# Spatial Production Concentration, Demand Uncertainty, and Multiple Markets

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## Abstract

The geographic concentration of production of main field crops in several growing regions is a distinctive feature of U.S. agriculture. Among many possible reasons for spatial concentration, I study here the effects of the distribution of end users and terminal markets on acreage allocation. The presence of multiple terminal markets in a growing area may allow for a more flexible marketing plan, along with introducing more idiosyncratic demand uncertainty associated with each consumption point. To take better advantage of future marketing opportunities, growers, depending on their location relative to terminal markets, may adjust the crop mix produced on the farm. I characterize the types of environments that lead to a spatial production concentration of a commodity in a growing area. I also analyze the equilibrium effects of an increase in transportation costs and a shift in acreage available for planting on spatial acreage allocation.



**Keywords:** commodity prices, location, marketing, production concentration, supermodularity, systematic risk.

# SPATIAL PRODUCTION CONCENTRATION, DEMAND UNCERTAINTY, AND MULTIPLE MARKETS

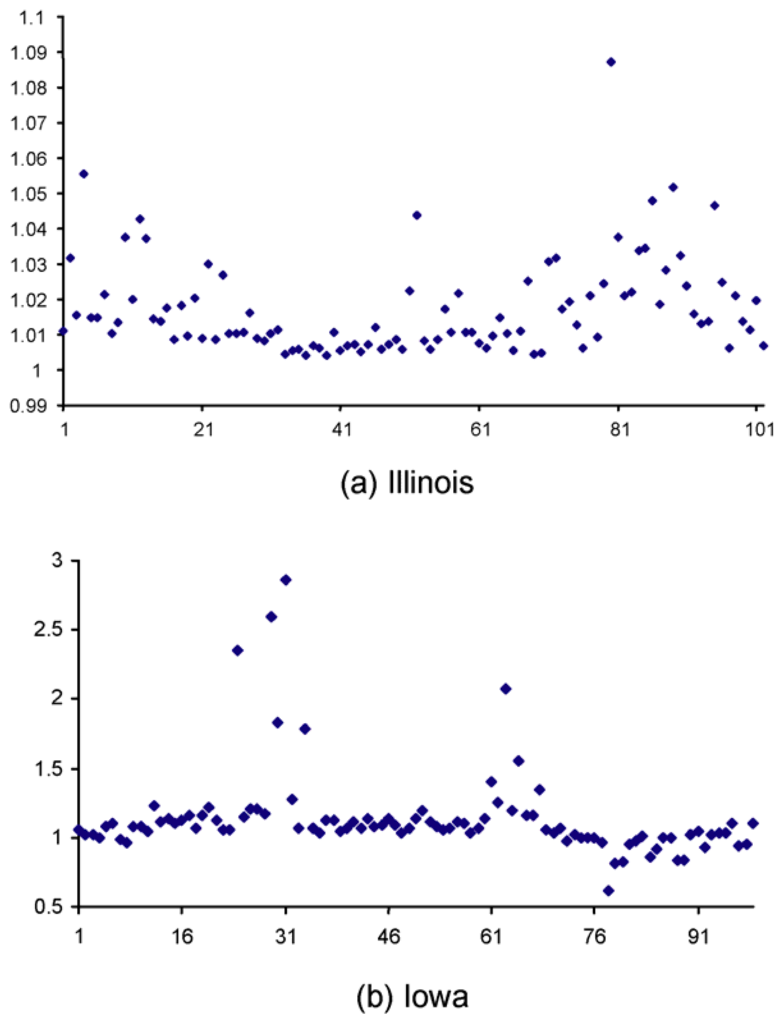
## Introduction

In recent years, agricultural market analysts have paid increasingly more attention to the spatial concentration of production in both animal and grain agriculture. In particular, the geographic concentration of production of main field crops in several growing regions is a distinctive feature of the U.S. agricultural landscape. Spatial production patterns are shaped by a host of factors, including agronomic considerations, proximity to input markets, vertical integration, farm size, and environmental regulations. I will focus here on another essential feature of the grower's decision environment: the presence of multiple end users and terminal markets in the growing area.

By allowing for a more flexible marketing plan, the ability to access multiple terminal markets and consumption points lowers the degree of systematic demand risk faced by growers at planting. To take better advantage of future marketing opportunities, growers, depending on their locations relative to terminal markets, may adjust the crop mix produced on the farm, which in turn affects the production concentration in the region. The goal of this paper is to develop a framework for studying the spatial concentration of production of a commodity in a growing area in the presence of multiple terminal markets.

The issue of land allocation among competing crops is important for policymakers because agricultural technologies and production practices may have significant environmental consequences, such as pesticide use patterns, pest resistance, impacts on biodiversity and beneficial insects, soil management, and other types of externalities (e.g., Feedstuffs 2002a,b). These environmental impacts are likely to differ across and within producing regions, in part because of the variation in the spatial production concentration (e.g., Feedstuffs 2002c). An understanding of how marketing conditions in the area interact with growers' planting decisions may help policymakers design better policies for promoting environmentally friendly farming practices. Also, regional production patterns are intimately connected with the demand for and development of transportation and grain handling infrastructures, which include in-land waterway systems, railroads, and trucks (McVey, Baumel, and Wisner 2000).

Figure 1 presents some evidence on the variation in production concentration within growing regions, using as an example the states of Illinois and Iowa. Relative production concentration is measured by the average ratios of acreages planted to corn and soybeans, the two main crops in Illinois and Iowa, across counties that are roughly ordered by geographic regions within a state (e.g., northeast, northwest). Examining the two graphs,




*Source:* U.S. Department of Agriculture 2002.

**FIGURE 1. Average ratios of corn and soybean acreages at the county level, 1998-2002**

corn and soybean crops appear to be considerably more evenly distributed across the growing area in Illinois than in Iowa; the standard deviations of the two series are, respectively, 0.014 and 0.32.

A number of factors influences acreage allocation decisions on a particular farm in a particular year. These may include soil characteristics, local weather conditions preceding and during the planting period, cost of inputs, various area-specific pest management issues, crop rotation benefits, cultivation practices, and government programs. Also, local demand conditions, transportation costs to the terminal markets, and the distribution of growers in the area have a bearing on the pattern of land allocation among different crops at a particular location. The prevalence of corn (soybean) acreage in some Iowa counties (see Figure 1) can be explained, at least partially, by a higher demand for a particular commodity for local uses rather than production-side considerations.

There are several indications, although indirect, that growers in Iowa routinely access a larger array of local markets and end users than do growers in Illinois. Commodity flow patterns suggest that growers in Illinois are more advantageously located to serve relatively more geographically concentrated export markets. Based on the estimates of the shares of transportation modes used for shipment in these two states, the share of production shipped by water for exports ("long-haul" movement) in Illinois exceeds that in Iowa (Berry, Hewings, and Leven 2003). On the other hand, a recent survey of Iowa farmers points out an increasing reliance on more flexible truck transportation and the ability of producers to deliver crops to multiple and diversified end users, such as feeding operations, processing facilities, and river markets (Baumel et al. 2001).

A greater number of geographically dispersed end users in the area not only expands the set of marketing strategies available to growers but also introduces more idiosyncratic demand uncertainty associated with each consumption point. My inquiry is into the role of multiple terminal markets, possessed of demand uncertainty that can be both commodity and market specific, on equilibrium acreage allocation in a growing region. Here, I examine the conjecture that the spatial distribution of terminal markets may cause production of certain commodities to concentrate in certain regions.

Note that producers in areas where transportation costs to different markets are similar more frequently market their crops based on the price differentials across markets than do producers who are closer to one particular market. This producer heterogeneity in the propensity to switch marketing outlets establishes the link between the extent of demand uncertainty when it takes the form of a commodity- and market-specific price volatility and spatial concentration of production in certain parts of the region.

The rest of the paper is organized as follows. First, a model is developed, and a general property of the equilibrium spatial acreage allocation is established. Then the notions of a spatial concentration of production and greater market-specific demand volatility are introduced. After investigating the quantitative effect of market-specific demand volatility on equilibrium acreage allocation, some determinants of the acreage allocation pattern in the spatial dimension are discussed. Next, sufficient conditions leading to a complete local spatial concentration of production are presented, and the equilibrium effects of an increase in transportation costs and a more spatially concentrated acreage distribution are investigated. The paper concludes with a discussion of possible lines of further inquiry.

