Can Spot and Contract Markets Co-Exist in Agriculture?

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Abstract

New production technologies, consumers who are more discriminating, and the need for improved coordination are among the forces driving the move from spot markets to contracts. Some worry that this tendency will result in the disappearance of spot markets, or at least that they will become too thin to be of help for an efficient price discovery process. Other authors point to the reduction in welfare of independent producers resulting from contracting in oligopsonistic industries. While a large body of literature is available tackling the contract versus spot market decision, much less is known about the reasons that lead to procurement in both markets. This paper provides a simple model to study how fundamental economic factors influence the contracting behavior of farmers and processors. In the model, processors contract upstream with price-taking farmers. Participation in both markets arises as a Nash equilibrium for a wide range of parameterizations. Numerical methods are used to examine the effects of fundamental economic factors on the relative size of the spot and contract markets.

Keywords: contract markets, contracting in agriculture, specialty grains, spot markets, yield risk.
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Introduction

A growing proportion of agricultural production is being raised and sold under contract arrangements. The literature is rich in reasons for the increasing use of contracts (see, for example, Barkema 1993; Drabenstott 1994; Dimitri and Jeenicke 2001; Featherstone and Sherrick 1992; Sykuta and Parcell 2003; Hennessy 1996; Hueth and Ligon 1999; Hennessy and Lawrence 1999). The move to production under contracts has some concerned about the viability of remaining spot markets and about the degree to which farmer welfare is negatively impacted by market power of processors (Smith 2001; Hayenga, Schroeder, and Lawrence 2002).

While a lot of research has been devoted to the spot market versus contracts question (starting with the work of Coase [1937]), much less is known about the motivations that lead to the co-existence of both spot and contract markets. It is somewhat surprising that agricultural economists have not explored the rationales for co-existence thoroughly, since many important sectors exhibit this feature. The first paper that explicitly models co-existence of spot and contract markets in agriculture is by Xia and Sexton (2004). The point made by these authors is that “top-of-the-market-pricing” (TOMP) clauses in cattle procurement result in reduced competition, when buyers can also influence the base price. The intuition is that as buyers have committed to buy output at a price tied to a spot price to be determined later on, they have incentives to compete less aggressively in that market. In their setting, because of externalities or coordination problems among themselves, sellers can be induced to sign contracts that are not in their collective best interest with little or no financial incentive.

In this paper, we study how fundamental economic factors (prices, production variability, competitive environment, and costs) influence the relative profitability of contract and spot production for farmers and processors. Our objective is to gain insight into how these factors help determine the incentives for participation in contract and spot
markets. For the sake of concreteness, this article focuses on the co-existence of spot and contract markets in the context of specialty grain production. However, the economic forces at work apply broadly (to any industry in which the same group of buyers and sellers participate in spot and contract markets and the buying side is concentrated).

**Specialty Grain Production and Marketing**

Increasing proportions of agricultural products being produced and sold under contracts have been reported not only on livestock (Lawrence and Hayenga 2002; Hayenga et al. 2000) and fruits and vegetables, but also on some grains and oilseeds. For example, the National Agricultural Statistics Service (NASS 2003) reports that 10.4 percent of all U.S. corn production, 8.6 percent of the soybeans, and 4.8 percent of the wheat was sold through marketing contracts in 2001. The value of the three crops marketed under contracts was about $2.25 billion, $1.25 billion, and $0.25 billion for corn, soybeans, and wheat, respectively. However, the proportion of specialty grains and oilseeds planted under contracts is much larger. Just to mention a few examples, the U.S. Grain Council (2001) reports that about 60-70 percent of waxy corn and 60-65 percent of white corn are grown under contract, with the remaining area produced speculatively for the spot market. The interest in planting and marketing specialty corn and soybeans is growing (Good, Bender, and Lowell 2000). These authors conducted a survey of grain handling firms in Illinois. In 1998, these firms obtained 85 percent, 87 percent, 59 percent, and 96 percent of the volumes handled of high oil, white, yellow food grade, and waxy corn, respectively, from contracts with farmers. The remaining product was procured mostly through spot markets. For soybeans, the surveyed firms reported that 97 percent, 80 percent, and 85 percent of the volumes handled of STS (Dupont herbicide-tolerant), tofu, and non-GMO soybeans, respectively, were obtained through contracts with farmers. Thus, the firms surveyed procure most of the input (for both types of crops) through contracts, using existing spot markets as residual suppliers.

Marketing contracts are most often used for procurement (NASS 2003; Boland et al. 1999). The market price plus a premium accounted for 60 percent of the contracts reported in a survey of specialty corn producers conducted by Ginder et al. (2000). A premium is paid over a reference price (yellow no. 2 corn), conditional on the crop
meeting certain quality specifications. Yield drags, higher variable costs of production (especially transportation and handling), and additional management time required are among the reasons that producers command premiums to plant specialty crops (Fulton, Pritchett, and Pederson 2003).

The situation modeled in this paper is similar to that in Xia and Sexton 2004 in the sense that we model an oligopsonistic industry that contracts with upstream, price-taking input providers, and the market for processed products (downstream) is perfectly competitive. The TOMP clause is also assumed to be used in our model. However, we depart from this setting by modeling a situation where the pricing arrangement embedded in the contract is not affected by the behavior of buyers in the spot market. That is, buyers do not have power in the market to influence the reference price to which the contract price is pegged, although they can influence price in the cash-market for the commodity procured. Four other distinctive features of our model are as follows: (a) we assume a harsher type of competition (in prices) in the spot market; (b) we include uncertainty as a factor inherent in agricultural activity; (c) we recognize that at any given point in time or growing season, there is only so much of a homogeneous output coming from a fixed pool of producers (i.e., supply in the cash market is fixed, influenced by decisions made possibly months in advance); and (d) processors are constrained in the amount of the agricultural produce they can process at any point in time. There are concessions, of course, that one has to make to model a more realistic setting. First, we lose some tractability, in the sense that we are not able to obtain analytic solutions for some cases. Second, an ad hoc assumption is needed to close the model in one of the possible scenarios (more on this to follow).

