

Copying and software pricing

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Summary: We analyze the effects of piracy on the pricing policy of a software publisher. Contrary to the existing literature, we focus on the consequences of cost randomness in the decision to use illegal packages of software and on the risk aversion of users and we characterize the optimal prices both under symmetric and asymmetric information. It clearly appears that piracy can have positive social effects in the short run, provided it does not provoke a market breakdown. In the long run, piracy is unambiguously detrimental because it limits the potential development of new products by the seller.

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1. Introduction

When manufacturers decide how to price their products, they take into account the observed or conjectured decisions of their competitors. But in some activities, competition mainly arises from outlaw agents. Indeed, piracy and counterfeiting are very active in data processing, textiles, music, etc. In the software production industry, they amount to 39 % of the turnover of the EU firms¹.

Copying software is easier and easier, as shown by the problems of file downloading on the Internet², the development of CD writers, and the more traditional method of software duplication on hard disks and floppies. The problem is not new since the industries of books, music and video have already been hit, but it is getting worse with digitalization. In the industries of information and communication, a second-hand copy is as good as the original while, with analogical duplication, for each new copy a decrease in quality was occurring because of cumulative noise. Nowadays each copier can become a competitor of the original producer by selling or loaning a perfect substitute. This piracy distorts the rules of competition and, content producers claim, devastates their profits.

Several types of software piracy are usually considered³:

- Softlifting: Purchasing a single licensed copy of software and loading it on several computers, contrary to the license. This includes sharing software with friends, co-workers and others.
- Internet Piracy: unlawfully transmitting software, or providing infringing material that enables users to violate copyright protection mechanisms in software (such as serial numbers and cracker utilities) over one of the Internet's components.
- Software counterfeiting: illegal duplication and sale of copyrighted software in a form designed to make it appear to be legitimate;
- Hard disk loading: whereby computer dealers load unauthorized copies of software onto the hard disks of personal computers, often as an incentive for the end user to buy the hardware from that dealer;

¹ "Follow-up to the Green Paper on combating counterfeiting and piracy in the single market", Communication from the Commission to the Council, the European Parliament and the Economic and Social Committee, COM (2000) 789, November 2000. According to the Software Publishers Association, piracy costs the industry over \$11 billion in 1997 alone (see www.spa.org). See also the Business Software Alliance (www.bsa.org).

² For example, see the rapid development of Napster during the year 2000. Napster is a company whose software allows its community of users to download music for free under the MP3 format. The aggressive legal attitude of big record companies has reduced Napster to virtually nothing in 2001. But new file-sharing networks are now operating that attract an increasing number of users, some of which become illegal sellers. According to the International Federation of the Phonographic Industry, over a third of all CDs and cassettes around the world are pirate copies (<http://www.ifpi.org>).

³ According the website of the Software & Information Industry Association: www.siiia.net/piracy/default.asp.

- Renting: unauthorized software renting for temporary use,
- Unbundling: selling stand-alone software that was intended to be sold packaged with specific accompanying hardware.

Concerning the struggle against illegal sellers of software, the problem is one of exclusive rights enforcement, which means market monitoring, and litigation or settlement⁴. By contrast, the analysis in this paper is devoted to softlifting by users whose behavior is not always observable from outside. For households or very small professional users, the problem is dichotomic: either to buy one unit of the original product or to acquire an illegal copy. Illegal behavior can consist in buying an obviously counterfeited copy at discounted price or in oneself copying the product using a model acquired legally, borrowed or rented⁵. Even if it can represent a large loss for the original producer, this type of copying is very difficult to control because the cost of enforcing the copyright against every individual user is higher than the lost revenue⁶. Rather than trying to fight this type of individual fraud, it is probably more efficient to compensate publishers for lost revenues by taxing the industry that produces devices to help the individual copiers: video recorders and video tapes, photocopiers, CD writers, blank CD, Napster-like software, etc.

For large firms, the problem is different. They buy software packages by hundreds, if not by thousands. And there is an obvious benefit in buying only a fraction of the total from an authorized seller and copying the remaining fraction. The number of personal computers installed at the industrial users' site can be a proxy information about the number of potential software packages needed. But, to confirm a fraud suspicion, it is necessary to check each individual PC to know whether the software is installed or not. It results that to be caught with illegal copies is only a random event. Consequently, the fraction of software packages that will be bought legally and the fraction that will be copied result from a trade-off between the certain price of the former and the random cost of the latter, which includes the fines in case of fraud discovery. One of the essential pieces of this decision process is the degree of risk aversion of the buyer.

To give a realistic view of the piracy problem, we should distinguish copies made within the user's firm and copies bought from a counterfeiter. Which type is more damageable for the original

⁴ The problem is similar to *ex post* patent protection (see Crampes and Langinier, 2000). Software is traditionally protected by copyrights, but one can now observe a switch towards software protection by patents. See Dam (1995, 2000).

In February 2002, the European commission has published a proposal for a Directive "on the patentability of computer-implemented inventions" (2002/0047 COD). See the text at http://europa.eu.int/eur-lex/en/com/pdf/2002/en_502PC0092.pdf

⁵ On the consequences of "small-scale sharing", see Besen and Kirby (1989), Bakos *et alii* (1999) and Varian (2000). They study under which circumstances the sharing of information goods within small social communities increase or decrease the profit of the seller.

producer? and for welfare? For example, illegal purchases cannot be registered in the accounts of the buyer without increasing the risk to be caught. The firm cannot deduce these purchases from its revenue before profit computation. It results an actual cost advantage of copies lower than the apparent one. From this point of view, inside copying at almost zero cost is better than illegal purchase. On the other hand, buying one authorized package for later copies has the adverse effect of drawing the attention of the copyright holder. It results a discontinuity in the cost/advantage relation of legal/illegal behaviors, and improving the monitoring technology can have the adverse effect to induce the illegal behavior of purchasing only illegal copies rather than mixing legal package purchases and inside copying.

In the economic literature on software piracy, the focus has been on production and copy costs (Besen and Kirby 1989, Varian 2000), aggregation and team diversity effects (Bakos *et alii* 1999) and network benefits (Conner and Rumelt (1991), Giron *et alii* (1995), Shy and Thisse (1999)). One important result is that piracy can be good for the seller of legal software because of network externalities that increase the willingness to pay of customers. None of these studies recognize the risky features of the illegal use of software. By contrast, in this paper we neglect network externalities and we insist on the consequences of cost randomness in the decision to use illegal packages of software. Clearly the random cost of illegal copies depends on the control process used by the software publishers and on the magnitude of fines imposed by courts. In this paper, we assume that the cost of illegal acquisition of software is a random exogenous variable. Because of this randomness, the risk aversion of users is an essential parameter of the analysis.

The paper is organized as follows. In Section 2, we describe the behavior of software buyers according to the intensity of their risk aversion. In Section 3, we specify a quadratic utility function for the software's users, from which we derive explicit demand functions for legal and illegal software. Section 4 is devoted to the pricing policy of a private monopolist that sells legal software when illegal copies can be substituted by users. In Section 5 we compare the quantities produced by the private monopolist with the quantities that maximize short run welfare, both when pirates are active. We observe that piracy is beneficial for efficiency when it creates some competition that obliges the monopolist to decrease its price and it provides some additional products to the market. But it can also provoke a total breakdown of the market. In Section 6, we consider long run constraints and we show how harmful piracy can be. The publisher has the obligation to earn enough profits for financing R&D and piracy obviously impedes it to extract high rents from the market. In Section 7, we consider the case where the software company is allowed to use non linear

⁶ Nevertheless, we can imagine a strategy of random lawsuits against individuals with intense publicity in order to

tariffs. Non-linear prices can be motivated by the allocation of R&D fixed costs among multi-unit buyers. But they also are proposed to large customers in order to limit their incentives to copy legal software. In this section, we suppose that the seller suffers from an informational gap concerning the degree of risk aversion of users. Section 8 concludes.

