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Economics Letters 85 (2004) 365–371

economics letters

www.elsevier.com/locate/econbase

Nash equilibrium in strictly competitive games: live play in soccer

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Received 10 February 2004; received in revised form 19 April 2004; accepted 17 June 2004
Available online 8 September 2004

Abstract

I model a scoring situation that arises frequently during soccer matches. The Nash equilibrium solution is shown to be broadly consistent with the conventional wisdom of experts. Data on goals scored over an entire season in Italy's soccer league provide statistical support for a prediction of the model's Nash equilibrium.

Keywords: Mixed strategies; Nash equilibrium; Soccer

JEL classification: C7

1. Introduction

As game theory increasingly is relied upon to model the ubiquitous strategic interactions that characterize many economic problems, economists have turned to games proper to validate a central tenet of game-theoretic explanations—the notion of Nash equilibrium. Walker and Wooders (2001) analyze data on first serves in 10 top tennis matches and find some support for the minimax hypothesis: the server's winning probabilities associated with the two actions being mixed (left and right serves) are statistically equal. Chiappori et al. (2002) turn to penalty kicks in professional soccer leagues and conclude that the data are consistent with the players optimally choosing mixed strategies. Palacios-Huerta (2003) also resorts to soccer penalty kicks to test two implications of Nash equilibrium in mixed

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0165-1765/$ - see front matter
doi:10.1016/j.econlet.2004.06.003

strategies. He finds that winning probabilities are statistically equal across strategies and that the players' choices are independent. These authors emphasize that one of the advantages of using sports data to test Nash equilibrium (relative to experimental data, for example) is that players are highly motivated and skilled at the game being analyzed. They also note that the strategic games analyzed are ideal for the purpose of statistical testing because of their simple structure. In particular, for both tennis first serves and soccer penalty kicks, the simultaneous interaction that is studied begins after play has stopped, and players have had time to collect their thoughts and concentrate on the task at hand.

Ease of testing the theory here comes at the cost of generality. It would be desirable to find support for Nash equilibrium in more complex situations. To that end, in this letter, I propose to analyze a strategic interaction that takes place during live soccer play, when the conditioning pressure of the environment of the game is higher. Specifically, the strategic setting I isolate concerns shots on goals taken from off-center positions in soccer matches. The conventional wisdom of experts here is that the goalkeeper should defend the near post, and the striker should try to score on the far post. As emphasized to me during my training for a U.S. Youth Soccer National coaching license, the rationale is that a shot to the far post is typically more difficult than a shot to the near post, and so the goalkeeper must deny the easy option to the striker (who, in turn, is then left with the far-post alternative).[1] Such precepts (and perhaps the need for a more sophisticated explanation) are clearly illustrated by the following quote from an international commentator during the Germany–Paraguay match of the 2002 World Cup.[2] The play sets up the German player, Schneider, against Paraguay's goalkeeper, Chilavert, and here is how ESPN's Seamus Malin[3] reviews the instant replay of this missed opportunity for Germany: "Oh, Schneider must do better with this . . . Neuville makes a good run down . . . that's a lovely ball inside. Schneider has got to pick out the far post. He is going that way. But give some credit to Chilavert here. He anticipates a little bit of that. He gave up his near post, and cheated a bit, but it paid off."

One may be skeptical of a literal interpretation of the soccer conventional wisdom concerning the near-post/far-post tradeoff, given its apparent reference to dominant (pure) strategies in a situation that likely requires players to be unpredictable. But it turns out that an orthodox interpretation is possible. To show that, I develop a simple explicit model that captures the fundamental nature of the strategic interaction between a striker and a goalkeeper in the described situation, and I find that the essential elements of the soccer experts' opinion on the matter can be consistent with the property of Nash equilibrium play. I also present original data from all goals scored in an entire season in a major professional soccer league (Italy's) and find that such data is consistent with the Nash equilibrium of the game structure that is postulated.

