State Trading Enterprises in a Differentiated Environment: The Case of Global Malting Barley Markets

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Abstract

The lack of transparency in the pricing and operational activities of state trading enterprises (STEs) has caused members of the World Trade Organization to express concern that certain countries’ STEs might circumvent Uruguay Round commitments on export subsidies, domestic support, or market access. The purpose of this study is to examine the market structure of the differentiated world malting barley market in which two STEs (the Canadian Wheat Board and the Australian Barley Board) maintain jointly a very large share of the export market. In particular, this study focuses on the exclusive procuring and pricing policies used by both STEs to test if these intra-country mechanisms can generate leadership and shift rent from other exporting countries. A conceptual and empirical framework is also provided to test if STEs set their initial payments at optimal levels. The study suggests that two STEs and other exporting countries were in Cournot competition. While some distortionary impacts from the STE prepayment systems are possible, it does not appear to be a tool that either STE employs. Empirical results from the precommitment stage show that the two STEs did not set their initial payments low enough to maximize their profits.

Keywords: malting barley, market structure, prepayment system, product differentiation, rent shifting, state trading enterprises (STEs).
STATE TRADING ENTERPRISES IN A DIFFERENTIATED ENVIRONMENT: THE CASE OF GLOBAL MALTING BARLEY MARKETS

As early as 1947, the General Agreement on Tariffs and Trade (GATT) acknowledged state trading enterprises (STEs) as legitimate participants in international trade. The World Trade Organization (WTO) defines STEs as “government and nongovernmental enterprises, including marketing boards, which have been granted exclusive or special rights or privileges, including statutory or constitutional powers, in the exercise of which they influence through purchases or sales the level or direction of imports or exports” (see USDA 1997). Because STEs may be privately owned, the defining consideration is thus not governance but exclusive privileges. State trading is more prevalent in agriculture than in other economic sectors. In 1995 and 1996, 32 countries notified the WTO of 96 agricultural enterprises or organizations operating as STEs. While STEs operate over a broad range of agricultural commodities, they are most active in grains and dairy products.

Given the exclusive or special rights of STEs, the potential to exert considerable influence on the world markets is certainly possible. Controversial issues such as price-pooling strategies and single-desk marketing functions of several large agricultural STEs (the Canadian Wheat Board [CWB]; the Australian Wheat Board [AWB], and the Australian Barley Board [ABB]) have been a major concern in the United States over the past decade (GAO 1995). In particular, questions have arisen as to whether the programs instituted by STEs could be tailored to circumvent the growing international commitments toward freer trade. Certainly, these are valid concerns: nations for centuries have tried to protect and promote politically powerful industries. Indeed, the reported objectives for operating STEs include protecting domestic markets from world market influence, maintaining a stable and adequate supply of key commodities for national defense purposes, and expanding and protecting export market shares (GAO 1995). Moreover, STEs oftentimes purposely operate under a shroud of government bureaucracy, which makes discerning their internal activities difficult.
The U.S. General Accounting Office (GAO) conducted two investigative studies covering a wide range of STE behavioral, organizational, and strategic issues (GAO 1995, 1996). Although STEs were found generally in compliance with WTO rules, some activities were considered potentially inconsistent with WTO law, such as export licenses, tax advantages, transportation subsidies, and delayed producer payments. The GAO study (1996) viewed the delayed payment system as a potential source of concern but primarily focused on its added flexibility in controlling internal budgets. Typically, the STEs pay upstream producers a below-market initial payment and then provide a lump-sum reimbursement after proceeds are generated in the downstream international markets. As a result, the prepayment approach is capable of creating a credible marginal cost advantage for the STEs in the international market and generating essentially the same rent-shifting effect as an export subsidy (Brander and Spencer 1985). In the case of STEs, Hamilton and Stiegert (2000) established the formal equivalence between the delayed producer payment system and these more familiar forms of precommitment. Hamilton and Stiegert (2002) empirically evaluated the rent-shifting hypothesis for a single STE operating in the international durum market. They found statistical support for the hypothesis that the CWB acted as a Stackelberg leader and derived its leadership role from its prepayment system.

The purpose of the research in this study is to evaluate the prepayment system used by STEs operating in the international malting barley market. Because of the major differences that exist in the market for malting barley as compared to durum, it was necessary to extend and evolve this research in ways different from that of Hamilton and Stiegert (2002). Malting barley markets have historically operated with two STEs (ABB and CWB) both of which maintain a similar initial payment structure. Malting barley maintains a sensitive product quality structure, and much of what is planted for malting markets ends up as lower-priced feed barley. Hamilton and Stiegert (2002) assumed durum was homogeneous across export nations. Thus, a novel theoretical and empirical approach is developed to include a market structure for two STEs that properly accounts for product differentiation while testing for rent shifting.

Much literature exists that examines STE trade impacts. First McCalla (1966) and then Alaouze, Watson, and Sturgess (1978) evaluated the international wheat market in terms of
its oligopolistic characteristics and keenly identified the role of STEs. Carter and Schmitz (1979) suggested that importer STEs had as much or more to do with the competitive structure in world wheat markets as did exporter STEs. While Carter (1993) found no evidence of imperfect competition in the international barley markets, Schmitz and Gray (2000) found that the CWB captured annually $72 million in noncompetitive rents. Kraft, Furtan, and Tyrchniewicz (1996) (in a study funded by the CWB) found that the CWB generated $19-$34/ton in benefits to farmers due to its single-selling authority system. Carter, Loyns, and Berwald (1998) (in a study funded by the Alberta Dept. of Agriculture) found completely the opposite result: that is, bureaucratic inefficiencies within the CWB generate $20-$37/ton in losses to Canadian farmers. Finally, general support for some form of STE leadership emerged from a variety of studies employing time-series analysis of international grain prices (Goodwin and Smith 1995; Smith, Goodwin, and Holt 1995; Spriggs, Kaylen, and Bessler 1982; and Goodwin and Schroeder 1991).

Both the CWB and ABB have made strong claims of net public benefits emanating from its single-desk status. In its 1995-1996 annual report, for example, the CWB literally crowed about the findings of Kraft, Furtan, and Tyrchniewicz (1996, p. 6), claimed it had monopoly power (p. 5) and the ability to price discriminate (p. 6). The ABB claimed to have gained between $19 million and $41 million per year in demonstrable market premiums between 1985/86 and 1994/95 and that it could price discriminate among countries. The single desk of the ABB also had the potential to extract premiums and economic rents as a monopoly seller to domestic maltsters. Researchers at the Center for International Economics (CIE) (1997) examined the claims of the ABB and found no evidence of any price premiums in export markets for malting barley exported by the ABB. The CIE also pointed out that the claims of the ABB were overstated, because the ABB did not consider other effects, such as different qualities of barley and services and different sale and shipping times, in its analysis.

