the impact of the other generator farms.

17. See Parker 2000 for a discussion of "unprotected" and "protected" borders in Case (a). The arbitrariness of assuming that land surrounding the region is in externality-generating use is particularly apparent in Case (c), since the distribution of the exogenous externality exposures across the region will then depend on the distribution of land uses outside the region.

18. A game is called "supermodular" if each player's objective function is supermodular in the player's own actions and actions of all other players. In other words, in a supermodular game, a "higher" equilibrium action by any player leads to an optimal equilibrium choice of "higher" actions by all other players.

19. This seems particularly realistic given the three-year transition period required to obtain an organic grower status. However, in other cases of interest, mixed strategies may be implemented through crop rotation beneficial in its own right.

20. The assumption that the lattice is square is important because neighborhoods of farms with compatible (similar) land use are formed in any equilibrium where both crops are produced. For example, if the lattice is one-dimensional so that $d_{ij} = |i - j|$ for any $i, j \in N$, it can be shown that the only PSNE are $\vec{g}^* = (0, ..., 0)$ or $\vec{g}^* = (1, ..., 1)$ for any externality-dissipation function $\gamma(.)$.

21. A set of farms $G \subset N$ is a 4 (8)-connected neighborhood, if for any $i, j \in G$ there is a sequence of neighbors that share a common border (border or corner) in the set, $f_1,..., f_k \in G$, such that $d_{f_t, f_{t+1}} \leq 1(\sqrt{2})$ for $t = 1,...k-1$, $f_1 = i$ and $f_k = j$. Unless specified otherwise, a "neighborhood" will refer to a 4-connected neighborhood.

22. These kinds of land-use patterns are obtained in Parker 2000 using a simulation approach in the case of "protected" borders.

23. These kinds of patterns are obtained in Parker 2000 using a simulation approach in the case of "unprotected" borders.

24. Function $round(x)$ rounds x to the nearest integer.

25. The convex hull of L is $Co(L) = \{(x, y) | (x, y) = \sum_t \alpha_t (x_t, y_t) \;\; \forall (x_t, y_t) \in L \;\; \forall \alpha_t \geq 0, \sum_t \alpha_t = 1\}$, i.e., the intersection of all (real) convex sets that contain L, or equivalently, "the smallest" convex set containing L.

26. This result can be easily generalized when there are multiple activities or multiple levels for an activity. Suppose that agents are located on a lattice and play a supermodular game with payoff functions of the form $\pi_i = f(g_i, \sum_{j \neq i} d_{ij} g_j)$, where $g \in \Re$ and $f(.,.)$ is supermodular (see Milgrom and Roberts 1990). Then, in the manner of Proposition 6, it can be shown that the Nash equilibrium profile $\{g(x_i, y_i) = g_i^*\}_{i=1}^{n^2}$ is a quasi-convex function on a lattice.

27. One way to assure this is to set $\gamma(0)$ (a self-imposed "externality") to equal some large number.

28. Formally, this amounts to removing constraints (ii) and (iii) in (6). Real-world examples are agreements between organic and conventional growers where conventional growers alter production practices or farming intensities so as to accommodate the needs of organic producers. Abatement measures also exist in the case of conflicts between livestock operations and residential properties due to odors.

29. Note that $\partial^2 \pi^r(e) / \partial e^2 = (c_{re}^r)^2 / c_{rr}^r - c_{ee}^r = -\partial^2 c^r(r^*(e, p^r), e) / \partial e^2$, where subscripts denote the second-order derivatives of $c^r(r, e)$ (cost minimization only implies that $c_{rr}^r \geq 0$).

30. The main application discussed in Yackel, Meyer, and Christou 1997 is partitioning multiple tasks among processors so as to minimize interprocessor communication. Because their interest is in the situations when the number of different tasks

(conflicting or distinct land uses) is large, they do not consider a special case with just two different tasks that is of primary interest in this paper.

31. A configuration with farms along all four of the region's boundaries cannot possibly be optimal since there must be a row (or a column) that contributes 2, which contradicts the necessary condition for optimality.