2. Model setting

We study the behavior of a business or industrial company that uses software services for its production of goods and services. Let q_0 be the number of original software packages bought at unit price p , and let q_c be the number of illegal copies installed at the user's location. The gross revenue from using q_0 and q_c is $R(q_0 + \mathbf{a}q_c)$ where \mathbf{a} represents the non random quality of illegal copies⁷. When the copying system is perfect so that the original and its copy are perfect substitutes, $\mathbf{a} = 1$. Imperfect substitutability ($\mathbf{a} < 1$) can result from the presence of some bugs in the copying process or from the impossibility to rely on the after sale services of the producer in case of breakdown⁸. The revenue function is continuous, concave and at least increasing for low values of $q = q_0 + \mathbf{a}q_c$. We also assume that $R'(0)$ is large. Consequently the business or industrial company is willing to install software, from any source. Concerning the illegal items, even if the technical cost of a copy is close to zero⁹, there remains the judicial cost of an inspection followed by penalties, plus some likely marketing losses due to bad publicity. Let \tilde{c} denote the random cost of a copy. We can write the (random) profit of the user as:

$$\tilde{\mathbf{p}} = R(q_0 + \mathbf{a}q_c) - pq_0 - \tilde{c}q_c$$

Her objective is to maximize expected utility $Eu(\tilde{\mathbf{p}})$ over q_0 and q_c , where $u(\cdot)$ represents her von Neumann - Morgenstern utility function. This function is increasing and, for a risk averse decision maker, strictly concave. The first order conditions of the maximization problem are

$$Eu'(\tilde{\mathbf{p}})[R'(q_0 + \mathbf{a}q_c) - p] \leq 0, \quad (= 0 \text{ if } q_0 > 0) \quad (1)$$

$$Eu'(\tilde{\mathbf{p}})[\mathbf{a}R'(q_0 + \mathbf{a}q_c) - \tilde{c}] \leq 0, \quad (= 0 \text{ if } q_c > 0) \quad (2)$$

We suppose that the problem is concave¹⁰, so that these conditions are sufficient to characterize the global maximum of the user's profit.

invest in toughness reputation.

⁷ The more general form $R(q_0, q_c)$ would allow to consider the case where q_0 and q_c are complements, for example because the user buys legal packages of operating systems but uses only counterfeited application programs.

⁸ We do not consider the possibility that some users with high technical skill can make α larger than 1.

⁹ As argued by Bakos *et alii* (1999, p.120) and Shy and Thisse (1999, footnote 2).

¹⁰ It follows, for example, from the concavity of $u(\cdot)$ and $R(\cdot)$ and from the fact that these functions are increasing in the relevant domains.

Let us denote the expected cost of copying $\mathbf{m} = E\tilde{c}$ and let $h(q_0, q_c) \stackrel{\text{def}}{=} \frac{\text{cov}(u', \tilde{c})}{\mathbf{a}Eu'} > 0$. To describe the general properties of the demand for software, we first establish the following lemma.

Lemma 1:

a) $q_0 = q_c = 0$ is never optimal.

b) $q_c = 0$ iff $p \leq \mathbf{m}/\mathbf{a}$.

c) $q_0 = 0$ iff $p \geq R'(\mathbf{a}\hat{q})$ where \hat{q} is defined by $R'(\mathbf{a}\hat{q}) = \frac{\mathbf{m}}{\mathbf{a}} + h(o, \hat{q})$.

Proof: See the appendix

Using this lemma, we can directly characterize the demand functions of the software user in the following Proposition.

Proposition 1: Demand functions for legal and illegal software packages by an individual user are given by

- $q_0 = 0, \quad q_c = \hat{q}$ for $p \geq R'(\mathbf{a}\hat{q})$
- $R'(q_0 + \mathbf{a}q_c) = p = h(q_0, q_c) + \mathbf{m}/\mathbf{a}$ for $R'(\mathbf{a}\hat{q}) \geq p \geq \mathbf{m}/\mathbf{a}$
- $q_0 = R^{-1}(p), \quad q_c = 0$ for $\mathbf{m}/\mathbf{a} \geq p$.

As a corollary to Proposition 1, one can notice that whenever $q_0 > 0$, $q_0 + \mathbf{a}q_c = R^{-1}(p)$ for all utility functions $u(\cdot)$. This means that when the user buys at least one original software package, the total number of "useful software" $q_0 + \mathbf{a}q_c$ she will install does not depend on her level of risk aversion. The latter influences only the intensity of her legal or illegal behavior. If copies are a perfect substitute for original software (i.e. if $\mathbf{a} = 1$), the gross total number $q_0 + q_c$ is also independent of the level of risk aversion. This last result is no longer true when $\mathbf{a} < 1$ since $q_0 + q_c = R^{-1}(p) + (1 - \mathbf{a})q_c$.

According to Proposition 1, since q_0 is zero (positive) and q_c is positive (zero) for a large (small) p , demand for legal (illegal) software is "globally decreasing" (increasing) with the price fixed by the software company. But for small changes in p , the demand functions can locally vary in the opposite direction, depending on the reaction of the user to an increase in risk, as established by the following Proposition.

Proposition 2: Under constant absolute risk aversion (CARA) or increasing absolute risk aversion

$$(IARA), \frac{dq_o}{dp} < 0 \text{ and } \frac{dq_c}{dp} > 0.$$

Under decreasing absolute risk aversion (DARA), for small values of q_o the signs are the same as under CARA and IARA, but for large values of q_o the signs can be reversed.

As usual, this result depends on the opposite forces of the substitution and income effects. When q_o is large, a decrease in p increases the wealth of the customer. If her coefficient of absolute risk aversion is a decreasing function of income, she is ready to take a more risky position, that is to buy more illegal copies and less legal packages. This will not occur if the enrichment is too weak or if the client is characterized by IARA or CARA.

Knowing the demand function for his software packages $q_o(p)$, the producer has to decide in the medium term how intensively to monitor the use of illegal copies and the government has to decide how toughly to punish illegal users. This policy affects the characteristics of the random variable \tilde{c} and the value of \mathbf{a} . In the short term, the only decision variable is the unit price of legal copies, p .

3. Demand specification

To get further insights into the consequences of piracy, we now suppose that the user maximizes the function

$$\mathbf{p} = R(q_o + \mathbf{a}q_c) - pq_o - \mathbf{m}q_c - \frac{1}{2} \mathbf{b}\mathbf{s}^2 q_c^2 \quad (3)$$

where $\mathbf{m} = E\tilde{c}$ is the expected value of the cost of copies, $\mathbf{s}^2 = E(\tilde{c} - \mathbf{m})^2$ is its variance and \hat{a} is an index that measures the intensity of risk aversion. As shown in Proposition A of the appendix, this quadratic form can be derived either from a quadratic utility function (under IARA) without any specification of the distribution for the cost of copying, or from a CARA utility function associated with a cost of copying normally distributed. With this specification, by Proposition 2 the demand for legal packages is a monotonically decreasing function of price.