2. The model

Because the strategic interaction of interest is fairly complex, the model I present is necessarily a simplification, but one that I believe captures the essential elements of the problem. Specifically, I

[1] Ancillary considerations on the near-post/far-post tradeoff are proffered by experts, but their importance is generally considered marginal and are ignored here.

[2] This round-of-16 match was played on June 15, 2002, and the event in question occurred at the 47th minute of play.

[3] Seamus Malin was Harvard's leading soccer scorer in the 1960s, coached for 15 years, and has been a soccer commentator for many years for major U.S. TV stations.

postulate that when the opportunity arises for a player (the striker) to take a shot at the opponent's goal from an off-center position, he has two actions available: he can choose to shoot to the near post (N) or to shoot to the far post (F). The goalkeeper's problem is represented as a choice of position between the near post and the far post. Specifically, the goalkeeper chooses a position $p \in [0,1]$, where p measures the distance from the far post. The game is best viewed as sequential. The goalkeeper selects his position p prior to the shot, and the striker chooses the direction of the shot (either N or F) having observed p. The outcome of the shot is stochastic; a shot aimed at the desired direction can miss. Specifically, we assume that a shot aimed at the near post follows its intended trajectory with probability $n \in (0,1)$, whereas a shot aimed at the far post reaches its intended target with probability $f \in (0,1)$. Fig. 1 illustrates the game.

The accuracy of a shot, along the intended trajectory, obviously depends on the skills of the striker. It also depends on the field position from which it is taken (angle of the striker's position relative to the goal, as well as distance from the goal). To allow for such effects, let $n=\theta a_{\mathrm{N}}(\lambda)$ and $f=\theta a_{\mathrm{F}}(\lambda)$, where $\theta \in (0,1)$ indexes the skill of the striker, $\lambda \in L \subset R_{+}^{2}$ denotes the location from where the shot originates, and a_i: $L \rightarrow [0,1]$, $i \in \{N, F\}$, are accuracy functions that depend on the location of the shot. Note that, with this parameterization, an increase in the striker's skill increases his accuracy for both far-post and near-post shots, but the ratio f/n is assumed invariant to skill. Because the probabilities n and f reflect the striker's accuracy, and in keeping with the experts' opinion discussed in the Introduction, I assume:

Condition 1. Near-post shots are more accurate than far-post shots, i.e., $a_{\mathrm{N}}(\lambda) > a_{\mathrm{F}}(\lambda)$, $\forall \lambda$.

Whether the shots that reach the goal actually score also depends on the keeper's ability (who can, in fact, react to the shot) and, more to the point, on the position p chosen by the goalkeeper. We model the scoring outcome as follows. A missed shot never scores (thus, with little loss of generality, we do not distinguish between shots that miss by going high or wide or towards the center of the goal). On the other hand, the probability that a correctly executed shot will score is inversely related to the goalkeeper's reaction ability, indexed by the parameter is $\kappa \in (0,1)$, and inversely related to how close the position p of the goalkeeper is to the shot direction chosen by the striker. This being a zero-sum game,

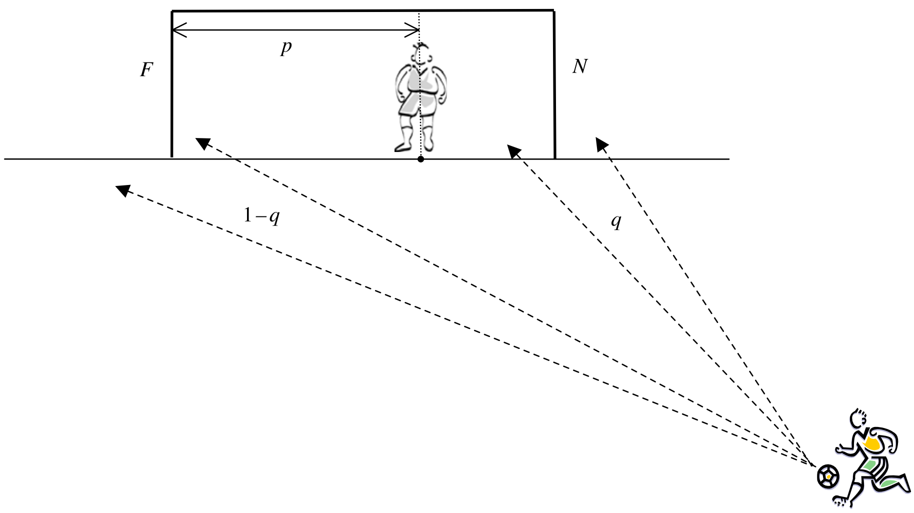

Fig. 1. The model.