The situation we examine is fairly typical in agriculture. Processors offer farmers contracts to purchase all production on a specified number of acres at a price pegged to a yet-to-be-realized market price. In other words, the pricing arrangement embedded in the contract is TOMP, or “market price plus a premium.” Each processor has a target amount of production to procure. Procurement in excess of this target—for example, when per-acre yields are very high—can be sold at some salvage price, which is assumed throughout to be the commodity price for the product. When contract supply is low, the processor can turn to the spot market to make up the difference, but only if farmers have
planted for the spot market. The price in the spot market clears the *ex post* (after random yields are realized) processor demand with the fixed supply of the product. Thus, stochastic yields lead to stochastic spot market prices. We account for such economic factors as the price the processor receives for output, salvage values of excess supply for the processor and the farmer, the farm-level cost of production, the number of farmers and processors, and the amount of yield variability. We find that purely financial reasons can explain the preference of contract and/or spot market procurement.

We begin by providing an overview of the problem to be addressed here and then present a formal presentation of a model that captures the profit incentives of processors and farmers. Numerical simulations show how the Nash solution reacts to changes in key economic parameters. These simulation results allow us to determine the key factors affecting the preference of farmers and processors for contract and spot production.

**Overview of the Problem**

Suppose there are $M$ input buyers (processors) in a geographic region who can enter into grower contracts that specify that the processor will purchase at a guaranteed premium over a yet to be determined market price all the production that comes off of contracted acres. There are $N$ growers. Each of the $M$ processors has a capacity constraint $Q$ that limits the amount of delivered input that can be used.\(^7\) Output technology for the processor is a fixed proportions technology $q = k(A_c y + x_s)$, where $q$ is output, $A_c$ is acreage contracted by each processor, $x_s$ is the amount bought on the spot market, and $y$ is the per-acre yield on all spot and contract acres in a given year.\(^8\) At no additional cost of generality, the conversion factor between raw input and output $k$ is set to one (by choosing units of measurement appropriately). A problem arises because per-acre yield on the contract acres is stochastic so that the total quantity of produced input from contracted acres will vary from the amount needed by the processor to achieve capacity production. Any excess production can be sold by the processor for a salvage price. Farmers have the option to plant additional acres of production without a contract. The amount of production depends on the expected spot price they will receive.

The *ex post* input price in the spot market equals the commodity price if there is excess supply of the input. If, *ex post*, there is excess demand of the input in the spot market, then
the spot price of the input (given that there are sufficient buyers) will be bid up to the point where profits for the processors equal zero. Under certain excess demand conditions, there is no equilibrium price that can be obtained. That is, there is no general solution to the problem of a limited number of buyers bidding for a fixed supply of the input. To get around this problem, we assume, for now, that the spot price in such excess demand conditions is midway between the excess demand and the excess supply prices.

The processor offers to identical risk-neutral farmers a premium, $\delta$, and a total number of contract acres, $A_c$. If this contract price exceeds or meets the opportunity cost of land, then the processor will find an excess demand for the contract acres and will not have a problem finding takers for the contracts. We assume throughout that farmers will not plant for the spot market if they know they will be obtaining the commodity price for their output. The processor’s capacity constraint and the number of contract acres determines the probability that production will be less than that needed to run at capacity. This creates the possibility that the processor will find it profitable to buy in the spot market. If this probability is high enough, this creates an incentive for farmers to produce for the spot market.

The processor can control the profitability of farmers growing for the spot market through the choice of $A_c$. That is, increases in $A_c$ decrease the profitability because there will be less total spot demand. There is a spot market supply curve $A_s = g(A_c)$ with $g_{A_c} < 0$ that captures farmers’ willingness to plant for the spot market.

Then, each processor $i$ has a demand for contract acreage function that results from the profit-maximization problem that depends on the number of spot acres and the number of acres contracted by its rivals: $A_{c,i} = h(A_{c,-i}, A_s)$. Vector $A_{c,-i}$ contains the acreage contracted by other processors in the industry. Presumably, $h_{A_c,-i} < 0; h_{A_s} < 0$ because an increase in the demand for contracted acres by the processor’s rivals increases the premium required to entice farmers to take contracts, and increases in spot market acreage decrease the payoff from additional contract acres. All of this is common knowledge for both farmers and processors.
Farmers and processors face a two-stage optimization problem. In the first stage processors decide simultaneously and independently the premium to pay for the input and how many contract acres to offer. The farmer’s choices are whether to take the contracts offered and how many acres to plant for the spot market. Contracts cannot be reoffered. Both decisionmakers face spot price uncertainty caused by supply uncertainty from random yields. They would also face uncertainty in the contract price, since the premium is tied to a yet-to-be-determined cash price, and premiums usually reflect quality differences. To keep things simple, however, we assume that the reference price and quality are fixed at their expected value. In the second stage, processors compete to buy the input they need if it turns out that the contracted input is not enough to work at capacity. The optimization problem is solved using backward induction. The next section formalizes the problem described so far. First, we analyze the optimization problem of processors; then, we analyze the problem farmers face.

The Model

The Processor Problem

The second stage of the processor problem occurs after harvest, so yield uncertainty has been resolved. *Ex post*, processors face a perfectly inelastic supply of the homogeneous input, given by the total acres planted for the spot market multiplied by its yield.

After observing yield $y$, processors decide whether or not to try to buy more input. The second-stage (*ex post*) demand for each processor is $x_r = (Q - A_r y)^{10}$, which can be negative if it turns out that the processors get more input than they need.

The price in the spot market, and hence the allocation of the rent among farmers and processors, will depend on the *ex post* relative “bargaining” power. Spot price is determined by demand and supply in the spot market, both of which are determined by planting and contracting decisions made in the first period. In this stage, processors bid simultaneously and independently to purchase the amount of input they need (if any) to work at capacity. Processors are engaging in a Bertrand type of competition with each other. After seeing the bids of each processor, farmers decide whether to sell their production to one of the processors (and to which one) or not to sell at all and obtain the
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salvage value for the production. For the allocation of the input, we assume that farmers will sell it first to the processor offering the highest price. This processor is able to buy all the input he or she needs (if less than the aggregate supply); then, farmers offer the input remaining to the second-highest bid, and so on. In case of a tie, the input is distributed evenly across processors (in the cases where there is excess demand). In short, we are using a “highest offer first” allocation rule (see Weninger and Reinhorn n.d., and endnote 18).