32. Note that the perimeter of the generator (recipient) neighborhood in Cases (a) and (c) is also equal to $2M((x,0),(0,y))$, where $M((x_1,y_1),(x_2,y_2)) = |x_1 - x_2| + |y_1 - y_2|$ is "Manhattan" or city-block distance.

33. As mentioned in the introduction, Helfand and Rubin (1994) identify a number of "technical" and "psychological" sources of nonlinearities in social welfare, environmental quality, and cost functions. They also discuss different circumstances that may lead to optimal spatial concentration or spreading of generated externality but in a setting with just two production sites. When the landscape consists of multiple sites, a more careful analysis of optimal location of externality-generating activity is needed.

34. This procedure follows the "partial square" construction described in Yackel, Meyer, and Christou 1997. The authors establish that the arrangements of farms that are square or nearly square (height and width differ by at most one) have a minimum perimeter among all arrangements with a given number of farms. They also demonstrate a procedure that can be used to construct all possible minimum-perimeter configurations for a given number of generators s.

Appendix

Proofs

Proof of Proposition 1

Part (i). If $2 < \hat{e}$ then, by Assumption 1, $\pi^r(e) > \pi^g(e)$ for $e \in \{0,1,2\}$ and farms located in the corners of the region must be recipients because $e_k \le \sum_{j \ne k} \gamma(d_{kj}) = 2$ and $\pi^r(e_k) > \pi^g(e_k)$ for $k = 1, n, n^2 - n + 1, n^2$. But this implies that any farms located in the cells adjacent to the corners must be recipients because we also have $e_k \le 2$ for $k = \{\, 2, n+1, n-1, 2n,\ n^2 - 2n + 1,\ n^2 - n + 2, n^2 - 1, n^2 - n \,\}$. But the farms adjacent to these cells may have at most two neighbors that are generators and thus must be recipients as well. Continuing in this manner, it follows that all farms must be recipients in the PSNE.

Part (ii). Observe that $1 < \hat{e} < 2$ implies that $\pi^g(e) < \pi^r(e)$ for $e \in \{0,1\}$ and $\pi^r(e) < \pi^g(e)$ for $e \in \{2,3,4\}$. Hence, in any PSNE where both crops are produced, each recipient has at most one generator neighbor while each generator has at least two generator neighbors: (a) $e_i^* \le 1$ if $g_i^* = 0$, and (b) $e_i^* \ge 2$ if $g_i^* = 1$. Suppose that there is a generator neighborhood that is not a rectangle with dimensions (the height and width of the smallest rectangle that contains the neighborhood) not less than 2 by 2. If one of the dimensions is equal to one then there are (two) generators (one at each end) in the generator neighborhood that have only one generator neighbor each. And so, condition (a) is violated. Otherwise, there exists a recipient that has two generator neighbors, which violates condition (b).

Part (iii). Clearly, if $\sum_i g_i^* = 0$ then $\pi_i^r = \pi^r(0) \ge \pi^g(0)$ so that $g_i^* = 0$ for all i is the PSNE. If $\sum_i g_i^* = n^2$ then $e_i \ge 2$ and $\pi^r(e_i) < \pi^g(e_i)$ for all i, hence, $g_i^* = 1$ for all

i is the PSNE. If $0 < \sum_i g_i^* < n^2$ then there exists a recipient i that has at least one generator neighbor. But this is impossible because, by assumption, $\pi^r(e) < \pi^g(e)$ for any $e \geq 1$.

Proof of Proposition 3

Part (i). If $4 < \hat{e}$ then $\pi^r(e) > \pi^g(e)$ for any $e \in \{0,...,4\}$. Then farms located in the corners of the region must be recipients because $e_i \leq 3$ for $i = \{1, n, n^2 - n + 1, n^2\}$. But then farms adjacent to the corner farms and located on the edge of the region, $i = \{$ 2, n+1, n-1, 2n, n^2-2n+1, $n^2 - n + 2, n^2 - 1, n^2 - n\}$, have at most four neighbors that can be generators. And so, they must be recipients as well. The same is true for all of the remaining edge cells, $i = 3,..., n-2, 2n+1,...,(n-3)n+1, 3n,...,$ $(n-3)n$, $(n-1)n+3,...,n^2-2$. Next, remove the (recipient) edge cells of the region and apply the same reasoning to the remaining cells. Continue in this manner until the region consists of the one cell $i = [n^2/2]+1$ when n is odd or the four adjacent cells in the middle, $i = \{(n-1)n/2$, $(n-1)n/2+1, (n-1)n/2+n, (n-1)n/2+n+1\}$, when n is even. Hence, all farms must be recipients.

