# Location and Marketing under Marketing Assistance Loan and Loan Deficiency Payment Programs

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## Abstract

Marketing assistance loan (MAL) and loan deficiency payment (LDP) programs differ in their treatment of basis. This paper analyzes marketing decisions under these programs when producers are differentiated by location with respect to the terminal market. The developed model may help explain the observed lack of an association between the county loan rate and the share of a county's production enrolled in MAL programs. Under certain conditions, multiple equilibria are shown to emerge. The effects of MAL and LDP programs on welfare and policy implications are discussed.

**Key words**: commodity prices, grain storage, price support programs.

# LOCATION AND MARKETING UNDER MARKETING ASSISTANCE LOAN AND LOAN DEFICIENCY PAYMENT PROGRAMS

## Introduction

As part of the governmental effort to support revenues for agricultural producers, the Federal Agriculture Improvement and Reform Act of 1996 (the 1996 FAIR Act) launched non-recourse marketing assistance loan (MAL) and loan deficiency payment (LDP) programs for the 16 major crops. These programs expanded the set of marketing strategies available to producers, who could participate in either one but not both programs. Under MAL programs, eligible producers receive a non-recourse loan by using the stored crop as collateral. The amount of the loan is equal to the value of the crop priced at the fixed loan rate adjusted by county. Farmers always have the option of repaying the loan by delivering the crop to the Commodity Credit Corporation at loan maturity. The rules for repaying the loan in cash are as follows. If the so-called posted county price (PCP) is less than the loan rate on the day of the final sale, then the loan can be repaid at that price; otherwise, producers must repay the principal plus the accrued interest and expenses. The PCP is a calculated daily price index intended to echo the actual market conditions in the county. While on any given day the PCP may differ from the prices offered by local elevators, the adjustments made to determine the PCP and the county loan rate are largely equivalent and can be thought of as the county basis for the crop in question (Babcock, Hayes, and Kaus). Under LDP programs, producers receive the difference between the county loan rate and the PCP on any date as long as they own the crop and the difference is positive.

In 1998, when for the first time since the beginning of the programs grain prices were lower than the loan rates, grain farmers were confronted with a choice between the two programs (Hayes and Babcock). In the period from 1999 to 2001, agricultural producers seemed consistently to favor LDP over MAL programs (see Table 1). For producers who sell their crop at harvest, the choice between the two programs is likely to

**TABLE 1. National ratios of quantities under LDP and MAL programs**

| Year | Corn | Soybeans | Wheat |
|---|---|---|---|
| 1999 | 0.19 | .12 | .07 |
| 2000 | 0.17 | .13 | .01 |
| 2001 | 0.19 | .12 | .27 |

Source: USDA, “Unofficial Price Support Programs Activity Reports.”

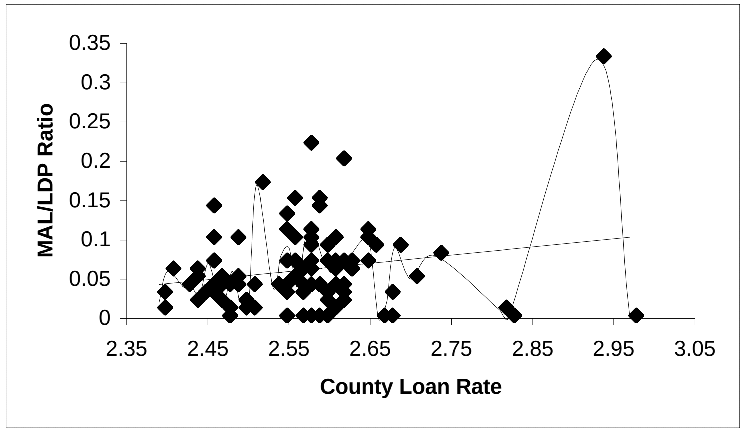
be immaterial or governed by administrative costs. However, producers who market their crop after the harvest weigh the LDP against the combined values of the loan and the call option embedded in MAL programs. While the two programs differ in many respects, this paper emphasizes the difference in the “spatial” dimension.

### County Basis Adjustment in Loan Deficiency Payment and Marketing Assistance Loan Programs

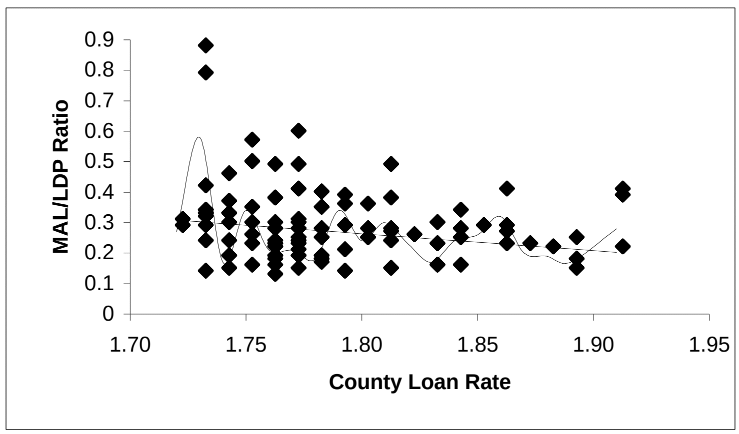
An important distinction between the effects of LDP and MAL programs on producer revenues arises because of the way the basis (or local market) conditions are incorporated into the respective payments. Babcock, Hayes, and Kaus point out that “if calculated properly, LDPs should be the same in every county in every state.” In contrast, of course by design, the loan rates for most agricultural commodities differ across counties. Therefore, growers in areas closer to terminal markets receive higher subsidies under MAL programs. This happens because the county adjustment factor vanishes from the amount repaid at loan maturity for precisely the same reason that the LDPs are invariant to location. Consequently, growers in areas with a lower basis are expected to use MAL programs more than are growers in areas far from terminal markets because the loan rates in these areas are lower.

### Contradicting Empirical Evidence

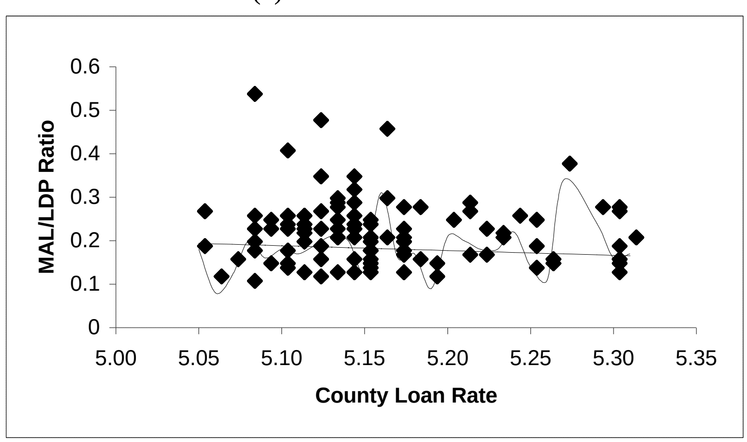
And so, the ratios of bushels under MAL and LDP programs in each county should be positively correlated with the county loan rate. However, this intuition runs counter to the empirical evidence presented in Figure 1, which plots the county ratios of bushels under MAL and LDP programs against the county loan rates for three selected crops. In each graph the smoothed dashed curve is the ratio averaged over counties with the same loan rate; the straight line is the fitted linear regression. Using ordinary least squares




(a) Kansas Wheat in 2000




(b) Iowa Corn in 2000




(c) Iowa Soybeans in 2000

Source: USDA, “Unofficial Price Support Programs Activity Reports.”

