

Carbon Sequestration in Agricultural Soils: Discounting for Uncertainty

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The study presents a conceptual model of an aggregator who selectively pays farmers for altering farming practices in exchange for carbon offsets that the change in practices generates. Under the assumption that the offsets are stochastic and that the aggregator maximizes the sum of the offsets from the purchase that he/she can rightfully claim with a specified level of confidence subject to a budget constraint, we investigate the optimal discounting of expected carbon offsets. We use the model to empirically estimate of the optimal discounting levels and costs for a hypothetical carbon purchasing project in the Upper Iowa River basin.

La présente étude porte sur un modèle théorique selon lequel un revendeur paie de façon sélective des producteurs pour qu'ils modifient leurs pratiques culturales en échange des contreparties de la fixation du carbone générées par ces changements. En supposant que les contreparties de la fixation du carbone sont stochastiques et que le revendeur maximise la somme des contreparties de l'achat qu'il réclame à juste titre avec un degré de confiance spécifié et sous réserve de restriction budgétaire, nous avons examiné l'escompte optimal des contreparties de la fixation du carbone prévues. Nous avons utilisé le modèle pour l'évaluation empirique des niveaux d'escompte et des coûts optimaux d'un projet hypothétique d'achat de carbone dans le bassin de la haute rivière Iowa.

INTRODUCTION

Agricultural communities in the United States, Canada, and in a number of other countries have been excited about the prospect of farmers selling credits for carbon sequestered in cropland soils as greenhouse gas emission offsets. However, one of the big practical issues hindering the potential carbon sales is the uncertainty associated with the offsets. From a buyer's point of view, future offsets are uncertain because carbon sequestration in agricultural soil is affected by a multitude of factors many of which, such as weather and solar radiation, are inherently stochastic.

While there is a growing soil science literature quantifying the uncertainty and its determinants, the topic received little attention in the economic analyses of carbon sequestration in cropland (Antle and McCarl 2002). Marland et al (2001) mention the importance of the uncertainty in the context of proposed carbon accounting protocols that allow the credits only if there is at least 95% certainty about their magnitude. McCarl et al (2004) also discuss the importance of the uncertainty and report the ensuing discounting of carbon offsets at the Chicago Climate Exchange at about 15%. Antle et al (2003) propose a sampling procedure to reduce the uncertainty about carbon sequestered,

yet neither study investigates how the presence of uncertainty alters economic agents' decision-making. This study attempts to fill in this gap by analyzing the mechanism of discounting carbon offsets for uncertainty.

The focus of the current study is an aggregator, the economic agent vital for carbon trading involving agriculture as large emitters of greenhouse gases usually need quantities of offsets much larger than any single farm can provide (Thomassin 2003; McCarl et al 2004). In a carbon sequestration program administered by a government agency the role of the aggregator is also important. In this case, the aggregator would assemble the offsets for reporting them to interested parties such as those representing taxpayers (for domestic programs) or to certifying international organizations (for international agreements). In any case, because of either market requirements or policy design stipulations, the aggregator may be seriously concerned about the uncertainty of the offsets being delivered and may adjust behavior accordingly.

This study presents a conceptual model of an aggregator selectively purchasing carbon offsets from the farmers who switch farming practices to those that increase soil carbon content. We assume that the aggregator facing a carbon price maximizes the sum of the offsets from the project that he/she can rightfully claim with a specified level of confidence, subject to a budget constraint. The model, which has similarities with the Capital Assets Pricing Model (CAPM) (see, e.g., Varian 1992),¹ builds on the earlier work on cost-efficiency of achieving probabilistic pollution reduction targets (see, e.g., Shortle and Horan 2001). Empirical applications of the approach have been limited and almost exclusively focused on water quality (Milon 1987; Lichtenberg et al 1989; Bystrom 1998; Shortle et al 1999; Bystrom et al 2000). In contrast, we build the model to specifically examine expected offset discounting arising because of uncertainty in the amount of carbon to be sequestered and the aggregator's concern about confidence bounds on the total offset. We apply the model to an empirical study of a hypothetical carbon sequestration project in the Upper Iowa River basin.

