The Calibration of Incomplete Demand Systems in Quantitative Analysis

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Abstract

We introduce an easily implemented and flexible calibration technique for partial demand systems, combining recent developments in incomplete demand systems and a set of restrictions conditioned on the available elasticity estimates. The technique accommodates various degrees of knowledge on cross-price elasticities, satisfies curvature restrictions, and allows the recovery of an exact welfare measure for policy analysis. The technique is illustrated with a partial demand system for food consumption in Korea for different states of knowledge on cross-price effects. The consumer welfare impact of food and agricultural trade liberalization is measured.

Keywords: calibration, exact welfare measure, incomplete demand systems, policy analysis.

THE CALIBRATION OF INCOMPLETE DEMAND SYSTEMS IN QUANTITATIVE ANALYSIS

Introduction

This paper is a methodological contribution to policy analysis and more particularly to the calibration of partial demand systems involving a subset of disaggregated goods. Our approach provides a feasible and suitable answer to the following generic problem. To quantify the impact of changing market conditions (e.g., policy shock or brand structure) on a subset of markets and consumer welfare, economic analysis often requires the calibration of disaggregated but partial demand systems and the recovery of a welfare measure associated with multiple price changes affecting these demands. We obviously have policy analysis in mind, but the approach applies to modeling other exogenous changes in markets (see Baltas 2002 for a business application).

We introduce an easily implemented and flexible calibration technique for partial demand systems combining recent developments in incomplete demand systems (LaFrance 1998) and a set of restrictions conditioned on the available elasticity estimates. The proposed technique accommodates various degrees of knowledge on cross-price elasticities, satisfies curvature restrictions, and allows the recovery of an exact welfare measure for economic analysis. The calibration technique is illustrated with an incomplete demand system for agricultural and food consumption in Korea and for different states of knowledge on cross-price responses. Then, we measure the consumer welfare impact of a policy shock and the trade liberalization of agricultural and food markets, and we assess the sensitivity of the welfare measure to the inclusion/deletion of cross-price effects.

Calibration, rather than econometric estimation, is the rule in quantitative policy analysis for several reasons. First, quantitative policy analysis typically occurs when data are not available to estimate a demand system (partial or full), or when the data are too old to make the analysis current and representative of current market conditions. In addition, to palliate the data availability problem, the econometric estimation of partial systems often relies on restricting assumptions on separability precluding welfare analysis because the recovery of an exact welfare measure is difficult or impossible (Moschini 2001).

Other considerations also matter. Typically, a small subset of markets is relevant for the analysis (e.g., food markets). However, these few markets have to be sufficiently disaggregated for the analysis to be meaningful and useful (e.g., dairy, livestock, grains, or oilseeds, as opposed to aggregate agriculture and food). This disaggregation requirement exacerbates the data availability problem. Timeliness is another important consideration. For example, congressional requests impose a tight schedule on policy analysts with the U.S. General Accounting Office or policy organizations. These tight deadlines exclude the collection of recent data and careful econometric estimation.

Calibration has its own drawbacks. It requires "finding" a large set of elasticities,[1] which may come from various sources or which may not exist. Most often, the set of elasticities is incomplete and ad hoc restrictions are added to palliate the lack of available estimates (e.g., OECD 2000). Particularly acute is the problem of unknown cross-price responses. Many applied researchers restrict unknown cross-price elasticities to zero (Roningen, Sullivan, and Dixit 1991; OECD 2000). From this ad hoc, incomplete demand system, one cannot recover an exact welfare measure. This shortcoming plagues well-known applied partial-equilibrium models. Other researchers force cross-price responses for all concerned goods to be positive, identical, or proportional to expenditure shares (Keller 1984; Moschini 1999; Selvanathan 1985). The latter two approaches lead to an exact welfare measure for the representative consumer but impose too much structure on key parameters (cross-price effects).

To summarize, could the calibration of the partial demand model generate calibrated estimates of missing cross-price responses, based on the few existing estimates of elasticities available to the analyst (typically, the own-price and income elasticities)? Further, could it lead to exact welfare measures, such as equivalent variation (EV) or compensating variation (CV)? Finally, could the calibration procedure be adaptable, as new econometric estimates become available for the missing cross-price elasticities?

The calibration method we propose provides a satisfactory answer to all three questions. The approach is flexible in the sense that it does not impose restrictions on available individual income response or cross-price effects. For example, complementarity between any two goods is easily accommodated. Finally, the approach satisfies curvature restrictions (concavity).[2]

The paper is organized as follows. First, we introduce incomplete demand systems. We follow with the presentation of the calibration method and the procedure to accommodate various cross-price effects. We provide sufficient conditions for concavity to be satisfied, which are defined over available elasticity estimates. Then, we follow with an illustration of welfare measurement of consumer price changes in Korea.

Incomplete Demand Systems

LaFrance (1985), LaFrance and Hanemann (1989), and LaFrance et al. (2002) proposed a methodology of identification and recovery of the structure of preferences for incomplete demand systems. The researchers obviously had econometric applications in mind, but as we show in the next section, the approach they used provides fruitful grounds for calibration exercises. The most recent development in incomplete demand systems is the LinQuad system, which is quadratic in price and linear in income (LaFrance 1998). LinQuad preserves the theoretical consistency of the previous incomplete demand systems but allows for more flexibility to reflect preferences underlying the demand system by including quadratic price terms in its specification.