Given the political sensitivity of STE activity and the subsequent debate in the academic literature, this study is motivated by the need for much greater understanding of STE activity in product differentiated, imperfectly competitive export markets. For internationally traded grains, attempts are usually made to standardize and/or grade and contract commodities to aid in transactions. There is, however, a fairly convincing and
growing literature that suggests a more differentiated market exists for most of these commodities (e.g., Stiegert and Blanc 1997; and Marsh 2003). In most cases, raw food commodities are differentiated by physical growing constraints, geographic origin, credit policies, delivery dates, and ancillary services.

A key point from the seminal article by Brander and Spencer (1985) is that rent-shifting is only possible when markets are imperfect and there exists some form of precommitment. Brander and Spencer demonstrated that this precommitment can occur when governments set a credible export subsidy in advance of the quantity decision by firms. However, the concept of Stackelberg leadership is most sensible in a situation when only one firm can precommit. If both governments could offer export subsidies, it is possible that both countries may be worse off as the result of a subsidy war and the rent-shifting outcome collapses to a classic prisoners’ dilemma (see Krugman 1989). With two STEs capable of precommitment, any rent shifted from other exporters would have to be shared, thus diminishing its incentive for use. The optimal strategic trade policy depends critically on details of the market (Eaton and Grossman 1986). Product differentiation creates other opportunities for market strategies that are not available when products are close substitutes (i.e., price discrimination, brand identity, etc.). The point here is that STEs have at their disposal a potential form of a precommitment mechanism. Exactly how that mechanism functions in a product-differentiated market is an interesting and important question to address.

The organization of the remaining chapters is as follows. First, we provide some information on STEs in the international malting barley market. Then we develop both theoretical and empirical models and discuss data issues. Finally, we discuss the empirical results and summarize our findings.

**State Trading Enterprises in World Malting Barley Markets**

The CWB and the ABB are the two major STEs operating in the international export market for malting barley. The CWB is a single-desk state trading agency responsible for marketing all wheat and barley sold for human domestic consumption and for export, with jurisdiction over areas that typically produce 95 percent of the Canadian barley crop. One of the major responsibilities of the CWB is to market wheat and barley in order to
maximize returns to prairie producers. At the beginning of each crop year, the government establishes initial producer payments for grain sold to the CWB. The initial payment is usually set low enough to avoid a deficit in the pool. Farmers get the initial payment upon delivery of the harvested crop. This acts as a price floor because the government will fund the pool should average market prices fall below the initial payment. Once the CWB has marketed all the grain in a particular pool, the revenue is pooled, and freight and handling charges are deducted. If returns to the pool exceed the sum of initial payments, then a final payment is distributed to each individual producer based on the relative producer share of grain in that particular pool.

The practice of pooling makes the final price paid to producers a blended price based on net revenue of all sales in foreign and domestic markets. The STEs pay producers the same return regardless of the time of delivery during the marketing year.

The ABB had the sole right to export barley grown in South Australia and Victoria, which produce over half of all barley grown in Australia. The ABB accounted for about 90 percent of malting barley exports from Australia in 1992/93 (CIE 1997). The domestic market for malting barley is effectively controlled through the single-desk power of the ABB. One of the objectives of the ABB is to maximize the net returns to growers who deliver barley or other grain to a pool of the board. The ABB’s prepayment system and operations are similar to those of the CWB. In 1999, the ABB was privatized and changed to ABB Grain Ltd. Its single-desk export rights for barley from South Australia and Victoria were eliminated in July 2001.

For marketing purposes, barley is classified into feed and malting varieties. Malting barley is simply high-quality barley that has the appropriate characteristics to produce good malt. The malting barley is further divided into two-row and six-row varieties, for which brewer demands differ. Breeding programs, agronomic practices, soil characteristics, climatic conditions, and expected price differentials determine the varieties of barley grown in different regions. Farmers in Canada grow both two-row and six-row varieties of barley. Since 1991, plantings of six-row white varieties have increased because of contracts for the U.S. market. Australian barley producers almost exclusively plant two-row varieties.
Conceptual Framework

This section describes the derivation of a theoretical framework to examine firms’ behavior in a differentiated product market.

Theoretical Model

As discussed above, the delayed payment approach has the potential of creating a credible marginal cost advantage because STEs pay less to acquire exportable products and this has the same effect as an export subsidy. Moreover, in the case of STEs, the final payment in a delayed producer payment system, which is typically delivered in a lump-sum fashion, provides an explicit method of transfer back to the input supplier that rationalizes the system. Therefore, the delayed producer payment structure is equivalent in this regard to a policy of direct export subsidization.

The analysis in this study will be conducted on global malting barley markets. The malting barley market is considered to consist of imperfect substitutes. Agronomic practices, soil characteristics, and climatic conditions determine barley varieties grown in different regions, and downstream brewers have specific quality requirements in terms of acceptable varieties, protein, plumpness, and germination. Trade practices such as credit terms, delivery dates, and ancillary services add to the overall product differentiation. Finally, consumer preferences vary by region, personal taste, and suppliers and lead to a derived demand for various sets of malt characteristics.

We begin with a theoretical model that proposes endogenous control of an upstream supply in that STEs choose the initial prices of their principal raw commodity and then compete in an international market of imperfect substitutes. We presume throughout that STEs and producers are vertically aligned and that the government grants the STE exclusive purchase rights of the raw commodity. The vertical structure analyzed here consists of two stages solved by backward induction. In the first stage (precommitment stage), both STEs simultaneously choose their initial payments for the material input. In this stage, we employ a subset of the output-stage results to characterize the value of the trade policy parameter associated with the optimal degree of rent shifting, which is consistent with the assumption that the government sets a subsidy level with the understanding of how it influences the output equilibrium. The second stage is an output stage, in which the STEs and other exporting firms maximize profits by choosing quantities and
maintain the ability to either store nonoptimal supplies or downgrade the quality of nonoptimal supplies for sale to a residual feed barley market. We estimate the output stage by considering STE trade policy as a given shift parameter in the domestic marginal cost function.