**FIGURE 1. The ratio of bushels placed under MAL and LDP programs for selected crops**

(OLS) allows us to reject the hypothesis of a positive (or negative) relationship between the ratio of counties' output enrolled in MAL and LDP programs and the county loan rates at the standard levels of statistical significance.[1] As evident from the graphs, this could be, at least in part, a result of a considerable amount of variation in the MAL/LDP ratios in counties with the same loan rate. However, fitting OLS regressions for aggregated counties (i.e., counties with the same loan rate) does not yield statistically significant estimates.

Several county or farm level attributes, other than location, are likely to affect the producer's decision to enroll in either program. For example, adjustment factors used to calculate the county loan rates are typically set once a year and are changed infrequently. However, the actual county basis fluctuates daily and may consistently deviate from the county adjustment factor. Therefore, if the actual basis is smaller than the basis assumed in the county adjustment factor, producers in that county will favor LDP programs over MAL programs. In addition, access to credit, on-farm and off-farm storage costs and capacity, and livestock production are also likely to affect marketing decisions. While carefully accounting for such idiosyncrasies at the county (or even farm) level may explain the spatial pattern of the split between the two programs in individual cases, a different approach is possible.

The goal of this paper is to reconcile the facts and the economic intuition by presenting a general argument demonstrating that location may play no role in the producer choice between the two programs. I provide a parsimonious explanation that does not involve any county-specific characteristics apart, of course, from the county loan rate. The explanation relies on endogenizing commodity prices by explicitly recognizing the supply and demand forces determining the intra-year dynamics and price paths. Because one of the conditions is that the share of the total production under MAL programs is relatively large, the suggested reasoning is more likely to apply in cases (b) and (c) in Figure 1 rather than in case (a). In panel (a), only a fraction of the area crop, wheat in Kansas, was enrolled in MAL programs in 2000 (less than 7 percent of the state production.)

The supply mechanism in geographically dispersed commodity markets is considered next. To focus on the role of location in marketing decisions, the following discussion assumes away any uncertainty characteristic of agricultural commodity markets that stems mainly from developments in international trade in a post-harvest environment.

## Prices and Marketing Patterns in Geographically Dispersed Markets

As is well known from the storage literature (e.g., Williams and Wright), in a competitive market, carry-over (ending) stocks exist when the discounted price in the next period warrants delaying the sale and incurring storage costs. Similarly, within the marketing year, the opportunity cost of a delayed sale explains why commodity prices must rise over time to stimulate storage. Benirschka and Binkley show that in a geographically dispersed market where suppliers of the terminal end-use market are differentiated by location, transportation costs determine the optimal pattern of shipping. Firms with the lowest transportation costs supply the market first because "a producer close to the market has a relatively high opportunity cost of storage, [and] he will store commodities only for a short time." Benirschka and Binkley also find that the market price grows at a rate smaller than the rate of interest because of the growing cost of shipment. This, of course, implies that the discounted market price falls as the marketing year progresses.

## Marketing Decisions under Marketing Assistance Loan and Loan Deficiency Payment Programs

Consequently, MAL programs may disrupt the optimal "sequential" marketing pattern because producers that are close to the market will choose to participate in MAL programs and defer sale. As an extreme case, imagine that all producers exclusively use MAL programs. Also, suppose that the adjustment factors used to determine the county loan rates and the PCPs coincide with the actual local basis. Then, after taking a loan, the profits of the MAL program participants who market their crop after harvest will no longer depend on location. Thus, the pattern of spatial supply (the order in which producers market their crop) is arbitrary in this case. This feature of MAL programs underlies the argument used to explain the empirical evidence presented earlier.

In contrast, LDP is a price support program that complements producer revenues without interfering with marketing decisions in the spatial dimension. This is so because

LDPs, at least theoretically, are the same for all producers, independent of their location. When intra-year prices are increasing over time, an LDP clearly has the greatest value at harvest. In that case, producers decide which program to use at harvest time because there is no value in postponing participating in either program. If both programs are in use, then producers closer to the market will take a loan and producers in more distant locations will take the relatively higher value LDP.

### Spatial and Temporal Commodity Markets with Endogenous Prices

While for most bulk agricultural commodities prices are determined in international markets, domestic supply, at least in part, must be a contributing factor. Consider equilibrium where price is determined in a central market that is supplied by spatially differentiated producers in each period between the two harvests. Imagine that there are periods when market price in equilibrium without any programs is sufficiently high (or, alternatively, let the loan rate unadjusted for county basis be relatively low.) Then, once the price support programs are introduced, MAL producers will have an incentive to store and market their crop after harvest, which will lower the post-harvest prices. In periods when MAL producers supply the market, the discounted market price must fall as fast as the discounted value of the repayment amount on the loans taken by MAL producers. Otherwise, MAL producers will not be indifferent between marketing their crop this period or the next period. On the other hand, LDP producers (in areas with lower loan rates) choose the marketing time based on their differences in transportation costs. Because transportation costs are lower than the loan rate, the discounted market price falls slower than the discounted value of the repayment amount on the loans taken by MAL producers when LDP producers supply the market. Therefore, in equilibrium, MAL producers have an incentive to wait and supply the market after LDP producers have taken action.

### Multiple Equilibria and Equilibrium Where Returns to Storage Are Independent of Location

In general, there may be multiple equilibrium time points when LDP producers give way to MAL producers in supplying the market. The set of such points is particularly large if, in the absence of the MAL program, in periods immediately following harvest, market price is high enough to warrant shipments by producers enrolling in the MAL program were such to become available. As the share of producers who opt for the MAL program increases, the discounted market price begins to fall faster. This makes the LDP option less attractive, and more producers (in the areas with higher loan rates) will have an incentive to switch to the MAL alternative. On the other hand, it may happen that the fraction of producers who use the MAL program and store their crop is small. The discounted market price then falls sufficiently slowly (following the "optimal" time pattern) to warrant the use of the LDP program by the majority of producers (in the areas with lower loan rates).

Conditions are provided on demand and supply environments such that only producers who market their crop at harvest use LDP programs. Then, only MAL program users engage in storage and supply the market during the rest of the year. Ignoring the differences in administrative costs, for producers who market their crop at harvest, the payments under the LDP and MAL programs are equivalent. Consequently, the choice between the two programs becomes invariant to location. This may help rationalize the apparent lack of a positive relationship found in Figure 1 in cases where the share of production enrolled in MAL programs is substantial.

The rest of the paper is organized as follows. First, a formal model with a fixed price sequence is developed. Then, marketing decisions under both programs, as well as the producer choice between the two programs, are analyzed. In the third section, an allowance is made for endogenous prices, and conditions sufficient for the existence of multiple equilibria are provided. The paper concludes with a discussion about the welfare implications of the analysis.