Part (ii). If $3 < \hat{e} < 4$ then $\pi^r(e) > \pi^g(e)$ for $e \in \{0,...,3\}$ and $\pi^r(e) < \pi^g(e)$ for $e \in \{4,...,8\}$. Hence, in any PSNE where both crops are produced, (a) $e_i \leq 3$ for each $g_i^* = 0$, and (b) $e_i \geq 4$ for each $g_i^* = 1$. It is clear that any octagon-shaped neighborhood consisting of 12 or more generators (with two or more generators along each side) satisfies conditions (a) and (b). Also, observe that only borders of generator neighborhoods that are formed by diagonal, vertical, or horizontal lines do not violate condition (a). However, in a neighborhood formed by the intersection of vertical and horizontal lines there exists a generator who has at most three other generator neighbors, which violates condition (b). Therefore, each generator neighborhood must have obtuse corners formed by the intersection of diagonal and vertical or diagonal and horizontal lines.

Part (iii). If $2 \leq \hat{e} < 3$, it follows that (a) $e_i \leq 2$ for each $g_i^* = 0$, and (b) $e_i \geq 2$ for each $g_i^* = 1$. Hence, the dimensions (height and width of the smallest rectangle that contains the neighborhood) of any neighborhood of generators cannot exceed two if condition (a) holds. On the other hand, the dimensions of any neighborhood of generators cannot be less than two if condition (b) holds. Therefore, generators must be arranged in square neighborhoods of exactly four generators, if $s(\vec{g}^*) \neq 0$ or n^2.

Part (iv). Clearly, if $\sum_i g_i^* = 0$ then $\pi_i^r = \pi^r(0) \geq \pi^g(0)$ so that $g_i^* = 0$ for all i is the PSNE. If $\sum_i g_i^* = n^2$ then $e_i \geq 3$ for each i. But, by assumption, $\pi^g(e) \geq \pi^r(e)$ for any $e \in \{2,...,8\}$, so that $g_i^* = 1$ for all i is the PSNE. If $\pi^r(1) < \pi^g(1)$ then $0 < \sum_i g_i^* < n^2$ is clearly impossible. So assume that $\pi^g(1) \leq \pi^r(1)$ and $\pi^r(2) < \pi^g(2)$. If $0 < \sum_i g_i^* < n^2$ then there must exist a recipient i with $\pi_i^r = \pi^r(1) \geq \pi^g(1)$ that has at least one generator neighbor j with $d_{ij} \leq \sqrt{2}$. Then all farms in the eight-neighborhood of generator j must be recipients (to assure that $\pi_k^r = \pi^r(1) \geq \pi^g(1)$ for all farms k with $d_{kj} \leq \sqrt{2}$ including farm i). Then we have $e_j = 0$, but this is impossible because farm j is generator with $e_j \geq 2$.

Proof of Proposition 5

In any PSNE with $1 \leq s \leq n^2 - 1$, there must exist farm i with $g_i^* = 0$ and farm j with $g_j^* = 1$ such that $d_{ij} = 1$. From the necessary equilibrium condition it follows that $e_i = \sum_{t \neq j} \gamma(d_{it}) g_t^* + \gamma(d_{ij}) \leq \sum_{t \neq j} \gamma(d_{jt}) g_t^* = e_j$. Rearranging this inequality yields $\gamma(1) \leq \sum_{t \neq j} [\gamma(d_{jt}) - \gamma(d_{it})] g_t^* \leq \sum_{t \neq j} [\gamma(d_{jt}) - \gamma(d_{jt} + 1)] g_t^* \leq -(s-1)\Delta\gamma(\hat{d})$, where $\Delta\gamma(d) = \gamma(d+1) - \gamma(d)$, $\hat{d} = 1$ if $\Delta^2\gamma \geq 0$ and $\hat{d} = (n-1)\sqrt{2}$ if $\Delta^2\gamma \leq 0$. The second inequality follows because $d_{ij} = 1$ implies that $|d_{jt} - d_{it}| \leq 1$ for any $t \in N$ and $\Delta\gamma \leq 0$. The third inequality uses the curvature of function γ to obtain an upper bound on the difference $\Delta\gamma(d)$ for $d \in [1, (n-1)\sqrt{2})$.