CONCEPTUAL MODEL

Assume there are N farms indexed by i that can potentially change their current farming practice to that sequestering carbon. Let \bar{x}_i denote the size of farm in acres and c_i be the per acre opportunity cost of changing practice (known to the aggregator). The farm i per acre offset generated by the change in practice, b_i , is stochastic, and because of varying natural conditions (soils and landscape characteristics, cropping history, etc.), the offset distributions differ potentially from farm to farm. Moreover, the b_i 's may be correlated across farms. The offsets on one farm may be positively correlated with those on another farm if, for example, the soils and other land characteristics are similar between the two farms, and weather tends to be similar on them as well. That is most likely the case for two farms located in close proximity to one another. In this case, if weather happens to be beneficial for carbon sequestration on one farm, it is likely to be similarly beneficial on the other farm, leading to positive correlation of offsets. However, the offsets may be negatively correlated when, for example, the weather patterns are very dissimilar (e.g., when it rains around one farm, it is customarily much drier at the other farm area). To reflect these possibilities, we assume that the b_i 's are jointly Normally distributed,

$$\begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_N \end{pmatrix} \sim N \left(\begin{pmatrix} \bar{b}_1 \\ \bar{b}_2 \\ \dots \\ \bar{b}_N \end{pmatrix}, \begin{pmatrix} \sigma_{11} & \sigma_{12} & \dots & \sigma_{1N} \\ \sigma_{12} & \sigma_{22} & \dots & \sigma_{2N} \\ \dots & \dots & \dots & \dots \\ \sigma_{1N} & \sigma_{2N} & \dots & \sigma_{NN} \end{pmatrix} \right) \tag{1}$$

where $\bar{b}_i \equiv E(b_i)$, $\sigma_{ii} \equiv \text{var}(b_i)$, $\sigma_{ij} \equiv \text{cov}(b_i, b_j)$, $i, j = 1, \dots, N$.

Aggregator is an economic agent who selectively offers farmers payments for switching the practices in exchange for carbon offsets that the change in practices generates. Aggregator faces a price, p , at which he/she can sell the offsets and has to cover the costs of the transaction by the proceeds.² For each farmer, i , the aggregator decides on the number of acres, x_i , $0 \leq x_i \leq \bar{x}_i$, on which to offer per acre payment c_i . We assume that as long as a farmer is offered the payment, he/she switches the practices and the aggregator acquires the offset.

Because of carbon market regulations (or those of the policy if the offset purchasing is done under auspices of a government-administered policy) the aggregator is concerned about the certainty of the total offset he/she is getting from the individual purchases. Specifically, we assume that the aggregator maximizes the amount of the aggregate offset that can be rightfully claimed with a confidence level α . The confidence level, α , is typically large and is greater than 0.5. Thus, the aggregator is maximizing the offset amount B defined by

$$\Pr \left\{ \sum_{i=1}^N b_i x_i \geq B \right\} = \alpha \tag{2}$$

Under the assumptions (1), the total carbon sequestered in the program, $\sum_{i=1}^N b_i x_i$, is normally distributed with the expected value $\sum_{i=1}^N \bar{b}_i x_i$ and variance $\sum_{i=1}^N \sum_{j=1}^N x_i x_j \sigma_{ij}$. Therefore, the deterministic equivalent of (2) is

$$B = \sum_{i=1}^N \bar{b}_i x_i - z_\alpha \sqrt{\sum_{i=1}^N \sum_{j=1}^N \sigma_{ij} x_i x_j}$$

where z_α is the number such that $\Pr\{Z \leq z_\alpha\} = \alpha$ and $Z \sim N(0, 1)$ (Charnes and Cooper 1963). Note that if the aggregator is indifferent between falling below and exceeding the total offset target B , then $\alpha = 0.5$ and $z_\alpha = 0$ meaning that aggregator is not making any adjustments to the uncertainty of the total offset and is simply maximizing the total expected value of the offsets purchased. However, $z_\alpha > 0$ as long as $\alpha > 0.5$, implying that in this case the aggregator is always *discounting* the expected value of the total offset purchased by the amount

$$A \equiv z_\alpha \sqrt{\sum_{i=1}^N \sum_{j=1}^N \sigma_{ij} x_i x_j} \tag{3}$$

As expected intuitively, the magnitude of discounting increases with the confidence level α and depends on the variability of offsets as described by their variance–covariance matrix.