Concerning marginal revenue from the use of software, we assume it is given by the linear function $R'(q) = a - q$ where $q = q_o + \mathbf{a}q_c$. Then, when the user buys both types of software, we can write from Proposition 1.

$$q_c(p) = \frac{\mathbf{a}p - \mathbf{m}}{\mathbf{g}} \text{ and } q_o(p) = a + \frac{\mathbf{a}\mathbf{m}}{\mathbf{g}} - p\left(1 + \frac{\mathbf{a}^2}{\mathbf{g}}\right) \quad (4)$$

where $\mathbf{g} \stackrel{\text{def}}{=} \mathbf{b}\mathbf{s}^2$ stands for the unit risk-penalty.

As already noticed, when $a = 1$ the price p determines the total number of software that will be installed. The average value and the variance of the copying cost, as well as the risk aversion coefficient, intervene only to determine how many legal and how many illegal items to install. This case is illustrated in Figure 1.

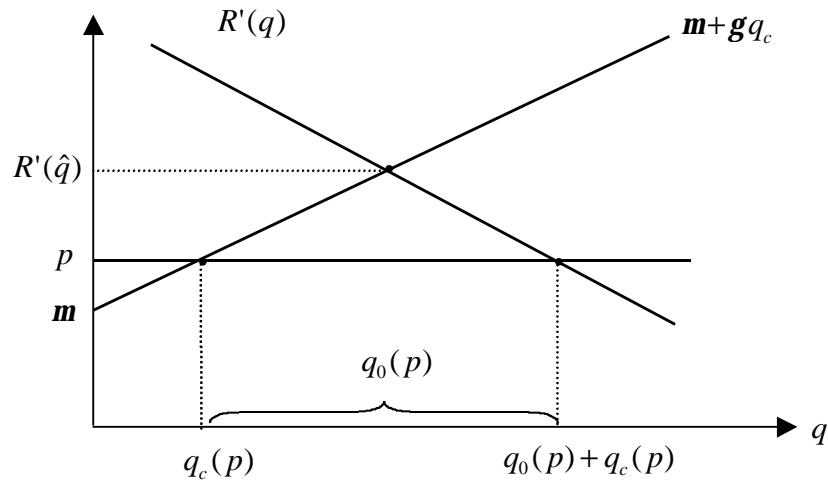


Figure 1 : Demand for legal and illegal software packages when they are perfect substitutes

Actually, the degree of substitution between legal and illegal copies a is strictly less than 1. The closer to 1, the more competitive the pirate software. Downgrading the value of a by proposing complementary services to legal buyers is one of the possible tools for the defense of the software company. In the simple case of $a = 1$, the impacts of an increase in the expected cost of illegal copies m and of an increase of the risk penalty $g = bs^2$ are illustrated in Figure 1.

4. Profit maximization

Using the model of Section 3, we consider now the case where there is only one type of software users, characterized by the same b and facing the same risk s^2 . Consequently they have to pay the same unit risk-penalty g . The demand for legal software packages is described by

$$q_0(p) = \begin{cases} a - p & \text{if } p \leq \frac{m}{a} \\ a + \frac{am}{g} - p\left(1 + \frac{a^2}{g}\right) & \text{if } \frac{m}{a} \leq p \leq \frac{am + ag}{a^2 + g} \\ 0 & \text{if } p \geq \frac{am + ag}{a^2 + g} \end{cases} \quad (5)$$

This function is graphed in Figure 2.

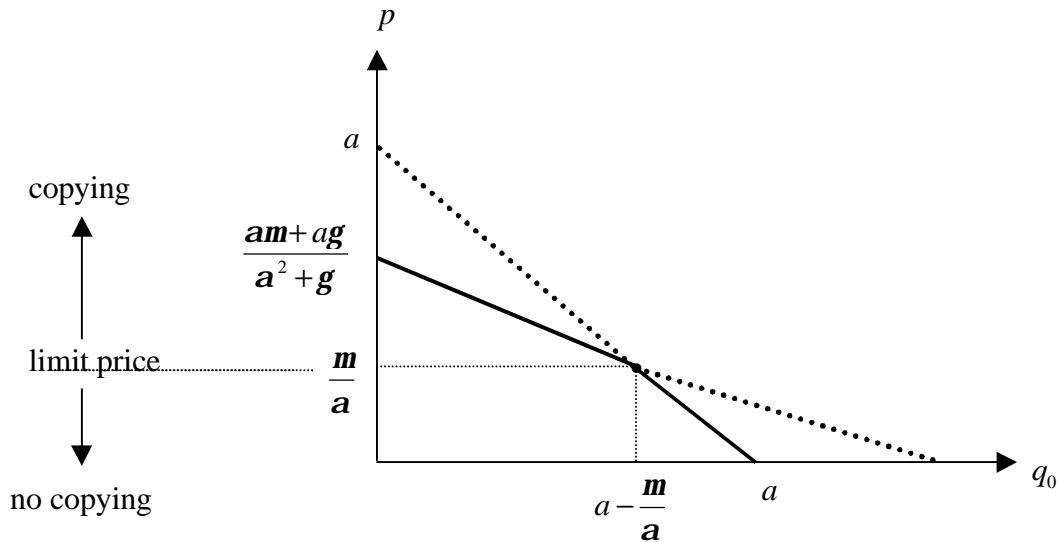


Figure 2 : The kinked demand function for original software

Denoting by c the marginal cost of software manufacturing, the objective of the producer is¹¹

$$\max_p F(q_0(p), p) \text{ where } F(q_0(p), p) = (p - c)q_0(p).$$

With a kinked demand function like the one chartered in Figure 2, there are three obvious candidates to be the solution with a strictly positive output:

- $p_1 = \frac{a+c}{2}$. It is the solution to $\max_p F_1$ where $F_1 = (p - c)(a - p)$, that is the monopolist's best choice when there is no threat of copy.
- $p_2 = \frac{ga + am}{g + a^2} + c$ which is the solution to $\max_p F_2$ where $F_2 = (p - c)(a + \frac{am}{g} - p(1 + \frac{a^2}{g}))$ is the profit from the residual demand not served by pirates. It takes into account the characteristics of the "competitor", that is a , m and g .
- $\bar{p} = \frac{m}{a}$. This price is the solution to $F_1 = F_2$. It corresponds to the kink of the demand function.

Depending on the characteristics a , m and g of the copying process, we can now establish the following:

¹¹ The software publisher also receives damages from illegal users when they are condemned or when they accept to settle. We assume that these inflows do not distort the pricing decision. In a more realistic setting, we should take into account the gains from penalties as well as the litigation and settlement costs. For an illustration in the case of patents, see Crampes and Langinier (2002)

Proposition 3: The price that maximizes profit is

- i) $p^* = p_1$ for $\frac{a(a+c)}{2} < m$
- ii) $p^* = \frac{m}{a}$ for $ac + \frac{ag(a-c)}{2(g+a^2)} < m < \frac{a(a+c)}{2}$
- iii) $p^* = p_2$ for $ac - \frac{g}{a}(a-c) < m < ac + \frac{ag(a-c)}{2(g+a^2)}$
- iv) $p^* \geq \frac{ga+am}{g+a^2}$ for $m < ac - \frac{g}{a}(a-c)$.

For an expected cost of copies $m \leq ac - \frac{g}{a}(a-c)$, the manufacturer charges any price that makes demand for legal software equal to zero (case *iv*). Indeed, with such a low m it is impossible to earn a positive margin. The manufacturer is impeded to produce by the lost revenue due to copying. This interval is increasing with c and a and decreasing with a and g . In this case, no software will be produced since copying is only possible when some production of original software is active: there occurs a breakdown of the market.