Integrability conditions establish the connection between a system of demands and a well-behaved expenditure function. These conditions ensure that the demands are consistent with well-behaved consumer preferences. Utility maximization subject to a budget constraint results in a complete set of demand functions with certain properties. If a subset of demands from this complete demand system is considered separately, its properties change only slightly. The key insight in this body of work is the development of a duality theory of incomplete systems, as explained next.

Consider a system of Marshallian demands:

$$\boldsymbol{x} = \boldsymbol{x}^{M}(\boldsymbol{q}, \boldsymbol{q}_z, R), \tag{1}$$

where $\boldsymbol{x}$=[x_1, ..., x_n]' is the vector of consumption levels for the commodities of interest to the modeler, $\boldsymbol{q}$= [q_1, ..., q_n]' is the corresponding price vector, $\boldsymbol{q}_z$=[q_{z1}, ...,q_{zm}]' is the corresponding price vector for the vector of consumption levels of all other commodities denoted by variable $\boldsymbol{z}$=[z_1, ..., z_m] with $m \geq 2$, and R is income. Commodities to be included in **x** are selected on a case-by-case basis depending on the policy problem to quantify.

Maximizing an increasing, quasi-concave utility function, $u(\boldsymbol{x}, \boldsymbol{z})$, with respect to consumption, under the budget constraint $\boldsymbol{q}'\boldsymbol{x} + \boldsymbol{q}_z'\boldsymbol{z} \leq R$ results in demands for the goods of interest with four properties: (*a*) the demands are positive valued, $\boldsymbol{x} = \boldsymbol{x}^M(\boldsymbol{q}, \boldsymbol{q}_z, R) > \boldsymbol{0}$; (*b*) the demands are zero degree homogeneous in all prices and income, $\boldsymbol{x}^M(\boldsymbol{q}, \boldsymbol{q}_z, R)$= $\boldsymbol{x}^M(t\boldsymbol{q}, t\boldsymbol{q}_z, tR)$ for all $t \geq 0$; (*c*) the $n \times n$ matrix of compensated substitution effects for $\boldsymbol{x}$, or Slutsky matrix $\boldsymbol{S} = \partial \boldsymbol{x}^M / \partial \boldsymbol{q}' + \partial \boldsymbol{x}^M / \partial R \, \boldsymbol{x}^{M\prime}$, is symmetric, negative semi-definite; and (*d*) total expenditure on the subset of the goods of interest consumed is strictly smaller than income, $\boldsymbol{q}'\boldsymbol{x}^M(\boldsymbol{q}, \boldsymbol{q}_z, R) < R$.

Complete and incomplete demand systems share the first three properties. The last property is specific to incomplete systems. A composite commodity including all other final goods establishes the link between complete and incomplete systems. The expenditure on this composite good is defined as $s = \boldsymbol{q}_z'\boldsymbol{z} = R - \boldsymbol{q}'\boldsymbol{x}$. With a properly defined utility function and the price of s innocuously normalized to one, duality applies to the incomplete system just as if it were a complete system (LaFrance et al. 2002). The four properties of the incomplete demand system and new budget identity are equivalent to the existence of an expenditure function,

$$e(\boldsymbol{q}, \boldsymbol{q}_z, u) = \boldsymbol{q}'\boldsymbol{x}[\boldsymbol{q}, \boldsymbol{q}_z, e(\boldsymbol{q}, \boldsymbol{q}_z, u)] + s[\boldsymbol{q}, \boldsymbol{q}_z, e(\boldsymbol{q}, \boldsymbol{q}_z, u)]. \tag{2}$$

By applying integrability conditions, the LinQuad demand system is generated from the following quasi-expenditure function:

$$e(\boldsymbol{q}, \boldsymbol{q}_z, \theta) = \boldsymbol{q}'\boldsymbol{\varepsilon} + \tfrac{1}{2}\boldsymbol{q}'\boldsymbol{V}\boldsymbol{q} + \delta(\boldsymbol{q}_z) + \theta(\boldsymbol{q}_z, u)e^{\boldsymbol{\chi}'\boldsymbol{q}}, \tag{3}$$

where $\boldsymbol{q}$ is the vector of prices; $\delta(\boldsymbol{q}_z)$ is an arbitrary real-valued function of $\boldsymbol{q}_z$; $\theta(\boldsymbol{q}_z, u)$ is the constant of integration increasing in u; and $\boldsymbol{\chi}$, $\boldsymbol{\varepsilon}$, and $\boldsymbol{V}$ are the vectors and matrix of parameters to be recovered in the calibration.

Hicksian demands, $\boldsymbol{x}$, are obtained by applying Shepherd's lemma to (3):

$$\boldsymbol{x} = \boldsymbol{\varepsilon} + \boldsymbol{V}\boldsymbol{q} + \boldsymbol{\chi}[\theta(\boldsymbol{q}_z, u)e^{\chi'\boldsymbol{q}}]. \tag{4}$$

The integrating factor, $e^{\chi'\boldsymbol{p}}$, makes the demand system an exact system of partial differential equations. The LinQuad expenditure function (3) provides a complete solution class to this system of differentials and represents the exhaustive class of expenditure functions generating demands for $\boldsymbol{x}$ that are linear in total income and linear and quadratic in prices for $\boldsymbol{x}$.