Mathematically, the aggregator’s problem is

$$\max_{x_1, \dots, x_N} \sum_{i=1}^N \bar{b}_i x_i - z_\alpha \sqrt{\sum_{i=1}^N \sum_{j=1}^N x_i x_j \sigma_{ij}} \tag{4}$$

subject to the budget constraint,

$$p \left(\sum_{i=1}^N \bar{b}_i x_i - z_\alpha \sqrt{\sum_{i=1}^N \sum_{j=1}^N x_i x_j \sigma_{ij}} \right) - \sum_{i=1}^N c_i x_i \geq 0,$$

the N land constraints, $\bar{x}_i - x_i \geq 0, i = 1, \dots, N$, and the N nonnegativity constraints, $x_i \geq 0, i = 1, \dots, N$.

The aggregator’s problem has similarities with the CAPM (e.g., Varian 1992). As in the CAPM, the aggregator is investing to get a return, which is in this problem measured in claimable units of carbon. There are several assets in which the aggregator can invest: a risk-free asset (the market) where \$1 brings a return of $1/p$ units of carbon, and some N risky assets represented by the individual farmers from which the aggregator can obtain carbon by paying them to change their farming practices. The return on an i th risky asset is b_i/c_i units of carbon per \$1 with the expected return of \bar{b}_i/c_i units of carbon per \$1 invested. Under this interpretation, the decision on which farmers to enroll in the program is then the decision on the optimal composition of an investment portfolio. Since the objective function in (4) is a function of the mean and the variance of the portfolio return only, similarly to the CAPM, the investor is minimizing the variance of the portfolio for a given level of expected return.

Model (4) differs from the classical versions of the CAPM in the existence of the constraints on the quantities of the individual risky asset available for investment (the land constraints, $\bar{x}_i - x_i \geq 0, i = 1, \dots, N$). If, however, the budget constraint is binding, and the land constraints are not binding at the solution to (4), i.e., if the optimal numbers of acres enrolled, $x_i^*, i = 1, \dots, N$, satisfy the inequalities $x_i^* < \bar{x}_i, i = 1, \dots, N$, then the analogy with the CAPM can be taken further. Specifically, as detailed in the Appendix, in this case

$$\frac{\bar{b}_i}{c_i} = \frac{1}{p} + \frac{\text{cov}\left(\frac{b_i}{c_i}, \frac{\tilde{B}}{C}\right)}{\text{var}\left(\frac{\tilde{B}}{C}\right)} \left(\frac{\tilde{B}}{C} - \frac{1}{p} \right) \tag{5}$$

for all i ’s for which x_i^* is positive. Here $\tilde{B} = \sum_{i=1}^N b_i x_i^*, C = \sum_{i=1}^N c_i x_i^*$, and \tilde{B}/C can be interpreted as the portfolio return; $\bar{\tilde{B}} = \sum_{i=1}^N \bar{b}_i x_i^*$ and $\bar{\tilde{B}}/C$ can be interpreted as the expected portfolio return. Thus, formula (5) conveys the well-known message of the CAPM: for any efficient portfolio of risky assets, the expected return on any asset is equal