When the expected cost of useful copies m/a is larger than the aforementioned value but still smaller than the monopoly price $(a+c)/2$, the producer is active but he has to take account of the pirates, which explains that his pricing policy is either accommodation or deterrence. In cases *ii*) and *iii*), the software producer is like facing competitors that sell at price m/a . In cases *iii*) where m/a is still low, pirates are in the market. The publisher behaves like a monopolist on the residual demand. He charges a reduced price when the risk penalty is low and the substitutability of copies is high. Remember that the technical cost of copying is very low, but the total cost (including potential fines) can be very high. By contrast, in case *ii*), by charging the limit price m/a , the monopolist can impede the entry of pirates. Finally, when copying is drastically inefficient ($\frac{m}{a} > \frac{a+c}{2}$), the software publisher can charge the monopoly price (case *i*).

Case *ii*) is illustrated by Figure 3, where the producer maximizes profits when fixing the "limit price" m/a which is totally determined by the expected cost of the pirate option.

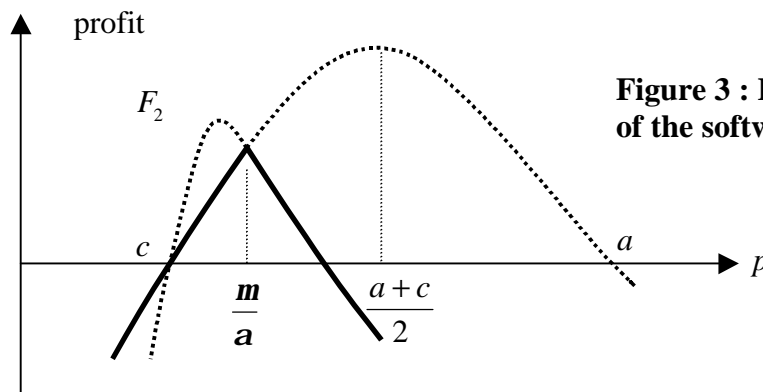


Figure 3 : Profit function of the software company

When legal and illegal software packages are in the market, that is when $p^* = p_2$, the quantities sold are respectively

$$q_o^* = \frac{a-c}{2} - \frac{\mathbf{a}(\mathbf{a}c - \mathbf{m})}{2\mathbf{g}} \quad \text{and} \quad q_c^* = \frac{\mathbf{a}^2(\mathbf{a}c - \mathbf{m}) + \mathbf{a}\mathbf{g}(a + c - 2\mathbf{m}/\mathbf{a})}{2\mathbf{g}(\mathbf{g} + \mathbf{a}^2)} \quad (6).$$

The number of legal software q_o^* is increasing in \mathbf{m}, \mathbf{g} and a and decreasing in c and \mathbf{a} . The number of copies q_c^* is increasing in \mathbf{a}, c and a and decreasing in \mathbf{m} and \mathbf{g} .

5. Welfare benefits of copying in the short run

In some models of software piracy, legal producers can be better off when pirates are active. The reason is that users benefit from network externalities which give them the incentive to buy more packages¹². By contrast, in our setting piracy is always detrimental for the software company. At best, it is facing competitors who prevent it from appropriating the whole monopoly rent. At worst, it can be unable to produce.

We now consider the effects of piracy on welfare. First, note that we assume perfect competition among pirates so that their net profit is zero¹³. Second, in the preceding Section we did not include penalties in the benefits of the software publisher. Consequently, to keep symmetry with this assumption, we assume that the penalties paid by illegal users do not distort the welfare function. In this framework, the first best allocation is the one that maximizes the sum of the certainty equivalent of the user's surplus and the producer's profit

$$W(q_o, q_c) = R(q_o + \mathbf{a}q_c) - cq_o - \mathbf{m}q_c - \frac{1}{2}\mathbf{g}q_c^2 \quad (7)$$

For the social planner, the problem is similar to combining two "technologies" like the individual user does, except that now the original software is valued at marginal cost c . If $c < \frac{\mathbf{m}}{\mathbf{a}}$, it is "cheaper" to manufacture all the packages in the industrial plant of the copyright owner. Copies are inefficient and we obtain the corner solution $q_o^o = a - c$, $q_c^o = 0$ which results from the equality between marginal utility $R'(q_o) = a - q_o$ and marginal cost c . In contrast when $c \geq \frac{\mathbf{m}}{\mathbf{a}}$, maximizing

$W(q_o, q_c)$ gives the optimal outputs $q_o^o = a - c - \frac{\mathbf{a}}{\mathbf{g}}(\mathbf{a}c - \mathbf{m})$ and $q_c^o = \frac{\mathbf{a}c - \mathbf{m}}{\mathbf{g}}$. The total quantity of

useful software $q_o^o + \mathbf{a}q_c^o$ is still equal to the optimal quantity planned without piracy $a - c$.

¹² Shy and Thisse (1999), Bakos *et alii* (1999), Varian (2000).

¹³ In poor countries, the social planner can be biased in favor of piracy because it generates a domestic positive surplus while legal producers are located abroad.

When $m \leq ac - \frac{g}{a}(a-c)$, local optimality commands $q_o^o = 0$. But as original software is indispensable for copying, this would result in $q_c^o = 0$. Therefore, the optimal plan is $q_o^o = 1$ if one original is sufficient for copies and $q_c^o = \frac{(a-c)}{a}$.

These outputs can be compared with the quantities that are produced by the private monopolist. Because $p^* > c$, there is an obvious underprovision of output by the monopolist. But we have to take into account the contribution of piracy to welfare. The total number of software packages sold in the economy is $q_0 + q_c$, but actually, in terms of "useful" packages, the number is only $q_0 + aq_c = a - p$. When the software company accommodates the presence of illegal products in the market (case *iii* in Proposition 3), at price $p^* = p_2$ one can compute

$$q_0(p_2) + aq_c(p_2) = \frac{a-c}{2} + (a - \frac{m}{a}) \frac{a^2}{2(g+a^2)}$$

In comparison with the first best output $a-c$, this total quantity is too small but it is larger than the output of the pure monopolist $q^m = \frac{a-c}{2}$. Actually illegal sellers contribute to welfare in two ways:

- first directly, since their own output $q_c(p_2)$ has some utility for customers as long as $a > 0$. As we can see in Figure 4, for $m < ac$ the monopolist produces less than without piracy threat ($q_o^* < \frac{a-c}{2}$), but the positive output of pirates more than compensates the reduction of output by the monopolist.
- second, in a reaction to piracy, the publisher is obliged to charge a price p_2 less than the monopolist price p_1 , so that he produces an extra quantity $q_0(p_2) - q^m = \frac{a(m-ac)}{2g}$ when $m > ac$. This indirect effect of piracy is positive even when pirates are voluntarily excluded from the market, that is when the monopolist charges the limit price $\frac{m}{a}$.

Consequently, in a purely static framework, piracy plays a positive social role on quantities since it obliges the monopolist publisher to decrease its price. But this is true only if the publisher is not excluded from the market by pirates. Indeed, if $0 \leq ac - \frac{g}{a}(a-c)$, for very small values of m there is no market at all while it would be efficient to produce software.