## Model

Following Benirschka and Binkley, consider a market for a single commodity (grain) that is supplied by producers differentiated by their location relative to the terminal market, $d \in [0,1]$. The focus is on the intra-year dynamics between the two harvests. Time is discrete and indexed by $t = 0,...,T+1$, where $t = 0$ represents the harvest time and $t = T$ is the end of the crop year. At $t = 0$, the distribution of grain over the uniform measure of producers is given by $F(d)$ where $F(0) = 0$, $F(1) = 1$, and $F$ is a continuous and differentiable function with $F'(d) = f(d) \geq 0 \;\; \forall d$. The per unit transport cost (identified with basis) is proportional to distance and is given by $d$.[2] If a producer chooses to sell his crop at any time after harvest, $t > 0$, then he incurs an additional storage cost $c > 0$ per unit of the crop. At time $t$, the inverse demand for grain at the terminal market is given by $p_t = P(s_t)$ where $P$ is a continuous and differentiable decreasing function with $P(0) = \infty$, and $s_t$ is the supply of grain. Function $P$ possesses a well-defined inverse denoted by $D$. There is no uncertainty, and producers discount future profits at $b = 1/(1+r) \in (0,1)$ where $r > 0$ is the per period risk-free interest rate.

### Competitive Equilibrium with Exogenous Prices

For now, let the price sequence $\{p_t\}$ be fixed and known to producers. At harvest, the present value of the unit profit of the producer located at $d$ who markets his crop at time $t$ is given by

$$p(d,t) = \begin{cases} p_0 - d, \text{if } t = 0 \\ b^t(p_t - d) - c, \text{if } t > 0 \end{cases}. \tag{1}$$

Each producer chooses the marketing time $t$ that maximizes the discounted (unit) profit

$$p(d) = \max_t p(d,t). \tag{2}$$

As a result, the equilibrium is characterized by the function $t^*(d)$ that specifies the optimal timing of marketing for each producer.[3]

RESULT 1. For any $\{p_t\}$, $t^*(d) \leq t^*(s)$ if $d < s$.

Also, Result 1 immediately follows from Theorem 2.8.2 in Topkis (p. 77) because $p(d,t)$ is supermodular in $(d,t)$: $\partial^2 p / \partial d \partial t = -b^t \ln b > 0$. Alternatively, the equilibrium can be described by the correspondences, $d^*(t) : t \rightarrow [0,1]$, that determine the set of producers that supply the terminal market at time $t$. Clearly, $d^*(t)$ is a convex interval because $p(d,t)$ is continuous and monotone in $d$ for each $t$. Let $d_t^u = \sup d^*(t)$ and $d_t^l = \inf d^*(t)$, i.e., $d_t^u$ and $d_t^l$ are the locations of the marginal producers that market their crop at $t$. Denote by $T^N = \{t : d^*(t) \text{ is not empty}\}$ the times when shipping takes place. Order elements in set $T^N$ in increasing order and number them $n = 1,.., N$

where $N \le T$ . Then we can further characterize the competitive equilibrium in terms of the following arbitrage conditions.

LEMMA 1. *For any* $\{p_t\}$, *pick* $t(i),\ t(i+1) \in T^N$ *where* $N \ge 2$. *Then*
$p_{t(i)} - d^u_{t(i)} = b^{\,t(i+1)-t(i)}(p_{t(i+1)} - d^u_{t(i)})$, $p_{t(i)} - d^l_{t(i+1)} = b^{\,t(i+1)-t(i)}(p_{t(i+1)} - d^l_{t(i+1)})$, *and*
$p_0 - d^u_0 = b^{\,t(2)}(p_{t(2)} - d^u_0) - c$, $p_0 - d^l_{t(2)} = b^{\,t(2)}(p_{t(2)} - d^l_{t(2)}) - c$, *if* $t(1) = 0$.

An immediate corollary is that in equilibrium $d^u_{t(i)} = d^l_{t(i+1)} = d^*_{t(i)}$. Now we can identify prices $\{p_t\}$ such that at each $t$ some shipping takes place, i.e., $N = T$. From Lemma 1 it follows that the equilibrium sequence $\{d^*_t\}$ is then given by

$$(p_0 - d^*_0) = b\,(p_1 - d^*_0) - c, \tag{3a}$$

$$(p_t - d^*_t) = b^{\,t}(p_{t+1} - d^*_t) - c,\ t = 1,\dots,T. \tag{3b}$$

Inspecting equations in (3) and using difference operators, $\Delta p_t = p_{t+1} - p_t$, gives:

RESULT 2. If and only if, for each $t > 0$, (a) $0 < p_0 - bp_1 - c$; (b) $\Delta p_0 > b\Delta p_1 + c$; (c) $\Delta p_t > b\Delta p_{t+1}$; (d) $p_T - bp_{T+1} < 1 - b$; then there exist unique strictly increasing $\{d^*_t\}$ given by (3).

Condition (c) in Result 2 holds when the price sequence increases at a decreasing rate. However, the price sequence that grows over time is not required for a “continuous” sequential supply in the competitive equilibrium unless the individual rationality (or participating) constraint is satisfied. If the option of staying out of production and earning zero profit is always available to producers, i.e.,

$$p\,(d) = \max_t[0, p\,(d,t)], \tag{4}$$

then condition $\Delta p_t > 0$ must, in fact, hold when shipping takes place each period.

COROLLARY 1. If in equilibrium equations (3) hold, and (i) $p\,(1) \ge 0$ or (ii) $p_T > 1$ then $\Delta p_t > 0 \ \ \forall t$.

Note that two properties of the equilibrium when market price is increasing and shipping takes place each period are interesting in light of some of the literature on storage in agricultural markets.

RESULT 3. Let conditions (a) – (d) in Result 2 hold. (i) Then we have $\Delta p_t / p_t < r$. (ii) If $\Delta p_t > 0 \ \ \forall t > 0$ then we have $\Delta(\Delta p_t / p_t) > 0$ for "small" $b$ and $\Delta(\Delta p_t / p_t) < 0$ for "large" $b \ \ \forall t > 0$.

Result 3 provides conditions such that the equilibrium (discrete) rate of growth of market prices, $\Delta p_t / p_t$, is less than the interest rate (see Benirschka and Binkley for economic intuition and econometric implications of this result). As the marketing year progresses, the rate of growth increases when the interest rate, $r$, is "large," and it decreases when $r$ is "small." While part (i) of the result is consonant with the analysis by Benirschka and Binkley, part (ii) extends their findings.

Our next task is to establish that the competitive equilibrium in (3) satisfies some efficiency criteria. Clearly, because a producer's dynamic maximization problem (2) is not subject to dynamic inconsistency and there are no market failures, competitive equilibrium must be socially optimal. We define the producers' welfare function as the sum of the discounted producer surpluses:

$$W(d_0,...,d_T) = (p_0 - d_0)F(d_0) + \sum_{t=0}^{T} \int_{d_{t-1}}^{d_t} (b^t (p_t - s) - c) dF(s). \tag{5}$$

RESULT 4. For any $\{p_t\}$, the competitive equilibrium supply pattern $\{d_t^*\}$ maximizes (5).

Now we are in a position to explore how two price support programs, LDP and MAL, interact with marketing decisions and welfare properties of the competitive equilibrium.