Let \( x_1, x_2, \) and \( x_3 \) represent total sales of malting barley to the world market by the CWB (1), ABB (2), and the other malting barley exporting countries (3), respectively, and denote the downstream inverse demand functions of malting barley marketed by the CWB, the ABB, and other exporting countries as \( P_1, P_2, \) and \( P_3, \) respectively. The country-specific inverse demand functions of malting barley are as follows:

\[
P_i = P_i(x_1, x_2, x_3; \Phi_i)
\]

where \( \Phi_i \) are exogenous variables. If barley varieties were perfect substitutes or homogeneous, all the prices would be equal, net of transport costs. Obviously, if barley varieties were imperfect substitutes, each demand change would generate a different impact on each price.

In the output stage, the STEs and the firms in other exporting countries choose their outputs to maximize profits by

\[
\begin{align*}
\max_{x_1} \pi_1(x_1) &= P_1x_1 - w_1x_1 \\
\max_{x_2} \pi_2(x_2) &= P_2x_2 - w_2x_2 \\
\max_q \pi_3(q) &= P_3q - c_3q
\end{align*}
\]

where \( w_1 \) and \( w_2 \) are initial payments set in the precommitment stage by the CWB and the ABB, respectively; and \( c_3 \) is the price received by farmers of other exporting countries. Here, we assume that there are \( n \) symmetric firms in the other exporting countries and thus \( q=(1/n)x_3. \) In this study, we choose outputs as strategic variables because the STEs and other exporting firms have the ability to either store nonoptimal supplies or down-
grade the quality of nonoptimal supplies for sale to a residual market or feed barley market and could subsequently maximize profits by choosing quantities.

Maximization of equations (4), (5), and (6) with respect to \( x_1, x_2, \) and \( q \), respectively, yield the first-order conditions:

\[
\begin{align*}
P_1 + x_1(P_{11} + \gamma_{12}P_{12} + \gamma_{13}P_{13}) - w_1 &= 0 \\
P_2 + x_2(\gamma_{21}P_{21} + \gamma_{22}P_{22} + \gamma_{23}P_{23}) - w_2 &= 0 \\
P_3 + x_3(\gamma_{31}P_{31} + \gamma_{32}P_{32} + \gamma_{33}P_{33}) - c_3 &= 0
\end{align*}
\]

where \( P_{ij} = \partial P_i / \partial x_j \leq 0 \) and \( \gamma_{ij} = \partial x_j / \partial x_i \). The \( \gamma_{ij} \) (i, j = 1, 2, 3, and i ≠ j) indicates firm j’s reaction or best response to the change of firm i’s quantity. For example, \( \gamma_{12} \) indicates the ABB’s reaction/best response to the output change of the CWB. The reactions of firms to other firms’ output changes provide an index of the degree of market power, consistent with behavior from price-taking to perfect collusion, by leading directly to the relevant first-order conditions for the various models.

When products are imperfect substitutes, the conditions for various market models are different from those under a homogeneous product scenario. In particular, each of the best-response parameters (the \( \gamma_{ij} \)’s) is weighted by the unique cross-price impacts \( (P_{ij}) \) that can limit or exasperate the degree of market power. For example, for the CWB, if the term in the parentheses in equation (7) is equal to zero (i.e., if \( P_{11} + \gamma_{12}P_{12} + \gamma_{13}P_{13} = 0 \)), then the CWB is a price taker. However, when market power is present and as products become more differentiated, cross-price effects dissipate and the own-price effect takes on more relative weight. For the homogeneous case with market power, each of the best-response parameters are equally weighted by the aggregate price effect \( (\partial P_i / \partial x_i) \), which can be easily reinterpreted as a market demand elasticity.

Let \( x_i(w_1, w_2; \Psi_i) \) represent the equilibrium levels of sales from country i in the output stage, given initial payments of \( w_1 \) and \( w_2 \). In the precommitment stage, the STEs select transfer prices, \( w_1 \) and \( w_2 \), so as to

\[
Max_{w_1} \pi_{1p} = P_1(x_1(w_1, w_2; \Psi_1), x_2(w_1, w_2; \Psi_2), x_3(w_1, w_2; \Psi_3))x_1(w_1, w_2; \Psi_1) - c_{x_1}(w_1, w_2; \Psi_1) - F_1
\]
Max \( \pi_{2p} = P_1(x_1(w_1, w_2; \psi_1), x_2(w_1, w_2; \psi_2), x_3(w_1, w_2; \psi_3))x_2(w_1, w_2; \psi_2) \) 
\[-c_a x_2(w_1, w_2; \psi_2) - F_2 \]  
(11)

where \( \pi_{1p} \) and \( \pi_{2p} \) are the profit of producers under the CWB and the ABB, respectively.

The variables \( c_c \) and \( c_a \) are the marginal production costs for producers in Canada and Australia, respectively. For simplification of the problem, production costs are assumed to be constant. The \( \psi_i \) are exogenous variables affecting supplies. Variables \( F_1 \) and \( F_2 \) are fixed costs that could include, respectively, marketing and administration costs incurred by the CWB and the ABB.

**Optimal Initial Payments**

Let the \( w_i^* \)'s denote the optimal initial payments. The first-order conditions of equations (10) and (11) are

\[
P_1 \frac{\partial x_1}{\partial w_1} + x_1 \left( \frac{\partial P_1}{\partial x_1} \frac{\partial x_1}{\partial w_1} + \frac{\partial P_1}{\partial x_2} \frac{\partial x_2}{\partial w_1} + \frac{\partial P_1}{\partial x_3} \frac{\partial x_3}{\partial w_1} \right) - c_c \frac{\partial x_1}{\partial w_1} = 0 
\]  
(12)

\[
P_2 \frac{\partial x_2}{\partial w_2} + x_2 \left( \frac{\partial P_2}{\partial x_1} \frac{\partial x_1}{\partial w_2} + \frac{\partial P_2}{\partial x_2} \frac{\partial x_2}{\partial w_2} + \frac{\partial P_2}{\partial x_3} \frac{\partial x_3}{\partial w_2} \right) - c_a \frac{\partial x_2}{\partial w_2} = 0. 
\]  
(13)