Solving the quasi-expenditure function (3) for $\theta(\boldsymbol{q}_z, u)e^{\chi'\boldsymbol{p}}$, and replacing the expenditure with R for income yields the LinQuad Marshallian demands:

$$\boldsymbol{x}^M = \boldsymbol{\varepsilon} + \boldsymbol{Vq} + \boldsymbol{\chi}(R - \boldsymbol{\varepsilon}'\boldsymbol{q} - \tfrac{1}{2}\boldsymbol{q}'\boldsymbol{Vq} - \times(\boldsymbol{q}_z)) \,. \tag{5}$$

The uncompensated own- and cross-price elasticities are

$$\eta_{ii} = [v_{ii} - \chi_i(\varepsilon_i + \textstyle\sum_j \boldsymbol{v_{ij}q_j})]q_i/x_i\,, \tag{6a}$$

and

$$\eta_{ij} = [v_{ij} - \chi_i(\varepsilon_j + \textstyle\sum_k \boldsymbol{v_{jk}q_k})]q_j/x_i. \tag{6b}$$

The corresponding Hicksian price elasticities are obtained from the Slutsky matrix $\boldsymbol{S}=\boldsymbol{V}+ (R\text{-}\boldsymbol{\varepsilon}'\boldsymbol{q}\text{-}0.5\boldsymbol{q}'\boldsymbol{Vq} - \delta(\boldsymbol{q}_z))\boldsymbol{\chi\chi}'$, which leads to own- and cross-price compensated elasticities,

$$\eta^h_{ii} = [v_{ii} + \chi^2_i(R\text{-}\boldsymbol{\varepsilon}'\boldsymbol{q}\text{-}0.5\boldsymbol{q}'\boldsymbol{Vq} - \delta(\boldsymbol{q}_z))]q_i/x_i, \tag{7a}$$

and

$$\eta^h_{ij} = [v_{ij} + \chi_i\,\chi_j(R\text{-}\boldsymbol{\varepsilon}'\boldsymbol{q}\text{-}0.5\boldsymbol{q}'\boldsymbol{Vq} - \delta(\boldsymbol{q}_z)))]q_j/x_i. \tag{7b}$$

The duality theory of incomplete demand systems allows exact welfare measures to be obtained from the quasi-indirect utility function. To derive the EV associated with the LinQuad demand system (5), the quasi-expenditure equation (3) is inverted with respect to θ after being set equal to income, R, or $\theta(\boldsymbol{q}, u, \boldsymbol{z}) = [R\text{-}\ \boldsymbol{q}'\boldsymbol{\varepsilon}\ \text{-}\ \tfrac{1}{2}\boldsymbol{q}'\boldsymbol{V}\,\boldsymbol{q}\text{-}\ \delta(\boldsymbol{q}_z)]e^{-\chi'q}$.

The EV identity becomes

$$[R+EV - \boldsymbol{q}^{0}{}'\boldsymbol{\varepsilon} - \tfrac{1}{2}\, \boldsymbol{q}^{0}{}'\boldsymbol{V}\boldsymbol{q}^{0} - \delta(\boldsymbol{q}_z)]e^{-\chi'q^{0}} = [R - \boldsymbol{q}^{1}{}'\boldsymbol{\varepsilon} - \tfrac{1}{2}\, \boldsymbol{q}^{1}{}'\boldsymbol{V}\boldsymbol{q}^{1} - \delta(\boldsymbol{q}_z)]e^{-\chi'q^{1}}, \qquad (8)$$

where $\mathbf{q}^0$ and $\mathbf{q}^1$ are vectors of prices of $\mathbf{x}$ before and after the policy shock inducing the price changes, respectively. The EV is

$$EV = [R - \boldsymbol{q}^{1}{}'\boldsymbol{\varepsilon} - \tfrac{1}{2}\boldsymbol{q}^{1}{}'\boldsymbol{V}\boldsymbol{q}^{1} - \delta(\boldsymbol{q}_z)]e^{\chi(q^{0}-q^{1})} - [R - \boldsymbol{q}^{0}{}'\boldsymbol{\varepsilon} - \tfrac{1}{2}\, \boldsymbol{q}^{0}{}'\boldsymbol{V}\boldsymbol{q}^{0} - \delta(\boldsymbol{q}_z)]. \qquad (9)$$

The CV measure can be obtained following similar steps.

Calibration

Our calibration approach builds on the LinQuad structure explained in the previous section as the foundation for the partial demand system. Then, it imposes a set of restrictions on the system conditioned on the available information and integrability to recover taste parameters. From the latter we generate values for missing elasticities and an exact welfare measure consistent with the initial price and income responses on hand. The necessary information set for the calibration is as follows: income and own-price elasticity estimates; levels of Marshallian demands x_i^M; level of income R; prices q_i or, alternatively, expenditure $(x_i\, q_i)$;[3] and, optionally, some cross-price elasticity estimates for good i and $j = 1,\ldots,n$.

More specifically, the calibration involves the recovery of elements of the n-vectors $\boldsymbol{\chi}$ and $\boldsymbol{\varepsilon}$, together with the elements of the $n\times n$ matrix $\boldsymbol{V}$ in equation (5). The calibration imposes symmetry and negative semi-definiteness of $\boldsymbol{S}$, the Hessian of e. Homogeneity of degree one in prices for e is imposed by deflating prices by a consumer price index serving as a proxy for the price of all other goods. Homogeneity in prices plays no role in the recovery of parameters in the calibration procedure.