## Loan Deficiency Payment Program

The LDP program is a payment scheme administered by the government in order to complement revenues for the chosen agricultural commodities. An LDP is the difference between the set "loan rate" price, $L(d) = L - d$, $L > 1$, adjusted by location, and the actual price on the date producers apply for the payment given that they still own the crop at that time, $p_t - d$. An LDP is then uniform across all producers who own the crop and is given by $\max[L - p_t, 0]$. Under LDP, producer $d$'s profit can be written as

$$p^{LDP}(d,t,q)=\begin{cases}\max[L,p_0]-d, \text{if } t=q=0\\ b^q\max[L-p_q,0]+b^t(p_t-d)-c, \text{if } 0\le q\le t\end{cases}. \tag{6}$$

Now the choice variables are the time of the marketing, $t$, and the time of exercising the right to receive an LDP, $q\le t$. If $t=q$, then an LDP is equivalent to establishing the floor price $L$. Let the optimal time of marketing and receiving of the LDP for a producer located at $d$ be given by $t_L^*(d)$ and $q^*(d)$, respectively.

RESULT 5. If $\{p_t\}$ is increasing, then $q^*(d)=0$ and $t_L^*(d)=t^*(d)$ $\forall d$.

Hence, when prices at the terminal market are growing over time, the LDP program provides a means of money transfer that does not interfere with marketing decisions.

## Marketing Assistance Loan Program

An alternative payment scheme is based on loan payments. At any time prior to selling the crop, producers are eligible to obtain a "marketing" loan equal to the value of the crop priced using the fixed loan rate adjusted for basis, $L(d)=L-d$. Producers have to repay the full value of the loan only if they sell the crop at a price higher than $L-d$. For simplicity, we hold that the interest rate in that case is zero. Otherwise, the loan is repaid at the sale price that may be less than the loan rate. Therefore, this payment scheme provides loans with a non-positive interest rate to the benefit of producers. Under the MAL program, producer $d$'s profit can be written as

$$p^{MAL}(d,t)=\begin{cases}\max[L,p_0]-d, \text{if } t=0\\ L-d+b^t\max[p_t-L,0]-c, t>0\end{cases}. \tag{7}$$

All producers who market their crop at time $t>0$ always choose to obtain a loan at $t=0$ because $p^{MAL}(d)\ge L-d+b^{t^*(d)}(p_{t^*(d)}-L)-c>p(d)$, where the last inequality follows because $L>1$.

Let $k(t)=b^t\max[p_t-L,0]-c$ denote the "storage" component of the discounted profits accrued to the MAL program participants. If $p_0\le L$ and $\max_t k(t)>0$, then all producers choose to market their crop at $t_M^* = \arg\max_t\{k(t)\}$. From Result 1 it follows that the timing of marketing for producers who took a MAL payment, $t_M^*$, depends positively on the loan rate, $L$ (note that $L$ plays the role of the basis $d$ in determining $t_M^*$).

The "disruption" of the optimal sequential supply pattern originates in the fact that loan rates are adjusted for local basis. For example, if the loan rate $L(d)$ is fixed for all producers at $L$, the optimal sequential pattern persists even though the exact timing may be different. Let the optimal time of marketing for a producer located at $d$ be given by $t_M^*(d)$.

RESULT 6. For any $\{p_t\}$ with $p_0 \geq L$ and $L(d) = L \ \ \forall d$, $t_M^*(d) \geq t^*(d) \ \ \forall d$.

The following example shows that condition $p_0 \geq L$ is necessary for the result to hold.

EXAMPLE. Let $T = 1$ and $\{p_t\} = (0, p_1)$ where $bp_1 > c$. Then in the absence of any programs we have $d_0^* = 0$. Under the MAL program with $p_1 - c/b < L < p_1$, $d_0^M = [c - b(p_1 - L)]/[1 - b] > 0 = d_0^*$, which violates $t_M^*(d) \geq t^*(d)$. For $d \in [0, d_0^M]$ we have $t_M^*(d) = 0 < t^*(d) = 1$.

Thus, in contrast to LDP, a MAL program generally leads to suboptimal marketing decisions. Let $W^{LDP} = \int_0^1 p^{LDP}(z)dF$ and $W^{MAL} = \int_0^1 p^{MAL}(z)dF$ where $p^{LDP}(d)$ and $p^{MAL}(d)$ are indirect profit functions. When

$$\max_t b^t (L - p_t) = L - p_0 \geq 0 \text{ and} \tag{8}$$

$$k(t) < 0 \text{ for all } t, \tag{9}$$

the MAL program is inferior to LDP as a means of money transfer using aggregate producer surplus as a welfare criterion, $W^{LDP} \geq W^{MAL}$.[4] This follows from observing that $p^{LDP}(d) \geq p^{MAL}(d)$ for all $d$ because no MAL producers store their crop. In a general case, the comparison between producer welfares under the two programs is ambiguous because the amount of transfers depends on the price sequence (its smallest and largest elements).

### The Choice between Marketing Assistance Loan and Loan Deficiency Payment Options

When both programs are in operation, producers choose among three alternative marketing strategies: sell at harvest for $\max[L, p_0] - d$, take an LDP and store, or take a loan and store. Consider price sequences such that condition (8) holds. Then the two price support schemes, MAL and LDP, converge at $t = 0$. Hence, at harvest the choice between the two programs is indeterminate. Consider the choice between participating in LDP and MAL programs for producers marketing at $t > 0$:

$$p^{MAL}(d) = L - d + k(t_M^*) > (\leq) L - p_0 + b^{t^*(d)}(p_{t^*(d)} - d) - c = p^{LDP}(d). \qquad (10)$$

If $k(t_M^*) < 0$, then selling at harvest is clearly a better option than taking a loan and storing until $t > 0$. In this case, all producers with $t^*(d) > 0$ use the LDP program. If $k(t_M^*) > 0$, the following result presents a characterization of the relationship between the program choice and location based on the strict single-crossing property of the profit differential, $p^{LDP}(d) - p^{MAL}(d)$.

RESULT 7. Let $k(t_M^*) > 0$, $p^{MAL}(0) > p^{LDP}(0)$, $p^{MAL}(1) < p^{LDP}(1)$, and condition (8) hold. There exists unique $d_{ML} \in [0,1]$ given by $p^{MAL}(d_{ML}) = p^{LDP}(d_{ML})$ such that $p^{MAL}(d) > p^{LDP}(d)$ for all $d \in [0, d_{ML})$ and $p^{MAL}(d) < p^{LDP}(d)$ for all $d \in (d_{ML}, 1]$.

Note that while $p^{LDP}(d)$, an indirect profit function, is decreasing and convex, $p^{MAL}(d)$ decreases linearly with $d$. Hence, the point of intersection must be unique and must be in the unit interval, so that MAL producers are located in the interval $[0, d_{ML}]$, and LDP producers are in $(d_{ML}, 1]$. Otherwise, only one program, MAL if $p^{MAL}(1) \geq p^{LDP}(1)$ or LDP if $p^{MAL}(0) \leq p^{LDP}(0)$, is used when crop is put in storage. The presence of both programs (given that the payment under the LDP program is positive) may resolve much of the “distortion” in the marketing pattern caused by the MAL program when $d_{ML}$ is low. If $k(t_M^*) = 0$ producers in $[0, d_{ML}]$ are indifferent between marketing at $t = 0$ under either program, or using the MAL program to store until $t_M^*$.

The previous analysis considered fixed output prices $\{p_t\}$. This restrictive assumption is relaxed in the following section.