Using backward induction from (7) and (8) and substituting \( P_i+\gamma_i P_{ii} = w_i(\gamma_{ij} P_{ij} + \gamma_{ik} P_{ik}) \) \((i,j=1,2, i \neq j; k=3)\) into (12) and (13), the optimal upstream prices set by the STEs are

\[
w_1^* - c_c = -x_1 \left[ P_{12} \left( \frac{\partial w_i}{\partial x_2} - \gamma_{12} \right) + P_{13} \left( \frac{\partial w_i}{\partial x_3} - \gamma_{13} \right) \right] 
\]  
(14)

\[
w_2^* - c_a = -x_2 \left[ P_{21} \left( \frac{\partial w_i}{\partial x_1} - \gamma_{21} \right) + P_{23} \left( \frac{\partial w_i}{\partial x_3} - \gamma_{23} \right) \right] 
\]  
(15)

where \( \partial x_i / \partial w_j \) \((i=1, 2, \text{ or } 3; \text{ and } j=1 \text{ or } 2)\) may be derived by taking total differential of equations (7), (8), and (9). The ratios of marginal effects in (14) and (15) can be expressed as
In the above equations, the $S_i$'s are the submatrices in the matrix $S$ defined as

$$
S = \begin{pmatrix}
S_1 & S_2 & S_3 \\
S_4 & S_5 & S_6 \\
S_7 & S_8 & S_9
\end{pmatrix}
$$

with the notation that $P_{ijk} = \frac{\partial^2 P_i}{\partial x_j \partial x_k}$ (i, j, k=1, 2, 3, respectively).

In equations (7), (8), and (9), the value of the $\gamma_{ij}$'s combined with the cross-price effects gives an illustration of the market structure and the degree of competition. Specifically, the departure of the $\gamma_{ij}$'s from zero is a logically consistent test of whether the Cournot-Nash model provides an accurate description of the industry equilibrium.
Rent Shifting

“Rent shifting” is a theoretical concept implying that governments can employ trade policy as a pre-commitment device to transfer profits from foreign to domestic markets. To test the hypothesis that the CWB and the ABB strategically utilize their pre-payment systems and product differentiation to shift rents from other foreign firms, the following formulas can be calculated:

\[
\frac{\partial \pi_i}{\partial w_j} = \sum_{k=1}^{3} \frac{\partial \pi_i}{\partial x_k} \frac{\partial x_k}{\partial w_j}. \tag{16}
\]

The expressions \(\frac{\partial x_k}{\partial w_j}\) has been defined earlier and \(\frac{\partial \pi_i}{\partial x_k}\) can be derived from equations (4), (5), and (6) by taking derivatives. If \(\frac{\partial \pi_i}{\partial w_1} < 0\) and \(\frac{\partial \pi_i}{\partial w_1} > 0\) \((i=2, \text{ or } 3)\), then by lowering its initial payments, the CWB could increase its profit and decrease firm \(i\)'s profits. In this case, the CWB strategically utilizes its pre-payment system to shift rents from country \(i\). Similar analysis could be applied to the ABB. Unlike in the homogeneous product market, rent shifting in the product-differentiated market depends not only on the market structure but also on cross-price effects, which indicate the degree of product differentiation.

Empirical Methods

In this section, an empirical methodology will be discussed to conduct empirically an examination and evaluation of what has been developed in the previous section.

Model Specification

To evaluate the degree of market power, it is necessary to identify \(\gamma_{ij}\)'s. Equations (7), (8), and (9) are expanded and rearranged as

\[
P_{1t} - w_{1t} = \lambda_{12}(P_{12}x_{1t}) + \lambda_{13}(P_{13}x_{1t}) - P_{11}x_{1t} \tag{17}
\]

\[
P_{2t} - w_{2t} = \lambda_{21}(P_{21}x_{2t}) + \lambda_{23}(P_{23}x_{2t}) - P_{22}x_{2t} \tag{18}
\]

\[
P_{3t} - c_{3t} = \lambda_{31}(P_{31}x_{3t}) + \lambda_{32}(P_{32}x_{3t}) - P_{33}x_{3t} \tag{19}
\]
The market power parameters in the above equations (\(\lambda_{ij}\)’s) are the negative counterparts of the response parameters in (7), (8), and (9). That is, \(\gamma_{ij} = -\lambda_{ij}\). To estimate the parameters (\(\lambda_{ij}\)’s) in this system, we must have estimates of the derivatives of prices with respect to quantities, which are \(P_{12}, P_{13}, P_{21}, P_{23}, P_{31},\) and \(P_{32}\). In this manner, we empirically allow for some degree of product heterogeneity (i.e., imperfect substitutes).

To identify the quantity derivatives of prices, an input distance function is specified for malt production and inverse demand equations are derived. From the inverse demand equations, the derivatives of prices with respect to quantities can be identified directly. The aggregate input distance function of malting barley by importing countries is defined as

\[
D = D(Q, Y) = \max_{d} \{d | F(Q/d) > Y\}
\]

where \(Q\) is an \((n \times 1)\) vector of input quantities; \(Y\) is a \((1 \times 1)\) scalar representing malt output; and \(F(Q/d)\) is the production technology. The behavioral assumption is to rescale all the input levels that are consistent with a target output level. Specifically, \(d\) is the largest scalar value that could be used to divide \(Q\) and still produce \(Y\). The distance function is assumed to be weakly separable in inputs by partitioning inputs into two subgroups of raw grain input and other inputs.

To complete the model specification, the distance function is assumed to take the form of a normalized quadratic distance function (Marsh and Featherstone 2003; Holt and Bishop 2002). The normalized quadratic distance function is a flexible functional form and is given by

\[
d^*(Q', Y) = b_y + \sum b_i x_i^* + \frac{1}{2} (\sum \sum b_{ij} x_i^* x_j^*) + b_Y Y + \sum b_{iy} x_i^* Y + \frac{1}{2} b_{yy} Y^2
\]

where \(d^*\) and \(x_i^*\) are normalized distance and input quantities, \(d^* = D/x_3\), and \(x_i^* = x_i/x_3\), respectively. The normalized quadratic distance function is linear homogeneous, concave, nondecreasing in inputs, and nonincreasing in output. The inverse demand functions for CWB and ABB are obtained using Gorman’s Lemma:

\[
P_i^* = b_1 + b_{1i} x_1^* + b_{2i} x_2^* + b_{3i} Y
\]
\[ P^*_2 = b_2 + b_{22}x^*_2 + b_{12}x^*_1 + b_{21}Y \]  

(23)

where \( P^*_1 \) is normalized input prices by cost \( P^*_i = P_i / \sum_{j=1}^{3} P_j x_j \). Consequently, the cost of producing the target level of output is unity. The third inverse demand function for other exporting countries is dropped to avoid singularity of the error covariance matrix. The derived inverse demand function is homogeneous of degree zero in inputs. Homogeneity is realized by the normalization process and symmetry is imposed by setting \( b_{ij} = b_{ji} \). Coefficients for the inverse demand response for other exporting countries are recovered using standard demand restrictions.

