Multiperiod Production with Forward and Options Markets

Sergio H. Lence,
Yong Sakong, and Dermot J. Hayes

Working Paper 93-WP 112
August 1993

Center for Agricultural and Rural Development
Iowa State University
Ames, Iowa 50011-1070

Journal Paper No. J-15188 of the Iowa Agriculture and Home Economics Experiment Station, Ames, Iowa, Project No. 2994.

Sergio H. Lence is a CARD postdoctoral research associate in economics; Yong Sakong is a postdoctoral research associate in economics; and Dermot J. Hayes is associate professor of economics and head of the Trade and Agricultural Policy Division of CARD.

MULTIPERIOD PRODUCTION WITH FORWARD AND OPTION MARKETS

Abstract

Production and hedging in both forward and options markets are analyzed for forward-looking firms that maximize expected utility. In the presence of unbiased forward and options prices, it is shown that such firms will use options as hedging instruments. This result contrasts with the conclusions from studies that assume myopic behavior, and occurs because forward-looking agents care about the effect of future output prices on profits from future production cycles. Simulations support the theoretical results and show how the introduction of an options market influences the optimal forward position.

MULTIPERIOD PRODUCTION WITH FORWARD AND OPTIONS MARKETS

Most of the literature on risk aversion and price uncertainty has incorporated Sandmo's implicit assumption that the firm is concerned with a single production cycle. This convention has been extended into the hedging literature (e.g., Holthausen; Feder, Just, and Schmitz; Anderson and Danthine 1980; Losq; Batlin; Benninga, Eldor, and Zilcha; Smith and Stulz; Paroush and Wolf; Kerkvliet and Moffett). However, output price changes that occur in one period will change price expectations for subsequent production periods. In addition, for industries with at least one fixed input, output supply changes will cause changes in input prices and optimal input use for subsequent production cycles. Firms operating and hedging in this environment will presumably have noticed the effects of changes in this period's price on next period's profitability and adjusted their hedging behavior accordingly.

This paper extends the number of production cycles to two and shows how optimal hedging depends on the perceived relationships between this period's output price and next period's input and output prices.[1] We focus on forward rather than futures markets to show that the main results do not depend on the existence of basis risk, and also to avoid the complexities that arise from the presence of basis risk. In addition, we extend the model to incorporate options and show a valid role for options as hedging instruments that has here-to-fore been lacking in the one-period hedging models (Lapan, Moschini and Hanson; Sakong, Hayes, and Hallam). The analysis by Lapan, Moschini, and Hanson reveals that myopic firms with nonstochastic production will use options only if futures and/or options prices are biased. Sakong, Hayes, and Hallam showed that production uncertainty will lead myopic firms to use options for hedging purposes. In this paper, we will show that options are used for hedging even if production is nonstochastic and prices are unbiased, as long as firms exhibit forward-looking behavior.

---

[1]Anderson and Danthine (1983), Hey, and Karp also analyzed dynamic hedging. Our model differs from that of Anderson and Danthine (1983) in that they considered hedging revisions within a single production cycle. It differs from that of Hey in that he assumed that prices were independently distributed and that agents had intertemporally additive utility functions. Karp's basic model is similar to that of Anderson and Danthine (1983), with the addition of stochastic production. None of these references includes options.

The first section of this paper develops the optimal options and forward positions for a firm that will be in operation for two periods. It is shown that the hedging decision made at the beginning of the first period will incorporate the effect of the first period's price changes on both the first and second periods' profits. The second period effect is caused by the impact of the first period price on expected input and output prices for the second period. One of the more interesting aspects of this second period effect is that the first period price will almost always have a nonlinear relationship with the profits from the second production cycle. Because the payoff from forward contracts is linear in prices, it is therefore not possible to fully hedge with forward contracts against this nonlinear relationship. Options offer a more flexible way of hedging against this nonlinearity and consequently options are shown to have a hedging role. The derivation of the optimal options position is complex because options truncate the distribution of expected returns. This derivation is the subject of the second section. In the third section, we perform simulations to calculate the expected-utility-maximizing forward and options positions for individuals with constant absolute risk aversion. These optimal positions are shown to depend on the characteristics of the utility and the production functions, and on the expected correlation among the first period's output prices and the second period's input and forward prices. These simulations are performed for an individual who has access to both forward and options markets and to a forward market alone.