## Competitive Equilibrium with Endogenous Prices

Throughout the rest of the paper, we hold that $p_t = P(s_t)$ with $s_t = \int_{X_t} dF$ where $X_t$ is the set of producers who market their crop at time $t$. From Result 1, it follows that

$X_t \subseteq [0, d_t]$ where $d_t = \sup\{d : t^*(d) = t\}$, i.e, we know that nobody at $d > d_t$ will choose to supply the terminal market at time $t$. However, we also know that, for example, $X_0 \subseteq [0, d_0]$ and $D_0 \cap D_1 = \varnothing$. Then, by induction, $X_t = (d_{t-1}, d_t]$, and hence, $s_t = F(d_t) - F(d_{t-1})$ where $s_0 = F(d_0)$ and $s_{T+1} = 1 - F(d_T)$. Observe that if no marketing takes place at time $t$, $d_t = d_{t-1}$ and $s_t = 0$, which cannot be an equilibrium outcome because $P(0) = \infty$. Hence, the equilibrium price path must satisfy conditions (a)-(d) in Result 2.

The profit of producer at $d$ who markets his crop at time $t$ is, therefore, given by

$$p(d,t) = \begin{cases} P(F(d_0)) - d, \text{if } t = 0 \\ b^t(P(F(d_t) - F(d_{t-1})) - d) - c, \text{if } t > 0 \end{cases}.$$

If a producer at $d$ optimally chooses to market his crop at time $t > 0$, then

$$b^t(P(F(d_t) - F(d_{t-1})) - d) - c \geq b^l(P(F(d_l) - F(d_{l-1})) - d) - c \quad \forall l > 0. \tag{11}$$

But consider $d = d_t$ and $l = t+1$:

$$P(F(d_t) - F(d_{t-1})) - d_t \geq b(P(F(d_{t+1}) - F(d_t)) - d_t). \tag{12}$$

Condition (12) must hold with equality because otherwise there exists a producer at $d \in (d_t, d_{t+1}]$ who can make a higher profit by marketing at time $t$ instead of time $t+1$. Also, from Lemma 1 it immediately follows that (12) must hold with equality because it must be that $s_t > 0 \ \forall t$. And so, the equilibrium sequence $\{d_t^*\}$ is given by

$$P(F(d_0)) - d_0 = b(P(F(d_1) - F(d_0)) - d_0) - c$$
$$P(F(d_t) - F(d_{t-1})) - d_t = b(P(F(d_{t+1}) - F(d_t)) - d_t), \ t = 1,...,T \tag{13}$$

where $d_{T+1} = 1$.

Note that the difference equation (13) has embedded in it the two initial conditions, needed to determine the solution. Namely, we postulate that $s_0 = F(d_0) - F(0) = F(d_0)$ and $s_T = F(1) - F(d_T) = 1 - F(d_T)$. This also implies that the solution — the equilibrium sequence $\{d_t^*\}$ that satisfies (13) — is unique because, by inspection, each given pair of $d_{t-1}$ and $d_t$ uniquely determines $d_{t+1}$. The condition that in equilibrium, each producer earns a positive discounted profit can always be satisfied only if the "terminating" condition $d_{T+1}$ or $T$ is allowed to vary. This is the case when producers can postpone marketing their crop until the following crop year. In a long-run equilibrium when production and marketing decisions are tied together, it seems plausible to require that all producers make non-negative profits. Clearly, this is always true if $p(1,T) = b^T(p_T - 1) - c > b^T(P(1) - 1) - c \geq 0$. Further analyzing (13) gives the following.

RESULT 8. If $P(1) > 1$ in the equilibrium with endogenous prices, $\Delta p_t > 0 \;\; \forall t$ and $\Delta s_t = \Delta^2 F(d_t) < 0$.

Next, we characterize equilibrium with the MAL program when prices are determined endogenously.

### Equilibrium with the Marketing Assistance Loan Program When Prices Are Endogenous

As was shown previously, the timing of marketing for individual producers is indeterminate under the MAL program because it does not depend on location (see (7)). Because all producers participate in the MAL program competitive arbitrage assures that the discounted profits, $k(t) = b^t(p_t - L) - c$, are equalized across time. Then there are two distinct equilibrium outcomes:

$$p_0 = L + k \text{, if } k(t) = k > 0 \text{ for all } t > 0 \text{, and} \tag{14}$$

$$p_0 \leq L \text{, if } k(t) = 0 \text{ for all } t > 0 . \tag{15}$$

Even though profits of each producer are higher in equilibrium (14), it may not be supportable if the total available supply is large. In what follows, equilibrium in (15) will be of most relevance because the LDP program will then be operative. In contrast, competitive equilibrium with the LDP program is given by (13), and, hence, producers' marketing decisions remain unchanged relative to equilibrium without any programs. Now we turn to an investigation of the properties of equilibrium under both programs.

### The Choice between Marketing Assistance Loan and Loan Deficiency Payment Options When Prices Are Endogenous

Because the equilibrium price sequence is increasing, producer's profits under different marketing strategies can be written as

$p(d,0) = \max[L, p_0] - d$, (sell at $t = 0$)

$p^{LDP}(d,t) = \max[L - p_0, 0] + b^t(p_t - d) - c$, (take LDP, store, and sell at $t > 0$)

$p^{MAL}(d,t) = L - d + k(t)$ (take loan, store and sell at $t > 0$ if $k(t) \geq 0$).

If $L \leq p_0$ then only the MAL program is used and equilibrium is given by (14). LDPs are zero at each period because market price rises over time to assure that there is some shipping each period.

If $L > p_0$, equilibrium prices must satisfy condition $k(t) \leq 0$ for all $t > 0$; otherwise, no producers will find it profitable to sell at $t = 0$. In this case, the presence of the LDP program may resolve some of the indeterminacy of the supply pattern under the MAL program alone. Suppose that all producers abandon the LDP program in favor of the MAL program. Then the market is supplied by MAL producers and $k(t) = 0$ for all $t > 0$. However, this cannot be in equilibrium unless the set of producers, $(d_{ML}, 1]$, who prefer to use the LDP program, is empty. This may happen only if the “switch” point, $d_{ML}$, is equal to 1. Then we have

$$p^{MAL}(d) = L - d \geq L - p_0 + b^t(L - d) = p^{LDP}(d) \text{ for all } d \text{ and } t,$$

where condition $k(t) = 0$ is used to substitute for market price. However, this cannot be true for all $d$ and $t$ if $p_0 < bL + 1 - b$, i.e., if the loan rate is sufficiently high.

Now suppose that in equilibrium, $k(t) = 0$ and $k(l) < 0$, where $l > t$; i.e., no producers who opted for the MAL program chose to supply the market at time $l$. Then the LDP program participants must supply the market because $P(0) = \infty$. But none of the participants will do so, as $p^{LDP}(d,t) > p^{LDP}(d,l)$ for any $d$. Therefore, in equilibrium it must be that $k(l) = 0$ for all $l > t$ if $k(t) = 0$. On the other hand, $k(t) < 0$ implies that $k(l) < k(t)$ for all $l < t$ in equilibrium because LDP producers supply the market in these periods. In other words, after harvest, the LDP program participants supply the market before the MAL program participants do. Consequently, in equilibrium we have $d_{ML} = d_0$ because only producers in $[0, d_0]$ may choose the MAL program (see Result 7).