## Model

Consider a region with $n$ terminal markets, indexed $i=1,...,n$, for two types of commodities $h$ and $l$. Producers at each location are differentiated by transportation costs to the terminal markets. Transportation costs per unit of distance are invariant across commodities, are normalized to 1, $e=1$, and are proportional to the corresponding distances to each market, $ed_i$, $d_i \in [0,1]$, $i=1,...,n$. Let $D_F \subset [0,1]^n$ denote the set of feasible distances to the terminal markets in the region. The number of acres at locations with transportation costs $(d_1,...,d_n) \in D_F$, or less, is given by the cumulative distribution function $F(d_1,...,d_n)$ with a strictly positive support on $D_F$ and the corresponding density function $f(d_1,...,d_n)$.[1] The total number of acres in the region is normalized to 1, $\int_{D_F} dF = 1$. The per acre costs of producing the two commodities, $c^h$ and $c^l$, are invariant across locations. The yields are certain, common across both varieties and locations, and are normalized to 1. There are two time points: the planting time when producers decide which variety to plant, and the harvest time when producers decide to which terminal market to ship their crop.

At terminal market $i$, inverse demands for the two commodities are given by $p_i^q = P^{q,i}(s_i^h, s_i^l, \theta_i^q, \omega)$, $q = l, h$, where $s_i^q$ is the quantity of commodity $q$ shipped to market $i$. Demand uncertainty is decomposed into uncertainty that is common across both markets and commodities, $\omega$, and commodity- and market-specific uncertainty, summarized by the parameter $\theta_i^q$. For example, the former type of uncertainty stems from news disseminated through public media about the end-use qualities of the two commodities. To concentrate on the effects of multiple markets on acreage allocation, I will ignore this type of uncertainty as it does not have much relevance to the analysis that follows.[2] The focus here is on the uncertainty that is related to local conditions, such as demand from adjacent feeding operations and processing plants for a particular commodity. Hold that $P_{s^q}^{q,i} < 0$ and $P_\theta^{q,i} \geq 0$ where the subscripts denote differentiation. Furthermore, assume that the degree of substitutability, if any, between commodities $h$ and $l$ in consumption is limited: $P^{h,i}(0, s^l, \theta) = \infty$ and $P^{l,i}(s^h, 0, \theta) = \infty$ for all $i = 1,...,n$. The joint cumulative probability distribution of the demand shocks is given by $G(\theta_1^h, \theta_1^l, ..., \theta_n^h, \theta_n^l)$ with support on $[\underline{\theta}, \overline{\theta}]^{2n}$.

### Analysis

At harvest time, producers of commodity $q$ located at $d_1, ..., d_n$ decide where to market their crop based on the relative prices net of transportation costs

$$\pi(d_1,..., d_N, q) = \max_{i \in \{1,...,n\}} [\, p_i^q - d_i \,]. \tag{1}$$

Therefore, commodity $q$ producers located in areas $S_i^q = \{d_1,..., d_n \in D_F : d_i - d_j \leq p_i^q - p_j^q, \forall j \neq i\}$ supply market $i$. In equilibrium, the relative distance, $d_i - d_j = d_{ij}^q$, at the locations of threshold producers that are indifferent between shipping commodity $q$ to market $i$ and market $j$ is equal to the price differential between markets $i$ and $j$, $p_i^q - p_j^q$:

$$P^{q,i}(s_i^h, s_i^l, \theta_i^q) - P^{q,j}(s_j^h, s_j^l, \theta_j^q) - d_{ij}^q = 0, \; q = l, h, \tag{2}$$

where $s_i^h = \int_{S_i^h} \alpha(z_1,...,z_n)dF$ , $s_i^l = \int_{S_i^l} (1-\alpha(z_1,...,z_n))dF$ , and $\alpha(d_1,...,d_n) \in [0,1]$ denotes the share of acres at locations with $d_1,...,d_n$ planted to variety $h$. Note that the assumptions made about the inverse demand functions rule out "corner" solutions by assuring that the amount of each commodity supplied to each market is strictly positive.

Let $\overline{d}_{ij}^q = \sup d_{ij}^q$ and $\underline{d}_{ij}^q = \inf d_{ij}^q$ denote the largest and smallest realization of the price differential between markets $i$ and $j$ for commodity $q$, $\overline{d}_{ij}^q = -\underline{d}_{ji}^q$. Then the locations of commodity $q$ producers that may deliver their crop to either of $M \subseteq \{1,...,n\}$ markets are $D_M^q = \{d_1,...,d_n \in D_F : \underline{d}_{ij}^q \le d_i - d_j \le \overline{d}_{ij}^q,\ d_i - d_k \le \underline{d}_{ik}^q,\ \forall i, j \in M,\ \forall k \notin M\}$. The purpose of defining such areas in the producing region will become apparent in Result 1. All commodity $q$ producers located in $D_{M=\{i\}}^q$ always ship their crop to market $i$. In contrast, commodity $q$ producers located in $D_M^q$, $M = \{i,..., j\}$ where $1 \le i < j \le n$ may ship their crop to any of the markets in $M$ depending on the price differential. Therefore, the growing region can be divided into two types of areas distinguished by the available marketing opportunities. In "arbitrage" areas, producers alternate between two or more markets in order to take advantage of the price differential. In "no-arbitrage" areas, producers always supply only one market because the price differential never covers the additional transportation expense.

At planting time, producers choose which variety to plant, anticipating the marketing opportunities available at harvest:

$$\pi(d_1,...,d_N) = \max_{q=\{l,h\}} E\pi(d_1,...,d_n,q) - c^q, \tag{3}$$

where $E$ is the expectation operator with respect to random variables $\theta_i^q$. Based on the limited degree of substitutability between commodities in consumption, $P^{h,i}(0,s^l,\theta) = \infty$ and $P^{l,i}(s^h,0,\theta) = \infty$, the following can be readily inferred about the acreage allocation pattern in terms of "no-arbitrage" and "arbitrage" areas.

RESULT 1. Suppose that $\Pr(d_{i1}^q = \underline{d}_{i1}^q, ..., d_{in}^q = \underline{d}_{in}^q) > 0$ for $i \in A \subseteq \{1,...,n\}$. Then any planting time equilibrium is characterized by

$$Ep_i^h - c^h = Ep_i^l - c^l, \ \forall\, i \in A, \tag{4}$$

and both commodities are produced in the areas near market $i$, where the "no-arbitrage" areas for both commodities overlap, $\int_{D_{\{i\}}^h \cap D_{\{i\}}^l} \alpha(z_1,...,z_n)dF \in (0,1)$, and only commodity $h$ $(l)$ is produced in the areas, where the "arbitrage" areas for commodity $h$ $(l)$ and "no-arbitrage" areas for commodity $l$ $(h)$ overlap, $\alpha(d_1,...,d_n) = 1(0)$ for all $(d_1,...,d_n)$ $\in D_{\{i\}\cup M}^h \cap D_{\{i\}}^l$ $(D_{\{i\}\cup M}^l \cap D_{\{i\}}^h)$, $\forall\, i \in A$, $M = \{k,..., j\}$ where $1 \le k < j \le n$.

Result 1 provides a partial characterization of the equilibrium acreage allocation in an environment in which planting decisions are governed by future spatial arbitrage considerations. Suppose that there is a positive probability that the price at market $i$ is low relative to prices at all other markets so that occasionally all producers who sometimes deliver to $i$ ship their crop to other markets. Then in the area "near" market $i$ but "away" from all other markets, $j \ne i$, the mix of the crops is produced. The areas located "near" market $i$ but "closer" to the other markets, which are better suited for taking advantage of the spatial price inequalities, will be planted to a commodity with "greater" spatial price differentials, $\bar{d}_{ij}^q$.

To simplify presentation, for the rest of the paper I consider a producing region with just two terminal markets, $i = 1,2$. In this case, the probability condition in Result 1 holds trivially because there are only two possible marketing outlets. The spatial acreage allocation pattern is illustrated in Figure 2 where the curve bounding areas A, B, C, D, and E correspond to points with $d_1 - d_2 = \underline{d}_{12}^q$ and $d_1 - d_2 = \bar{d}_{12}^q$, $q = l, h$.[3] In Figure 2, in the areas where both commodities are always shipped to the closest market, A $= D_{\{1\}}^h \cap D_{\{1\}}^l$ and D $= D_{\{2\}}^h \cap D_{\{2\}}^l$, the mix of the two is produced. On the other hand, in areas B $= D_{\{1,2\}}^h \cap D_{\{1\}}^l$ and C $= D_{\{1,2\}}^l \cap D_{\{2\}}^h$, one commodity is always shipped to the closest market while the marketing plan for the other one depends on the price

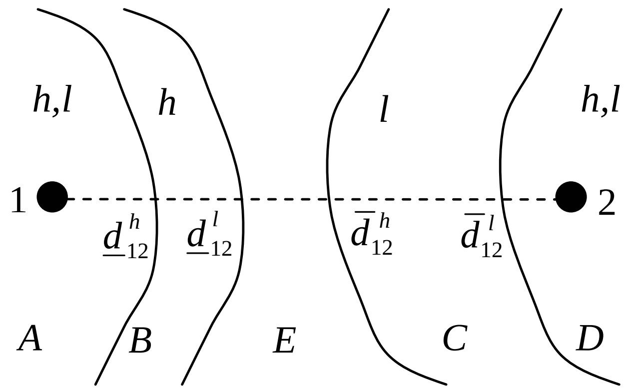



**FIGURE 2. Spatial acreage allocation pattern**

differential. In equilibrium, only the latter commodity will be produced in those areas: commodity $h$ in area B and commodity $l$ in area C.