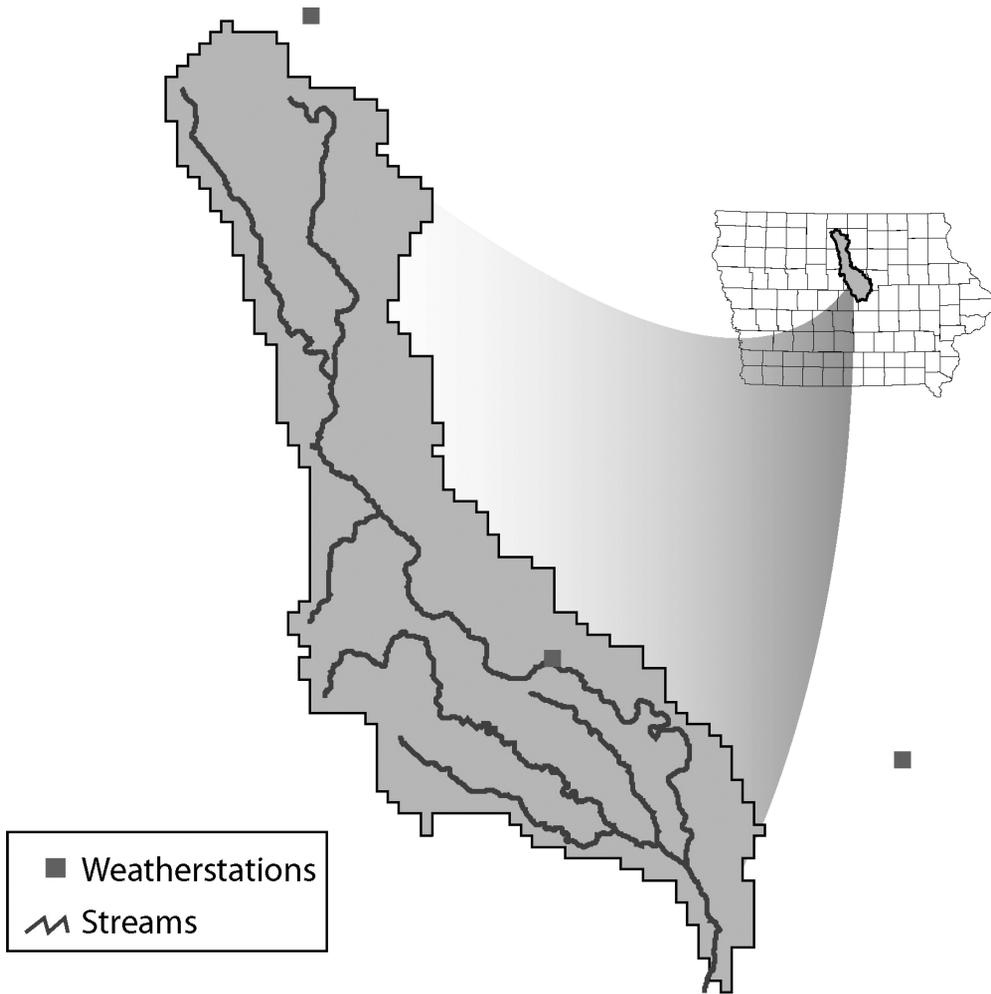


Figure 1. Upper Iowa River basin and the location of weather stations

to the risk-free return plus a risk premium that depends on the covariance of the asset's return with the efficient portfolio of risky assets and is proportional to the difference between the expected return on the portfolio and that on the risk-free asset.

Next section presents an empirical application of the model to the analysis of the expected offset discounting for a hypothetical carbon purchase project in an agricultural production area in the United States.

EMPIRICAL APPLICATION

The empirical study region is the Upper Iowa River basin defined as U.S. Geological Survey 8-digit Hydrologic Cataloging Unit watershed 7080207 (Seaber et al 1987)(Figure 1). We investigate a hypothetical carbon project that pays farmers for retiring land from

Table 1. Data summary, 346 NRI points

Variable	Notation	Sample min	Sample average	Sample max
Expected carbon offset, metric tons C per acre	\bar{b}_i	0.014	0.643	2.004
Cost of retiring land from production, \$ per acre	c_i	81.7	130.5	188.6
NRI expansion acres	\bar{x}_i	100	2004	3200
Variance of carbon offset, metric tons C squared per acre squared	σ_{ii}	0.030×10^{-3}	2.493×10^{-3}	86.055×10^{-3}

crop production and placing it under permanent grass cover in the Conservation Reserve Program (CRP). We consider the uncertainty of offsets resulting from uncertainty in weather, which is known to significantly affect carbon sequestration of CRP (Bruce et al 1999; Follett et al 2001; Paustian et al 2001).

The basic data for simulations come from 1997 National Resources Inventory (NRI) (Nusser and Goebel 1997); each NRI point is treated as representing a farm with the size equal to the number of acres represented by the point (the NRI expansion factor). Some $N = 346$ NRI data points in the basin representing 693,400 acres of cropland are used for the analysis. The estimates of opportunity costs of retiring land from production, c_i , come from Kurkalova et al (2004), who followed the approach of Smith (1995) to measure the opportunity cost of land retirement via cropland cash rental rates. Given that the area is a part of prime agricultural land, it's not surprising that the costs of land retirement are very high: they average over \$130 per acre (Table 1).