Next we show how the possibility of multiple equilibria arises. Observe that, in the absence of the MAL program, in equilibrium, $k(l) > k(t)$ for any $l > t$. Let $\hat{t} = \inf\{t : k(t) \geq 0\}$ denote the first time after harvest when MAL producers would supply the market in the equilibrium given by (13), i.e., in the absence of the MAL program. Here, only cases with $\hat{t} \leq T$ are of interests because otherwise equilibrium is always given by (13) and only producers marketing their crop at harvest may use the MAL program. If $\hat{t} \leq T$, equilibrium under both programs is characterized by some time $z \geq \hat{t}$, not necessarily $z = \hat{t}$, such that $z = \inf\{t : k(t) = 0\}$. This is illustrated in Figure 2.

In Figure 2, discounted prices and loan payments are plotted against time. Line A depicts the path of discounted prices in the absence of the MAL program; line B is the discounted loan payment (at its face value) plus the storage cost (per bushel). When the MAL program is in place, line A cannot be in equilibrium because $k(t) > 0$ for $t > \hat{t}$, which means that no producers will supply the market at harvest. An equilibrium price path may look like line C; it lies below the discounted loan payment curve for $t < z$ and coincides with it starting at $t = z$. Note that condition $L > p_0$ continues to hold in any equilibrium with $z \geq \hat{t}$. This is because discounted price falls slower when LDP

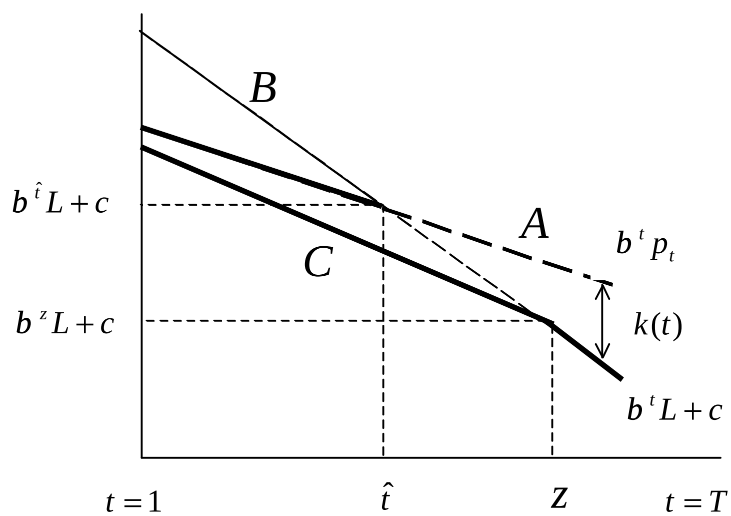



**FIGURE 2. Multiple equilibria**

producers supply the market. At $t = 0$, equilibrium is given by $p_0 = bp_1 + (1-b)d_0 - c$. Substituting condition $k(1) = b(p_1 - L) - c < 0$ yields $p_0 < bL + (1-b)d_0 < L$ because $L > 1 \geq d_0$.

As was explained earlier, because $k(t)$ is not a function of $d$, the spatial supply pattern for the MAL program participants is indeterminate. Let $a_t \in [0,1]$ denote the share of the MAL producers who store and market their crop at time $t$. Then the quantities supplied are given by $s_0 = (1-a)F(d_0)$, and $s_t = F(d_t) - F(d_{t+1}) + a_t F(d_0)$ where $a = \sum_{t=1}^{T} a_t$, and $p^{LDP}(d_t, t) = p^{LDP}(d_t, t+1)$ if $d_t \in (0,1)$ and $d_t = d_{t-1}$ otherwise. Producers in $[0, d_0]$ prefer marketing their crop at $t = 0$ (using either program) or taking a loan, storing and marketing at any $t > 0$ with $k(t) = 0$, to the LDP option. Then, for a fixed time $z \geq \hat{t}$, equilibrium with both programs is given by sequences $\{d_t^*\}$ and $\{a_t^*\}$ such that

$$P((1-a^*)F(d_0^*)) - d_0^* = b(P(F(d_1^*) - F(d_0^*)) - d_0^*) - c \text{ for } t = 0, \tag{16a}$$

$$P(F(d_t^*) - F(d_{t-1}^*)) - d_t^* = b(P(F(d_{t+1}^*) - F(d_t^*)) - d_t^*) \text{ for } t = 1,...,z-1, \tag{16b}$$

$$P(F(d_z^*) - F(d_{z-1}^*)) - d_z^* = b(L + cb^{-z} - d_z^*), \text{ and}$$
$$b^z(P(1 - F(d_z^*) + a_z^* F(d_0^*)) - L) - c = 0 \text{ for } t = z, \tag{16c}$$

$$b^t(P(a_t^* F(d_0^*)) - L) - c = 0,\ d_t = d_z \text{ for } t = z+1,...,T, \tag{16d}$$

where $a^* = \sum_{t=z}^{T} a_t^*$, $b^t(P(F(d_t^*) + F(d_{t-1}^*)) - L) - c < 0$, and $a_t^* = 0$ for $t = 1,...,z-1$.

In general, in equilibrium with $z \leq T$, the supply pattern (timing of marketing) for producers in $[0, d_0]$ is indeterminate because they are indifferent between marketing their crop at $t = 0$ and taking a loan and storing until $t \geq z$. Starting at $t = 1$, LDP producers in $(d_{t-1}, d_t]$ market their crop at time $t$ until $t = z-1$. At time $z$, producers in $(d_{z-1}, 1]$ market their crop along with some MAL producers in $[0, d_0]$. Starting at $t = z+1$, only MAL producers supply the market.

Consider an extreme case with $z = \hat{t} = 1$. Then MAL producers must be indifferent to marketing their crop in any period. The case of equilibrium where LDP producers supply the market (along with MAL producers) only at $t = 0,1$ is given by

$$P((1-a^*)F(d_0^*)) - d_0^* = b(L - d_0^*)\text{, for } t = 0 \tag{17a}$$

$$b(P(1-(1-a_1^*)F(d_0^*)) - L) - c = 0\text{, for } t = 1 \tag{17b}$$

$$b^t(P(a_t^* F(d_0^*)) - L) - c = 0\text{, for } t = 2,...,T\text{ .} \tag{17c}$$

The following result is obtained.

RESULT 9. If (a) $L$ is large (i.e., $L > p_0$); and (b) $D(1-b+bL) + \sum_{t=1}^{T} D(L + cb^{-t})$ $< 1$, then the LDP program must be used in equilibrium.

In contrast, if condition (b) in the result does not hold, and equilibrium with $z = 1$ is realized, then no LDP producers choose to store their crop because $p(d,0) = L - d >$ $p^{LDP}(d,t)$ for any $t > 0$ and $d$ . In this equilibrium, even though both programs are available, the pattern of spatial supply (the order in which producers supply the market) cannot be ascertained for any producer.

In the absence of any programs, the marketing pattern $\{d_t^*\}$ in competitive equilibrium maximizes social welfare measured by the sum of discounted producer and consumer surpluses for $t = 0,...,T$ . Therefore, the LDP program is a socially desirable means of income transfer because it does not interfere with marketing and consumption decisions. In general, equilibrium "switching times", $\hat{t} \le z \le T$ , cannot be ranked in terms of social welfare unless some assumptions are made about the composition of $a_t F(d_0)$, i.e., the order of marketing among the MAL producers. However, it seems plausible that equilibrium with $z = T$ may be socially preferable to equilibrium with $z < T$ because the amount of "unordered supply", $a^* F(d_0)$, is decreasing with $z$ .