We now come back to the case where there exists an active market. The static positive effects of piracy can be confirmed if we study the effects of an increase in the cost of copying on the private gains and on the welfare function. Let us denote by $\Pi^* = \Pi(q_o^*, q_c^*)$ the expected utility of the user and by $F^* = F(q_o^*)$ the profit of the publisher when the latter fixes the price that maximizes profits. We can compute that a change in the average production cost of copies m has the following effects:

$$\frac{d\Pi^*}{dm} = -q_c^* - q_o^* \frac{dp^*}{dm} < 0, \quad \frac{dF^*}{dm} = (p^* - c) \frac{\partial q_o^*}{\partial m} > 0 \text{ when copies are in the market and}$$

$$\frac{d\Pi^*}{dm} = -\frac{1}{a} \left(a - \frac{m}{a} \right) < 0, \quad \frac{dF^*}{dm} = \frac{1}{a} \left(a + c - 2 \frac{m}{a} \right) > 0 \text{ under the limit pricing policy.}$$

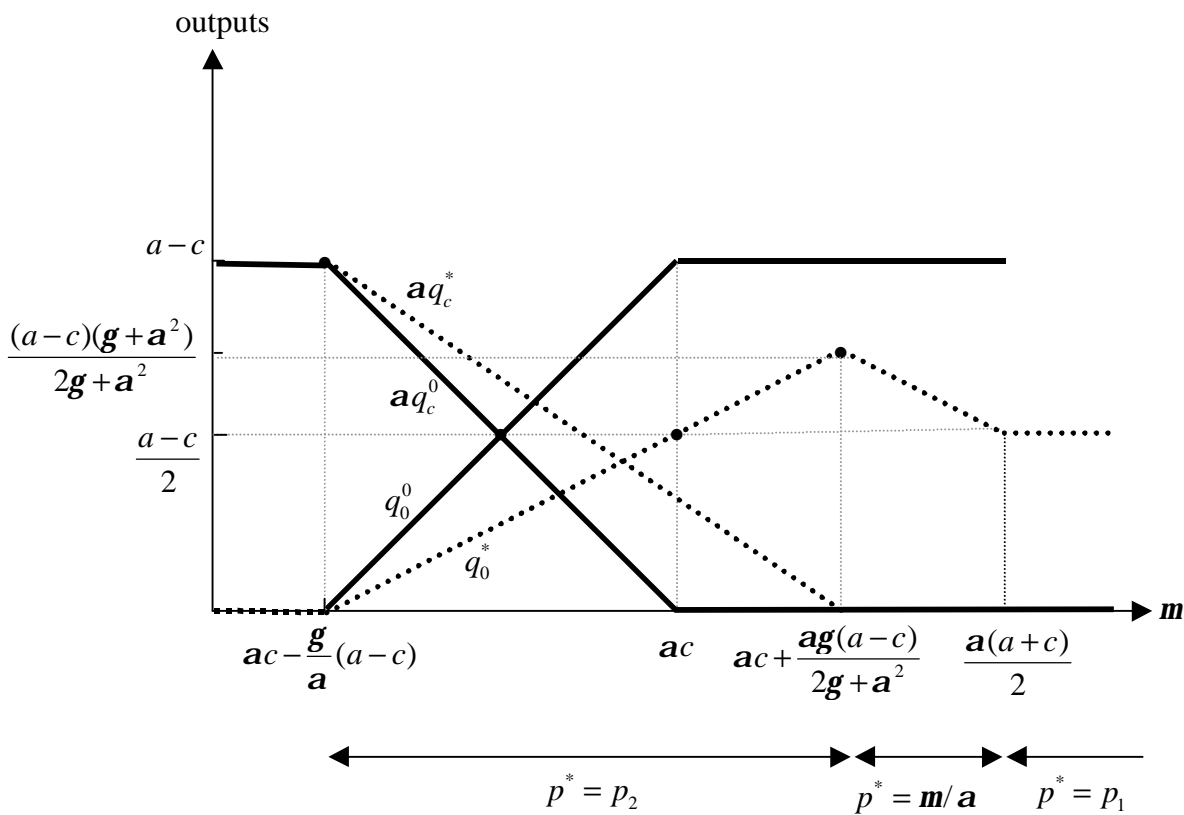


Figure 4: Legal output and copy output as functions of the average cost of copying

It results that an increase in the average cost of producing copies is detrimental for the user who suffers from both a higher cost on each illegal copy she buys and from the increase in the price of the legal commodity which is made feasible because competition is softer. This second effect is obviously beneficial for the publisher who is facing a higher residual demand. Denoting by $W^* = W(q_o^*, q_c^*)$ the social performance under private pricing, for active piracy as well as for

deterred entry we have $\frac{dW^*}{d\mathbf{m}} = \frac{d\Pi^*}{d\mathbf{m}} + \frac{dF^*}{d\mathbf{m}} < 0$, which means there is no social incentive to increase

the cost of copying. On the contrary, the short term social optimum is to decrease the cost of copies in order to allow pirates to compete fiercely with the software company.

At this stage, we have to be more precise in the definition of the copy cost. Assume that \mathbf{m} is composed of two terms: $\mathbf{m} = c_0 + \mathbf{d}$, where c_0 is the marginal cost of producing copies and \mathbf{d} is the penalty cost, which does not correspond necessarily to a social cost. In the preceding paragraph, what we had in mind was $\Delta\mathbf{m} = \Delta c_0$, rather than $\Delta\mathbf{m} = \Delta\mathbf{d}$. Indeed, the effect of an increase in the fine \mathbf{d} would be more complex to assess because we should take into account the government as a separate agent. The benefits of the government, equal to the difference between the revenues from the fines and the cost of monitoring and punishing pirates, should be added to the welfare of the legal publisher and of the software user.

6. Harmful piracy in the long run

By contrast, because illegal competition decreases the profits that the software company can dedicate to R&D, piracy has strong adverse effects in the long run. The competitive advantages of piracy and its negative effects in terms of R&D are an illustration of the conflict between intellectual property rights (authors and innovators must be protected) and antitrust rules (barriers to entry are bad for static efficiency).

To model the problem, we can note that it is isomorphic to the case of a public monopoly obliged to balance its budget. Here, because the software developer would be dissuaded to launch new products absent some guarantee to at least recoup the R&D expenditures, the optimal production of original and copied software that should be decided by the social planner is closer to the second best than to the first best. Denoting by D the R&D budget that must be financed, the welfare problem is

$$\max_{q_0, q_c} W(q_0, q_c) \text{ subject to } [p_0(q_0) - c]q_0 \geq D$$

where $p_0(q_0)$ stands for the marginal utility of legal software. When the constraint is binding, the second best price solution \hat{p} as given by

$$(\hat{p} - c)q_0(\hat{p}) - D = 0$$

where $q_0(p)$ is defined in (5). We derive that

$$\hat{p} = \frac{A + Bc - \sqrt{(A - Bc)^2 - 4BD}}{2B}$$

where $A = a + \frac{\mathbf{am}}{\mathbf{g}}$ and $B = 1 + \frac{\mathbf{a}^2}{\mathbf{g}}$.

It is easy to check that $\hat{p} > c$ as long as $D > 0$, which is the usual effect of a budget constraint on the optimal price. And the higher D , the larger the difference between \hat{p} and c . But note

that $\frac{d\hat{p}}{dm} = \frac{a}{2gB} \left(1 - \frac{(A-Bc)}{\sqrt{(A-Bc)^2 - 4BD}}\right) < 0$, which means that the second best distortion can be

alleviated by an increase in the expected cost of copies. Actually, when D is large the software company needs to sell large quantities in order to break even. In this case we can have

$\frac{d\hat{W}}{dm} = \frac{d\hat{\Pi}}{dm} = -\hat{q}_c - \hat{q}_o \frac{d\hat{p}}{dm} > 0$. This means that there may exist a strictly positive optimal value of

the fines incurred for illegal use. This value is the one that equates marginal social benefits $\frac{d\hat{W}}{dm}$

with the marginal social cost of implementing a more repressive policy against piracy.