Areas with small differences in transportation costs to markets 1 and 2, such as area E in Figure 2, are dominant in terms of the spatial arbitrage opportunities. Producers of both commodities located in these areas determine their marketing plan based on relative prices. Next, I investigate the determinants of the planting decisions in such overlapping "arbitrage" areas. For producers in areas $D_{\{1,2\}}^{h} \cap D_{\{1,2\}}^{l} = \{d_1, d_2 :$ $\max[\underline{d}^{h}, \underline{d}^{l}] < d_1 - d_2 < \min[\overline{d}^{h}, \overline{d}^{l}]\}$, the expected incremental return from switching from variety $l$ to $h$ is given by[4]

$$\Delta_q E\pi(d_1, d_2, q) = E\pi(d_1, d_2, h) - c^h - (E\pi(d_1, d_2, l) - c^l) \tag{5}$$

$$= E\{\max[p_1^h - p_2^h, d_1 - d_2] - \max[p_1^l - p_2^l, d_1 - d_2]\},$$

where condition (4) was used. Differentiating (5) with respect to $(d_1 - d_2)$ yields $\partial \Delta_q E\pi(d_1, d_2, q) / \partial(d_1 - d_2) = E\{1_{p_1^h - p_2^h \le d_1 - d_2} - 1_{p_1^l - p_2^l \le d_1 - d_2}\} = \Pr(p_1^h - p_2^h \le d_1 - d_2) -$ $\Pr(p_1^l - p_2^l \le d_1 - d_2)$.

In general, the incremental profit (5) may rise or fall as the relative distance (transportation costs) to the markets, $d_1 - d_2$, increases. Suppose that the market price differentials for commodities $h$ and $l$ are ordered by the first-degree stochastic dominance, so that $\Pr(p_1^h - p_2^h \le p) \le (\ge) \Pr(p_1^l - p_2^l \le p)$ for all $p \in [-1,1]$. If the price differential between markets 1 and 2 for commodity $h$ is likely to be larger (smaller) than that for commodity $l$, the equilibrium acreage allocation pattern takes the following form. Commodity $h\,(l)$ is produced in the areas with small $d_1 - d_2$ (near market 1 and away from market 2), while commodity $l\ (h)$ is produced in the areas with large $d_1 - d_2$ (near market 2 and away from market 1). Next, I consider conditions on the price differentials that lead to spatial production patterns that are of central interest in this study.

Suppose that there exists a $\hat{p} \in (-1,1)$ such that for $p > \hat{p}$, $\Pr(p_1^h - p_2^h \le p)$ $\le \Pr(p_1^l - p_2^l \le p)$ and $\Pr(p_1^h - p_2^h \le p) \ge \Pr(p_1^l - p_2^l \le p)$ for $p \le \hat{p}$. As is well known, this type of single-crossing condition may imply the second-degree stochastic dominance. Then the incremental profit, $\Delta_q E\pi(d_1, d_2, q)$, increases with the transportation cost differential, $d_1 - d_2$, in the areas "closer to market 1 than to market 2," where $d_1 - d_2 \le \hat{p}$, and decreases in the areas "closer to market 2 than to market 1," where $d_1 - d_2 > \hat{p}$. Therefore, in equilibrium, commodity $h$ will be produced somewhere in the "middle" of the "arbitrage" region, while commodity $l$ will be produced outside of the "middle" area. In other words, the production of a commodity characterized by a greater extent of market-specific price volatility is relatively more geographically concentrated in the "middle" of the "arbitrage" area. Next, I make more precise the notions of market-specific price volatility and geographic concentration and investigate the connection between them.

### Market-Specific Demand Uncertainty and Production Concentration: Definitions

A further inquiry into the determinants of acreage allocation patterns warrants the following.

ASSUMPTION 1. (*i*) $G(\theta_1^h, \theta_1^l, \theta_2^h, \theta_2^l) = G^h(\theta_1^h, \theta_2^h) G^l(\theta_1^l, \theta_2^l)$, $G^h(\theta_1, \theta_2) = G^h(\theta_2, \theta_1)$, $G^l(\theta_1, \theta_2) = G^l(\theta_2, \theta_1)$; (*ii*) $F(d_1, d_2) = F(d_2, d_1)$ for all $d_1, d_2 \in [0,1]$; (*iii*) $P^{q,1}(s^h, s^l, \theta^q) = P^{q,2}(s^h, s^l, \theta^q) = P^q(s^h, s^l, \theta^q)$, $q = l, h$; (*iv*) $P^h_{s^l} = 0$, $P^l_{s^h} = 0$.

Parts (*i*), (*ii*), and (*iii*) assure that markets 1 and 2 are "symmetric." Conditions (*i*) allow us to better focus on the forces behind the concentration of production of one commodity in a particular area other than *ex ante* asymmetries in demand across markets. Demand shocks for commodities $h$ and $l$ are independent, and demand shocks for each commodity at markets 1 and 2 are exchangeable random variables (see endnote 2). According to condition (*ii*), the distribution of acreage in the region is symmetric, which, for example, prevents an asymmetric concentration of acreage near one of the markets. Part (*iv*) is another simplifying assumption that excludes any cross-price effects, so that the only interaction between the crops is on the supply side.

In addition, I make the following behavioral assumption about the equilibrium spatial distribution of acres between the two commodities.

ASSUMPTION 2. For any $(d_1, d_2)$ and $(d_1', d_2')$, if $\pi(d_1, d_2) + d_1 = \pi(d_1', d_2') + d_1'$ or $\pi(d_1, d_2) + d_2 = \pi(d_1', d_2') + d_2'$ then $\alpha(d_1, d_2) = \alpha(d_1', d_2')$.

This assumption assures that it is only the location of producers relative to markets 1 and 2 that influences their planting decisions. Suppose that the expected revenues of producers after compensating them for the cost of transportation to one of the markets are identical. Then the crop mix at each location is also identical. Assumption 2 amounts to taking all other factors determining the acreage allocation to be invariant across locations and the number of producers at each point in the region to be large. The invariance of the "transportation cost compensated" revenues across locations implies either the lack of "material" differences between the locations of the two producers or that the producers never alternate between marketing outlets. Write the "market 1 transportation cost compensated" revenues for producers with $(d_1, d_2)$ and $(d_1', d_2')$ as follows:

$$\pi(d_1, d_2) + d_1 = \max_q [E \max[p_1^q, p_2^q + d_1 - d_2] - c^q],$$

$$\pi(d_1', d_2') + d_1' = \max_q [E \max[p_1^q, p_2^q + d_1' - d_2'] - c^q].$$

By inspection, the left-hand sides are equal in one of two instances. The differences in transportation costs may be the same, $d_1 - d_2 = d_1' - d_2'$. Then the optimal marketing decisions at harvest are trivially the same for these producers, and hence their planting decisions must conform. Otherwise, it must be that at any harvest time equilibrium, $p_1^q - d_1 \geq p_2^q - d_2$ and $p_1^q - d_1' \geq p_2^q - d_2'$ for $q = l, h$ (see equation (4)). In other words, for any realization of uncertainty, producers of both commodities located at $(d_1, d_2)$ and $(d_1', d_2')$ always ship their crop to market 1.

Assumption (*ii*) results in symmetric spatial acreage allocation when the prices are exchangeable random variables (i.e., the *ex ante* prices are symmetric across markets). The spatial acreage allocation is determined only up to the relative distance, $d_1 - d_2$, in the sense that $\alpha(d_1, d_2) = \alpha(d_1', d_2')$ for all $| d_1 - d_2 | = | d_1' - d_2' |$, and hence, $\bar{d}^q = -\underline{d}^q$, $q = l, h$. In particular, from Result 1 and using Assumption 2, it follows that $\alpha(d_1, d_2) = \alpha \in (0,1)$ on $(D_{\{1\}}^h \cap D_{\{1\}}^l) \cup (D_{\{2\}}^h \cap D_{\{2\}}^l) = \{d_1, d_2 \in D_F : | d_1 - d_2 | \geq \max[\bar{d}^h, \bar{d}^l]\}$. Finally, observe that in a case when the realizations of demand shocks are always common across markets, Assumptions 1 and 2 imply that in equilibrium, $\alpha(d_1, d_2) = \alpha \in (0,1)$ for all $d_1, d_2 \in D_F$ so that production of each commodity is evenly distributed everywhere in the region. And so, given these assumptions, any concentration of production is caused precisely by the market-specific uncertainty that differs across commodities.