The calibration is done sequentially. First, point estimates of derivatives of demand with respect to income are obtained from the known income elasticity estimates. Then, income response parameters $\boldsymbol{\chi}$ are substituted into equations (5) and (6). Next, price responses are recovered from the point estimates corresponding to the available price elasticities, evaluated at the reference level of the data. Then, all price responses, together with restrictions on ***S*** from integrability, and the observed demand levels are used to estimate the parameters of the model.

Derivation of Income Responses χ

From the available income elasticity estimates of demand x_i^M, η_{iI}, we derive the vector of parameters $\boldsymbol{\chi}$, the vector of partial derivatives of the Marshallian demands with respect to income, $\chi_i = x_i^M{}_i\eta_{iI} / R$.

Integrability Conditions and Derivation of Parameters ε and V

Symmetry of ***V*** is sufficient to ensure the symmetry of Slutsky matrix ***S***. Symmetry of ***S*** implies that $v_{ij}=v_{ji}$. This is imposed by choosing a preferred cross-price elasticity η_{ij}, if Marshallian cross-price responses are available, to be substituted in (6b) and then identifying a single v_{ij} as explained in what follows. Then the symmetric element v_{ji} is set equal to the identified v_{ij}. If no estimate of η_{ij} is available then we set $v_{ij}=v_{ji}=0$ in (6b), and the unavailable η_{ij} becomes the unknown variable of interest in this case.

Regarding curvature, ***S*** should be negative semi-definite to satisfy quasi-concavity of the utility function. We distinguish two cases. The first case refers to the simple situation in which only own-price and income elasticities are available (i.e., $v_{ij}=v_{ji}=0$). We derive a sufficient condition for the concavity of the calibrated demand system, which applies to the available elasticity estimates. The condition is based on strict diagonal dominance and the Gerschgorin-Hadamard theorems (Lascaux and Théodor 1986, Theorems 53 and 57 and corollary 63). These theorems, applied to any real symmetric matrix with positive diagonal terms, say that if the absolute value of each diagonal term of such a matrix is larger (at least as large) as the sum of the individual absolute values of the off-diagonal terms of the corresponding row or column, then the matrix is positive (semi-)definite. These theorems are applied to $-\boldsymbol{S}$, which should be positive semi-definite for (quasi-) concavity to be satisfied. The dominance condition for any Slutsky matrix is

$$\left| -v_{ii} - \chi_i^2[R - \boldsymbol{q^0}'\boldsymbol{\varepsilon} - \tfrac{1}{2}\, \boldsymbol{q^0}'\boldsymbol{V}\boldsymbol{q^0} - \delta(\boldsymbol{q}_z)] \right| \geq \sum_{j\neq i} \left| -v_{ij} - \chi_i\, \chi_j\, [R - \boldsymbol{q^0}'\boldsymbol{\varepsilon} - \tfrac{1}{2}\, \boldsymbol{q^0}'\boldsymbol{V}\boldsymbol{q^0} - \delta(\boldsymbol{q}_z)] \right|. \quad (10)$$

Recall that in this first calibration case, off-diagonal terms of ***S*** are made just off the income effect in the Slutsky decomposition, $\chi_i\chi_j[R - \boldsymbol{q^0}'\boldsymbol{\varepsilon} - \tfrac{1}{2}\, \boldsymbol{q^0}'\boldsymbol{V}\boldsymbol{q^0} - \delta(\boldsymbol{q}_z)]$, since $v_{ij}=0$.

In order to transform inequality (10) in elasticity terms, we momentarily normalize prices $\boldsymbol{q}$ to one by appropriate choice of units and without any loss of generality. Diagonal dominance condition (10) is preserved by adding on both sides the income effect of good i ($\chi_i x_i$).[4] In elasticity form, the dominance condition becomes

$$\left| -\eta_{ii} \right| \geq \sum_{j\neq i} \left| -\chi_i \chi_j [R - \boldsymbol{q}^{\mathbf{0}}{}'\boldsymbol{\varepsilon} - \tfrac{1}{2}\, \boldsymbol{q}^{\mathbf{0}}{}'\boldsymbol{V}\boldsymbol{q}^{\mathbf{0}} - \delta(\boldsymbol{q}_z)]/x_i \right| + \chi_i. \tag{11}$$

Next, we substitute income R for $[R - \boldsymbol{q}^{\mathbf{0}}{}'\boldsymbol{\varepsilon} - \tfrac{1}{2}\, \boldsymbol{q}^{\mathbf{0}}{}'\boldsymbol{V}\boldsymbol{q}^{\mathbf{0}} - \delta(\boldsymbol{q}_z)]$ in (11), which reinforces the inequality. It leads to the following sufficient condition for concavity in terms of available information on the Marshallian own-price elasticities, income elasticities, and expenditure shares:

$$\left| -\eta_{ii} \right| - \sum_{j\neq i} \left| -\eta_{iI}\, \eta_{jI}\, \alpha_j \right| - \alpha_i\, \eta_{iI}, \geq 0 \tag{12}$$