*Ex post*, processors will find themselves in one of four possible scenarios regarding their demand for additional production. The first scenario examined is when yields are high enough so that processors exceed their capacity constraint with contracted production. This occurs when $y \geq \frac{Q}{A_c}$. Any surplus production on contract acres will be sold at the going salvage or commodity price. In this case, profits for the processor are

$$\pi_1 = pQ - \left( r_i + \delta \right) A_c y - r_i \left( Q - A_c y \right),$$

or

$$\pi_1 = \left( p - \left( r_i + \delta \right) \right) A_c y + \left( p - r_i \right) \left( Q - A_c y \right)$$

where $p$ is the price of output (net of processing costs)$^{11}$, and $r_i + \delta$ is the per-unit price of contracted acreage. Here, $x_s = (Q - A_c y) \leq 0$, and the firm is getting some money back for the excess input. Note, however, that processors lose money on each bushel contracted in excess of their target. They would have contracted less acres had they known that such a realization of $y$ was going to occur. This situation happens with probability $\Pr(y \geq \frac{Q}{A_c}) = \Pr(y \geq v) = 1 - F(v)$, where $v = \frac{Q}{A_c}$, and $F(\cdot)$ is the cumulative distribution function of yield.

The second scenario occurs when *ex post* demand by processors is positive but there is still excess supply in aggregate. That is, $\sum_{i=1}^{M} \left( Q - A_c y \right) \leq \bar{A}_s y$, where $\bar{A}_s$ is total acreage planted for the spot market by all farmers. Thus $\bar{A}_s y$ is *ex post* aggregate, fixed supply in the spot market. The range of yields for this situation is given by
\[
\frac{MQ}{(\bar{A}_s + \sum_{i=1}^{M} A_{ci})} \leq y \leq \frac{Q}{A_c}.
\]

Because there is still aggregate excess supply, an offer by processors of \( r_i = r_1 \) for spot production (where \( r_1 \) is the salvage value for spot production) constitutes a pure strategy Nash equilibrium. This is what Sexton and Zhang (1996) observed in the market for California iceberg lettuce. Processors do not need to bid the price up to get all the input they need to work at capacity.

Again processors can work at capacity, and profits are

\[
\pi_2 = pQ - (r_1 + \delta)A_c y - r_1(Q - A_c y),
\]

or

\[
\pi_2 = \left( p - (r_1 + \delta) \right)A_c y + (p - r_1)(Q - A_c y).
\]

This case happens with probability

\[
\Pr \left( \frac{MQ}{(\bar{A}_s + \sum_{i=1}^{M} A_{ci})} \leq y \leq \frac{Q}{A_c} \right) = \Pr(u \leq y \leq v) = F(v) - F(u),
\]

where \( v \) is defined as above, and

\[
u = \frac{MQ}{(\bar{A}_s + \sum_{i=1}^{M} A_{ci})}.
\]

The third scenario is when there is excess demand \textit{ex post} but only one processor would not have enough production to run at capacity. One would think that in an excess demand condition, processors would bid up the spot price to the point in which all their rents are dissipated. But if only one processor does not have enough input to run at capacity, then there is an incentive for this processor to strategically underbid its rivals for the residual supply. This is the case in which a Nash equilibrium in pure strategies fails, in general, to exist. Edgeworth cycles may arise in this case (following a loose dynamic argument). For example, the price may rise as the processors try to increase their share in the input market, until the profit of doing so is lower than the one resulting from offering the reservation price of the farmers (or commodity price) and keeping the residual supply. Once the price is at the reservation price of the farmers (or low enough), processors will find it profitable to bid \( \varepsilon \) above their rivals and work at capacity.
This case implies \[ \frac{(M-1)Q}{(\overline{A}_s + \sum_{i=1}^{M-1} A_{ci})} \leq y \leq \frac{MQ}{(\overline{A}_s + \sum_{i=1}^M A_{ci})} \]. Because we are assuming a symmetric solution in which processors get an even distribution of the fixed supply (i.e., each processor is able to buy \( \frac{\overline{A}_s y}{M} \) in the spot market), processors will not be able to work at capacity. Profits in this case are

\[ \pi_3 = \left( p - (r_i + \delta) \right) A_{s} y + \left( p - r_j \right) \frac{\overline{A}_s}{M} y, \]

where \( r_s \) is the resulting input price in the spot market.

In this case we cannot know with certainty what \( r_s \) will be. The Nash equilibrium for this situation will be in mixed strategies, implying that the payoff function will not be continuous.\(^\text{15}\) To close the model, we set it equal to the average of the marginal valuations of the processors and the farmers. This case happens with probability

\[ \Pr \left[ \frac{(M-1)Q}{(\overline{A}_s + \sum_{i=1}^{M-1} A_{ci})} \leq y \leq \frac{MQ}{(\overline{A}_s + \sum_{i=1}^M A_{ci})} \right] = \Pr \left( s \leq y \leq u \right) = F(u) - F(s), \]

where \( u \) is defined as above and \( s = \frac{(M-1)Q}{(\overline{A}_s + \sum_{i=1}^{M-1} A_{ci})} \).

The fourth and final scenario is when yield is so low (given the areas contracted and planted for the spot) that at least one processor would be left out of the market if that processor tries to underbid his or her rivals. For example, the second-to-last processor finds some supply but it is not enough to work at capacity. In this case, there is no room for strategic underbidding by the last processor. If the last processor tries to underbid his or her rivals, that processor will be left out of the market. This will happen if

\[ \sum_{i=1}^{M-1} (Q - A_{ci}y) \geq \overline{A}_s y, \]

or, equivalently, \( y \leq \frac{(M-1)Q}{(\overline{A}_s + \sum_{i=1}^{M-1} A_{ci})} \). The pure strategy Nash equilibrium here is for processors to offer their marginal valuation \( (p) \) for the input. Processors will again split the input evenly and will not be able to work at capacity. Profits in this case are
\[ \pi_s = pA_s y + p \frac{\bar{A}_s y}{M} - (r_i + \delta) A_s y - p \frac{\bar{A}_s y}{M} = \left( p - (r_i + \delta) \right) A_s y. \]

This will occur with probability
\[
\Pr \left( y \leq \frac{(M-1)Q}{\bar{A}_s + \sum_{i=1}^{M-1} A_{ci}} \right) = \Pr(y \leq s) = F(s)
\]
where \( s \) is defined as above.