It is valuable to notice that the necessity to collect costly funds for research also applies when the software monopoly is a public firm. Consequently, the gap between the second best pricing policy of a public firm under the obligation to raise funds for R&D or subsidized with costly public funds on one hand and the private monopolist pricing policy on the other hand can be very narrow. In any case, the development of new software is slowed down by piracy¹⁴.

7. Second degree price discrimination

Software producers often propose non linear prices, mainly unit prices decreasing with the order size. One possible explanation is based on accounting principles: unit prices are decreasing because the average cost of production is a decreasing function due to huge fixed costs. An alternative explanation is market-based: it is always in the interest of the monopolist to price discriminate, even when fixed costs are nil.

If the software company is allowed to use non linear tariffs, it can extract the whole rent from the consumer, so that its interest is to produce the first best quantity. But this first degree price discrimination necessitates complete information about the characteristics of the user. Actually, there is an informational gap between the seller and the user, in particular concerning the degree of risk aversion \mathbf{b} .

Assume that the software user's type \mathbf{b} is distributed according to the absolutely continuous function $F(\mathbf{b})$ with density $f(\mathbf{b})$ on the interval $[\underline{\mathbf{b}}, \bar{\mathbf{b}}]$. The user knows the exact value of \mathbf{b} while the software company only knows the distribution function. The user remains free to choose

¹⁴ Takeyama (1997) considers the additional negative effect provoked by the durability of software: "The presence of copying results in a greater reduction in future prices beyond that which would normally occur without copying ... This

how many illegal copies to use. Consequently, the seller is facing both an adverse selection problem and a moral hazard problem.

Let $q_o(\tilde{\mathbf{b}}), t(\tilde{\mathbf{b}})$ stand for the contract offered to the user if she declares she is of type $\tilde{\mathbf{b}}$: she will be sold $q_o(\tilde{\mathbf{b}})$ units of software at the total expense of $t(\tilde{\mathbf{b}})$. Denoting by \mathbf{b} the true index of risk aversion, her illegal behavior is given by

$$\max_{q_c} \mathbf{p}(q_o(\tilde{\mathbf{b}}), t(\tilde{\mathbf{b}}), q_c, \mathbf{b})$$

where
$$\mathbf{p}(q_o(\tilde{\mathbf{b}}), t(\tilde{\mathbf{b}}), q_c, \mathbf{b}) = R(q_o(\tilde{\mathbf{b}}) + \mathbf{a}q_c) - t(\tilde{\mathbf{b}}) - \mathbf{m}q_c - \mathbf{b} \frac{\mathbf{s}^2 q_c^2}{2}.$$

Note that this function is strictly concave in q_c . From the first order condition

$$\frac{\partial \mathbf{p}}{\partial q_c} = 0 \quad (8)$$

and using $R' = a - q_o - \mathbf{a}q_c$, we obtain the demand for pirate software

$$q_c(q_o(\tilde{\mathbf{b}}), \mathbf{b}) = \begin{cases} \frac{\mathbf{a}(a - q_o(\tilde{\mathbf{b}})) - \mathbf{m}}{\mathbf{a}^2 + \mathbf{b}\mathbf{s}^2} & \text{if } q_o(\tilde{\mathbf{b}}) \leq a - \frac{\mathbf{m}}{\mathbf{a}} \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

The surplus of a \mathbf{b} -user who claims to be a $\tilde{\mathbf{b}}$ -type is

$$U(q_o(\tilde{\mathbf{b}}), t(\tilde{\mathbf{b}}), \mathbf{b}) = \mathbf{p}(q_o(\tilde{\mathbf{b}}), t(\tilde{\mathbf{b}}), q_c(q_o(\tilde{\mathbf{b}}), \mathbf{b}), \mathbf{b}) \quad (10)$$

It is important to note that $\partial U / \partial q_o = R'(q_o + \mathbf{a}q_c)$. Consequently

$$\frac{\partial^2 U}{\partial q_o \partial \mathbf{b}} = \mathbf{a} R'' \frac{\partial q_c}{\partial \mathbf{b}} > 0 \quad \text{by (2) and } R'' < 0.$$

It results that the Spence-Mirrlees condition is satisfied: the software seller can propose a separating contract.¹⁵

To design the contract, the first constraint to respect is the participation constraint

$$\hat{U}(\mathbf{b}) = U(q_o(\mathbf{b}), t(\mathbf{b}), \mathbf{b}) \geq 0 \quad \text{for all } \mathbf{b}. \quad (11)$$

The second constraint is the incentive compatibility constraint

$$U(q_o(\mathbf{b}), t(\mathbf{b}), \mathbf{b}) \geq U(q_o(\tilde{\mathbf{b}}), t(\tilde{\mathbf{b}}), \mathbf{b}) \quad \text{for all } \mathbf{b} \text{ and } \tilde{\mathbf{b}}.$$

Alternatively, since the Spence-Mirrlees condition is satisfied, it can be written $\partial U(\cdot) / \partial \tilde{\mathbf{b}} = 0$ at $\tilde{\mathbf{b}} = \mathbf{b}$ for all \mathbf{b} .

reduction will also necessitate a corresponding reduction in all earlier prices via the time consistency constraint" (p.513).

¹⁵ Therefore, we are in a remarkable case where a mixed model (moral hazard and adverse selection) is amenable to a pure adverse selection problem. See Laffont and Martimort (2002) chapter 7 for a discussion of the difficulties of mixed models.

Therefore, $\frac{d\hat{U}(\mathbf{b})}{d\mathbf{b}} = \frac{\partial U(\cdot)}{\partial \mathbf{b}}$ or, with our specification,

$$\frac{d\hat{U}(\mathbf{b})}{d\mathbf{b}} = -\frac{\mathbf{s}^2[q_c(q_o(\bar{\mathbf{b}}), \mathbf{b})]^2}{2} \quad (12)$$

Now, we can write the problem of the software seller as

$$\max_{q_o(\cdot), t(\cdot)} \int_{\underline{\mathbf{b}}}^{\bar{\mathbf{b}}} [t(\mathbf{b}) - cq_o(\mathbf{b})] dF(\mathbf{b}) \quad \text{subject to (11) and (12).}$$

Using the definition of $U(\cdot)$, the objective function of the seller becomes

$$\int_{\underline{\mathbf{b}}}^{\bar{\mathbf{b}}} \left[R(q_o + \mathbf{a}q_c) - cq_o - \mathbf{m}q_c - \frac{\mathbf{b}\mathbf{s}^2q_c^2}{2} - \hat{U}(\mathbf{b}) \right] dF(\mathbf{b}).$$

But

$$\begin{aligned} \int_{\underline{\mathbf{b}}}^{\bar{\mathbf{b}}} \hat{U}(\mathbf{b}) dF(\mathbf{b}) &= \int_{\underline{\mathbf{b}}}^{\bar{\mathbf{b}}} \int_{\underline{\mathbf{b}}}^{\bar{\mathbf{b}}} \frac{\mathbf{s}^2q_c^2}{2} d\mathbf{b} dF(\mathbf{b}) + \hat{U}(\bar{\mathbf{b}}) \quad \text{by (12)} \\ &= \int_{\underline{\mathbf{b}}}^{\bar{\mathbf{b}}} \frac{\mathbf{s}^2q_c^2}{2} \frac{F(\mathbf{b})}{f(\mathbf{b})} dF(\mathbf{b}) + \hat{U}(\bar{\mathbf{b}}) \quad \text{after integration.} \end{aligned}$$