with parameters α_i denoting the total expenditure share of good i. Hence one can check right away if a chosen set of available estimates of elasticities satisfies dominance if it satisfies sufficient condition (12). It is not a necessary condition and it is slightly stronger than the dominance condition (11) since income R is larger than the income term in the demand $[R - \boldsymbol{q}^{\mathbf{0}}{}'\boldsymbol{\varepsilon} - \tfrac{1}{2}\, \boldsymbol{q}^{\mathbf{0}}{}'\boldsymbol{V}\boldsymbol{q}^{\mathbf{0}} - \delta(\boldsymbol{q}_z)]$. If the data on elasticity values do not satisfy either condition (10) or (12) the off-diagonal terms are then scaled down in absolute value by increasing constant $\delta(\boldsymbol{q}_z)$ until diagonal dominance is achieved, ensuring the proper curvature. Given that parameters $\chi_i \chi_j$ are typically six to eight orders of magnitude smaller than the diagonal terms, concavity is satisfied without having to rescale the off-diagonal terms in most cases we encountered. The intuition of condition (10) is that the aggregate magnitude of substitution (complementarity) effects should not be bigger than the own-price effects such that across-the-board price cuts increase. We illustrate this condition in our Korean food demand application in the application section.

If the sufficient condition for diagonal dominance is met, we set parameter $\delta(\boldsymbol{q}_z)$ equal to zero in equations (5), (7), and (9). This procedure is virtually innocuous because it has little impact on the value of the elasticities derived from the demand system. This normalization of $\delta(\boldsymbol{q}_z)$ to zero is used in many econometric investigations of demand because this parameter δ is practically unidentified in many econometric investigations of the almost ideal demand system and LinQuad demand system (e.g., Deaton and

Muellbauer 1980; Fang and Beghin 2002). Large variations in its value have little bearing on the values of ε and $\boldsymbol{V}$ and the exact welfare measure.[5] Other values for δ are obviously defensible.

In the second case related to concavity, estimates for some cross-price effects are available. Typically, the degree of knowledge and confidence of the analyst on these cross-price effects is limited. Our approach is to leave income and own-price responses unchanged and to scale down cross-price effects if conditions for concavity are not met. We scale the cross-price effects in absolute value until the concavity sufficient condition is satisfied either through diagonal dominance (10) or through Cholesky factorization (Lau 1978).[6] In this second case, the scaling affects mostly the cross-slope coefficients v_{ij} and then the intercept terms ε_i, which in turn affect the values of the own-price responses in the Slutsky matrix via feedback on the income term $[R - \boldsymbol{q^0}'\boldsymbol{\varepsilon} - \frac{1}{2}\, \boldsymbol{q^0}'\boldsymbol{V}\,\boldsymbol{q^0} - \delta(\boldsymbol{q}_z)]$ in Marshallian demands. One could check sequentially if own-price and income elasticities are consistent as a separate set of estimates using the first-case approach, then move to the second case and use the additional estimates of cross-price effects to constrain the whole set of available elasticity estimates.

With $\boldsymbol{\chi}$ being identified in the previous step, its values are then combined with available information on the level of demand (equation (5)) and elasticities (6), (own-price elasticities, and if available, cross-price elasticity estimates) to recover structural parameters ε_i and v_{ij}. This step leads to a system of 3 x n equations. The system of equations is linear in unrestricted parameters ε_i, v_{ij} and unknown cross-price responses $\partial x^M_i/\partial q_j$:

$$\begin{cases} \dfrac{x_i^M - \chi_i R}{\chi_i} = \left((\dfrac{1}{\chi_i} - q_i) \right) \varepsilon_i + \left((\dfrac{q_i}{\chi_i} - \tfrac{1}{2} q_i^2) \right) v_{ii} - \sum_{j\neq i} \varepsilon_j q_j - \sum_{j \neq i} \left((v_{ij} \dfrac{q_j}{\chi_i} - \tfrac{1}{2} q_i q_j) \right) - \tfrac{1}{2} \sum_{j\neq i} \sum_{k \neq i} v_{jk} q_j q_k \\ \dfrac{\partial x_i^M}{\partial q_j} \dfrac{1}{1+a_{ij}} = v_{ij} - \chi_i \varepsilon_j - \chi_i \sum_{all\, k} v_{jk} q_k \\ \dfrac{\partial x_i^M}{\partial q_i} = v_{ii}(1 - \chi_i q_i) - \chi_i \varepsilon_i - \chi_i \sum_{i \neq j} v_{ij} q_j, \end{cases} \qquad (13)$$

In the above system (13), whenever cross-prices effects $\partial x_i^M/\partial q_j$ are unknown, parameter v_{ij} in $\boldsymbol{V}$ is restricted to zero, which implies that $\partial x_i^M/\partial q_j = -\chi_i (\varepsilon_j + v_{jj} q_j)$. With scaling parameters a_{ij} set to zero, the system of equations in $\boldsymbol{\varepsilon}$ and $\boldsymbol{V}$ is exactly identified. To impose curvature restrictions the scaling parameters a_{ij} are set non-negative and chosen by minimizing the sum of corrections $\sum_i \sum_j a_{ij}$, which satisfies system (13) and condition (10). The non-negative constraint preserves the sign of the estimates of the substitution/complementarity effects.

With the calibrated values of the elements of $\boldsymbol{V}$ and ε, the EV is calibrated, and welfare analysis of price changes is possible. We use the GAMS DNLP solver, which handles absolute values.