we can focus on the payoff to the striker (his scoring probability), which, based on the foregoing, we write as:

$$\pi(N,p|\theta,\kappa,\lambda) = \theta a_{\mathrm{N}}(\lambda)(1-p)(1-\kappa) \tag{1}$$

$$\pi(F,p|\theta,\kappa,\lambda) = \theta a_{\mathrm{F}}(\lambda)p(1-\kappa) \tag{2}$$

We can immediately deduce that there exist no pure strategy Nash equilibria. The reasons for that are obvious. The goalkeeper's best response to the striker choice of F is p=0, and the goalkeeper's best response to the striker choice of N is p=1. But there does exist a Nash equilibrium where the striker mixes between the near and the far post. To characterize this equilibrium, let $q \in [0,1]$ denote the striker's probability of using action N [such that (1-q) is the probability of using action F].[4] The pair of strategies (p*, q*) that constitutes the Nash equilibrium of the game is then easily computed:

$$p^* = \frac{n}{f+n} = \frac{a_{\mathrm{N}}(\lambda)}{a_{\mathrm{N}}(\lambda)+a_{\mathrm{F}}(\lambda)} \tag{3}$$

$$q^* = \frac{f}{n+f} = \frac{a_{\mathrm{F}}(\lambda)}{a_{\mathrm{N}}(\lambda)+a_{\mathrm{F}}(\lambda)} \tag{4}$$

It is readily verified that this equilibrium is unique, and it is also subgame perfect. Note that, in equilibrium, the optimal strategies of both striker and goalkeeper depend on the location λ but are independent of the striker's skill parameter θ and of the goalkeeper's skill parameter κ. Furthermore, given Condition 1, that is $f(\lambda) < n(\lambda)$, we have:

Result 1. Under the assumption that the near-post shot is easier (Condition 1), the striker favors the far post (i.e., $q^* < 1/2$), and the goalkeeper favors the near post (i.e., $p^* > 1/2$).

Thus, the conventional wisdom that the striker should favor the far post and the goalkeeper favors the near post is vindicated (although the explanation provided here is somewhat more sophisticated than that articulated by soccer practitioners).

It is of some interest to note the comparative statics implications of our Nash equilibrium. Specifically, $\partial p^*/\partial n > 0$, $\partial p^*/\partial f < 0$, $\partial q^*/\partial n < 0$, and $\partial q^*/\partial f > 0$. Thus, a higher striker's accuracy in aiming at a given post in equilibrium induces the goalkeeper to cover that post more, but results in the striker favoring the other post. The (seemingly counterintuitive) result for the striker highlights a standard feature of mixed strategy equilibria, which arises because the choice of an agent's equilibrium strategy must have the property that it keeps the opponent indifferent between pure strategies (it denies a dominant pure strategy). It is also noteworthy that the equilibrium strategies p* and q* are independent of the skill parameters of the striker and goalkeeper. Remarkably, what is a good rule for amateurish soccer players may remain a good rule for professionals, which would make it easier for the rule to be learned as a player develops his skills.

[4] The model presented can alternatively be viewed as one with simultaneous moves, where the goalkeeper plays a role similar to the receiver in the tennis model of Walker and Wooders (2001), so that p defines the mixing probabilities for the two actions available to him (prepare for a near-post or a far-post shot).