Finally, the seller's program reduces to

$$\max_{q_o(\cdot), \hat{U}(\bar{\mathbf{b}})} \int_{\underline{\mathbf{b}}}^{\bar{\mathbf{b}}} \left[R(q_o + \mathbf{a}q_c) - cq_o - \mathbf{m}q_c - \mathbf{b} \frac{\mathbf{s}^2q_c^2}{2} - \frac{\mathbf{s}^2q_c^2}{2} \frac{F(\mathbf{b})}{f(\mathbf{b})} \right] dF(\mathbf{b}) - \hat{U}(\bar{\mathbf{b}}) \quad \text{s.t. (11)}$$

The first obvious result is $\hat{U}(\bar{\mathbf{b}}) = 0$. The seller will extract all the rent from the most risk averse user. Given (12), this means that the users more inclined toward illegal behavior will keep some positive net surplus. Given the reaction function (9), the first order condition with respect to $q_o(\cdot)$ is

$$R'(q_o + \mathbf{a}q_c) - c + \frac{F(\mathbf{b})}{f(\mathbf{b})} \frac{\mathbf{a}\mathbf{s}^2}{\mathbf{a}^2 + \mathbf{b}\mathbf{s}^2} q_c = 0.$$

It results that, except at $\underline{\mathbf{b}}$, $R'(q_o + \mathbf{a}q_c) < c$ so that $q_o + \mathbf{a}q_c$ is larger than the first-best total number of useful software. Denoting by $\mathbf{k}(\mathbf{b}) = \frac{\mathbf{a}\mathbf{s}^2}{\mathbf{a}^2 + \mathbf{b}\mathbf{s}^2} \frac{F(\mathbf{b})}{f(\mathbf{b})}$ the unit informational bias, we can write the quantity of software sold under second degree price discrimination as

$$q_o^{sd} = a - c - \frac{\mathbf{a} - \mathbf{k}(\mathbf{b})}{\mathbf{b}\mathbf{s}^2 + \mathbf{a}\mathbf{k}(\mathbf{b})} (\mathbf{a}c - \mathbf{m})$$

It is easy to check that this quantity is larger than the first-best output $q_o^o = a - c - \frac{\mathbf{a}(\mathbf{a}c - \mathbf{m})}{\mathbf{b}\mathbf{s}^2}$ except for the less averse user $\underline{\mathbf{b}}$ for whom the two quantities are equal.

To understand the distortion that asymmetric information entails, it is enough to note that the expected rent of a user is an increasing function of the quantity of illegal copies bought. (This in turn is due to the fact that asymmetric information is about the index of risk aversion and that any lie about \mathbf{b} has a higher impact on the agent's utility, the greater is its purchase of illegal copies).

Since the seller wishes to decrease the information rent he gives up to users, he wishes to decrease the quantity of illegal copies. Given the user's behavior this requires an increase of the purchase of legal copies (and this calls for a decrease of the marginal price of legal copies).

Summing up, asymmetric information about risk aversion leads to a higher level of legal copies, because it makes more costly for the seller the presence of illegal copies, who is then more determined to compete them away.

8. Conclusion

We have analyzed the effects of piracy on the pricing policy of a software publisher. Contrary to the existing literature, we have focused on the consequences of cost randomness in the decision to use illegal packages of software and on the behavior of users in terms of their risk aversion. It clearly appears that piracy can have positive social effects in the short run, but in the long run it is severely detrimental because it limits the possibility of the seller to develop new products.

Several extensions can be considered:

- The acquisition cost of illegal software depends on the control process used by the software publisher and on the amount of fines imposed by courts. In this paper, we have assumed it is a random exogenous variable. We should now go one step backward to design mechanisms aimed at limiting fraud both by the publisher and the government.
- Because of network externalities, it can be in the interest of the publisher to accept piracy for a limited period, and then to take advantage of the large base of users by means of a more repressive policy. A dynamic model would allow to analyze the effects of this differentiated policy.
- When there is competition between several legal publishers, the intensity of the punitive policy against pirates can be viewed as a quality attribute since it will prevent some potential users from buying the software. How piracy and defense against piracy affect the equilibrium prices calls for a careful analysis.
- Fighting piracy is a public good for software publishers and some can have the incentive to free-ride their competitors.

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Appendix

Proof of Lemma 1.

a) Suppose the user buys nothing. From condition (1) in the text, we would obtain $R'(0) \leq p$, which is excluded by hypothesis. *QED*

b) First note that since $\text{cov}(u', \tilde{c}) = Eu' \tilde{c} - Eu' \mathbf{m}$ (where $\mathbf{m} = E\tilde{c}$) and since $\frac{\partial u'}{\partial \tilde{c}} = -q_c u'' \geq 0$, we have $Eu' \tilde{c} > Eu' \mathbf{m}$ for $q_c > 0$ and $Eu' \tilde{c} = Eu' \mathbf{m}$ for $q_c = 0$.

- Necessity:

When $q_c = 0$, from part a) of the Lemma, $q_0 > 0$ so that $R' = p$. Then (2) reads $\mathbf{a} p Eu' \leq Eu' \tilde{c} = Eu' \mathbf{m}$ so that $p \leq \mathbf{m}/\mathbf{a}$.

- Sufficiency:

* If $\mathbf{a} p < \mathbf{m}$, then $\mathbf{a} p Eu' < \mathbf{m} Eu' \leq Eu' \tilde{c}$. From (1) and (2) we know that

$Eu' R' \leq \min\left(Eu' p, \frac{Eu' \tilde{c}}{\mathbf{a}}\right)$. We deduce that $Eu' R' < \frac{Eu' \tilde{c}}{\mathbf{a}}$ and, consequently, $q_c = 0$.

* If $p = \mathbf{m}/\mathbf{a}$, suppose $q_c > 0$. Then $Eu' R' = Eu' \tilde{c}/\mathbf{a}$ by (2).

$\Rightarrow \mathbf{a} Eu' R' = \text{cov}(u', \tilde{c}) + Eu' \mathbf{m} = \text{cov}(u', \tilde{c}) + Eu' p \mathbf{a}$. Therefore $Eu' R' > Eu' p$ which violates (1).

We conclude that $q_c > 0$ is impossible. *QED*.

c)

- Necessity: $q_0 = 0 \Rightarrow q_c > 0$ by part a) of the Lemma $\Rightarrow Eu' R' = Eu' \tilde{c}/\mathbf{a}$ by (2)

$\Rightarrow R' Eu' = \text{cov}(u', \tilde{c})/\mathbf{a} + Eu' \mathbf{m}/\mathbf{a} \Rightarrow R'(\mathbf{a} q_c) = h(o, q_c) + \mathbf{m}/\mathbf{a}$ where $h(q_0, q_c) \stackrel{\text{def}}{=} \frac{\text{cov}(u', \tilde{c})}{\mathbf{a} Eu'}$ > 0 .

Let $\hat{q} = \arg \left\{ R'(\mathbf{a} \hat{q}) = h(o, \hat{q}) + \frac{\mathbf{m}}{\mathbf{a}} \right\}$. Therefore $q_0 = 0 \Rightarrow q_c = \hat{q}$, which implies $p \geq R'(\mathbf{a} \hat{q})$ by (2).