Application to Korea

We now turn to our illustration for the case of Korea. We look at an incomplete food demand system for a representative Korean agent consuming the following commodities: rice, barley, wheat, corn, soybean, dairy, beef, pork, and poultry. Korea provides a good illustration because consumer prices are distorted and induce large consumer welfare losses. We have various income and price elasticity estimates available, including six cross-price effects between the three cereals and the three meats. The sources are various and are detailed in Beghin, Bureau, and Park (2002). Table 1 summarizes the available information on elasticity values, consumption levels, and relative prices in 1995 won.

We start with the first case in which we assume that only own-price and income

TABLE 1. Available data for calibration of a partial demand system in Korea (2000 data)

Goods	Quantities	Domestic Prices (q_0)	World Prices (q_1)	Own-Price Elasticity η_{ii}	Income Elasticities η_{iI}
Rice	5126.00	1657.55	259.88	-0.20	0.12
Barley	467.00	417.08	133.19	-0.60	0.24
Wheat	3173.32	182.42	182.00	-0.40	0.18
Corn	9425.38	153.51	152.68	-0.45	0.43
Soybean	1815.00	358.21	213.58	-0.32	0.32
Milk	2753.00	497.51	131.56	-0.57	0.57
Beef	585.00	6348.26	1914.96	-0.80	0.54
Pork	1012.00	1961.03	1215.87	-0.89	0.73
Poultry	427.00	1692.37	1199.11	-0.70	0.37

Notes: Income = 475.830 billion won (1995 prices); cross-price elasticities: $\eta_{rice\ wheat} = 0.08$; $\eta_{barley\ wheat} = 0.21$; $\eta_{barley\ corn} = 0.15$; $\eta_{beef\ pork} = 0.22$; $\eta_{beef\ poultry} = 0.04$; $\eta_{pork\ poultry} = 0.04$.

elasticities are available to the researcher. These elasticity values and implied expenditure shares satisfy the dominance condition (12) (for rice, 0.19647; barley, 0.59665; wheat, 0.39736; corn, 0.44338; soybeans, 0.31533; milk, 0.56150; beef, 0.79068; pork, 0.87920; and poultry, 0.69459). Hence, no correction is required, and parameter $\delta(\boldsymbol{q_z})$ is set equal to zero. Table 2 shows the implied Hicksian price-elasticity values implied by the calibration for the diagonal case. The Hicksian cross-price response elasticities generated by the calibration procedure are small but fully consistent with an integrable demand system and lead to an exact welfare measure. They are positive as expected because all goods are normal in this illustration. Indeed, when any v_{ij} is restricted to be equal to zero, then the sign of the product of the income responses for good i and j, $\chi_i \chi_j$, determines the sign of the substitution effect between goods i and j. The implied Marshallian elasticities are shown in Table 3. Marshallian cross-price effects are negative because the correction for the income effect is larger than the small positive substitution effect.

In the second calibration case, we make use of all available cross-price elasticities. The diagonal dominance condition requires that we scale down by 26.9 percent the wheat-rice cross-price effect. Table 4 gives the Slutsky price responses, and Table 5 gives the implied Marshallian elasticities. We also show the corresponding results when the curvature restriction is imposed via Cholesky factorization (Tables 6 and 7), which implies scaling the wheat-rice price effect by 12.9 percent. Results are qualitatively similar between the two approaches to impose curvature. Diagonal dominance induces a slightly larger adjustment of the estimate of the cross-price response between wheat and rice than does the Cholesky factorization. This is expected since the former method is a sufficient but not necessary condition, whereas the latter is necessary and sufficient to establish positive semi-definiteness of a symmetric real matrix.

Next, we simulate a large policy shock equivalent to full trade liberalization and measure the EV corresponding to the price changes from domestic prices to border prices. We do so for the two calibration cases (no off-diagonal information, the polar case with information on six cross-price responses and curvature restrictions under both diagonal dominance and Cholesky factorization). Table 8 shows the three EV estimates. As shown in the table, the EV measures do change somewhat but the order of magnitude of the impact of the price shock does not. The three EV measures are between 13.7 and 14 billion won, and income is 476 billion won. Hence, we conclude the EV measure is robust to the inclusion or absence of available estimates of cross-price effects.

TABLE 2. Hicksian price elasticities without information on off-diagonal elasticities

	Rice	Barley	Wheat	Corn	Soybean	Milk	Beef	Pork	Poultry
Rice	-0.19786	0.00001	0.00003	0.00015	0.00005	0.00019	0.00049	0.00035	0.00006
Barley	0.0005	-0.5999	0.00005	0.0003	0.0001	0.00038	0.00097	0.0007	0.00013
Wheat	0.00037	0.00002	-0.39978	0.00023	0.00008	0.00028	0.00073	0.00053	0.0001
Corn	0.00089	0.00004	0.00009	-0.44869	0.00018	0.00068	0.00175	0.00126	0.00023
Soybean	0.00066	0.00003	0.00007	0.0004	-0.31956	0.00051	0.0013	0.00094	0.00017
Milk	0.00118	0.00005	0.00012	0.00072	0.00024	-0.56836	0.00231	0.00167	0.00031
Beef	0.00111	0.00005	0.00011	0.00068	0.00023	0.00085	-0.79579	0.00158	0.00029
Pork	0.00151	0.00007	0.00015	0.00092	0.00031	0.00115	0.00296	-0.88696	0.0004
Poultry	0.00076	0.00004	0.00008	0.00047	0.00016	0.00058	0.0015	0.00109	-0.69944