3. Empirical evidence

The foregoing results could be used in a positive framework to investigate whether the notion of Nash equilibrium, and the particular parameterization of the game that I have developed, can in fact be supported by observation. Ideally, one would want to record all shots on goal taken, determine whether the striker had a choice between near and far posts, decide whether the shot taken were aimed at the near post or at the far post, and note whether or not a goal was scored. Such data would allow a direct test of Nash equilibrium in terms of the success probabilities of the pure strategies that are used in equilibrium. But that is not practical in our context for at least two reasons. First, it would require monitoring entire games, which is extremely time consuming and also limiting, because only a selected few games are generally shown in their entirety on television. Second, the angle offered by television images often does not allow a conclusive observation of whether a particular shot was directed to the near or the far post.

The alternative tack used here is to consider as data only shots that result in outright goals. The advantage of this approach is that, for major soccer leagues, all goals are typically shown by various television broadcasts, and goals are shown from multiple viewpoints, thus making it easier to determine the direction of the shot. Let g_{N} and g_{F} denote the probability of observing a goal scored at the near and far posts, respectively, given that a shot to either post is taken. In equilibrium, therefore, $g_{\mathrm{N}}^{*}=E[q^{*}(1-p^{*})(1-\kappa)\theta n(\lambda)]$ and $g_{\mathrm{F}}^{*}=E[(1-q^{*})p^{*}(1-\kappa)\theta f(\lambda)]$. Hence,

$$g_{\mathrm{F}}^{*}-g_{\mathrm{N}}^{*}=\int_{A}\frac{\theta a_{\mathrm{N}}(\lambda)a_{\mathrm{F}}(\lambda)(1-\kappa)}{\left[a_{\mathrm{N}}(\lambda)+a_{\mathrm{F}}(\lambda)\right]^{2}}[a_{\mathrm{N}}(\lambda)-a_{\mathrm{F}}(\lambda)]dH(\theta,\kappa,\lambda) \tag{5}$$

where $H(\theta, \kappa, \lambda)$ denotes the distribution function of skill and shot location parameters, and $A\equiv(0,1)\times(0,1)\times L$.

Result 2. Given Condition 1, in a Nash equilibrium, one should observe more goals scored at the far post than at the near post (i.e., $g_{\mathrm{F}}^{*}>g_{\mathrm{N}}^{*}$).

Of course, this prediction of the model characterizes a necessary (given Condition 1) but not sufficient condition for outcomes to be from Nash equilibrium play. As such, it is not as sharp, in relation to the notion of Nash equilibrium, as the equality of success probabilities across strategies that was tested by the studies cited earlier. It does however carry some interest. To check the prediction of Result 2, I undertook to classify all goals scored in the Italian Serie A in the season 2002–2003. Serie A is considered one of the top three professional soccer leagues in the world, together with Spain's Liga and the English Premier League. Eighteen teams took part in this league, each playing every other team twice (at home and away). Thus, the season involved a total of 304 matches grouped in 34 match days played over a 9-month period, from September 15, 2002 to May 24, 2003. All goals scored were viewed from television broadcasts[5] and classified according to the mode of scoring. In this process, the first question was to decide whether the striker objectively has a meaningful choice of whether to direct the shot at the near or at the far post. For a number of goals scored (such as those scored from a penalty, from a header, or directly from a free kick), the choice clearly does not apply. In addition, goals scored on a live-action

[5] These include "La Giostra del Gol," "Novantesimo Minuto," and "La Domenica Sportiva" all available to North American viewers.

shot were deemed not to present a meaningful choice, for our purposes, when the goal involved a short-range deflection or tap-in, or when the shot was taken from a central position (relative to the goal), such that no obvious near or far posts could be identified. Goals for which a meaningful choice existed were then classified according to whether they were scored at the near post or at the far post.