Next, I formalize the concepts of spatial production concentration and market-specific uncertainty (the extent of systematic demand risk across markets).

DEFINITION 1. (Spatial Production Concentration). Spatial distribution of shares of acres under commodity $h$, $\alpha(d_1, d_2)$, is said to undergo a (symmetric) increase in concentration around the center, denoted by $\alpha(d_1, d_2) \leq_{spc} \alpha'(d_1, d_2)$, if

$$\int_0^1 \int_0^1 \alpha(z_1, z_2) dF = \int_0^1 \int_0^1 \alpha'(z_1, z_2) dF \text{ and } \int_0^1 \int_{z_2 - d}^{z_2 + d} \alpha(z_1, z_2) dF \leq \int_0^1 \int_{z_2 - d}^{z_2 + d} \alpha'(z_1, z_2) dF \text{ for all}$$

$d \in [0,1]$.

An increase in spatial production concentration for commodity $l$ can be defined analogously by replacing $\alpha(d_1, d_2)$ with $1 - \alpha(d_1, d_2)$. Of course, an increase in the concentration around the center of the acreage planted to commodity $h$ implies a decrease in the concentration for commodity $l$. Note that while an increase in the concentration, under Assumptions 1 and 2, is a mean-preserving contraction (mpc) in the sense of Rothschild and Stiglitz (1970), the converse may not be true. Also, the introduced notion of spatial production concentration is distinct from the diversification of production in the area measured by a Shannon's entropy index, $H = -\int_0^1\int_0^1 p(z_1, z_2)$ $\ln p(z_1, z_2) dz_1 dz_2$ where $p = \alpha(z_1, z_2) f(z_1, z_2) / \int_0^1\int_0^1 \alpha(z_1, z_2) dF$. This measure of production diversification (crop mix) does not take into account the location of acres, unlike Definition 1, which is better suited to model the response of equilibrium acreage allocation that differs across locations. However, starting with a uniform spatial acreage allocation (no production concentration), $\alpha(d_1, d_2) = \alpha$ for all $d_1, d_2 \in D_F$, an increase in spatial production concentration implies a decrease in the measure of diversification of production in the region.

To measure systematic risk or the extent of positive dependence present in the system, the following notion is commonly used (Shaked and Shanthikumar 1994).[5]

DEFINITION 2. (The Supermodular Stochastic Order). A bivariate probability distribution $G(\theta_1, \theta_2)$ is said to be smaller than the probability distribution $G'(\theta_1, \theta_2)$ in the supermodular stochastic order (denoted by $\leq_{sm}$) if $\int \phi(\theta_1, \theta_2) dG \leq \int \phi(\theta_1, \theta_2) dG'$ for all supermodular functions $\phi$ for which the expectations exist.

A function $\phi$ is called supermodular (submodular) if $\phi(\max[x_1, \hat{x}_1], \max[x_2, \hat{x}_2])$ $+ \phi(\min[x_1, \hat{x}_1], \min[x_2, \hat{x}_2]) \geq (\leq) \phi(\max[x_1, \hat{x}_1], \min[x_2, \hat{x}_2]) + \phi(\min[x_1, \hat{x}_1],$ $\max[x_2, \hat{x}_2])$ for any $x_1, \hat{x}_1, x_2, \hat{x}_2$ in the domain. This property is equivalent to the "increasing (decreasing) differences" property: $\Delta_i^{\varepsilon} \Delta_j^{\delta} \phi(x_1, x_2) \geq (\leq) 0$ for $i, j = 1,2$, $i \neq j$, $\varepsilon > 0$, and $\delta > 0$, where $\Delta_1^{\varepsilon} \phi(x_1, x_2) = \phi(x_1 + \varepsilon, x_2) - \phi(x_1, x_2)$. The supermodular stochastic order adequately captures the strength of positive dependence: "big (small) values of $\theta_1^q$ go with big (small) values of $\theta_2^q$." Furthermore, the ordering is possible only if the joint distributions are possessed of the same marginals. According to Definition 2, the extent of market-specific uncertainty regarding the demand for commodity $q$ decreases under the map $G^q \to G^{q,'}$ such that $G^q \leq_{sm} G^{q,'}$.

**Merged Markets, Acreage Allocation, and Demand Uncertainty: Quantitative Effect**

In general, the effect of an increase in systematic demand risk across markets (a decrease in market-specific volatility) on production concentration can be decomposed into two effects: (*a*) the change in the total share of acres under a commodity, and (*b*) the change in the spatial distribution of acreage. First, I isolate the quantitative effect on the equilibrium acreage allocation. To do so, I consider a special case with $\int_B dF = 1$, where $B = \{d_1, d_2 :| d_1 - d_2 |= 0\}$, that is, all acres for planting are concentrated in the area in the middle of the region. Note that such a distribution of acreage also arises if the two markets merge. The proximity of markets 1 and 2 can be modeled analogously to model an increase in dependence. As markets 1 and 2 move closer together, the number of producers that are either close to or far away from both markets increases, while the number of producers that are close to one market and far away from the other market decreases. Formally, this can be written as $F = F^1(\max[d_1, d_2]) \leq_{sm} F'$ for any $F'$ corresponding to a growing region where markets are more distant from each other, where $F^1$ is the marginal spatial distribution of acres relative to market 1. Alternatively, the analysis to follow adheres if there is no transportation cost for shipping commodities from any point in the region, $e = 0$.

In either of those cases, prices will be equalized across markets for any realization of demand uncertainty (see equation (2)):

$$P^q(s_1^q, \theta_1^q) = P^q(s_2^q, \theta_2^q) \text{ for any } \theta_1^q, \theta_2^q \in [\underline{\theta}, \overline{\theta}],\ q = l, h, \tag{6}$$

where $s_1^h + s_2^h = \alpha^*$, and $s_1^l + s_2^l = 1 - \alpha^*$. Here, $\alpha^*$ is the unique equilibrium share of acres allocated to commodity $h$ that is invariant across the region (Assumption 2) and is determined at planting by equation (4). Suppose that there is an increase in the dependence of the market-specific demand shocks for commodity $q$, $G^q \leq_{sm} G^{q,'}$, $q = l, h$.

Then in the new equilibrium the share of acres planted to commodity $q$ adjusts upward or downward depending on whether $\partial^2 P^q / \partial\theta_1^q \partial\theta_2^q \geq (\leq) 0$. The appendix shows that there are sufficient conditions such that the supermodularity (submodularity) of the market price $p_i^q$ in $(\theta_1^q, \theta_2^q)$ holds. Summarizing gives the following.

RESULT 2. Let Assumptions 1 and 2 hold, and $\int_B dF = 1$, where $B = \{d_1, d_2 : | d_1 - d_2 | = 0\}$. Then acres planted to commodity $q$ increase (decrease) with an increase in the dependence of the market-specific demand shocks for the commodity depending on whether $P_{11}^q \leq (\geq) 0$ and $P_{1\theta}^q \leq (\geq) 0$ for all $s_i^q$ and $\theta_i^q$, $q = l, h$.

An increase in dependence (an increase in systematic risk) among the demand shocks at markets 1 and 2 for commodity $h\,(l)$ in fact dampens the volatility of the "arbitraged" shipment, $\delta^h = | s_i^h - 0.5\alpha^* |$ ($\delta^l = | s_i^l - 0.5(1 - \alpha^*) |$). As market-specific demand shocks become more dependent, that is, when it is more likely that both of them are either "high" or "low" simultaneously, there is less likelihood that $\delta^q$ will deviate from zero. The curvature conditions on the inverse demand functions assure that the expectation of the price, $Ep_i^q$, varies in a monotone manner with the volatility of the "arbitraged" shipment, $\delta^q$, and that they have a standard interpretation.

**Equilibrium Acreage Allocation and Demand Uncertainty: Spatial Pattern**

Next, I consider the effects of market-specific demand volatility and spatial acreage allocation on the volatility of the price differential between markets (or the "arbitraged" shipment) in a more general case, where not all producers are equally distanced from both markets. Let $\leq_{mpc}$ denote a mean-preserving contraction of a probability distribution in the sense of Rothschild and Stiglitz (a special case of the second-order stochastically dominating shift).