**Processor Expected Profits**

Now we are ready to write the first-stage objective function of a representative processor. Bearing the rules that will arise in the second stage in mind, processors independently and simultaneously make a decision as to how many contracts to offer and price premiums to pay in order to maximize their expected payoff. That is, each processor chooses \( A_{ci} \) and \( \delta \) to maximize its expected profits, which are defined as
\[
E(\pi_i | A_{c_i, \bar{A}_s}) = E\left( \left( p - (r_i + \delta) \right) A_s y + \left( p - r_i \right) \left( Q - A_s y \right) | u \leq y \right) \Pr(y \geq u)
\]
\[
+ E\left( \left( p - (r_i + \delta) \right) A_s y + \left( p - (p + r_i)/2 \right) \bar{A}_s y | s \leq y \leq u \right) \Pr(s \leq y \leq u)
\]
\[
+ E\left( \left( p - (r_i + \delta) \right) A_s y | s \leq y \right) \Pr(y \leq s)
\]
subject to the constraint that the contracts need to be accepted by farmers. This objective function can be rewritten as follows:
\[
E(\pi_i | A_{c_i, \bar{A}_s}) = \left( p - r_i \right) \left( A_s \int_0^u ydF(y) + Q \left( 1 - F(u) \right) + \frac{\bar{A}_s y}{2M} \int_s^u ydF(y) \right) - \delta A_s E(y)
\]
(1)
where \( f(y) \), \( 0 \leq y \leq y_m \) is the density function for the random yield and \( E(y) \) is the expected value of the yield. Formally, the problem is
\[
\max_{A_{ci} \geq 0, \delta \geq 0} E(\pi_i | A_{c_i, \bar{A}_s})
\]
subject to
\[
(p - \delta) E(y) \geq C'(a_s + a_c).
\]
The marginal cost for farmers of planting an acre of the crop is $C'(a_x + a_c)$. The number of acres planted for the cash market and contract market by a farmer are $a_x$ and $a_c$, respectively. The farmer’s problem will be presented subsequently. The constraint indicates that the expected marginal revenue of a contracted acre cannot be lower than the marginal cost of planting that extra acre. In other words, the premium and number of acres offered to a farmer have to be consistent with his or her supply schedule if the contract is going to be accepted.\textsuperscript{16} The constraint has to be binding at any optimum for this problem (if any positive number of contracts is offered). Otherwise, processors can reduce the financial incentive offered and still entice acceptance of the contracts by farmers. This observation allows us to subsume the constraint into the objective function and perform the optimization only with respect to the number of contracts offered.\textsuperscript{17}

The first-order condition for a maximum is found by differentiating equation (1) with respect to $A_c$ using the Leibnitz rule. After imposing symmetry and using some algebra, we get

$$\frac{\partial E(\pi | A_{-i}, A_i)}{\partial A_i} = (p - r)\left(\int_0^u x dF(y) + \frac{A}{MQ} \left[f(u)u^3 + f(s)s^3\right]\right) - E\left(E\left(\frac{\partial \delta}{\partial a_x} \frac{\partial a_x}{\partial A_i} A_i + \delta\right)\right) \leq 0 \quad (2)$$

with equality if $A_{ij} > 0$, $i = 1, ..., M$. Second-order sufficient conditions are presented in the Appendix.

Processors take into account that they can affect the probability of being in each of the situations described. They realize, for example, that if they increase the contracted area, it is less likely that they will have to buy in the spot market. Of course, the magnitude of the marginal effect each processor has decreases as the number of processors increases. Before exploring the ramifications of this assumption, we will first examine the problem of farmers.

**Farmer Decisions**

Farmers have rational expectations, share common beliefs, and take contract area and contract price as given in solving their optimization problem. They decide whether to take a processor’s offer of acreage and price or whether to plant for the spot market. If they are indifferent between taking and rejecting the contracts, they are assumed to accept
the processors offer. Assume a large number \( N \) of identical farmers. The large number assumption is not crucial. The important assumption is that farmers take prices and aggregate acreages as given. Thus, they do not act as if they can affect the probability of *ex post* spot market demand. Here, \( a_{ij} \) is the acreage contracted by farmer \( j \), \( a_{s} \) is the area planted for the spot market by farmer \( j \), and \( C(a_{c} + a_{s}) \) is the cost of production.

Because it is assumed that farmers are identical, the subscript \( j \) will be dropped. The expected profit of a farmer thus can be written as

\[
E(\pi^F) = E\left( (r_1 + \delta) a_c y + r_1 a_s y - C(a_c + a_s) \right).
\]

Again, four different scenarios may arise in the spot market. These scenarios are the same as the ones described in the processor problem and for that reason we will only state here what farmers expect, that is, their payoff function and their probability of being in each scenario. In the first scenario, there is no demand in the spot market. Here, farmers do not receive any bid for their output and therefore the value of the crop equals the commodity price. The profit function for this scenario is

\[
\pi_1^F = (r_1 + \delta) a_c y + r_1 a_s y - C(a_c + a_s),
\]

which, as before, occurs with probability

\[
Pr\left( y \geq \frac{Q}{A_c} \right) = Pr\left( y \geq v \right) = 1 - F(v).
\]

In the second scenario, there is demand for the input in the spot market. However, aggregate supply exceeds aggregate demand. Profits and probability of occurrence for this scenario are

\[
\pi_2^F = (r_1 + \delta) a_c y + r_1 a_s y - C(a_c + a_s)
\]

and

\[
Pr\left( \frac{MQ}{\tilde{A}_s + \sum_{i=1}^{M} A_{ci}} \leq y \leq \frac{Q}{A_c} \right) = Pr\left( u \leq y \leq v \right) = F(v) - F(u).
\]

The third scenario corresponds to the one where only \( M - 1 \) processors can work at capacity. Farmer profits and probability of occurrence for this scenario are

\[
\pi_3^F = (r_1 + \delta) a_c y + r_1 a_s y - C(a_c + a_s)
\]

and
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And the last scenario is as before, where at least one processor would be left out of the market if that processor tries to underbid his or her rivals so that the spot price is bid up to \((p)\), the processor’s marginal valuation for the input. Profits and probability of occurrence for this case are

\[
\pi^F = (r_i + \delta) a_i y + pa_s y - C(a_c + a_s)
\]

and

\[
\Pr\left( y \leq \frac{(M-1)Q}{\bar{A}_s + \sum_{i=1}^{M-1} A_{ci}} \right) = \Pr(s \leq y \leq u) = F(u) - F(s).
\]

Now we can write the expected profit of farmers as

\[
E(\pi^F) = (r_i + \delta) a_i E(y) + a_s \left( \int_r^y yf(y)dy + \int_y^u yf(y)dy + \int_0^s pydy \right) - C(a_c + a_s).
\]  

(3)

The first-order condition for a maximum is obtained by differentiating equation (3) with respect to \(a_s\):

\[
\frac{\partial E(\pi^F)}{\partial a_s} = r_i \int_r^y yf(y)dy + r_s \int_y^u yf(y)dy + p\int_y^s yf(y)dy - C'(a_c + a_s) \leq 0,
\]  

(4)

with equality if \(a_s > 0\). Second-order conditions trivially hold for any convex cost function.