The derivatives of prices with respect to quantities \( (\partial P_i / \partial x_j) \) could be expressed by normalized input quantities and parameters in (22) and (23). Then, equations (17), (18), and (19) could be expressed as follows:

\[ P^*_1 - w^*_1 = (\lambda_{12}b_{12} - \lambda_{13}b_{11}x^*_1 - \lambda_{13}b_{12}x^*_2 - b_{11})x^*_1 \]  

(24)

\[ P^*_2 - w^*_2 = (\lambda_{21}b_{12} - \lambda_{23}b_{12}x^*_1 - \lambda_{23}b_{22}x^*_2 - b_{22})x^*_2 \]  

(25)

\[ P^*_3 - c^*_3 = -\lambda_{31}(b_{11}x^*_1 + b_{12}x^*_2) - \lambda_{32}(b_{12}x^*_1 + b_{22}x^*_2) - (b_{11}x^*_1 + b_{12}x^*_2)x^*_1 - (b_{12}x^*_1 + b_{22}x^*_2)x^*_2 \]  

(26)

where \( w^*_1 \) are the normalized initial payments by cost (see the Appendix for derivation).

In all, the empirical model consists of a system of five equations: two inverse demand equations [(22) and (23)] and three equations for estimating market power parameters [(24), (25), and (26)].

**Data**

International statistics, such as the Food and Agriculture Organization Yearbook and World Grain Statistics, report barley trade aggregately instead of separating it into feed and malting barley. Consistent data for malting barley export quantities and prices were available only for Canada and Australia. Multiple data sources include but are not limited to the CWB annual report (various), the ABB annual report (various), ABARE’s *Australian Commodity Statistics* (various), CIE 1997, Schmitz and Koo 1996, USDA 1997, and
Agriculture and Agri-Food Canada’s *Bi-weekly Bulletin* (various). Considering data availability and status change of the ABB, the data range is set between 1975/76 and 1997/98. A thorough description of the procedures for obtaining and developing data for the study may be obtained from the authors.

**Estimation Results and Discussion**

The empirical analyses were carried out using both a Bayesian inference framework with restrictions (e.g., Geweke 1986) and a generalized likelihood framework (Gasmi, Laffont, and Vuong 1992). The Bayesian framework allowed parametric restrictions on the $\lambda_{ij}$’s and other parameters, as well as imposition of general demand conditions. Moreover, it was a convenient framework from which to obtain bootstrapped confidence intervals for levels of initial payments defined in (14) and (15) and rent shifting defined in (16). Alternatively, the generalized likelihood framework enabled us to set up several different market structures and draw inferences from nonnested hypothesis tests. In addition to testing alternative market structures, the generalized likelihood results provided a comparison to the Bayesian results.

**Bayesian Approach**

In this study, the Bayesian approach is applied because of its advantage in drawing finite sample inferences concerning nonlinear functions of parameters and imposing economic restrictions. Let $L(Y, X|\beta, \Sigma)$ be the likelihood function summarizing all the sample information. Then, applying Bayes’ theorem, the posterior distribution becomes

$$f(\beta, \Sigma|Y, X) \propto L(Y, X|\beta, \Sigma)p(\beta, \Sigma)$$

(27)

where $\propto$ means “is proportional to.” In (27), $\beta$ is a vector of model parameters, $\Sigma$ denotes the covariance matrix, and $Y, X$ represent data observations. The posterior joint density function is $f(\beta, \Sigma|Y, X)$ for $\beta$ and $\Sigma$, given observed random variables $Y$ and $X$, or revised beliefs about the distribution of $\beta$ and $\Sigma$ after observing the data. The prior density function $p(\beta, \Sigma)$ summarizes the nonsample information about $\beta$ and $\Sigma$.

Following Judge et al. (1985, pp. 478-80), and assuming a multivariate normal, the likelihood function compatible with the seemingly unrelated regression model is given by
\[ L(Y, X | \beta, \Sigma) \propto |\Sigma|^{-N/2} \exp[-0.5tr(R^*\Sigma^{-1})] \] (28)

where \( tr \) denotes the trace operator, which is the sum of the diagonals of a square matrix; \( R \) is a symmetric estimated covariance matrix; and \( N \) is the number of observations. A noninformative or diffuse prior is used for \( \beta \) and \( \Sigma \):

\[ p(\beta, \Sigma) = p(\beta) p(\Sigma) I(\beta \in h_s) \quad s=1,2 \] (29)

where \( p(\beta) \propto \) constant and \( p(\Sigma) \propto |\Sigma|^{-(I+1)/2} \) is in the form of a Wishart distribution. In (29), \( h_s \) is the set of permissible parameter values when constraint information is \((s=2)\) and is not \((s=1)\) available. The \( I(\_\_) \) is an indicator function that takes the value 1 if the argument is true (Griffiths, O’Donnell, and Cruz 2000). The posterior density under the noninformative prior is

\[ f(\beta, \Sigma | Y, X) \propto [ |\Sigma|]^{-(N+I+1)/2} \exp[-0.5tr(R^*\Sigma^{-1})]I(\beta \in h_s) \quad s=1,2. \] (30)

Techniques of Markov Chain Monte Carlo (MCMC) simulation estimation (i.e., the Metropolis-Hastings algorithm) are applied to carry out the Bayesian estimation. The Metropolis-Hastings algorithm can draw samples from a marginal probability density indirectly without having to derive the density itself. The algorithm allows the imposition of curvature, monotonicity, and bounds on market power parameters during the sample drawing process. It imposes curvature restrictions locally with computational advantages over importance sampling (Chib and Greenberg 1996).