Proof of Proposition 6

There are two steps. First, we prove that the convex hull of L^* contains no recipients. Then we show that L^* is a connected neighborhood.

Step 1. Let $Co(L^*) = \{(x,y) \mid \sum_i \delta_i (x_i, y_i) g_i^* \ \ \forall \delta_i \geq 0, \sum_i \delta_i g_i^* = 1\}$ denote the convex hull of the set of generator farms. Prove by contradiction. Suppose that $g_i^* = 0$ and $(x_i, y_i) \in Co(L^*)$. By definition, we have $(x_i, y_i) = \sum_k \alpha_k (x_k, y_k) g_k^*$ for some $\alpha_k \geq 0$ and $\sum_k \alpha_k g_k^* = 1$. Then externality received by farm i is $e_i = \sum_{j \neq i} \gamma(d_{ij}) g_j^*$

$= \sum_{j\neq i} \gamma(d((x_i, y_i),(x_j, y_j))g_j^* = \sum_{j \neq i} \gamma(d(\sum_k \alpha_k (x_k, y_k) g_k^*, (x_j, y_j))g_j^*$

$\geq \sum_{j \neq i} \gamma(\sum_k \alpha_k d((x_k, y_k),(x_j, y_j))g_k^*)g_j^* \geq \sum_j \sum_k \alpha_k \gamma(d((x_k, y_k),(x_j, y_j))g_k^* g_j^*$

$= \sum_k \alpha_k g_k^* [\sum_{j \neq k} \gamma(d((x_k, y_k),(x_j, y_j))g_j^* + \gamma(0)] = \sum_k \alpha_k g_k^* e_k + \gamma(0) > e_m$, where $e_m = \min_k \{e_k \mid \alpha_k g_k^* > 0\}$ and $d((x_i, y_i),(x_j, y_j)) = d_{ij}$. The first and second inequalities follow from convexity of $d((x_i, y_i),(x_j, y_j))$ and concavity of $\gamma(d)$, respectively. The last equality and inequality follow because $\sum_k \alpha_k g_k^* = 1$ and $\gamma(0) \geq 0$, respectively. But this cannot be in the Nash equilibrium if Assumption 1(a) holds.

Step 2. To complete the proof we need to show that L^* is an eight-connected neighborhood. By Step 1, $Co(L^*) = L^*$ (i.e., only farms in L^* belong to the convex hull of L^*). It is not hard to demonstrate that each farm in (any isolated neighborhood of) L^* must have at least three generator neighbors that are mutual border or corner neighbors (i.e., four of them form a 2×2 square) when $\gamma(d)$ is concave ($n > 3$). This can be checked using a simple test function $\gamma^k(d) = \min[k, \bar{d} - d]$ for any $k > 0$, where $\bar{d} \geq (n-1)\sqrt{2}$, since any decreasing concave function can be obtained as a linear combination of γ^k. But if any neighborhood of L^* has a thickness of at least two at every cell (every generator belongs to some 2×2 square of other generators), then $Co(L^*)$ can be shown to be connected (see Chaudhuri and Rosenfeld 1998 for details).

Proof of Proposition 7

The proof proceeds in two steps. In Step 1, we develop a tight lower bound for $E^*(s)$. In Step 2, we use Theorem A to find arrangements that achieve the lower bound.

Step 1. Note that a generator farm imposes a cost on a recipient farm if and only if they share a common border. Consequently, it is always possible to arrange generators and recipients so that each row and column of the region contributes at most 1 to $E^*(s)$. However, the total length of the border separating $n^2 - s$ recipients from s generators cannot be less than the minimum number of rows and columns of the region that contain both types of farms. Equivalently, the total border cannot be less than the total number of rows and columns ($2n$) minus the maximum number of rows and columns that contain only one type of farms.