The empirical distributions of offsets, b_i , are obtained at each data point using simulation model EPIC (Williams 1990) as follows. First, we use EPIC to generate $M = 49$ random weather patterns from the distribution of weather patterns as recorded by the three weather stations in the region. Next, we run $2M$ 30-year simulations at each data point: M assuming conventional tillage practices and M assuming land retirement. Then, we compute M estimates of carbon sequestration potential as the difference in soil carbon content after 30 years under land retirement and that under tillage, divided by 30, each time pairing the simulations corresponding to the same weather pattern. Finally, the resulting $N \cdot M$ estimates (N points times M weather patterns) are used to compute sample means \bar{b}_i , variances σ_{ii} , and covariances σ_{ij} , $i, j = 1, \dots, N$. The average of the expected per acre offsets in the sample, $1,587 \text{ kg C ha}^{-1} \text{ year}^{-1}$, compares favorably with the estimates for this region (Follett et al 2001; Paustian et al 2001). Summary statistics on the data used in simulations are given in Table 1.

Given the data on c_i , \bar{b}_i , σ_{ii} , and σ_{ij} , $i, j = 1, \dots, N$, the aggregator's problem (4) is solved for three levels of carbon prices, \$80/tonne, \$90/tonne, and \$100/tonne.³ For each of the prices, three confidence levels, $\alpha = 0.90, 0.95$, and 0.99 are analyzed. For comparison purposes, we also report results for the case of $\alpha = 0.50$ corresponding to maximizing total expected offset (and no discounting).

Table 2. Simulations results

	Carbon price, \$ per tonne											
	80				90				100			
	0.50	0.90	0.95	0.99	0.50	0.90	0.95	0.99	0.50	0.90	0.95	0.99
Confidence level, α												
Carbon claimable, 1,000 tonnes	51.9	44.0	42.3	41.1	92.0	76.4	72.5	65.9	152.6	130.3	123.7	114.4
Total expected carbon discount, percent	0	2.3	2.9	3.9	0	2.5	3.1	4.2	0	3.6	4.3	5.2
Percent area enrolled in purchase	4.9	4.3	4.1	3.9	9.5	8.1	7.7	7.1	16.6	14.6	14.0	13.0
Cost of purchase, \$ million	4.2	3.5	3.4	3.2	8.3	6.8	6.5	5.9	15.2	13.0	12.4	11.4
Payment for discounting, percent of purchase cost	0	5.7	6.4	7.5	0	5.7	6.8	8.8	0	5.3	6.3	7.7

RESULTS

To simplify comparisons across budgets and confidence levels, the estimated expected offset discounting is reported in Table 2 in relative terms, i.e., as a percentage of the corresponding expected offset, i.e., the total expected offset discount is reported as $A^*/\sum_{i=1}^N \bar{b}_i x_i^*$. The results of estimation suggest that weather uncertainty as simulated is consistent with the total expected carbon discounting in the range of 2.3 to 5.2%. Interestingly, for a given confidence level, the relative magnitude of discounting increases as carbon price increases. For example, when offset target is to be achieved with the confidence level of $\alpha = 0.99$, the total expected offset discounting was found to increase from 3.9% at the price \$80/tonne to 4.2% at the price \$90/tonne and to 5.2% at the price \$100/tonne.

Results of estimation clearly demonstrate that the economic feasibility of sequestration in agricultural soils should be addressed with the confidence levels taken into account. For example, if offset price is set at \$100 and the confidence level is of no concern, purchasing 0.153 million metric tons of offset is profitable in this area. But if the confidence level for the offsets were to increase to 99%, the quantity of economically feasible carbon offsets from the area declines some 25% to 0.144 million metric tons. Thus, ignoring the confidence levels may lead to unrealistically optimistic estimates of economic feasibility of carbon sequestration in agricultural soils.

To monetize the effect of offset discounting in an alternative way, we estimate the additional expenses the aggregator incurs because of purchasing the claimable offset $B^* = \sum_{i=1}^N \bar{b}_i x_i^* - A^*$, with a specified confidence level as opposed to purchasing the same offset B^* with the confidence level $\alpha = 0.50$, which does not involve discounting.

These additional expenses expressed as the percentage of the total cost of purchase are reported as “payment for discounting” in Table 2. We found that a sizable share of the total cost of purchase, from 5.3 to 8.8%, may be spared for nothing but making sure that the total offset is claimable with the specified confidence level.

CONCLUDING COMMENTS

The study presents a model of discounting expected carbon sequestration offsets for uncertainty and estimates that weather variability is consistent with up to 5% discounting of expected offsets from retiring land from agricultural production in the Upper Iowa River basin for the carbon prices of \$80 to \$100/tonne and offset confidence levels of 90 to 99%. We found that nearly 9% of the cost of purchase may be used exclusively to ensure the specified confidence levels of the offsets. The results underscore the importance of incorporating uncertainty and offset confidence levels in the economic assessments of carbon sequestration potential of agricultural soils. Ignoring the uncertainty may lead to overly optimistic conclusions about economically feasible carbon sequestration levels.