RESULT 3. Let Assumptions 1 and 2 hold. (*a*) The volatility of the price differential between markets 1 and 2, $d^q$, decreases with an increase in dependence among market-specific demand shocks for commodity $q$, that is, $G^q \leq_{sm} G^{q,'}$ implies $d^q(G^q)$ $\leq_{mpc} d^q(G^{q,'})$, if $P_{11}^q = 0$ and $P_{1\theta}^q = 0$ for all $s_i^q$ and $\theta_i^q$, and $\alpha_1^* / \alpha^* \leq -f_1 / f$ for $q = h$ and $\alpha_1^* /(1-\alpha^*) \geq f_1 / f$ for $q = l$ for all $d_1, d_2 \in D_F$. (*b*) The volatility of the differential for commodity $h$ ($l$) decreases (increases) with an increase (decrease) in the spatial production concentration of commodity $h$ ($l$) around the center, that is, $\alpha(d_1, d_2)$ $\leq_{spc} \alpha'(d_1, d_2)$ implies $d^h(\alpha) \leq_{mpc} d^h(\alpha')$ and $d^l(\alpha') \leq_{mpc} d^l(\alpha)$.

In light of Result 2, the linearity of the inverse demand functions required in part (*a*) allows a better isolation of the role of spatial dispersion of producers (and markets) by removing the “quantity” effect. The conditions imposed on the spatial acreage allocation are satisfied if, for example, there is no production concentration (in the sense of the entropy index), $\alpha^*(d_1, d_2) = \alpha$, and the acres available for planting are distributed evenly in the region, $f(d_1, d_2) = 1$ for all $d_1, d_2 \in D_F$. Note that the nature of the shifts of the distributions of $d^q$ caused by an increase in demand shock dependence and production concentration is distinct. While an increase in demand shock dependence transforms the probabilities of the price differential, $d^q$, a greater production concentration shifts the map of realizations of the price differentials “closer” to (“away from”) zero.

Under certain conditions, a decrease in dependence among market-specific demand shocks and an increase in the concentration of commodity $h$ production have the opposite effects on the expected incremental profit (5), which is convex in the price differentials $d^h$ and $d^l$. And so, Result 3 seems to indicate that an increase in the production concentration for a commodity is a feasible equilibrium response to a greater extent of market-specific demand volatility for that commodity. To verify this conjecture, I introduce a slightly weaker version of Definition 1.

DEFINITION 3. (Local Production Concentration). Spatial distribution of shares of acres under commodity $h$, $\alpha(d_1, d_2)$, is said to undergo a local (symmetric) increase in concentration in the region $| d_1 - d_2 | \leq d_c$, around the center, denoted by

$\alpha(d_1,d_2) \leq_{lpc} \alpha'(d_1,d_2)$ if $\int_0^1 \int_{z_2-d_c}^{z_2+d_c} \alpha(z_1,z_2)dF = \int_0^1 \int_{z_2-d_c}^{z_2+d_c} \alpha'(z_1,z_2)dF$ and

$\int_0^1 \int_{z_2-d}^{z_2+d} \alpha(z_1,z_2)dF \leq \int_0^1 \int_{z_2-d}^{z_2+d} \alpha'(z_1,z_2)dF$ for all $d \in [0,d_c]$.

To proceed, consider equilibrium with "complete" production concentration in the sense of Definition 3, where only one commodity is produced in the "middle" of the area. For concreteness, suppose that equilibrium acreage allocation is $\alpha^*(d_1,d_2)=1$ for $|d_1-d_2|<\bar{d}^h$ and $\alpha^*(d_1,d_2)=\alpha$ for $|d_1-d_2|\geq \bar{d}^h$.[6] Furthermore, let conditions on the inverse demand functions in part (*a*) of Result 3 hold for commodity $l$.

Now consider a decrease in dependence among market-specific demand shocks for commodity $l$, $G^l \to G^{l,'} \leq_{sm} G^l$. Then, by part (*a*) of Result 3 and the linearity of the inverse demand function, the expected profit differential (5) weakly decreases for producers with $|d_1-d_2|<d^l$, $d^l(G^{l,'})\geq 0$. Suppose that, by further decreasing the degree of dependence, the expected incremental profit becomes strictly negative. Then the new equilibrium acreage allocation, $\alpha'(d_1,d_2)$, must be such that $\alpha'(d_1,d_2) \leq_{lpc} \alpha^*(d_1,d_2)$ for some $d_c>0$. This is because from the assumed linearity of the inverse demand function, $P^l(s_i^l,\theta_i^l)$, it follows that $d^l$ is directly proportional to

$\int_0^1 \int_{z_2-d^l}^{z_2+d^l} (1-\alpha^*(z_1,z_2))dF$.

Suppose that $\int_0^1 \int_{z_2-d^l}^{z_2+d^l} (1-\alpha^*(z_1,z_2))dF \leq \int_0^1 \int_{z_2-d^l}^{z_2+d^l} (1-\alpha'(z_1,z_2))dF$ for all $d^l$. Then, analogous to part (*b*) of Result 3, it can be shown that $d^l(\alpha^*) \leq_{mpc} d^l(\alpha')$, and the expected profit differential (5) weakly decreases. But this is impossible because there is no commodity $l$ produced in the middle of the region while the profit from producing commodity $l$ is strictly larger than that for commodity $h$ for some producers with $|d_1-d_2|<\bar{d}^l$. Therefore, it follows that $\int_0^1 \int_{z_2-d^l}^{z_2+d^l} (1-\alpha^*(z_1,z_2))dF$ $> \int_0^1 \int_{z_2-d^l}^{z_2+d^l} (1-\alpha'(z_1,z_2))dF$ for some $d^l$. Because $\alpha^*(d_1,d_2)=1$ for $|d_1-d_2|<\bar{d}^h$ there exists a $d_c>0$ such that $\alpha'(d_1,d_2) \leq_{lpc} \alpha^*(d_1,d_2)$. Summarizing gives the following.

RESULT 4. Let Assumptions 1 and 2 hold, $P_{11}^{l}=0$ and $P_{1\theta}^{l}=0$ for all $s_i^l$ and $\theta_i^l$, and $\alpha_1^* f \geq f_1(1-\alpha^*)$ for all $d_1, d_2 \in D_F$. Furthermore, suppose that $\alpha^*(d_1,d_2)=1$ for $|d_1-d_2|<\bar{d}^h$ and $\alpha^*(d_1,d_2)=\alpha\in(0,1)$ for $|d_1-d_2|\geq\bar{d}^h$.[7] Then a decrease in dependence among the market-specific demand shocks for commodity $l$ implies a decrease in the local production concentration of commodity $h$ around the center, $\alpha'(d_1,d_2)\leq_{lpc}\alpha^*(d_1,d_2)$ for some $d_c>0$.

In the next section, I consider in greater detail some determinants of equilibrium characterized by a complete (local) production concentration around the center.

### Sufficient Conditions for Complete Spatial Production Concentration

Let demand shocks for one of the commodities, for example, commodity $l$, be possessed of the greatest degree of systematic risk, that is, let it be perfectly correlated. Formally, this can be written as $G^l(\theta_1^l,\theta_2^l)=G_1^l(\min[\theta_1^l,\theta_2^l])\geq_{sm} G^{l,'}(\theta_1^l,\theta_2^l)$ for any other $G^{l,'}(\theta_1^l,\theta_2^l)$.[8] In other words, $\Pr(\theta_1^l=\theta_2^l)=1$, and barring any *ex ante* asymmetries between the two markets (excluded by Assumptions 1 and 2), at harvest time no commodity $l$ producers have an incentive to ship their crop to a relatively more distant market. The prices of commodity $l$ are always common across markets, $p_1^l=p_l^2$ and $\bar{d}^l=\sup|d^l|=0$, for any realization of uncertainty. In contrast, hold that $\int_B dG^h>0$ for some $B=\{\theta_1,\theta_2:\theta_1\neq\theta_2\}$, so that not all demand risk for commodity $h$ is systematic.