Note that farmers take the aggregate amount planted for the spot as given. Hence, we do not use the Leibnitz rule here. Farmers do not realize they can change the probability of being in the scenarios described (they take \(\bar{A}_s\) as given). This is because individual farmer output is assumed to be too small to affect the aggregate spot supply. Recall that farmers are assumed to have rational expectations. Farmers believe the aggregate acreage for the spot will be \(\bar{A}_s\), and in equilibrium their expectations are realized.
Equilibrium

In this section, we characterize the equilibrium acreages for both the farmer and processor problems. An equilibrium for this model is conformed by two main components corresponding to the processor’s and farmer’s problem. A pure strategy Nash equilibrium for the processor’s game is a number of contracted acres $A_i^*$, $i = 1, \ldots, M$ such that no processor can benefit from unilateral deviations, for a given number of acres planted for the spot market. That is to say that in equilibrium, a processor cannot increase its profit by unilaterally choosing to contract a different number of acres. Farmers take the number of contract acres and the price offered as given, and, based also on their beliefs regarding the aggregate spot acreage, they choose the number of acres to plant for the spot market in order to maximize profits. It cannot be overemphasized that farmers’ decisions are the result of an optimization problem and do not arise from strategic interactions with processors or other farmers. However, the discussion will proceed as if farmers were “reacting” optimally to processors’ offers, and Nash equilibrium will be used in a loose way to refer to the equilibrium of the model.

Three additional conditions have to hold in equilibrium. First, beliefs regarding aggregate spot acreage are confirmed: $\bar{A}_s = Na_s$. Second, aggregate demand and supply for contracts are equalized: $Na_c = MA_c$. Finally, farmers’ profits are nonnegative. In short, any equilibrium has to satisfy equations (5) and (6), which obtain from imposing the equilibrium conditions to equations (2) and (4), respectively:

\[
(p-r_1)\left(\int_0^uydF(y) + \frac{\bar{A}_s}{2MQ}\left(\frac{f(u)u^3}{M} + \frac{f(s)s^3}{(M-1)}\right)\right) - E(y)\left(\frac{\partial \delta}{\partial A_i} \frac{\partial a_s}{\partial A_c} A_i + \delta\right) \leq 0, \quad (5)
\]

with equality if $A_c > 0$;

\[
r_1\int_y^u yf(y)dy + r_s\int_y^u yf(y)dy + p\int_y^s yf(y)dy - C\left(A_c \frac{M}{N} + a_s\right) \leq 0,
\]

with equality if $a_s > 0$, for $v = \frac{Q}{A_c}$, $u = \frac{MQ}{(Na_s + MA_c)}$, $s = \frac{(M-1)Q}{(Na_s + (M-1)A_c)}$, (6)

and equation (3) $\geq 0$. 
The equilibrium for a particular environment can be predicted using the best-response functions of processors, \( A_i = h(A_c) \), and farmers, \( a_{ij} = z(A_c) \) (for \( z(A_c) = g(A_c)/N \), as previously introduced in the problem overview section. These functions are defined implicitly by equations (5) and (6), respectively. The equilibrium is determined by the intersection of the best-response functions or, equivalently, by solving equations (5) and (6) simultaneously for \( A_i^* \) and \( a_{ij}^* \). Unfortunately, we cannot obtain closed solutions for this model. However, numeric techniques can be used to characterize the predicted Nash equilibrium as well as responses to changes in the environment. In what follows, we provide a characterization of the equilibrium for a particular calibration of the model (hereupon referred to as the benchmark case). The parameter values assumed for the benchmark case are shown in Table 1. The parameters were chosen in a way such that if the yield were to be fixed at its expected value (which we fix at \( E(y) = 100 \)), there would be no acres planted for the spot market in equilibrium; that is, processors would be fully contracted. To see this, suppose that processors offer a number of contracts such that they will not obtain enough input to work at capacity. Since there is no uncertainty, both farmers and processors know at planting time whether the spot price will be \( p \) or \( r \), given a positive expected aggregate spot acreage. If the cash market price will be \( p \), farmers will refuse to take any contract that offers \( r_i + \delta < p \), leaving zero profits for processors who in turn will not want to offer those contracts (they would be better off by being fully contracted and not participating in a cash market). If the cash price will be \( r \), individual farmers will not plant for that market, and expectations will not be confirmed. If, on the other hand, processors offer a number of contracts such that they will not need any extra input, expected aggregate spot acreage is zero, and farmers

<table>
<thead>
<tr>
<th>Parameter</th>
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<tr>
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<td>( y_m )</td>
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</tr>
<tr>
<td>( N )</td>
<td>10</td>
<td>( \alpha )</td>
<td>1.5</td>
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<tr>
<td>( M )</td>
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<td>( r_i )</td>
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know that the cash market price will be \( r_i \), farmers will take the contracts and confirm expectations by not planting for the spot market.

The problem for each processor reduces to the classical symmetric Cournot game with \( M \) players. That is, \( \max_{A_i} \left( p - r_i - \delta \right) A_i E(y) \), subject to \( \left( r_i + \delta \right) E(y) \geq C^i(a_i) \), where \( a_j = \left( 1/N \right) \left( \sum_{i=1}^{M} A_{ij} \right) \), \( j = 1, \ldots, N \). Using the cost function introduced in what follows, the solution is easily checked to be \( A_i^* = N \left( pE(y) - d \right) / b(M + 1) \). Plugging in the parameters from Table 1, we find that \( A_i^* = 50 \), implying that processors can work at capacity with the contracted acres (or that they are fully contracted). Note that the parameters presented in Table 1 are also consistent with the observation that producers will not plant for the spot market if they will obtain the commodity price with certainty.

To solve the model, we need to assume a probability distribution for the random yield and a functional form for the farmers’ cost function. The probability distribution for the random yield is assumed to be a three-parameter beta distribution, commonly used to model yield risk (see Babcock and Hennessy 1996), which has the following density function:

\[
    f(y) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \left( \frac{y-m}{\Delta} \right)^{\alpha-1} \left( m-y \right)^{\beta-1}, \quad 0 \leq y \leq y_m.
\]

The parameters for the density function presented in Table 1 imply that \( E(y) = 100 \). We will assume that the functional form of the cost function is the following:

\[
    C \left( a_s + a_c \right) = d \left( a_s + a_c \right) + \frac{b}{2} \left( a_s + a_c \right)^2.
\]

The first term captures the factors of production that can be easily obtained to increase the area planted, whereas the quadratic term represents the increasing costs associated with less easily adjustable factors. An example of such a factor is the managerial capacity of the farmer. Note that an implication of the functional form assumed to describe farmers’ costs is that increasing the number of acres planted for the spot market increases the marginal costs of contract production, shifting contract supply to the left. That is, higher premiums will be needed to entice farmers to take a given number of contracts when they also plant for the spot market. This specification
effectively links the supply for contract and spot production. The structure of the model implies that in any co-existence equilibrium, expected marginal revenues for contract and spot acres are equalized (this is also true in Newbery 1998).