And so, the lower bound is given by (x and y are non-negative integers)

$$E(s) \geq \min_{x,y} \begin{cases} x + y, \text{if } x < n, y < n \\ x + y - \lfloor t/n \rfloor, \text{if } \max[x, y] = n \end{cases} \tag{A.1}$$

subject to $t \leq xy$, $t = \min[s, n^2 - s]$.

An equivalent formulation of the lower bound uses the maximum number of rows and columns that contribute 0 to $E^*(s)$ given that there are s generator and $n^2 - s$ recipient farms in the region:

$$E(s) \geq 2n - \max_{r,c} \begin{cases} x + y, \text{ if } x > 0, y > 0 \\ x + y + n - \lceil t/n \rceil, \text{ if } xy = 0 \end{cases} \tag{A.2}$$

subject to $t \geq nx + ny - ry$, $t = \max[s, n^2 - s]$.

The second line in (A.2) states that, given that all rows (columns) contain both types of farms ($y = 0$ or $x = 0$), that the number of columns (rows) that contain only one type of farm cannot be greater than $n - 1$ if $s/n \neq [s/n]$ or n, otherwise. Note that $nx + ny - xy$ (see the constraint in [A.2]) is the number of farms (generators) in the rows and columns that contribute 0 to $E^*(s)$. It is straightforward to check the equivalence between (A.1) and (A.2). Formulation (A.1) is used in Step 2.

The objective function in the minimization problem in (A.1) is the shortest possible length of the border separating generators and recipients. It is equal to the smallest number of rows and columns that contribute 1 (contain both types of farms and exactly one border shared by farms of different types) given that farms in all other rows and columns are all of the same type and therefore contribute 0. The second line in (A.1) states that, given that all rows (columns) contain both types of farms, the number of columns (rows) that contain both types of farms cannot be less than 1 if $s/n \neq [s/n]$ (or 0, otherwise). This is obvious and easy to show by contradiction. Note that the number of farms in any set that is intersected by y distinct rows and x columns cannot exceed xy (hence, the constraint in [A.1]).

Step 2. The non-linearity in (A.1) indicates that there are two distinct candidate arrangements that may achieve the lower bound. (i) An arrangement G of $\min[s, n^2 - s]$ farms that minimizes the sum of rows and columns where each row and column contributes exactly 1 to $E^*(s)$. (ii) An arrangement G that maximizes the number of rows (or columns) that contribute zero to $E^*(s)$ given that each column (or each row) of G contributes at most 1 to $E^*(s)$. Case (ii) adheres when it is optimal to set $\max[x, y] = n$ in (A.1). Otherwise, an arrangement in Case (i) assures that (A.1) holds with equality.

Next, we determine what constitutes optimal arrangements in Case (i). By Theorem A, a rectangle with the sides of length $\lceil t^{1/2} \rceil$, or $\lceil t^{1/2} \rceil$ and $\lfloor t^{1/2} \rfloor$, has the minimum perimeter (the smallest number of rows and columns). Remove $\lceil t^{1/2} \rceil^2 - t$ (or $\lceil t^{1/2} \rceil \lfloor t^{1/2} \rfloor - t$) cells from an outer row or column of the rectangle starting with a corner cell (a cell that has two border neighbors from outside the neighborhood).[34] The resultant configuration of s farms has no gaps in rows or columns and "straight" sides. Hence, by fitting this arrangement in a corner of the square region (note that the "length" of each side is less than n since $\lceil t^{1/2} \rceil < n$), we obtain an arrangement of t cells with the minimum number of rows and columns each contributing exactly 1 to $E^*(s)$, so that, by

Theorem A, $E^*(s) \le P^*(t)/2$. Note that because $P^*(s)$ increases with s, we have $E^*(s) \le P^*(s)/2$ if $s \le n^2/2$ and $E^*(s) \le P^*(n^2 - s)/2$ if otherwise.