From Result 1 and Assumptions 1 and 2, it follows that equilibrium is characterized by $\alpha^*(d_1,d_2)=1$ for $|d_1-d_2|<\bar{d}^h$ and $\alpha^*(d_1,d_2)=\alpha\in(0,1)$ for $|d_1-d_2|\geq\bar{d}^h$. Here, a single commodity is produced in the middle of the region and the mix of commodities is produced outside of that region. Only commodity $h$ producers may deliver their crop to a more distant market to take advantage of the price differential (e.g., producers with $d_1<d_2$ shipping their crop to market 2.) The quantities of each commodity supplied to the markets at harvest are $s_1^h(d^h)=\int_0^1\int_{z_2-\bar{d}^h}^{z_2+d^h}dF+\alpha\int_0^1\int_0^{z_2-\bar{d}^h}dF$ ,

$s_2^h(d^h) = \int_0^1 \int_{z_2+d^h}^{z_2+\bar{d}^h} dF + \alpha \int_0^1 \int_{z_2+\bar{d}^h}^1 dF$ , $s_1^l = (1-\alpha)\int_0^1 \int_0^{z_2-\bar{d}^h} dF$ , and $s_2^l = (1-\alpha)\int_0^1 \int_{z_2+\bar{d}^h}^1 dF$ .
According to Definition 3, this spatial acreage allocation dominates any other allocation in terms of the local concentration of commodity $h$ production in the area around the center with $d_c = \bar{d}^h$. The largest (smallest) amount of commodity $h$ is shipped to a market when the realization of market-specific demand shocks has the most dispersion: $\theta_1^h = \bar{\theta}$ , $\theta_2^h = \underline{\theta}$, or $\theta_1^h = \underline{\theta}$, $\theta_2^h = \bar{\theta}$ :

$$P^h(\bar{s}^h, \bar{\theta}) - P^h(\underline{s}^h, \underline{\theta}) - \bar{d}^h = 0 , \qquad (7)$$

where $\bar{s}^h = s_1^h(d^h = \bar{d}^h)$ and $\underline{s}^h = s_1^h(d^h = -\bar{d}^h)$. Summarizing gives the following.

DEFINITION 4. (Equilibrium with Complete Local Production Concentration). Symmetric equilibrium with a complete local production concentration of commodity $h$ around the center is given by $d^* = \bar{d}^h$, the size of the concentration area, and $\alpha^* = \alpha$, the share of commodity $h$ produced outside of the concentration area, such that equations (2), (4), and (7) hold. Some mathematical exposition presented in the appendix establishes the uniqueness of this equilibrium.

RESULT 5. Let Assumptions 1 and 2 hold, $G^l(\theta_1^l, \theta_2^l) = G_1^l(\min[\theta_1^l, \theta_2^l])$, and $\int_B dG^h > 0$ for some $B = \{\theta_1, \theta_2 : \theta_1 \neq \theta_2\}$. Then there is unique equilibrium with a complete local production concentration of commodity $h$ around the center, $(\alpha^*, d^*)$ with $d^* \in (0,1)$.

Now I investigate how the size of the concentration region and the split of acreage outside of the region respond to the cost of transportation per unit distance and to the distribution of acreage available for planting. For example, a "global" increase in transportation costs may be a result of higher fuel prices or the abandonment of a local railroad or an inland waterway system. The distribution of acreage available for planting is influenced by a multitude of factors, including various alternative uses, environmental regulations, and the proximity to urban centers.

**Equilibrium Effects of Transportation Cost and Acreage Distribution**

Next I consider the effect of a uniform increase in transportation cost, $e$, on the equilibrium acreage allocation with a complete concentration of commodity $h$ in the middle of the region (Definition 4). Equilibrium conditions are now given by equation (4) and

$$P^q(s_1^h, \theta_1^h) - P^q(s_2^h, \theta_2^h) - ed^h = 0, \tag{2a}$$

$$P^h(\bar{s}^h, \bar{\theta}) - P^h(\underline{s}^h, \underline{\theta}) - e\bar{d}^h = 0. \tag{7a}$$

From these equations, it follows that the size of the concentration ("arbitrage") area must decline with the transportation cost, $e$. Suppose that, on the contrary, the size of the concentration area, $d^*$, increases. Inspecting equation (7a), it is clear that the share of commodity $h$ outside of the concentration area, $\alpha^*$, must adjust, because otherwise the difference, $P^h(\bar{s}^h, \bar{\theta}) - P^h(\underline{s}^h, \underline{\theta})$, decreases, which contradicts the assumption. Suppose that $\alpha^*$ increases. Then equation (4) no longer holds because supply of commodity $l$ decreases while that of commodity $h$ increases for all realizations of uncertainty. Suppose that $\alpha^*$ decreases. Then (7a) implies that $\bar{s}^h = \int_0^1 \int_{z_2-d^*}^{z_2+d^*} dF + \alpha^* \int_0^1 \int_0^{z_2-d^*} dF$ must decrease because $\underline{s}^h = \alpha^* \int_0^1 \int_0^{z_2-d^*} dF$ is clearly smaller than before. Hence, the total supply of commodity $l$ increases, which is impossible if Condition 4 continues to hold. Therefore, the size of the concentration area $d^*$ must decrease with the transportation cost $e$. The effect of the transportation cost on the share of commodity $h$ outside of the concentration area $\alpha^*$ is ambiguous, as illustrated using a special case in Result 4, part (*a*).

Next, I inquire into the effects of the spatial acreage distribution in the region on the equilibrium distribution of acreage allocation. Consider a shift of acreage available for planting toward the middle of the region, $F \leq_{spc} F'$ (use Definition 1 with $\alpha(d_1, d_2) = \alpha'(d_1, d_2) = 1$.) Inspecting equation (7a), it follows that in the new equilibrium, $d^*$ must adjust downward because $s_1^h(F') = \int_0^1 \int_{z_2-d^*}^{z_2+d^*} dF' + \alpha^*(F') \int_0^1 \int_0^{z_2-d^*} dF'$ $\geq \int_0^1 \int_{z_2-d^*}^{z_2+d^*} dF + \alpha^*(F) \int_0^1 \int_0^{z_2-d^*} dF = s_1^h(F)$, as long as $\alpha^*(F) \geq \alpha^*(F')$. In general, the effect of the acreage shift on $\alpha^*$ is ambiguous. This is demonstrated using a two-point distribution of demand shocks for commodity $h$, $g^h(\underline{\theta},\underline{\theta}) = \pi_{\underline{\theta},\underline{\theta}}$, $g^h(\bar{\theta},\bar{\theta}) = \pi_{\bar{\theta},\bar{\theta}}$, $g^h(\underline{\theta},\bar{\theta}) =$ $g^h(\bar{\theta},\underline{\theta}) = \pi_{\underline{\theta},\bar{\theta}}$. Here, the probability distribution of $d_m^h$ is concentrated at just three points (at most), $\Pr(d^h = -\bar{d}^h) = \Pr(d^h = \bar{d}^h) = \pi_{\underline{\theta},\bar{\theta}}$, and $\Pr(d^h = 0) = \pi_{\underline{\theta},\underline{\theta}}$ $+\pi_{\bar{\theta},\bar{\theta}}$. Therefore, for the purposes of investigating the effects of a greater acreage concentration on equilibrium allocation, any shift of the acreage into the middle of the region, $F \to F' \geq_{spc} F$, given the symmetry of acreage distribution, can be summarized as follows: $\int_{|d_1-d_2|\geq d^*} dF' = \int_{|d_1-d_2|\geq d^*} dF - \varepsilon$, and $\int_{|d_1-d_2|<d^*} dF' = \int_{|d_1-d_2|<d^*} dF + \varepsilon$, $\varepsilon \geq 0$. Some further exposition shown in the appendix leads to the following:

RESULT 6. Let Assumptions 1 and 2 hold, $G^l(\theta_1^l,\theta_2^l) = G_1^l(\min[\theta_1^l,\theta_2^l])$, and $g^h(\underline{\theta},\bar{\theta}) =$ $g^h(\bar{\theta},\underline{\theta}) = 1/2$. Then the size of the concentration area, $d^*$, decreases, and the share of commodity $h$ outside of the concentration area $\alpha^*$ decreases or increases, depending on $(1-\alpha^*)EP_1^l(s_i^l,\theta_i^l) + (1-\alpha^*/2)P_1^h(\bar{s}^h,\bar{\theta}) - (\alpha^*/2)P_1^h(\underline{s}^h,\underline{\theta}) \leq (>) 0$, with (*a*) a small increase in transportation cost, $e \to e' > e$; or (*b*) a small increase in acreage concentration around the center, $F \to F' \geq_{spc} F$.

## Conclusions

The model developed in this paper is used to study the effects of the distribution of terminal markets (or the concentration of end users) in a growing area on equilibrium of spatial acreage allocation. Here, I find that the multi-market environment may have an immediate impact on spatial acreage allocation. In particular, I inspect the possibility of a spatial concentration of production driven by commodity- and market-specific price volatilities. I discuss the effects of a “greater” spatial concentration of production and “greater” market-specific demand volatility on the probability distribution of the price differential between markets. I also establish sufficient conditions for a complete local spatial concentration of production and explore the equilibrium effects of acreage distribution and transportation costs in this case.

A similar analysis of spatial acreage allocation patterns may also apply in the case of commodities differentiated based on the extent of genetic modification (GM) present in the seed. In this case, the degree of substitutability in consumption of GM and non-GM varieties and the different costs of processing warrant modifications of the demand side of the model. In addition, much greater care needs to be exercised when decomposing demand uncertainty into systematic and commodity- and market-specific components.