A total of 783 goals were scored in this 2002–2003 Serie A season, for an average of 2.58 goals per match. All considered, every goal was allocated into one of seven categories according to the mode of scoring. The seven categories (and the corresponding number of goals) were as follows: (1) own goal (22 goals); (2) penalty (88 goals); (3) free kick (25 goals); (4) header (158 goals); (5) near post (64 goals); (6) far post (131 goals); and (7) other goals, such as short-range deflections, tap in, and shots from a central position (295 goals). Thus, 195 goals (about one-fourth of the total) were deemed to have been scored in a situation where the striker had a meaningful choice of shot direction. The numbers of goals scored on the near and far posts for each match day are reported in Table 1.

It appears that, of these goals of interest, about one-third were scored at the near post, and two-thirds were scored at the far post. This provides prima facie evidence in support of Result 2. To consider the question from a statistical viewpoint, let x_N and x_F denote the total number of goals scored on the near post and far post, respectively. Given that a binomial distribution is postulated as a data-generating process, and that x_N+x_F is fairly large, the following statistic is distributed as a standard normal (cf., Freund, 1992):

$$\hat{z} = \frac{x_N - (x_N + x_F)\phi}{\sqrt{(x_N + x_F)\phi(1-\phi)}} \tag{6}$$

where ϕ is the probability of a goal scored on the near post (conditional on having observed a goal at either the near or far post). The hypothesis $H_0{:}\phi=1/2$ can then be tested against the one-sided alternative $H_1{:}\phi<1/2$, and I find that H_0 is decisively rejected ($\hat{z}=-4.798$, p value$=8.02\times10^{-7}$). Thus, as predicted

Table 1
Goals scored in Italy's Serie A soccer matches, 2002–2003 season

Match day	Near-post goals	Far-post goals	Match day	Near-post goals	Far-post goals
1	2	6	18	2	5
2	1	6	19	5	6
3	0	5	20	1	2
4	2	1	21	1	3
5	1	4	22	2	3
6	4	2	23	3	4
7	1	2	24	1	3
8	0	2	25	1	3
9	3	3	26	0	7
10	3	8	27	3	3
11	1	2	28	1	2
12	4	3	29	4	2
13	0	2	30	2	7
14	4	4	31	1	5
15	1	3	32	2	5
16	3	4	33	0	6
17	2	2	34	3	6

by the Nash equilibrium of the model, the fraction of goals scored at the near post is significantly less than 1/2.

Having found that the distribution of goals scored on the near and far posts is consistent with the Nash equilibrium of the model, we can probe this hypothesis a bit further. Under Nash equilibrium play, the expected proportion of goals scored on the near post to those scored either way should be the same for all match days [assuming that the same distribution function $H(\theta, \kappa, \lambda)$ applies], a hypothesis that can be tested. Let $\hat{\phi}=x_N/(x_N+x_F)=64/195$, and let x_N^i and x_F^i denote the number of goals scored in match day i at the near post and far posts, respectively. Then the goodness-of-fit statistic

$$\hat{Q} = \sum_{i=1}^{d} \left\{ \frac{[x_N^i - \hat{\phi}(x_N^i + x_F^i)]^2}{\hat{\phi}(x_N^i + x_F^i)} + \frac{[x_F^i - (1-\hat{\phi})(x_N^i + x_F^i)]^2}{(1-\hat{\phi})(x_N^i + x_F^i)} \right\} \tag{7}$$

is distributed as $\chi^2(d-1)$, where d=34 is the number of match days (cf., Freund, 1992). From the data in Table 1, I obtain $\hat{Q}$=28.75 (p value=0.679), such that the hypothesis that the individual match day outcomes all derive from the same Nash equilibrium solution cannot be rejected at any reasonable probability level.

4. Conclusion

One implication of Nash equilibrium for the model, that strictly fewer goals should be scored at the near post than at the far post, is statistically supported by the data. Because the data cannot be used to test equality of success probabilities across strategies however, the analysis admittedly has low power vis-à-vis the notion of Nash equilibrium. The possibility remains, for example, that the goalkeepers overprotect the near post. The investigation of such issues, and the exploration of a possible normative role for game theory in soccer, must await further research.

Acknowledgements

The author thanks Harvey Lapan for commenting on an earlier draft, and Daniele Moro for his help in collecting and validating the data.

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