Figure 1 shows aggregate best-response functions for farmers and processors. Since all processors are identical, the aggregate best-response function is obtained by multiplying the individual function by the number of processors. The same rule applies to the best-response function for farmers.

The equilibrium for the model is where both curves intersect. The curve labeled as “processors” indicates the equilibrium aggregate number of contracts offered (and hence taken) in the game played between processors for every aggregate spot area. The curve labeled as “farmers” denotes the aggregate acreage planted for the spot market, consistent with any given contract offer by processors. Equilibrium for the model occurs, of course, where both curves intersect. At that point, the processors’ best-response functions intersect, and beliefs about aggregate spot acreage are confirmed.

Note that the introduction of yield uncertainty results in a quite different equilibrium. Each processor reduced the number of contracts offered (from 50 to 27 acres), and room was created for production for the open market (39 acres in aggregate), as a complement to the contract market. Recognizing the presence of uncertainty as a fact of agricultural

Figure 1. Farmer and processor best-response functions for the benchmark case
production yields a very different outcome than that in a deterministic model. In particular, co-existence of contract and spot production results in equilibrium.

The intuition behind this result is simple. With uncertainty, the possibility of overbuying the input arises. The more contract acres are offered, the higher the likelihood that a fraction of the input cannot be processed and has to be sold at a loss at the ongoing commodity price. As processors offer fewer contracts, the probability that they will need additional input (and the amount needed) \( \text{ex post} \) to work at capacity increases, along with the expected spot price.

Introduction of uncertainty benefits processors and harms farmers in this setting. Yield randomness allows processors to reduce the number of contracts offered (and hence the premiums needed to get farmers to accept the contracts) while creating an expected spot price that induces farmers to plant for the cash market. Profits for farmers decrease from $3,125 to $1,509, whereas processors’ profits increase from $2,500 to $3,826.

Introduction of uncertainty allows processors to lower the contract premiums from \( \bar{\delta} = 2.5 \) to \( \delta^* = 1.74 \), where the bar on \( \delta^* \) indicates that yield is fixed.

As mentioned before, there is a growing concern that spot markets are thinning and that farmers are more exposed to the market power of processors. The model presented allows us to comment on those concerns. Suppose for a moment that processors are effectively able to discourage production for the spot market.\(^{21}\) In that case, farmers can be enticed to take contracts offering lower premiums. This clearly benefits processors and harms farmers (profits now are predicted to be $4,048 and $1,382, respectively).

However, this is not an equilibrium, if processors compete \( \text{ex post} \) as noted in the model.\(^{22}\) Although comforting, the previous argument points to the need for authorities to be vigilant concerning the behavior of spot buyers.\(^{23}\)

**Equilibrium Responses to Changes in Economic Parameters**

In this section, we examine how changes in production uncertainty, the price of the processed good, the reference price, the number of processors, and the number of farmers affect the optimal choices of farmers and processors. The first variable we examine is production uncertainty. The effects of a mean-preserving spread in yields are illustrated
by solving the model for coefficients of variation (CV) of yields ranging from 0.1 to 1. The parameters of the yield distribution for the benchmark case represent a CV of 0.5.

Figure 2a depicts the Nash equilibria for the model for the different amounts of yield uncertainty. As Figure 2a makes clear, increasing (decreasing) the level of production uncertainty confirms the reduction (increase) in the number of acres contracted. The same intuition holds. A mean-preserving spread in yield risk increases the expected excess supply of input, conditional on yields being high enough to cover all input demand. This effect tends to decrease contract acres because processors will want to reduce the amount of excess supply they have to sell at a loss. But, a mean-preserving spread in risk also increases the yield shortfall conditional on being low enough to generate positive ex post demand. However, processors know that if they marginally reduce their offer of contract acres, farmers will marginally increase their supply of spot acres. This substitution, which reduces the risk of a yield shortfall, is observed for low levels of yield uncertainty. As processors keep lowering their contract offers, farmers stop increasing their spot acreage. We speculate that this effect is due to a decrease in the premium offered with fewer contracts (see Figure 2b). Farmers’ expected marginal revenues from contracting decrease, and this slows the expansion in spot area. Recall that by the structure of the model, farmers’ expected marginal revenue in spot and contract production have to be equalized in equilibrium.

![Figure 2](image)

**Figure 2.** Effects of yield uncertainty (CV) on (a) equilibrium acres in contract and spot markets and (b) contract premium
Overall, an increase in yield uncertainty will tend to increase the relative importance of spot market activity, as processors try to avoid situations of excess supply from contracted acres. This suggests that spot markets will be more prevalent in situations where yield risk is relatively large.

Figure 3 shows how output profitability (measured by the per-unit margin) affects the set of Nash equilibrium. Higher profit margins result in more contracts offered and less area planted for the spot market. This result is very intuitive for processors—higher margins imply less willingness to operate at less than capacity. However, this result is less obvious for farmers. For the farmer, there are two forces acting in opposite directions. On the one hand, increases in the output price result in a higher return in the spot market in the case in which processors have to bid their marginal valuation to get the input. On the other hand, this situation of excess demand is less likely to arise because of the reduced expected demand in the spot market. Figure 3 shows that the latter force dominates farmers’ decisions for low output price, but for higher prices the forces even out or the former force dominates slightly.

When the output price is low enough, \( p = 2 \), the market for contracts disappear. Running at less than capacity is less expensive (in terms of foregone profits) for small margins. Also, adding premiums over the base price reduces the margin even further. Processors also recognize that if they decrease slightly the number of contracts offered,
farmers will increase their spot acreage, responding to stronger ex post demand expectations.

Analogous results apply for changes in the commodity price. Increasing the reference price increases the expected spot price for any given contract acreage, providing incentives for farmers to plant more area for that market. Contracting becomes more expensive for processors, leading them to reduce the number of contracts offered.