TABLE 3. Marshallian elasticities without information on off-diagonal price responses

	Rice	Barley	Wheat	Corn	Soybean	Milk	Beef	Pork	Poultry
Rice	-0.2	-0.00004	-0.00012	-0.00021	-0.00011	-0.00016	-0.00045	-0.00015	-0.00012
Barley	-0.00379	-0.6	-0.00024	-0.00043	-0.00023	-0.00031	-0.0009	-0.0003	-0.00023
Wheat	-0.00284	-0.00006	-0.4	-0.00032	-0.00017	-0.00023	-0.00067	-0.00022	-0.00018
Corn	-0.00679	-0.00014	-0.00043	-0.45	-0.00041	-0.00056	-0.00161	-0.00053	-0.00042
Soybean	-0.00505	-0.0001	-0.00032	-0.00057	-0.32	-0.00042	-0.0012	-0.0004	-0.00031
Milk	-0.009	-0.00018	-0.00057	-0.00102	-0.00054	-0.57	-0.00213	-0.00071	-0.00056
Beef	-0.00853	-0.00017	-0.00054	-0.00096	-0.00051	-0.0007	-0.8	-0.00067	-0.00053
Pork	-0.01153	-0.00023	-0.00073	-0.0013	-0.00069	-0.00095	-0.00273	-0.89	-0.00071
Poultry	-0.00584	-0.00012	-0.00037	-0.00066	-0.00035	-0.00048	-0.00139	-0.00046	-0.7

TABLE 4. Hicksian price elasticity estimates, diagonal dominance condition

	Rice	Barley	Wheat	Corn	Soybean	Milk	Beef	Pork	Poultry
Rice	-0.19786	0.00001	0.06321	0.00015	0.00005	0.00019	0.00049	0.00035	0.00006
Barley	0.0005	-0.5999	0.21029	0.15073	0.0001	0.00038	0.00097	0.0007	0.00013
Wheat	0.92778	0.07076	-0.39978	0.00023	0.00008	0.00028	0.00073	0.00053	0.0001
Corn	0.00089	0.02029	0.00009	-0.44869	0.00018	0.00068	0.00175	0.00126	0.00023
Soybean	0.00066	0.00003	0.00007	0.0004	-0.31956	0.00051	0.0013	0.00094	0.00017
Milk	0.00118	0.00005	0.00012	0.00072	0.00024	-0.56836	0.00231	0.00167	0.00031
Beef	0.00111	0.00005	0.00011	0.00068	0.00023	0.00085	-0.79579	0.22225	0.04082
Pork	0.00151	0.00007	0.00015	0.00092	0.00031	0.00115	0.4159	-0.88696	0.04111
Poultry	0.00076	0.00004	0.00008	0.00047	0.00016	0.00058	0.20978	0.1129	-0.69944

Note: The wheat-rice cross-price effect is scaled down by 26.9%.

TABLE 5. Marshallian elasticity estimates, diagonal dominance condition

	Rice	Barley	Wheat	Corn	Soybean	Milk	Beef	Pork	Poultry
Rice	-0.20000	-0.00004	0.06306	-0.00021	-0.00011	-0.00016	-0.00045	-0.00015	-0.00012
Barley	-0.00379	-0.60000	0.21000	0.15000	-0.00023	-0.00031	-0.00090	-0.00030	-0.00023
Wheat	0.92456	0.07068	-0.40000	-0.00032	-0.00017	-0.00023	-0.00067	-0.00022	-0.00018
Corn	-0.00679	0.02011	-0.00043	-0.45000	-0.00041	-0.00056	-0.00161	-0.00053	-0.00042
Soybean	-0.00505	-0.00010	-0.00032	-0.00057	-0.32000	-0.00042	-0.00120	-0.00040	-0.00031
Milk	-0.00900	-0.00018	-0.00057	-0.00102	-0.00054	-0.57000	-0.00213	-0.00071	-0.00056
Beef	-0.00853	-0.00017	-0.00054	-0.00096	-0.00051	-0.00070	-0.80000	0.22000	0.04000
Pork	-0.01153	-0.00023	-0.00073	-0.00130	-0.00069	-0.00095	0.41020	-0.89000	0.04000
Poultry	-0.00584	-0.00012	-0.00037	-0.00066	-0.00035	-0.00048	0.20689	0.11135	-0.70000

Note: The wheat-rice cross-price effect is scaled down by 26.9%.

TABLE 6. Hicksian elasticity estimates, Cholesky factorization

	Rice	Barley	Wheat	Corn	Soybean	Milk	Beef	Pork	Poultry
Rice	-0.19786	0.00001	0.07100	0.00015	0.00005	0.00019	0.00049	0.00035	0.00006
Barley	0.00050	-0.59990	0.21029	0.15073	0.00010	0.00038	0.00097	0.00070	0.00013
Wheat	1.04211	0.07076	-0.39978	0.00023	0.00008	0.00028	0.00073	0.00053	0.00010
Corn	0.00089	0.02029	0.00009	-0.44869	0.00018	0.00068	0.00175	0.00126	0.00023
Soybean	0.00066	0.00003	0.00007	0.00040	-0.31956	0.00051	0.00130	0.00094	0.00017
Milk	0.00118	0.00005	0.00012	0.00072	0.00024	-0.56836	0.00231	0.00167	0.00031
Beef	0.00111	0.00005	0.00011	0.00068	0.00023	0.00085	-0.79579	0.22225	0.04082
Pork	0.00151	0.00007	0.00015	0.00092	0.00031	0.00115	0.41590	-0.88696	0.04111
Poultry	0.00076	0.00004	0.00008	0.00047	0.00016	0.00058	0.20978	0.11290	-0.69944

Note: The wheat-rice cross-price effect is scaled down by 12.9%.