To focus on the role of location and market-specific demand uncertainty on planting decisions, among other issues, I ignored the temporal dimension of the marketing plan that spans the period between the two harvests (Benirschka and Binkley 1995; Frechette and Fackler 1999). In the case of a single terminal market, quality-differentiated commodities, and a multi-period marketing environment, acreage allocation decisions may vary across locations (Saak 2003). Therefore, it is of interest to understand how planting and marketing decisions are made under more realistic circumstances, including the case in which plural terminal markets and marketing periods and market-specific demand uncertainty are present.

## Endnotes

1. In general, the distribution of acres will not have a strictly positive support everywhere on $[0,1]^n$ because some combinations of distances to markets may not be possible. In particular, $f(0,...,0) > 0$ implies that all markets are located at the same point; otherwise, it must be that $f(0,...,0) = 0$.

2. For example, the effects of this type of uncertainty on acreage allocation between genetically modified and standard varieties are studied in Saak and Hennessy 2002.

3. The distances to markets need not correspond to Eucledian distances in a rectangular coordinate system.

4. Because there are only two markets, I drop the subscripts on $\bar{d}_{12}^q$ and $\underline{d}_{12}^q$, and take $\bar{d}^q = \bar{d}_{12}^q$ and $\underline{d}^q = \underline{d}_{12}^q$.

5. Hennessy and Lapan (2003) provide a fundamental treatment of the concept of "more systematic risk" and its formalizations in a broad economic setting with some applications. The supermodular stochastic order is used widely in insurance and financial management literatures.

6. In the next section, I consider environments where such patterns exist in equilibrium. For example, using Result 1, along with Assumptions 1 and 2, this spatial pattern emerges if the price of commodity $l$ does not vary across markets, $\bar{d}^l = 0$. However, note that the lack of variation is not necessary for the spatial allocation pattern where only one commodity is produced in the "middle" of the region.

7. We ignore the derivative condition $\alpha_1^* f \geq f_1(1-\alpha^*)$ on the set of zero measure $|d_1 - d_2| = \bar{d}^h$.

8. This is a well-known property of the Frechet upper bound for distribution $H(x,y) \leq \min[H_x(x),\ H_y(y)]$ with marginals $H_x(x), H_y(y)$. The random variables with distribution function $H(x,y) = \min[H_x(x),\ H_y(y)]$ are called co-monotonic.

# Appendix

## Proofs of Results

### *Proof of Result 1*

Without loss of generality, suppose that in equilibrium $D^h_{\{i\}\cup M} \bigcap D^l_{\{i\}} \neq \varnothing$ for some $i$ and $M = \{i,..., j\}$ where $1 \leq i < j \leq n$. Then it must be that $\alpha(d_1,..., d_n) > 0$ for some $(d_1,...., d_n) \in D^h_{\{i\}}$. Otherwise, when at harvest time equilibrium the price differential is such that $p^h_i - p^h_j = \underline{d}^h_{ij}$ for all $j \neq i$, there are no commodity $h$ producers supplying market $i$, which cannot be in equilibrium. Note that the expected profits for producers of both commodities in areas $D^q_{\{i\}}$ are $\pi(d_1,..., d_n, q) = Ep^q_i - d_i - c^q$ because they are located in the "no-arbitrage region." Hence, in planting time equilibrium it must be that $Ep^h_i - c^h \geq Ep^l_i - c^l$ so that some commodity $h$ is produced in that region. But this implies that all producers in areas with $D^h_{\{i\}\cup M} \bigcap D^l_{\{i\}}$ prefer to grow commodity $h$ because for them $\pi(d_1,..., d_n, h) = E \max_{j\in M\cup i}[p^h_j - d_j] - c^h >$ $Ep^h_i - d_i - c^h \geq Ep^l_i - d_i$ $- c^l = \pi(d_1,..., d_n, l)$ as they may switch from supplying market $i$ to supplying market $j \in M$ to their advantage. And so, $\alpha(d_1,..., d_n) = 1$ for all $(d_1,..., d_n) \in D^h_{\{i\}\cup M} \bigcap D^l_{\{i\}}$. Next, note that it cannot be that $Ep^h_i - c^h > Ep^l_i - c^l$ so that $\alpha(d_1,..., d_n) = 1$ for all $(d_1,...., d_n) \in D^h_{\{i\}}$. Because then, when at harvest time equilibrium the price differential is such that $p^l_i - p^l_j = \underline{d}^l_{ij}$ for all $j \neq i$, there is no commodity $l$ producers supplying market $i$, which cannot be in equilibrium. Hence, equilibrium is characterized by

$Ep_i^h - c^h = Ep_i^l - c^l$ and $\int_{D_{\{i\}}^h \cap D_{\{i\}}^l} \alpha(z_1,...,z_n)dF \in (0,1)$, and $\alpha(d_1,...,d_n) = 1(0)$ for all $(d_1,...,d_n) \in D_{\{i\}\cup M}^h \cap D_{\{i\}}^l$ $(D_{\{i\}\cup M}^l \cap D_{\{i\}}^h)$, $i = 1,...,n$.

*Proof of Result 2*

Substitute the expressions $s_2^h = \alpha^* - s_1^h$ and $s_2^l = (1-\alpha^*) - s_1^l$ in equation (6) in the text. Differentiating $P^q(s_1^q, \theta_1^q)$ twice yields $\partial^2 P^q / \partial\theta_1^q \partial\theta_2^q$
$= P_{11}^q(s_1^q, \theta_1^q)[\partial s_1^q / \partial\theta_1^q]\,[\partial s_1^q / \partial\theta_2^q] + P_{11}^q(s_1^q, \theta_1^q)[\partial^2 s_1^q / \partial\theta_1^q \partial\theta_2^q] + P_{1\theta}^q(s_1^q, \theta_1^q)$ $[\partial s_1^q / \partial\theta_2^q]$. Using condition (6) to find $\partial s_1^q / \partial\theta_i^q$ and $\partial^2 s_1^q / \partial\theta_1^q \partial\theta_2^q$, write $\partial^2 P^q / \partial\theta_1^q \partial\theta_2^q = -(P_{11}^{q,1}P_1^{q,2} + P_{11}^{q,2}P_1^{q,1})P_\theta^{q,1}P_\theta^{q,2} / A^3 + (P_{1\theta}^{q,1}P_1^{q,2}P_\theta^{q,2} + P_{1\theta}^{q,2}P_1^{q,1}P_\theta^{q,1}) / A^2$, where $A = P_1^{q,1} + P_1^{q,2}$ and the superscripts "$q,i$" denote the type of commodity and market where it is sold. Hence, it follows that $\partial^2 P^q / \partial\theta_1^q \partial\theta_2^q \geq (\leq) 0$ when $P_{11}^q \leq (\geq) 0$ and $P_{1\theta}^q \leq (\geq) 0$.

*Proof of Result 3*

($a$) Differentiating (2) twice shows that the conditions stated in the result suffice to assure that the price differential for commodity $q$ is submodular in demand shocks, $\partial^2 d^q / \partial\theta_1^q \partial\theta_2^q \leq 0$. To show that when the submodularity condition holds, an increase in dependence implies that $d^q(G^q) \leq_{mpc} d^q(G^{q,'})$, consider the following. The symmetry imposed by Assumption 1 assures that $\int d^q dG^q = \int d^q dG^{q,'} = 0$. Next, consider a twice differentiable function $H(d^q)$, where $H' \geq 0, H'' \geq 0$. Differentiation yields $\partial^2 H / \partial\theta_1^q \partial\theta_2^q = H''[\partial d^q / \partial\theta_1^q]\,[\partial d^q / \partial\theta_2^q] + H'[\partial^2 d^q / \partial\theta_1^q \partial\theta_2^q] \leq 0$. Note that $G^q \leq_{sm} G^{q,'}$ implies that $\int H(d^q(\theta_1^h, \theta_2^h))dG^q \leq \int H(d^q(\theta_1^h, \theta_2^h))dG^{q,'}$. Because $H$ is an arbitrary increasing and convex function, the last condition implies that $d^q(G^q) \leq_{mpc} d^q(G^{q,'})$.