The empirical model linked to the theory consists of a system of five equations: two inverse demand equations (22) and (23); and three equations for estimating market power parameters (24), (25), and (26). The Metropolis-Hastings algorithm on this system of equations is carried out in several steps:

Step 1: Specify an arbitrary starting value \( \beta^0 \) that satisfies the constraints of curvature, monotonicity, and bounds on market power parameters (and set iteration \( i=0 \))

Step 2: Given the current value \( \beta^i \), use a symmetric transition density \( q(\beta^i, \beta^i) \) to generate a value as the next candidate in the sequence.
Step 3: Use the candidate value $\mathbf{\beta}^c$ to evaluate the curvature, monotonicity, and bounds on market power parameters constraints. If any constraints are violated, then reject $\mathbf{\beta}^c$.

Step 4: Let $u(\mathbf{\beta}, \mathbf{\beta}^c) = \min(g(\mathbf{\beta}^c)/g(\mathbf{\beta}), 1)$, where $g(\mathbf{\beta})$ is the kernel of the marginal density $f(\mathbf{\beta} | Y, X)$ obtained by integrating $\Sigma$ out of the posterior function (Judge et al.):

$$f(\mathbf{\beta} | Y, X) \propto | R |^{-N/2} I(\mathbf{\beta} \in h_2) = g(\mathbf{\beta}).$$

Step 5: Generate an independent uniform random variable $U$ from the interval $[0,1]$.

Step 6: Let $\mathbf{\beta}^{i+1} = \mathbf{\beta}^c$ if $U < u(\mathbf{\beta}, \mathbf{\beta}^c)$ or $\mathbf{\beta}^{i+1} = \mathbf{\beta}^i$ if $U \geq u(\mathbf{\beta}, \mathbf{\beta}^c)$. Set $i = i + 1$ and return to step 2.

Additional assumptions and parameters are needed to specify completely the MCMC process. Chib and Greenberg (1996) provide a complete theoretical overview, while Griffiths, O’Donnell, and Cruz (2000) provide details specific to the estimation of the linear seemingly unrelated regression model with MCMC. The burn-in period for the empirical applications was set at 300,000 iterations, which was sufficient to ensure the elimination of the starting value influence and the convergence of the MCMC chain to a stationary distribution. The post burn-in sample size $m$ was set to 300,000 iterations. The iteration process generates a chain with the property that for large $i$ $\mathbf{\beta}^{i+1}$ is an effective sample from the posterior joint density. Consequently, the sequence $\mathbf{\beta}^{i+1}, \ldots, \mathbf{\beta}^{i+m}$ can be regarded as a sample from $f(\mathbf{\beta} | Y, X)$ that satisfies the constraints of curvature, monotonicity, and bounds on market power parameters. In step 3, the concavity constraint is evaluated by using the maximum eigenvalue of the estimated Hessian matrix. Starting values were chosen that satisfied economic constraints. The choice of transition density $q(\mathbf{\beta}, \mathbf{\beta}^c)$ is arbitrary, but it is commonplace to use a multivariate normal distribution (with mean $\mathbf{\beta}$ and covariance matrix from the unrestricted nonlinear seemingly unrelated regression estimator). In order to manipulate the rate at which the candidate $\mathbf{\beta}^c$ is accepted as the next value in the sequence, a tuning constant was used to multiply the covariance matrix. Based on trial and error, the tuning constant is set at 0.01 to make the acceptance rate of approximately 0.50.
Parameter Estimates

Confidence intervals for parameter estimates are constructed after the burn-in period. The 90 percent confidence interval for each parameter was constructed by the percentile method, which requires ranking the estimated parameters and then selecting the 15,000th (5 percent of total iterations) outcome as the lower critical value and the 285,000th (95 percent of total iterations) outcome as the upper critical value. If the confidence interval for a parameter estimate contains zero, then the parameter value is not considered significant from zero at the 10 percent level. The parameter estimates, along with the upper and lower bounds of the 90 percent confidence intervals for the Bayesian system, are reported in Table 1. Both $b_{11}$ and $b_{22}$ are significant at the 10 percent level and negative because of the curvature constraint set during the estimation process. Both output parameters ($b_{1y}$ and $b_{2y}$)

<table>
<thead>
<tr>
<th>Table 1. Estimations by Bayesian approach</th>
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<tbody>
<tr>
<td><strong>Estimations</strong></td>
</tr>
<tr>
<td><strong>Upper Critical Value</strong></td>
</tr>
<tr>
<td>$b_1$</td>
</tr>
<tr>
<td>$b_2$</td>
</tr>
<tr>
<td>$b_{11}$</td>
</tr>
<tr>
<td>$b_{12}$</td>
</tr>
<tr>
<td>$b_{22}$</td>
</tr>
<tr>
<td>$b_{1y}$</td>
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<td>$\lambda_{12}$</td>
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<td>$\lambda_{21}$</td>
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<tr>
<td>$\lambda_{23}$</td>
</tr>
<tr>
<td>$\lambda_{31}$</td>
</tr>
<tr>
<td>$\lambda_{32}$</td>
</tr>
</tbody>
</table>

*Note: Burn-in period=300,000. Sample size=300,000.*
are negative and only $b_{1y}$ is significant. This shows that output of malt has a significant effect on the price of malting barley from the CWB and has a insignificant effect on the price of malting barley from the ABB. However, the significant effect is very small. Cross-effect parameter $b_{12}$ is insignificant, which suggests that the substitution effects among malting barley from different origins are not significant. Besides product differentiation, one possibility for disguising substitution effects is the effect of geographic distance.

Market power parameters ($\lambda_{ij}$) are constrained to lie between -1 and 1. Significance or insignificance of response parameters describes the conduct of STEs and firms in the world malting barley market. If the market power parameter $\lambda_{ij}$ is not significant, then country $i$ does not consider country $j$’s output change when $i$ makes its decision. If both $\lambda_{ij}$ and $\lambda_{ji}$ are not significant, then the two countries are in Cournot competition. The results show that all market power parameters are not significant, which suggests that the CWB, the ABB, and the other exporting countries are in Cournot competition with each other.

**Initial Payment**

Using equations (14) and (15), we tested the two STEs to see if they had set their initial payments at optimal levels. With linear inverse demand functions, all second derivatives of prices with respect to quantities are zero, which greatly simplifies the matrix $S$. By testing the null hypothesis that the optimal markups (right-hand sides of equations [14] and [15]) were equal to the true values of the markups, $w_i-c_i$, which is the same as testing $H_0: w_i^*-w_i=0$, it could be determined statistically whether the CWB and the ABB set their initial payments at optimal levels. Table 2 contains the bootstrapped estimates of the differences between optimal initial payments and actual payments, along with the upper and lower bounds of the 90 percent confidence interval.