In Case (ii), because the region is square and symmetric, without loss of generality, let $G = \{i : i \le s\}$, so that $E^*(s) \le \min[n, s+1]$ if $s/n = [s/n]$ and $E^*(s) \le \min[n, s] + 1$ if otherwise. Hence, the constructed arrangements in Cases (i) or Case (ii) achieve the lower bound depending on whether $\min_{\{x,y:xy \le t\}} x + y \le (\ge) n + 1_{s/n > [s/n]}$. Using the formula for minimum-perimeter arrangements in Theorem A to calculate the value of the minimization problem in (A.1), we obtain $E^*(s) = \min[P^*(\min[s, n^2 - s])$ $/2, n + 1_{s/n > [s/n]}]$.

Proof of Proposition 8

Part (i). Consider a candidate arrangement $g_i^* = 1_{(x_i, y_i) \le (x, y)}$ for all $i \in N$. By Proposition 1(ii), this arrangement constitutes strict PSNE when $1 < \hat{e} < 2$. On the other hand, by Theorem B we have $P^*(s)/2 = x + y$. Therefore, this arrangement is EA because, by assumption, $P^*(s)/2 \le n + 1_{s/n > [s/n]}$ and $s < n^2/2$ (see Case (i) of Proposition 7).

Part (ii). Consider a candidate arrangement $g_i^* = 1_{i \le s}$ for all $i \in N$. By Proposition 1(ii), this arrangement is strict PSNE when it forms a rectangle of generators, i.e., $s/n = [s/n]$ and $1 < \hat{e} < 2$. By Proposition 7, Case (ii), the proposed arrangement is also EA because, by assumption, $P^*(\min[s, n^2 - s]) \ge n$.

Proof of Corollary 1

Let $\{g_i^e\}_{i \in N}$ denote a self-enforcing EA, and let $e_i^e = \sum_{j \ne i} 1_{d_{ij} \le 1} g_j^e$ represent externality exposures associated with it. Then for any $\{g_i'\}_i$ where $g_i' \in \{0,1\}$ and $\sum_i g_i' = s$, we have $\sum_{i=1}^k z_{(i)}^e \le \sum_{i=1}^k z_{(i)}'$ for $1 \le k \le n^2$, where $z_i^e = e_i^e(1 - g_i^e)$ and $z_i' = e_i'(1 - g_i')$ and $z_{(1)}^e \ge \ldots \ge z_{(n^2)}^e$ and $z_{(1)}' \ge \ldots \ge z_{(n^2)}'$. This follows because, by Proposition 7, we have $\sum_{i=1}^{n^2} z_i^e \le \sum_{i=1}^{n^2} z_i'$, and by Proposition 8, $z_i^e \in \{0,1\}$ for all $i \in N$, so a self-enforcing EA has the least possible amount of dispersion compared with any sequence of n^2 non-negative integers that sum to $E^*(s) = \sum_{i=1}^{n^2} z_i^e$. Therefore, by Definition 3, we have $\{e_i^e(1-g_i^e)\}_{i \in N} \prec_w \{e_i'(1-g_i')\}_{i \in N}$. Because $E(s) = \sum_i f(e_i)$ $(1-g_i)$ and, by assumption, f is an increasing and convex function, the result follows.

Proof of Corollary 2

If $s = a(a+1)/2$, in a triangular arrangement, we have $E^t(s) = (a-1) f(2)$ $+ 2f(1) + (n^2 - s - a + 1)(0)$. If $s = n^2 - a(a+1)/2$, in a triangular arrangement, we have $E^t(s) = af(2) + (n^2 - s - a) f(0)$. The result is obtained by comparing $E^t(s)$ with $E(s) = \sum_{i \in N} f(\sum_{j \neq i} 1_{d_{ij} \leq 1} g_j^e)(1 - g_i^e)$, where $\{g_i^e\}_{i \in N}$ is the assignment of generators in EA under Assumptions 2–5, and observing that, in EA under Assumptions 2–5, the number of recipients with two generator neighbors is at most one if $s \leq n^2/2$ or two if $E^*(s) < n$ and $s > n^2/2$.

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