- Sufficiency:

$p \geq R'(\mathbf{a} \hat{q}) \Rightarrow p > R'(\mathbf{a} \hat{q} + q_0) \quad \forall q_0 > 0$ since $R'' < 0$. Consequently, when $p \geq R'(\mathbf{a} \hat{q})$, $q_0 > 0$ would violate (1). *QED*

Proof of Proposition 1. The first part follows directly from parts a) and c) of Lemma 1. The last part follows from parts a) and b) of Lemma 1. To obtain the middle part, note that when $q_0 > 0$ and $q_c > 0$, both (1) and (2) are equalities.

We now prove another lemma which will be used in the proof of Proposition 2.

Lemma 2. Under Decreasing Absolute Risk Aversion (alt. IARA), $Eu''(\tilde{p})(aR' - \tilde{c}) > 0$ (alt. < 0)

Proof of Lemma 2. From the first order condition (2) in the text, we can write

$$\int_{\tilde{c} < aR'} u'(\tilde{p})(aR' - \tilde{c})dF(\tilde{c}) = \int_{\tilde{c} > aR'} u'(\tilde{p})(\tilde{c} - aR')dF(\tilde{c})$$

Under DARA, $-u''/u'$ is an increasing function of \tilde{c} , so that

$$\int_{\tilde{c} < aR'} \left(-\frac{u''}{u'} \right) u'(\tilde{p})(aR' - \tilde{c})dF(\tilde{c}) < \int_{\tilde{c} > aR'} \left(-\frac{u''}{u'} \right) u'(\tilde{p})(\tilde{c} - aR')dF(\tilde{c}) \text{ which results in } Eu''(\tilde{p})(aR' - \tilde{c}) > 0.$$

And the opposite is true under IARA.

Proof of Proposition 2. Let $g_0(q_0, q_c, p) = 0$ and $g_c(q_0, q_c, p) = 0$ respectively stand for the first order conditions (1) and (2) in the text when $q_0 > 0$ and $q_c > 0$. With obvious notations, the second order conditions

$$E\{u'R'' + u''(R' - p)^2\} < 0, \quad \left| \begin{array}{cc} R''Eu' + (R' - p)^2Eu'' & aR''Eu' + (R' - p)Eu'' \cdot (aR' - \tilde{c}) \\ aR''Eu' + (R' - p)Eu'' \cdot (aR' - \tilde{c}) & a^2R''Eu' + Eu'' \cdot (aR' - \tilde{c})^2 \end{array} \right| > 0$$

read $g_{cc} < 0$ and $g_{00}g_{cc} - g_{0c}g_{c0} > 0$.

Differentiating the first order conditions, we can easily establish that

$$\text{sign of } \frac{dq_0}{dp} = \text{sign of } (g_{cp}g_{0c} - g_{0p}g_{cc})$$

$$\text{sign of } \frac{dq_c}{dp} = \text{sign of } (g_{c0}g_{0p} - g_{00}g_{cp})$$

Dividing by $g_{0p} = -Eu' < 0$, the sign of $\frac{dq_0}{dp}$ is given by $g_{cc} - \frac{g_{cp}g_{0c}}{g_{0p}}$. Because of the second order

conditions, the first term is negative: it is the substitution effect. The second term, the income effect, reads

$$\frac{-g_{cp}g_{0c}}{g_{0p}} = -q_0 aR'' Eu''(\tilde{p})(aR' - \tilde{c}) \quad (A1)$$

In the same way, one can check that the sign of $\frac{dq_c}{dp}$ is given by $-g_{c0} + \frac{g_{00}g_{cp}}{g_{op}}$. The substitution effect $-g_{c0} = -\mathbf{a}R''Eu' > 0$ shows that the client has an incentive to use more copies when the price of the legal item is increased. But the income effect is

$$\frac{g_{00}g_{cp}}{g_{op}} = q_0R''Eu''(\tilde{\mathbf{p}})(\mathbf{a}R' - \tilde{c}) \quad (\text{A2})$$

Therefore, to determine the sign of the income effects (A1) and (A2) we have to analyze the sign of $Eu''(\tilde{\mathbf{p}})(\mathbf{a}R' - \tilde{c})$. From Lemma 2, under IARA, (A1) is negative and (A2) is positive so that

$dq_0/dp < 0$ and $dq_c/dp > 0$. Under DARA, (A1) is positive so that $\text{sign } \frac{dq_0}{dp} = (-) + q_0(+)$ and A2

is negative so that $\text{sign } \frac{dq_c}{dp} = (+) + q_0(-)$.

To complete the proof, note that under CARA there is no income effect.

Proposition A. For a user either characterized by a CARA utility function and facing a normal distribution of the cost of copying or characterized by a quadratic utility function, the objective is to maximize the certainty equivalent $\mathbf{p} = R(q_0 + \mathbf{a}q_c) - pq_0 - \mathbf{m}q_c - \frac{1}{2}\mathbf{b}\mathbf{s}^2q_c^2$, where $\mathbf{m} = E\tilde{c}$, $\mathbf{s}^2 = E(\tilde{c} - \mathbf{m})^2$ and \hat{a} is an index of risk aversion.

Proof of Proposition A. Taking the mathematical expectation of a quadratic utility function $u(\tilde{\mathbf{p}}) = \tilde{\mathbf{p}} - \frac{\mathbf{b}(\tilde{\mathbf{p}} - E\tilde{\mathbf{p}})^2}{2}$ where $\mathbf{b} > 0$ and $\tilde{\mathbf{p}} = R(q_0 + \mathbf{a}q_c) - pq_0 - \tilde{c}q_c$, it is straightforward to derive the above formula which is true for any distribution function of the cost of copying \tilde{c} .

Consider now the case of a CARA utility function $u(\tilde{\mathbf{p}}) = -e^{-\mathbf{b}\tilde{\mathbf{p}}}$ and suppose that \tilde{c} follows a normal distribution with the mean \mathbf{m} and the variance \mathbf{s}^2 . The expected utility is

$$Eu(\tilde{\mathbf{p}}) = -\int e^{-\mathbf{b}(R(q_0 + \mathbf{a}q_c) - pq_0 - \tilde{c}q_c)} \frac{e^{-\frac{(\tilde{c} - \mathbf{m})^2}{2\mathbf{s}^2}}}{\mathbf{s}\sqrt{2\pi}} d\tilde{c}. \text{ Using the properties of the reduced normal}$$

distribution and integrating one readily obtains $Eu(\tilde{\mathbf{p}}) = -e^{-\mathbf{b}(R(q_0 + \mathbf{a}q_c) - pq_0 - \mathbf{m}q_c - \frac{1}{2}\mathbf{b}\mathbf{s}^2q_c^2)}$. QED

Proof of Proposition 3.

In part i), because attracting demand would give a negative margin $(p - c)$ for any q_0 , the best choice is to fix any price such that $q_0 = 0$. When the monopolist produces a positive quantity, he

can fix the monopoly price $p_1 = \frac{a+c}{2}$ as long as copying is drastically inefficient, that is $p_1 < \frac{m}{a}$ (part iv of the Proposition). Otherwise, the software company is facing potential competitors. Copying can be either deterred by charging the limit price $\bar{p} = \frac{m}{a}$ so that $q_c(\frac{m}{a}) = 0$, or accommodated by fixing price p_2 . In the set of sub-optimal prices ($p < p_1$) the higher, the better. Consequently, the monopoly will charge $\max(p_2, \frac{m}{a})$, which corresponds to cases ii) and iii).