**Table 2. Estimates for hypothesis test $H_0: w_i^*-w_i=0$ by Bayesian method**

<table>
<thead>
<tr>
<th></th>
<th>Mean ($/1,000$ tonnes)</th>
<th>90% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Upper Critical Value</td>
</tr>
<tr>
<td>$w_1^*-w_1$</td>
<td>698.333727</td>
<td>279.425133</td>
</tr>
<tr>
<td>$w_2^*-w_2$</td>
<td>854.334639</td>
<td>697.353257</td>
</tr>
</tbody>
</table>
for the Bayesian system. Both the CWB and the ABB set their initial payments considerably higher than optimal levels. This implies that while some rent shifting was possible, there is not much support for the conclusion that the prepayment system is operating as an effective strategic tool.

**Rent Shifting**

A null hypothesis test that STEs could shift rents from other exporting countries, which was based on equation (16), was also conducted by the bootstrap method. Table 3 shows the test results of rent shifting. All values are insignificant. Therefore the hypothesis that STEs could not utilize their initial payments to shift rent cannot be rejected. Combined with the earlier bootstrapped results, a fairly strong conclusion emerges. It does not appear that the prepayment system can be used to shift rent, and, even if it could, it is currently being strictly underutilized.

**The Wilcoxon Signed Rank Test**

To see if the findings from the bootstrap procedures hold up to additional testing, a Wilcoxon signed rank test was conducted. With 23 pairs of observations of the ranked data, the Wilcoxon signed rank statistic was -4.19726, which has an absolute value greater than the critical value at the 5 percent level for standard normal distribution for both the CWB and the ABB. Therefore, the null hypothesis that there were no differences between optimal and observed initial payments (H₀: wᵢ* - wᵢ = 0) should be rejected. Consequently, the left-tail alternative, which observed that initial payments were higher than optimal levels (H₁: wᵢ* - wᵢ < 0), could be accepted. Therefore, the Wilcoxon signed rank test suggests that both STEs set their initial payments at higher-than-optimal levels.

| Table 3. Hypothesis test that STEs could shift rents from other exporting countries by Bayesian method |
|----------------------------------------------------------|----------------------------------------------------------|----------------------------------------------------------|
| ∂π₂/∂w₁                                                  | Mean                      | 90% Confidence Interval                  |
| -36.456899                                               | 143.316877                | -302.079605                               |
| ∂π₃/∂w₁                                                  | -182.203001               | 293.554632                                | -839.634976                               |
| ∂π₁/∂w₂                                                  | -414.891691               | 608.167068                                | -1044.391131                              |
| ∂π₃/∂w₂                                                  | -752.743429               | 2094.303887                               | -2564.479655                              |
Generalized Likelihood Tests

Following Gasmi, Laffont, and Vuong (1992), we set up several plausible market structures and evaluated them using a normalized likelihood ratio (LR) test for nonnested models. The test is based on the generalized LR principle and is designed to test the null hypothesis that two competing models adjust the data equally well versus the alternative hypothesis that one model fits better. Four market structures were imposed:

Model 1: CWB Lead / ABB follow / other exporting countries follow.
Model 2: ABB Lead / CWB follow / other exporting countries follow.
Model 3: ABB and CWB jointly lead / other exporting countries follow.
Model 4: ABB, CWB, and other exporting countries are in Cournot equilibrium.

For each pair of models ($M_f, M_g$) ($f, g = 1, 2, 3, \text{ or } 4; \text{ and } f \neq g$), we calculated the likelihood ratio statistic normalized by

$$N^{-\frac{1}{2}} \hat{r}_n = \frac{1}{2} \left[ \sum_{t=1}^{N} (\mu_f' \Sigma_f^{-1} \mu_f - \mu_g' \Sigma_g^{-1} \mu_g)^2 \right]^{-\frac{1}{2}}$$

where $\mu_s$ and $\Sigma_s$ are the estimated residuals and covariance matrix for model $M_s, s = f, g$.

To take into account the difference in the number of estimated parameters in the models, we adjusted the LR statistic using penalties, proposed by Schwarz (1978), which give the highest penalty for the number of estimated parameters (Gasmi, Laffont, and Vuong 1992). The correction factor is $-0.5k \log N$, where $k$ is the difference in the number of parameters in $M_f$ and $M_g$. The resulting normalized statistic is asymptotically normally distributed under the null hypothesis of equal fit. Given a critical value $c$ from the standard normal distribution at some significance level, if the normalized LR statistic is smaller than $c$ in absolute value, then the null hypothesis cannot be rejected and we conclude that the data do not enable us to discriminate between the two models; if the normalized LR statistic is smaller than $-c$, then we conclude that $M_g$ is significantly better; and if it is greater than $+c$, then we conclude that $M_f$ is significantly better. The estimation was conducted by using the iterative nonlinear seemingly unrelated regression (SUR) method. The statistical tests based on the normalized LR statistics are given in Table 4 for each pair-wise comparison. Table 4 shows that model 1 provides a statistically worse fit than both model 2 and model 4. It also shows that model 4 is significantly
TABLE 4. Adjusted LR statistics for model selection

<table>
<thead>
<tr>
<th>$M_f$</th>
<th>$M_2$</th>
<th>$M_3$</th>
<th>$M_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>1.127174</td>
<td>-1.451088</td>
<td>-7.018934 *</td>
</tr>
<tr>
<td>$M_2$</td>
<td>-2.053776 *</td>
<td>-3.570608 *</td>
<td>-3.570608 *</td>
</tr>
<tr>
<td>$M_3$</td>
<td>-2.053776</td>
<td>-3.570608 *</td>
<td>-2.053776 *</td>
</tr>
</tbody>
</table>

* Significant at the 5% level in a one-sided test and at the 10% level in a two-sided test.

better than all other models, while model 1 and model 3, or model 2 and model 3 cannot discriminate against each other. Therefore, the conclusion from the Bayesian method that the CWB, the ABB, and the other exporting countries were in Cournot competition is supported by the generalized LR tests.³