($b$) It must be shown that an increase in the production concentration around the center in the region for commodity $h$, $\alpha(d_1, d_2) \leq_{spc} \alpha'(d_1, d_2)$ implies that $d^h(\alpha)$

$\leq_{mpc}\ d^h(\alpha')$. Note that the absolute value of the price differential between markets 1 and 2 decreases, $|P^h(s_1^h(\alpha),\theta_1^h) - P^h(s_2^h(\alpha),\theta_2^h)| \geq |P^h(s_1^h(\alpha'),\theta_1^h) - P^h(s_2^h(\alpha'),\theta_2^h)|$ because $s_1^h(\alpha) = \int_0^1\int_0^{d^h+z_2}\alpha(z_1,z_2)dF \leq (\geq) \int_0^1\int_0^{d^h+z_2}\alpha'(z_1,z_2)dF = s_1^h(\alpha')$ and $s_2^h(\alpha) = \int_0^1\int_{d^h+z_2}^1\alpha(z_1,z_2)dF \geq (\leq) \int_0^1\int_{d^h+z_2}^1\alpha'(z_1,z_2)dF = s_2^h(\alpha')$ depending on whether $d^h \geq (\leq) 0$. Therefore, it must be that $|d^h(\alpha',\theta_1^h,\theta_2^h)| \leq |d^h(\alpha,\theta_1^h,\theta_2^h)|$ for each $\theta_1^h,\theta_2^h$, in order for the harvest time equilibrium to be restored. Hence, write $\Pr(d^h(\alpha',\theta_1^h,\theta_2^h) \leq d) \leq (\geq) \Pr(d^h(\alpha,\theta_1^h,\theta_2^h) \leq d)$ for $d \geq (<) 0$, which implies that $d^h(\alpha) \leq_{mpc} d^h(\alpha')$. Also, by symmetry, it follows that $d^l(\alpha') \leq_{mpc} d^l(\alpha)$.

### *Proof of Result 5*

Without loss of generality, let $\int_B dG^h > 0$ for $B = \{\theta_1 = \overline{\theta}, \theta_2 = \underline{\theta}\}$. Differentiating (7) in the text yields $\partial\overline{d}^h/\partial\alpha = -A/B$, where $A = [P_1^h(\overline{s}^h,\overline{\theta}) - P_1^h(\underline{s}^h,\underline{\theta})] \int_0^1\int_0^{z_2-d_p} dF$,

$B = P_1^h(\overline{s}^h,\overline{\theta})[\int_0^1 f(z_2+\overline{d}^h,z_2)dz_2 + (1-\alpha)\int_0^1 f(z_2-\overline{d}^h,z_2)dz_2]$

$+ P_1^h(\underline{s}^h,\underline{\theta})\alpha\int_0^1 f(z_2+\overline{d}^h,z_2)dz_2 - 1$. Differentiating (2) yields $\partial d^h/\partial\alpha = -C/D$,

$C = [P_1^h(s_1^h,\theta_1^h) - P_1^h(s_2^h,\theta_2^h)][\int_0^1\int_0^{z_2-\overline{d}^h} dF + (1-\alpha)\int_0^1 f(z_2-\overline{d}^h,z_2)dz_2[\partial\overline{d}^h/\partial\alpha]$,

$D = (P_1^h(s_1^h,\theta_1^h) + P_1^h(s_2^h,\theta_2^h))\int_0^1 f(z_2+d^h,z_2)dz_2 - 1$. Differentiating $E[P^h(s_1^h,\theta_1^h)]$ with respect to $\alpha$ yields $\partial EP^h(s_1^h,\theta_1^h)/\partial\alpha = E\{P_1^h(s_1^h,\theta_1^h)\,[\int_0^1\int_0^{z_2-\overline{d}^h} dF +$

$(1-\alpha)\int_0^1 f(z_2-\overline{d}^h,z_2)dz_2\ (\partial\overline{d}^h/\partial\alpha) - \int_0^1 f(z_2+d^h,z_2)dz_2(\partial d^h/\partial\alpha)]\}$. Note that the sign of the expression in the square brackets is positive because substituting $\partial d^h/\partial\alpha = -C/D$ yields the following $\{(\int_0^1\int_0^{z_2-\overline{d}^h} dF + (1-\alpha)\int_0^1 f(z_2-\overline{d}^h,z_2)dz_2[\partial\overline{d}^h/\partial\alpha])\ \{(P_1^{h,1}+P_1^{h,2})$

$\int_0^1 f(z_2+d^h,z_2)dz_2 - 1\} + \int_0^1 f(z_2+d^h,z_2)dz_2[P_1^{h,1} - P_1^{h,2}]\,[\int_0^1\int_0^{z_2-\overline{d}^h} dF\ + (1-\alpha)$

$\int_0^1 f(z_2 - \bar{d}^h, z_2)dz_2 \ (\partial\bar{d}^h / \partial\alpha)]\}/ D = \{2(\int_0^1\int_0^{z_2-\bar{d}^h} dF + (1-\alpha)\int_0^1 f(z_2 - \bar{d}^h, z_2)dz_2$

$[\partial\bar{d}^h / \partial\alpha])\ \{(P_1^{h,1}\int_0^1 f(z_2 + d^h, z_2)dz_2 - 1/2\}\ / D > 0$. Therefore, $\partial EP^h(s_1^h, \theta_1^h)/\partial\alpha \ < 0$.

Finally, differentiating $E[P^l(s_i^l, \theta_i^l)]$ with respect to $\alpha$ yields $\partial EP^l(s_1^l, \theta_1^l)/\partial\alpha$

$= E\{P_1^l(s_1^l, \theta_1^l)\ \ [-\int_0^1\int_0^{z_2-\bar{d}^h} dF - (1-\alpha)\ \int_0^1 f(z_2 - \bar{d}^h, z_2)dz_2(\partial\bar{d}^h / \partial\alpha)]$. Note that the sign of the expression in the square brackets is negative, as substituting $\partial\bar{d}^h / \partial\alpha = -A/B$ yields $-[\int_0^1\int_0^{z_2-\bar{d}^h} dF\{P_1^h(s_1^h, \bar{\theta})\ [\int_0^1 f(z_2 + \bar{d}^h, z_2)dz_2 \ + (1-\alpha)\int_0^1 f(z_2 - \bar{d}^h, z_2)dz_2]$

$+ P_1^h(s_2^h, 0)\alpha\int_0^1 f(z_2 + \bar{d}^h, z_2)dz_2 - 1\} - (1-\alpha)\int_0^1 f(z_2 - \bar{d}^h, z_2)dz_2[P_1^h(s_1^h, 1) - P_1^h(s_2^h, 0)]$

$\int_0^1\int_0^{z_2-\bar{d}^h} dF]/ B = -[\int_0^1\int_0^{z_2-\bar{d}^h} dF\{[P_1^h(s_1^h, 1) + P_1^h(s_2^h, 0)]\ \int_0^1 f(z_2 + \bar{d}^h, z_2)dz_2 \ - 1\}]\ / B < 0$.

Therefore, $\partial EP^l(s_1^l, \theta_1^l)/\partial\alpha > 0$. This implies that there exists a unique $\alpha^*$ that solves equation (4). The observed monotonicity of the left-hand sides of equations (2) and (7) in, respectively, $d^h$ and $\bar{d}^h$ guarantees the uniqueness of equilibrium.

### *Proof of Result 6*

It must be shown that $\partial d^* / \partial\varphi < 0$ and $\partial\alpha^* / \partial\varphi < (\geq)0$ depending on $(1-\alpha)EP_1^l(s_1^l, \theta_i^l)\ + (1-\alpha/2)P_1^h(\bar{s}_1^h, \bar{\theta})\ \ - (\alpha/2)P_1^h(\underline{s}_1^h, \underline{\theta})\ \ (\leq) > 0$, where $\varphi = \varepsilon, e$. The conditions of the result imply that equilibrium $d^*$ and $\alpha^*$ is given by two equations: $P^h(\underline{s}^h, \underline{\theta}) + (1/2)d^* - EP^l(s_1^l, \theta_1^l)\ - (c^h - c^l) = 0$, and $P^h(\bar{s}^h, \bar{\theta}) - P^h(\underline{s}^h, \underline{\theta}) - ed^* = 0$,

where $s_1^l = (1-\alpha^*)\int_0^1\int_0^{z_2-d^*} dF - (1-\alpha^*)\varepsilon/2$, $\bar{s}^h \ = \ \int_0^1\int_{z_2-d^*}^{z_2+d^*} dF + \alpha^*\int_0^1\int_0^{z_2-d^*} dF$

$+ \varepsilon(1-\alpha^*/2)$, and $\underline{s}^h = \alpha^*\int_0^1\int_0^{z_2-d^*} dF - \alpha^*\varepsilon/2$. Straightforward differentiation shows that the Jacobian determinant for this system of equations is strictly negative if $\int_{|d_1-d_2|\geq d^*} dF - \varepsilon > 0$. Then differentiating the two equations with respect to $\varepsilon$ and $e$, and using the implicit function theorem establishes the result.

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