Figure 4 shows the effects of an increase in the number of processors, holding per-processor demand constant. This situation simulates the effects of an increase in total demand for the input, holding the number of farmers constant. An increase in demand increases the probability that processors will have to purchase in the spot market, thereby raising the input price. Ceteris paribus, this induces processors to offer more contract acres, to avoid a fierce ex post competition with other buyers. However, the increased likelihood of a strong spot demand also induces farmers to increase the area planted for the spot market, which is illustrated in Figure 4. Additionally, with increasing marginal costs, it is more difficult for individual processors to find takers for any fixed number of contracts, as the number of processing firms in the market increases. The forces just described act to reduce the number of contract acres offered by individual processors,

![Figure 4. Equilibrium responses to changes in the number of processors (M) (per-processor capacity held constant) (spot acres are per farmer; contract acres are per processor)]
though this is not enough to reduce aggregate contract offers. As a whole, the processing industry offers more contracts as it becomes larger (for a given pool of farmers) in an attempt to reduce competition in the spot market. Thus, an increase in demand, holding the number of suppliers constant, results in more contract acres.\textsuperscript{24}

Similar results are found by holding total market demand constant but increasing the number of farmers. The effect of such an increase depends on the form of the farm-level marginal cost function. If marginal costs increase with output (as in our case), then an increase in the number of (identical) farmers will reduce the premium needed to entice farmers to take a fixed contract acreage as each farmer is offered fewer contracts. This creates an incentive for processors to expand the area procured under contract, which is what we observe in Figure 5. Also, for any given number of per-farm spot acres, supply for the cash market increases, leading to a lower expected price, which creates incentives for individual farmers to reduce speculative production. However, this reduction is not as strong as reduction of the aggregate spot area.

Concluding Remarks

In this study we develop a simple theoretical model that, suitably parameterized, allows for the co-existence of contracts and spot markets in agriculture. The presentation
here focused on the production of a specialty crop. However, suitable parameterizations would afford the study of a wide variety of agricultural production processes. Numerical simulations were conducted to study the impacts of fundamental economic factors on the equilibrium outcomes. Our results suggest that for a wide range of distinct parameters, participation in both markets constitutes a Nash equilibrium for the model. This result would indicate that the fact that a growing proportion of agricultural raw products are transacted by means other than cash markets does not necessarily imply that spot markets for these sectors will disappear altogether. There is a balance to be attained between the sizes of the markets. Because the model assumes that all the market participants are risk neutral, the equilibrium outcomes are the results of purely financial considerations.

The predictions of the model make clear the substitutability between both systems of production (when the market structure remains unchanged). It is worth emphasizing, that co-existence of both markets only arose in the presence of uncertainty. For both specifications of the cash market supply function, specialization in contract markets is obtained absent uncertainty. This implies that because of its strong qualitative implications, uncertainty has to be explicitly accounted for in any modeling situation in which contracts and spot markets co-exist and production outcomes are subject to randomness. This is quite important in agriculture, for which biological uncertainty and weather variability (among other sources of randomness) play a major role.

The model demonstrates the antagonistic interests of farmers and processors concerning the relative size of the spot and contract markets. With increasing marginal costs of contracting (affected by spot acreage), processors would prefer specialization in contracting. Farmers prefer the equilibrium predicted by the model (co-existence). The fact that processors would like to be able to discourage production for the spot market makes clear that safeguards should be established to ensure vigorous competition when cash markets arise. In the current setting, producers have the option of planting without a contract (and a spot market is likely to arise). This constrains the behavior of processors since contract offers have to provide sufficient financial inducement, relative to expected spot prices, to entice farmers to take them.

The role of spot markets as a complement to contract production is illustrated. The term “complementing” is used here in the sense that the spot market serves as a residual
market in which buyers can make up for any difference in input needs, relative to their target procurement levels. Farmers can plant (speculatively) additional acres for spot markets at expected spot prices.

Although the model presented here may not capture many aspects of contracting decisions in agriculture, it is a reasonable starting point. We discuss the basic elements and indicate points at which assumptions may yield different qualitative predictions. The model could be modified in several directions to tackle different complexities that commonly arise in the markets. Examples are the introduction of quality issues or transaction costs related to the contracted versus spot market procurement. A different direction in which this research could be extended is to explore the efficiency of the contract observed relative to other coordination mechanisms, and whether co-existence also attains for other contractual arrangements.
Endnotes

1. Some examples are hogs, corn, soybeans, wheat, and cattle.

2. For an example of a model where co-existence may result in equilibrium in a non-agricultural setting, see Newbery 1998. This author models the British electricity market focusing on the use of contracts as entry-deterring devices.

3. Within value-enhanced corn, average premiums were highest for white corn and lowest for hard endosperm varieties over the 1996-2001 period (Stewart 2003).

4. For example, most specialty grain contracts use commodity prices as references. Good, Bender, and Lowell (2000) found that virtually all specialty grain handlers (corn and soybeans) in Illinois pay premiums based on cash or futures markets for commodities. Thus, it is reasonable to assume that they cannot manipulate the base price.

5. This is also recognized by Weninger and Reinhorn (n.d.), and by Sexton and Zhang (1996).

6. Note that we are not making any claim about the efficiency of this contract (observed) relative to other mechanisms. Incentives that shape the form of the contract (e.g., related to quality assurance) are taken as given and not modeled formally.

7. We motivate the target amount of input to procure as given by the capacity constraint. However, that target could also be the result of the processor’s commitments with downstream customers. For example, Good, Bender, and Lowell (2000) report that 66 percent of all firms handling white corn in Illinois contract with sellers based on acres, and 61 percent contract with buyers based on bushels. These
percentages are 80 percent and 48 percent for waxy corn, and 87 percent and 47 percent for STS soybeans.


9. This is consistent with the survey findings previously reported.

10. The resulting demand is rectangular. Processors will demand $x_s$ as long as the spot price does not exceed their marginal valuation of the input.

11. This can be written as $p = P - C$, where $P$ denotes the output price (taken as given by the processor) and $C$ represents the per-unit processing costs.

12. One could argue that there is not much strategic interaction in this scenario and that simple supply and demand analysis would yield the same outcome, since it is optimal for a processor to bid the reservation price of farmers no matter what other processors do. However, we can also say that bidding $r_i$ is a dominant strategy for all processors and we can analyze all scenarios using the same framework. This situation would be representative of what happened in the 1999 crop year for white corn. A combination of higher-than-normal yields, combined with a substantial increase in the number of acres planted by speculators (or “wildcatters”), led to excess supply conditions (Boland et al. 1999).