**Summary and Conclusions**

The lack of transparency in the pricing and operational activities of STEs has caused WTO members to express concern that certain countries’ STEs could circumvent Uruguay Round commitments on export subsidies, domestic support, or market access. Most previous studies have either examined single STE markets or evaluated an STE in isolation from other STEs. Furthermore, in most empirical work the important distinctions between homogeneous and differentiated goods are typically ignored. These are potentially very important issues because strategic trade policy is likely to be quite sensitive to specific market details. In this study, we examined a dual STE market structure of the differentiated world malting barley market in which two STEs (the CWB and ABB) maintained jointly a very large share of the export market. A conceptual two-stage model and an empirical framework were developed to evaluate the market structure and to examine possibilities of rent shifting. In addition, the model provides a framework to test if STEs set their initial payments at optimal levels within the context of their differentiated product. The theoretical model in the study proposed endogenous control of an upstream supply in that STEs chose the initial prices of their raw commodities given that they competed in a downstream market of imperfect substitutes. The decision sequence consisted of a precommitment stage in which STEs chose initial prices followed by an output stage that determined prices, quantities, and the trade flows for the two STEs and a group of other exporters.
Based on the conceptual model framework, data, and subsequent empirical results, important conclusions were reached. First, the STEs did not have market leadership in the differentiated global malting barley market. Both STEs and other exporting countries were in Cournot competition. Hamilton and Stiegert (2002) also found that the CWB was in Cournot competition with the other export sector in a homogeneous market. But unlike this study, they found support for rent-shifting and leadership outcomes for the STE. With product differentiation, firms rationally ignore rival behavior more than when products are the same, and we would naturally tend to observe the Cournot–Nash equilibrium in such cases.

Second, both STEs were not setting their initial payments at optimal levels and did not shift rent from other exporting countries by utilizing a prepayment system as a precommitment. We found that both STEs set their initial payments higher than profit-maximization levels, which may in part be attributable to a political constraint; that is, low initial payments are probably difficult to justify to producers. However, if the market is highly differentiated, then it may not make much sense to push this as a strategic policy tool compared to other practices such as price discrimination and developing long-term customer relations. In addition, the effect of rent shifting by lowering initial payments was not significant. In a product-differentiated environment, the realization of rent shifting depends not only on the presupposition of Cournot competition but also on the degree of product differentiation. In the world malting barley market, because of such things as product differentiation, geographic effects, and output shocks from weather conditions, the rent-shifting effects by using an initial payment as a precommitment mechanism were dampened. Therefore, there is no urgent need to impose disciplines on the prepayment system of STEs.
End Notes

1. Other forms of rent-shifting are certainly possible, e.g., Fershtman and Judd (1987) demonstrate market rent shifting is possible through internal incentive systems.

2. In the preliminary analysis, a host of different MCMC chains with alternative starting values were used to check convergence of the parameter estimates.

3. To further test the robustness of the Bayesian results, an iterative nonlinear SUR estimation procedure was used on the five-equation model (equations [22]-[26]). Nonlinear SUR and the Bayesian results were similar in the sense that all market power parameters were insignificant, optimal initial payments were lower than observed values, and rent-shifting effects were insignificant. However, they differ in magnitudes and significance of some parameter estimates (full details are available from the authors).
Appendix

Derivation of $\frac{\partial P_i}{\partial x_j}$ from the Inverse Demand Function

From the inverse demand function, the price flexibilities are

$$f_{ij} = \left. \frac{\partial P_i}{\partial x_j} \right|_{P^*} = \frac{b_j x_j^*}{P^*_i} = \frac{\partial P_i}{\partial x_j} \frac{x_j}{P_i}$$

$$\Rightarrow P_j = \frac{\partial P_i}{\partial x_j} = \frac{b_j x_j^*}{P^*_i} x_j = b_j \frac{x_j}{P_i} \frac{P_i}{x_j} = b_j \frac{\text{cost}}{x_j} \quad (i, j=1, 2).$$

From homogeneity, $f_{13} = -f_{11} - f_{12}$:

$$f_{13} = \frac{\partial P_i}{\partial x_3} = \frac{\partial P_1}{\partial x_3} = \frac{f_{13}}{x_3} = \frac{P_1}{x_3} = (-f_{11} - f_{12}) \frac{P_1}{x_3}$$

$$= (-b_{11} x_1^* - b_{12} x_2^*) \frac{P_1}{x_3}$$

$$= (-b_{11} x_1^* - b_{12} x_2^*) \frac{\text{cost}}{x_3}.$$

Similarly,

$$P_{23} = \frac{\partial P_2}{\partial x_3} = f_{23} \frac{x_3}{P_2} = (-f_{21} - f_{22}) \frac{P_2}{x_3} = (-b_{21} x_1^* - b_{22} x_2^*) \frac{\text{cost}}{x_3}.$$

Apply Young’s theorem to the normalized quadratic distance function,

$$P_{31} = \frac{\partial P_3}{\partial x_1} = \frac{\partial P_1}{\partial x_3} = P_{13},$$

$$P_{32} = \frac{\partial P_3}{\partial x_2} = \frac{\partial P_2}{\partial x_3} = P_{23}.$$

Since
\[ f_{33} = -f_{31} - f_{32} = -f_{13} P_3^* \frac{x_1^*}{P_3} - f_{23} P_2^* \frac{x_2^*}{P_3} \]

\[ = (b_{11} x_1^* + b_{12} x_2^*) \frac{x_1^*}{P_3} + (b_{12} x_1^* + b_{22} x_2^*) \frac{x_2^*}{P_3} \]

\[ = (b_{11} x_1^* + b_{12} x_2^*) \frac{x_1^*}{P_3} + (b_{12} x_1^* + b_{22} x_2^*) \frac{x_2^*}{P_3}, \]

we are able to derive \( P_{33} \) from \( f_{33} \):

\[ f_{33} = \frac{\partial P_3}{\partial x_3} P_3 \Rightarrow P_{33} = \frac{\partial P_3}{\partial x_3} = \frac{f_{33}}{P_3} x_3 \]

\[ = [(b_{11} x_1^* + b_{12} x_2^*) \frac{x_1^*}{P_3} + (b_{12} x_1^* + b_{22} x_2^*) \frac{x_2^*}{P_3}] \frac{P_3}{x_3} \]

\[ = [(b_{11} x_1^* + b_{12} x_2^*) x_1^* + (b_{12} x_1^* + b_{22} x_2^*) x_2^*] \frac{\text{cost}}{x_3}. \]

Substituting \( P_{ij} \) (i,j=1,2,3) into (17), (18), and (19), dividing both sides by cost, and combining \( x_1 \) with \( x_3 \) into \( x_1^* \) yields (24), (25), and (26).
References