## Basis and County Loan Rate Adjustment Factors

In the previous analysis, the adjustments in the county loan rates, $d$ , were held equal to the actual transportation costs (basis). In this section, this assumption is dropped and its implications for the marketing decisions are illustrated. Let the actual basis in location (county) $d$ be given by $b(d)$. Then producer profits are given by

$p(d,0) = \max[L, p_0] - b(d)$, (sell at $t = 0$)

$p^{LDP}(d,t) = \max[L - p_0, 0] + b^t(p_t - b(d)) - c$ (take LDP, store, and sell at $t > 0$)

$p^{MAL}(d,t) = L - d + b^t(\max[p_t - L, 0] - b(d) + d) - c$ (take loan, store, and sell at $t > 0$).

It is straightforward to see that this now makes possible the "alternating" marketing pattern, where as the adjustment factor, $d$, increases, producers switch between the two programs several times. For example, we have $p(d,0) > (\le) p^{MAL}(d,t)$ as $d - b(d) > (\le)$ $(b^t \max[p_t - L, 0] - c)/(1 - b^t)$.

## Discussion

The major limitation of the analytical approach used in this paper is ignoring the effects of uncertainty on marketing decisions. Broadly speaking, this is likely to bias the analysis in favor of the LDP program because the "time value" arising because of price volatility of a call option inherent in MAL programs is set at zero. On the other hand, LDP programs can also be thought of as offering producers a long position in a put option that pays off when commodity prices are low. However, in the case of LDP programs, the value of the put option needs to be counterbalanced with the opportunity cost of foregoing the sale, as was explained in the introduction. A distinct source of uncertainty that may warrant inquiry is the potential discrepancy between the PCP and the actual prices offered by elevators and grain processors in the county. Discrepancies occur when local market conditions temporarily deviate from adjustment factors, used to determine the PCP based on prices in the selected major grain markets.

Nevertheless, the main message of the paper is not likely to change when a "small" amount of uncertainty is introduced. LDP programs appear to be a welfare-enhancing means of income transfer because they do not entail any changes in marketing decisions. In contrast, MAL programs strip producers of any incentive to supply the market in a sequence that constitutes the optimal spatial pattern. The fact that farmers were so eager to embrace LDP alternatives when the harvest-time prices triggered these programs provides some empirical support for the spatial arbitrage argument used in this paper.

Namely, LDP programs preserve the spatial price structure countenanced by producers, and they induce producers to choose marketing time to capitalize on their differences in transportation costs.

## Endnotes

1. Maximum likelihood estimation that accounts for spatial autocorrelation among adjacent counties reduces the statistical significance of the estimates. Because the spatial autocorrelation coefficient is positive, OLS yields downward-biased estimated variances when applied to spatial data (e.g., Benirschka and Binkley.)
2. Note that the maximum transportation cost (or basis) and the total amount of production are both normalized to 1.
3. Proofs are provided in the Appendix.
4. For example, condition (8) is satisfied if market price rises over time.

## Appendix

*Proof of Result 1*. From (2) in the text it follows that $p(d, t^*(d)) \geq p(d, t)$ for any $t$. In particular, for $t = t^*(g)$, where $g \in [0,1]$,

$$b^{t^*(d)}(p_{t^*(d)} - d) - \hat{c}(t^*(d)) \geq b^{t^*(g)}(p_{t^*(g)} - d) - \hat{c}(t^*(g)), \tag{A1}$$

where $\hat{c}(t) = 0$ if $t = 0$ and $\hat{c}(t) = c$ if $t > 0$.

Similarly, we can write

$$b^{t^*(g)}(p_{t^*(g)} - g) - \hat{c}(t^*(g)) \geq b^{t^*(d)}(p_{t^*(d)} - g) - \hat{c}(t^*(d)). \tag{A2}$$

Summing (A1) and (A2) yields

$$(b^{t^*(g)} - b^{t^*(d)})d \geq (b^{t^*(g)} - b^{t^*(d)})g .$$

Hence, the result follows.

*Proof of Lemma 1*. Imagine that

$$p_{t(i)} - d^u_{t(i)} > b^{t(i+1)-t(i)}(p_{t(i+1)} - d^u_{t(i)}) .$$

Then $d^u_{t(i)}$ cannot be an equilibrium marginal producer who markets her crop at $t(i)$ because for some producers in $d \in (d^u_{t(i)}, d^u_{t(i+1)})$ it is more profitable to market their crops at $t(i)$ as well. Similarly,

$$p_{t(i)} - d^u_{t(i)} < b^{t(i+1)-t(i)}(p_{t(i+1)} - d^u_{t(i)})$$

cannot hold in equilibrium because then some producers in $d \in (d^l_{t(i)}, d^u_{t(i)})$ will increase their profits by marketing their crop at $t(i+1)$. The same reasoning delivers the other statements.

*Proof of Result 2*. Conditions (a)–(d) assure that there exist unique $z_t \in (0,1)$ such that equations (3) hold and $z_{t-1} < z_t$ for each $t > 0$:

$$p_0 - z_0 = b(p_1 - z_0) - c \tag{A3a}$$

$$p_t - z_t = b(p_{t+1} - z_t) \text{ for } t > 0 . \tag{A3b}$$

Now we show that $\forall d \in (z_{t-1}, z_t)$ $p(d,t) > p(d,l)$ for $l \neq t$. From (A3) it follows that for any $d < z_t$, $p(t,d) > p(t+1,d)$. But because we know that $z_t < z_l$ for any $l > t$ and $d < z_t$, it follows that $p(t,d) > p(l,d)$. Also, from (A3) it follows that for any $d > z_{t-1}$ we have $p(t-1,d) < p(t,d)$. But because we know that $z_l < z_t$ for any $l < t$, it also follows that $p(l,d) > p(t,d)$ for any $d > z_{t-1}$. Hence, we establish that $z_t =$ $\sup\{d : p(t,d) > p(l,d)\ \forall l \neq t\} = \inf\{d : p(t+1,d) > p(l,d)\ \forall l \neq t+1\}$, which implies that $d_t^* = z_t$. This proves the sufficiency part.

The necessity part follows from solving equations (A3) for $d_t^* = z_t$ and verifying that conditions (a) – (d) must hold if $0 < d_t^* < d_{t+1}^* < 1$ for each $t$, where $d_0^* = (p_0 - bp_1 + c)/(1-b)$, and $d_t^* = (p_t - bp_{t+1})/(1-b)$ for $t > 0$. For example, from $d_t^* < d_{t+1}^*$ it follows that condition (c) must hold: $\Delta p_t = p_{t+1} - p_t >$ $b(p_{t+2} - p_{t+1}) = b\Delta p_{t+1}$.