While the numerical estimates of the optimal discounting levels and costs may not be immediately transferable to other regions and farming practices, the modeling framework presented can be applied to study the effects of other sources of offset uncertainty. A particularly fascinating extension of this work would be to model and estimate the discounting due to uncertainty about the permanence of the offsets. In this case, the assumption on Normality of the distributions of farm-level offsets would probably have to be replaced with that of a more suitable distribution thus requiring alternative derivation or estimation of the certainty equivalent of the probabilistic definition of offset target.

NOTES

¹The author is thankful to an anonymous reviewer for contributing this observation.

²A government-administered carbon sequestration program may have the budget constraint imposed by a fixed program cost rather than the carbon price. The model presented can be modified to accommodate such setting.

³While the price of carbon is yet to be determined by developing carbon markets, the prices chosen for the analysis fall in the range regarded as reasonably realistic in the literature (see, e.g., the discussion in Williams et al 2005).

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APPENDIX

The Lagrangian to problem (4) can be written as

$$L = \sum_{i=1}^N \bar{b}_i x_i - z_\alpha \sqrt{\sum_{i=1}^N \sum_{j=1}^N x_i x_j \sigma_{ij}} - \lambda \left(\sum_{i=1}^N c_i x_i - p \left(\sum_{i=1}^N \bar{b}_i x_i - z_\alpha \sqrt{\sum_{i=1}^N \sum_{j=1}^N x_i x_j \sigma_{ij}} \right) \right) - \sum_{i=1}^N \theta_i (x_i - \bar{x}_i)$$

where λ and $\theta_i, i = 1, \dots, N$, are the Lagrange multipliers. The nonbinding land constraints imply that the optimum values of the Lagrange multipliers associated with the land constraints are equal zero for all i . The binding budget constraint implies that the optimum value of the Lagrange multiplier associated with the budget constraint, λ^* , is different from zero.

Let N^* be the set of indexes i for which $0 < x_i^* < \bar{x}_i$. Let $\tilde{B} = \sum_{i=1}^N b_i x_i^*$, $C = \sum_{i=1}^N c_i x_i^*$, and $\tilde{\bar{B}} = \sum_{i=1}^N \bar{b}_i x_i^*$. Then $\text{var}\left(\frac{\tilde{B}}{C}\right) = \frac{1}{C^2} \sum_{i=1}^N \sum_{j=1}^N \sigma_{ij} x_i^* x_j^*$ and $\text{cov}\left(\frac{b_i}{c_i}, \frac{\tilde{B}}{C}\right) = \frac{1}{c_i C} \sum_{j=1}^N \sigma_{ij} x_j^*$. From the first-order Kuhn–Tucker conditions,

$$\bar{b}_i - z_\alpha c_i \frac{\text{cov}\left(\frac{b_i}{c_i}, \frac{\tilde{B}}{C}\right)}{\sqrt{\text{var}\left(\frac{\tilde{B}}{C}\right)}} = \frac{\lambda}{1 + \lambda p} c_i, \quad \text{for all } i \in N^* \tag{A.1}$$

Multiplying (A.1) by x_i^* and summing over all i we get

$$\tilde{\bar{B}} - z_\alpha \sqrt{\text{var}\left(\frac{\tilde{B}}{C}\right)} = \frac{\lambda}{1 + \lambda p} C$$

Combining the last equation with (A.1), we obtain

$$\frac{\bar{b}_i}{c_i} - z_\alpha \frac{\text{cov}\left(\frac{b_i}{c_i}, \frac{\tilde{B}}{C}\right)}{\sqrt{\text{var}\left(\frac{\tilde{B}}{C}\right)}} = \frac{\tilde{\bar{B}}}{C} - z_\alpha \sqrt{\text{var}\left(\frac{\tilde{B}}{C}\right)}, \quad \text{for all } i \in N^* \tag{A.2}$$

Finally, note that the binding budget constraint imply

$$z_\alpha \sqrt{\text{var}\left(\frac{\tilde{B}}{C}\right)} = \frac{\tilde{\bar{B}}}{C} - \frac{1}{p}$$

Substituting the last expression in (A.2), we obtain (5).