TABLE 7. Marshallian elasticity estimates, Cholesky factorization

	Rice	Barley	Wheat	Corn	Soybean	Milk	Beef	Pork	Poultry
Rice	-0.20000	-0.00053	0.07016	-0.00165	-0.00135	-0.00175	-0.00199	-0.00235	-0.00214
Barley	0.00015	-0.60000	0.20990	0.15041	-0.00015	0.00004	0.00055	-0.00003	-0.00026
Wheat	1.04218	0.07074	-0.40000	0.00017	0.00003	0.00023	0.00067	0.00024	0.00003
Corn	-0.00062	0.01989	-0.00090	-0.45000	-0.00084	-0.00073	-0.00003	-0.00110	-0.00138
Soybean	0.00004	-0.00014	-0.00050	-0.00015	-0.32000	-0.00009	0.00056	-0.00023	-0.00051
Milk	-0.00056	-0.00041	-0.00111	-0.00081	-0.00096	-0.57000	0.00025	-0.00116	-0.00157
Beef	-0.00250	-0.00089	-0.00170	-0.00240	-0.00218	-0.00247	-0.80000	0.21724	0.03703
Pork	-0.00023	-0.00040	-0.00125	-0.00062	-0.00090	-0.00050	0.41383	-0.89000	0.03921
Poultry	0.00028	-0.00011	-0.00051	0.00001	-0.00020	0.00010	0.20918	0.11182	-0.70000

Note: The wheat-rice cross-price effect is scaled down by 12.9%.

TABLE 8. Equivalent variation for the removal of price distortions

Without information on off-diagonal	13.95086
With information and diagonal dominance	13.70318
With information and Cholesky	13.70355

Note: Units are in billion won at 1995 prices.

Conclusions

This paper is a methodological contribution to quantitative economic analysis and more particularly to the calibration of partial systems involving a subset of disaggregated goods. We propose and illustrate an easily implemented and flexible calibration technique for partial demand systems, combining recent developments in incomplete demand systems and a set of restrictions conditioned on the available elasticity estimates and integrability.

The technique accommodates various degrees of knowledge on cross-price elasticities and allows the recovery of an exact welfare measure. It generates values for missing cross-price elasticities, which are consistent with the available estimates. The approach is illustrated with a partial demand system for food consumption in Korea for different states of knowledge on cross-price effects. The consumer welfare impact of food and agricultural trade liberalization is measured and is shown not to be sensitive to the inclusion or deletion of available estimates of cross-price effects.

Curvature restrictions are imposed using alternative approaches (diagonal dominance and Cholesky factorization). Diagonal dominance provides a sufficient condition for concavity of utility, which can be expressed in terms of available estimates of Marshallian own-price and income elasticities. This condition provides a direct and convenient check of the estimates available to the policy analyst. The drawback of the diagonal dominance approach is that it might impose adjustments in estimates that are larger than what is necessary to satisfy curvature. Cholesky factorization does not allow for a "quick" check of available estimates of own-price and income elasticities. However, it does provide minimum adjustments in estimates that are necessary for curvature restrictions to be satisfied. In our calibration illustration, the two methods for imposing proper curvature yield very close estimates of preferences parameters and EV measures for the policy changes.

Endnotes

1. For n goods, the number of price elasticities to estimate is equal to $\{n(n+1)/2\}$, assuming symmetry is imposed in a calibration using deflated prices; n income elasticities have to be found as well.

2. We define concavity (quasi-concavity) of utility with the condition that the Slutsky matrix of compensated price responses of the demand system is negative definite (negative semi-definite).

3. If only expenditures are known, quantity units for each good are redefined so that the associated price is equal to 1 per unit.

4. We rule out Giffen goods (income term smaller in absolute value to the Hicksian price term in absolute value in the Slutsky decomposition).

5. The sensitivity of EV with respect to $\delta(\boldsymbol{q}_z)$ ($dEV/d\delta(\boldsymbol{q}_z)$) is 0.007 (an additional 1 million won in the income argument via $\delta(\boldsymbol{q}_z)$ induces 7,000 won of variation in EV, which is of the order of 14 billion won for the price change considered in the illustration).

6. The Cholesky factorization decomposes minus the Slutsky matrix $-\boldsymbol{S}$ into $-\boldsymbol{S}=\boldsymbol{L_l D L_h}$, where $\boldsymbol{D}$ is a diagonal matrix constrained to have nonnegative elements D_{ii} for quasi-concavity of the utility function, $\boldsymbol{L_l}$ is a unit lower triangular matrix, and $\boldsymbol{L_h}$ is the transpose of $\boldsymbol{L_l}$ (Lau). We use a similar scaling approach for the Cholesky factorization as for the diagonal-dominance approach. Scaling factors are applied to the slope estimates of the Marshallian cross-price effects to satisfy curvature restrictions (D_{ii}) positive.

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