*Proof of Corollary 1.* Let condition (i) hold. Equations (3) in the text give

$$\Delta p_0 = (1-b)(p_1 - d_t) + c = (1-b)(\Delta p_0 + p_0 - d_t) + c ,$$

$$\Delta p_t = (1-b)(p_{t+1} - d_t) = (1-b)(\Delta p_t + p_t - d_t) \quad \forall t > 0 .$$

Solving for $\Delta p_t$ yields

$$\Delta p_t = \frac{1-b}{b}(p_0 - d_0) + \frac{c}{b} = \frac{1-b}{b} p(d_0) + \frac{c}{b} \geq \frac{1-b}{b} p(1) + \frac{c}{b} > 0 ,$$

$$\Delta p_t = \frac{1-b}{b}(p_t - d_t) = \frac{1-b}{b^{t+1}} p(d_t) + \frac{1-b}{b} c \geq \frac{1-b}{b^{t+1}} p(1) + \frac{1-b}{b} c > 0 \quad \forall t > 0 . \tag{A4}$$

Now let condition (ii) hold. From condition (d) in Result 2 it follows that $p_{T+1} > (p_T - 1 + b)/b > p_T$ where the last inequality holds when (ii) is satisfied. From condition (c) in Result 2 it follows that $\Delta p_t > 0$ if $\Delta p_T > 0$.

*Proof of Result 3.* From the conditions imposed on market prices it follows that $p_t > b\, p_{t+1}$ $\forall t > 0$. Rewriting this inequality with $b = 1/(1+r)$ yields $(1+r)p_t > p_{t+1}$, or $\Delta p_t / p_t < r$. From equation (A4) in the proof of Corollary 1 it follows that $\Delta p_t > 0$ implies $p_t > d_t$ $\forall t > 0$. Also using (A4), we can write

$$\frac{\Delta p_{t+1}}{p_{t+1}} - \frac{\Delta p_t}{p_t} = \frac{1-b}{b}\left(\frac{d_t}{p_t} - \frac{d_{t+1}}{p_{t+1}}\right) = \frac{1-b}{b}\left(\frac{d_t}{bp_{t+1} + (1-b)d_t} - \frac{d_{t+1}}{p_{t+1}}\right).$$

Observe that the denominator in the first term of the last difference converges to $p_{t+1}$ ($d_t$) as $b \to 1$ ($b \to 0$). This, in combination with $p_{t+1} > d_{t+1} > d_t$, completes the proof.

*Proof of Result 4.* Note that function (5) in the text is separable in $(d_0, ..., d_T)$. The FOC and SOSC for the maximization problem are given by

$$\frac{\partial W}{\partial d_t} = f(d_0)(p_0 - d_0 - (b(p_1 - d_0) - c)) \le 0, \tag{A5}$$

$$\frac{\partial W}{\partial d_t} = f(d_t)(b^t(p_t - d_t) - c - (b^{t+1}(p_{t+1} - d_t) - c)) \le 0,\ t = 1,...,T, \tag{A6}$$

$$\frac{\partial^2 W}{\partial d_t^2} = f(d_t)b^t(-1+b) \le 0,\ t = 0,...,T. \tag{A7}$$

Comparing (3) in the text with (A5) and (A6) proves the result.

*Proof of Result 5.* The case with $p_0 > L$ is trivial. Consider $p_0 < L$. Then all producers optimally choose $q(d) = 0$. This follows from differentiating $b^t \max[L - p_t, 0]$ with respect to $t$. Alternatively, the technique used in proving Result 1 can be applied. Next, we show that the optimal timing of shipping is not affected. The equilibrium $t_L^*(d)$ is characterized by the condition

$$L - p_0 + b^{t_L^*(d)}(p_{t_L^*(d)} - d) - \hat{c} \ge L - p_0 + b^{t_L^*(l)}(p_{t_L^*(l)} - d) - \hat{c}\ \ \forall l. \tag{A8}$$

Canceling $L - p_0$ confirms the equivalence of (A8) and (A1).

*Proof of Result 6.* Clearly, for any $t \in R = \{t : p_t \le L, 0 < t < T\}$ no sales take place because discounted profits can be increased by waiting until $t$ such that $p_t > L$ or $t = T$. Rewrite $p^{MAL}(d,t)$ for $t \notin R$

$$p^{MAL}(d,t) = \begin{cases} \max[L, p_0] - d, \text{if } t = 0 \\ L + b^t(p_t - (L+d)) - c, \text{if } t \notin R \end{cases}.$$

The equilibrium $t_M^*(d) > 0$ is characterized by conditions

$$p_0 - (d+L) \ge b^{t_M^*(g)}(p_{t_M^*(g)} - (d+L)) - c, \text{ if } t_M^*(d) = 0 \tag{A9a}$$

$$L + b^{t_M^*(d)}(p_{t_M^*(d)} - (L+d)) - c \ge L + b^{t_M^*(g)}(p_{t_M^*(g)} - (L+g)) - c \quad \forall g \in [0,1]. \tag{A9b}$$

Inspecting (A9) and (A1), we conclude that $t_M^*(d) = t^*(d+L) \ge t^*(d)$ for all $d$.

*Proof of Result 7.* Note that it is enough to show that if $p^{MAL}(d) \le p^{LDP}(d)$ for some $d$, then $p^{MAL}(g) < p^{LDP}(g)$ for any $g > d$. By definition of $p(g)$, we have

$$p(g) \ge b^{t^*(d)}(p_{t^*(d)} - g) - c.$$

By assumption, the following holds for $d$:

$$b^{t^*(d)} p_{t^*(d)} - c > p_0 - (1 - b^{t^*(d)})d + k(t_M^*).$$

Combining the last two inequalities gives

$$p(g) > p_0 - (1 - b^{t^*(d)})d + k(t_M^*) - b^{t^*(d)} g > p_0 - g + k(t_M^*).$$

Substituting $p^{LDP}(g) = L - p_0 + p(g)$ confirms the statement. Observe that condition $k(t_M^*) > 0$ is only needed to avoid the indeterminacy of the producer's choice at $t = 0$.

*Proof of Result 8.* Because $P(1) > 1$, it follows that for any $d \in (d_{t-1}, 1]$ $P(F(d) - F(d_{t-1})) > d$ for each $d_{t-1} < 1$. Using the technique from Corollary 1 gives

$$\Delta p_0 = \frac{1-b}{b}(P(F(d_0) - d_0) + \frac{c}{b} > 0 \text{ and } \Delta p_t = \frac{1-b}{b}(P(F(d_t) - F(d_{t-1})) - d_t) > 0.$$

This implies that $s_t < s_{t-1}$ $\forall t$ and thus

$$\Delta^2 F(d_t) = F(d_{t+1}) - F(d_t) - (F(d_t) - F(d_{t-1})) = \Delta s_t < 0 .$$

*Proof of Result 9*. Note that the LDP program may not be used in equilibrium with $L > p_0$ only if $z = t_M = 1$. Hence, we focus on this case. Solving equations (17b) and (17c) in the text yields

$$a_1^* = 1 - (1 - D(L + c / b)) / F(d_0) ,$$

$$a_t^* = D(L + cb^{-t}) / F(d_0) ,\ t = 2,...,T .$$

Hence, we can write

$$a^*(d_0) = 1 + [\sum_{t=1}^{T} D(L + cb^{-t}) - 1] / F(d_0) .$$

Note that $\partial a^* / \partial d_0 = (1 - a^*) f(d_0) / F(d_0)$. Differentiating $P((1 - a^*(d_0))F(d_0))$ with respect to $d_0$ yields

$$\frac{\partial P((1 - a^*(d_0))F(d_0))}{\partial d_0} = P'[-(1 - a^*) f(d_0) + (1 - a^*) f(d_0)] = 0 .$$

Then (17a) has a unique solution $d_0^* \in (0,1)$ if

$$P(1 - a^*(1)) < 1 - b + bL .$$

Substituting for $a^*(1)$ and inverting completes the proof.

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