13. However, Sexton and Zhang’s approach differs in that they treated ex post supply and demand as being exogenous.

14. Kreps and Scheinkman (1983), Tasnadi (1999), and Levitan and Shubik (1972) addressed this problem for capacity-constrained duopolists. Weninger and Reinhorn (n.d.), studying a more closely related problem (oligopsonists facing a fixed supply), concluded that a pure strategy Nash equilibrium for this problem exists if the number of buyers is sufficiently large and the price space is discrete.
15. The mixed strategy equilibrium for $M=2$ is presented in the Appendix.

16. Here we are interpreting the supply function as giving the minimum price per unit at which firms are willing to sell any given quantity.

17. This can be written as

$$\delta(A_y) = \frac{C'(a_+ + a_-)}{E(y)} - r_i,$$

with primes indicating first derivatives.

18. These assumptions regarding the farmer’s problem closely follow Newbery 1998. Xia and Sexton (2004), stress the importance of considering rational agents on both sides of the market when modeling procurement of raw product inputs.

19. Keep in mind that when reaction functions for farmers are mentioned, those are not “true” reaction functions.

20. Note that there has to be excess demand in order for the spot market price to be between $p$ and $r_i$. This situation is ruled in the absence of uncertainty.

21. Either they are able to commit not to buy or they collude implicitly not to bid aggressively on the open market.

22. Though not presented, if the marginal costs of contract and spot production were independent, processors strictly prefer co-existence of the two markets. This result is driven by the fact that the spot market acts as a complement to the contract market without increasing the costs of contract production.

23. Note also that processors’ profits decrease and farmers’ profits increase when spot markets are viable. This may provide yet another rationale for processors to integrate backwards into farm production, as has been observed, for example, in the hog industry. See, for example, Hennessy 1996, Murray 1995, and Perry 1978 for other rationales.

24. Alternatively, if we increase the number of processors while holding aggregate capacity constant, the same pattern of reduction in contract offers by individual
processors is observed, without increasing aggregate contracted area. Actually, the total number of contracted acres declines slightly. This scenario simulates the effects of a decrease in the market power of processors, as each holds a smaller share of industry capacity (hence, their individual decisions have smaller marginal impacts). Growers also receive improved price premia in this setting, which is consistent with the findings of Elbehri and Paarlberg (2003).
References


Appendix

Mixed Strategy Equilibrium for Scenario 3 (M=2)

The argument presented here closely follows that of Levitan and Shubik (1972). A mixed-strategy equilibrium for this game is a pair of probability distributions over each player’s strategy space. These probability distributions must have the property that any strategy chosen by a player with positive probability must be optimal against the other player’s probability distribution.

The amount that firm $i$ is able to procure as a function of both offered prices in general is

$$x_{si} = \begin{cases} 
\min \left\{ \max(\bar{A}_i y - (Q - A_i y), 0), (Q - A_i y) \right\} & \text{if } r_i < r_j \\
\frac{\bar{A}_i y}{2} & \text{if } r_i = r_j \\
\min \left\{ (Q - A_i y), \bar{A}_i y \right\} & \text{if } r_i > r_j 
\end{cases}$$

However, since this scenario arises when $\frac{Q}{(\bar{A}_s + A_s)} \leq y \leq \frac{2Q}{(\bar{A}_s + 2A_s)}$, or $(Q - A_i y) \leq \bar{A}_i y \leq 2(Q - A_i y)$, and $(Q - A_i y) = x_i$ we can focus on the following case

$$x_{si} = \begin{cases} 
\bar{A}_i y - x_i & \text{if } r_i < r_j \\
\frac{\bar{A}_i y}{2} & \text{if } r_i = r_j \\
x_i & \text{if } r_i > r_j 
\end{cases}$$

Since both prices will coincide with zero probability, the expected amount procured in the spot can be written as $E\left(x_{si}\right) = (1 - \phi_i (r_i))\left(\bar{A}_i y - x_i\right) + \phi_i (r_i) x_i$, and expected profits, $\pi_i (r) = (p - r)\left(\left(1 - \phi_i (r)\right)\left(\bar{A}_i y - x_i\right) + \phi_i (r) x_i\right)$. The cumulative distribution function of player $j$ is $\phi_j (r)$. From this we obtain $\phi_j$ as
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\[ \phi_j(r) = \frac{(\bar{A}_s - x_s)(p - r) - \pi_i(r)}{(p - r)(\bar{A}_s - 2x_s)} \]

with support \([\underline{r}, \bar{r}] \subseteq [p, r_i]\). If processor \(i\) is willing to randomize, it must be the case that \(\pi_i\) is constant on \([\underline{r}, \bar{r}]\). If firm \(i\) offers \(\underline{r}\), it will be overbid with probability one.

Since it will be the residual claimant for the input, it will be optimal to bid \(r_i\) (if firm \(i\) knows it will be overbid, its payoff function is monotonically decreasing in the offer price, indicating that it is optimal to bid the reservation price of sellers). To pin down \(\bar{r}\) we use the fact that \(\pi_i(r) = \pi_i(\underline{r}) = \pi_i(\bar{r})\). It follows that \((\bar{A}_s y - x_s)(p - r_i) = x_s (p - \bar{r})\).

Solving for \(\bar{r}\) we get

\[ \bar{r} = p - \frac{(\bar{A}_s y - x_s)(p - r_i)}{x_s} \]

and

\[ \phi_j(r) = \frac{(\bar{A}_s y - x_s)(r_i - r)}{(p - r)(\bar{A}_s y - 2x_s)}. \]

**Second-Order Sufficient Conditions for the Processor’s Problem**

\[
\frac{\partial^2 \pi}{\partial A_i^2} = \left( \frac{\partial f(u)}{\partial A_i} \right) \left( \frac{\partial u}{\partial A_i^2} \right) + f(u) \frac{\partial^2 u}{\partial A_i^2} \left( u \left( \frac{\bar{A}_s}{2M} + A_i \right) - Q \right) \\
+ f(u) \frac{\partial^2 u}{\partial A_i^2} \left( \frac{\bar{A}_s}{2M} + A_i \right) + 2u \right) \\
- \left( \frac{\partial f(s)}{\partial s} \right)^2 s + f(s) \frac{\partial^2 s}{\partial A_i^2} s + f(s) \frac{\partial^2 s}{\partial A_i^2} \left( \frac{\bar{A}_s}{2M} \right) \\
- \frac{E(v)}{(p - r_i)} \left( \frac{\partial^2 \delta}{\partial A_i^2} + \frac{\partial^2 \delta}{\partial A_i^2} \right) \leq 0
\]

\(i = 1, \ldots, M\).