The Theoretical Model

Consider a decision maker characterized by a twice differentiable utility function of terminal wealth $U(W_T)$ such that $U' > 0$ and $U'' < 0$. Terminal wealth is defined as initial wealth at the current date plus the cash flows arising up to and including the terminal date:[2]

(1.1) $W_T = W_0 + c_1 + c_2 + \ldots + c_{T-1} + c_T,$

[2]To simplify the exposition, interest rates are omitted in (1.1). The omission does not affect in any meaningful way the main conclusions of the study, as long as interest rates are nonstochastic and there are no constraints regarding the amounts of money that can be borrowed or lent at the prevailing interest rate.

where W_t denotes monetary wealth at the end of trading date t and c_t is cash flow at time t.

The agent can produce output Q_t from input I_{t-1} by means of the transformation function $Q_t = q(I_{t-1})$, where $q(\cdot)$ is such that $q' > 0$, $q'' < 0$, $q(0) = 0$, and $q'(0) \to \infty$.[3] The time subscripts indicate that it takes one production cycle (from t-1 to t) to transform input into output. Production is assumed to be nonstochastic to emphasize that the role of options as hedging instruments is not due to production uncertainty (Sakong, Hayes, and Hallam).

It is also assumed that the agent has access to forward and options markets for the final good. Only forward and put markets are considered, however, because any combination of forward, put, and call contracts can be replicated by any two of these financial instruments. To see why this assertion is true, examine the upper panel of Figure 1. This position diagram shows the payoff to an investor who has written a put (dashed line) and who has sold a forward contract (solid line). The addition of the two payoffs (i.e., the sum of the vertical distances from the horizontal axis) yields the net payoff shown in the lower panel of Figure 1. This net position is identical to the payoff of a written call. In fact, any two of forward, put, and call contracts can be used to replicate the third. This allows us to ignore calls in the analysis that follows without precluding call-like solutions.

At date t, the individual sells X_t forward contracts for delivery at t+1 at the forward price F_t. At date t+1, the individual repurchases the X_t forward contracts at the prevailing cash price P_{t+1}. The cash flow from these two operations in the forward market occurs at date t+1 and is equal to $[(F_t - P_{t+1}) X_t]$. Similarly, the agent at date t buys Z_t puts with strike price F_t at a price R_t, and at t+1 obtains a revenue of zero if P_{t+1} exceeds the strike price F_t, or a gain of $[(F_t - P_{t+1}) Z_t]$ otherwise.[4] Given this setting, the cash flow at each decision date is given by

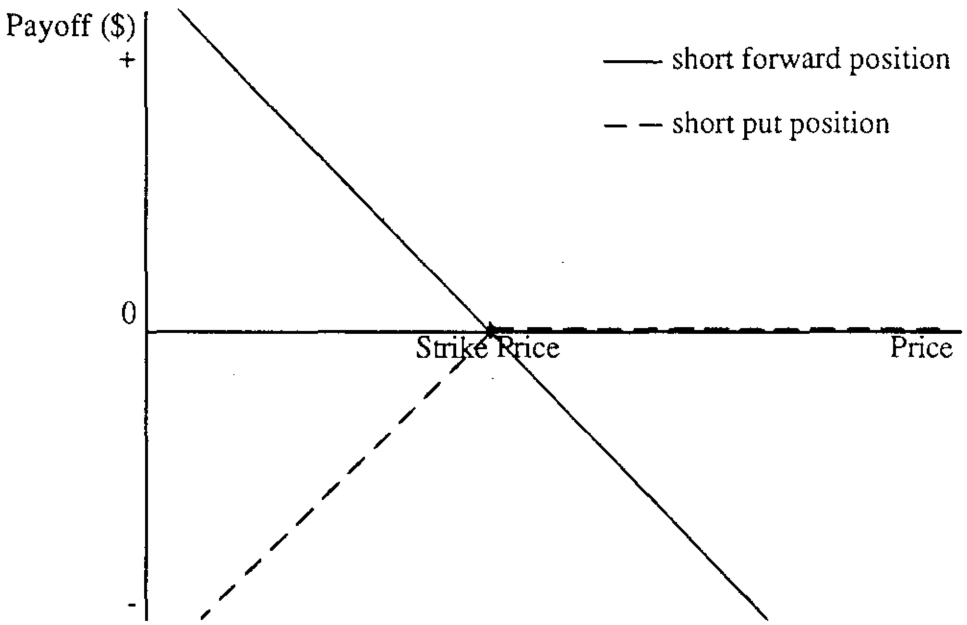
$$(1.2) \quad c_t = P_t Q_t + (F_{t-1} - P_t) X_{t-1} - R_t Z_t + (F_{t-1} - P_t) L_t Z_{t-1} - H_t I_t \qquad \text{s.t.} \quad Q_t = q(I_{t-1})$$

---

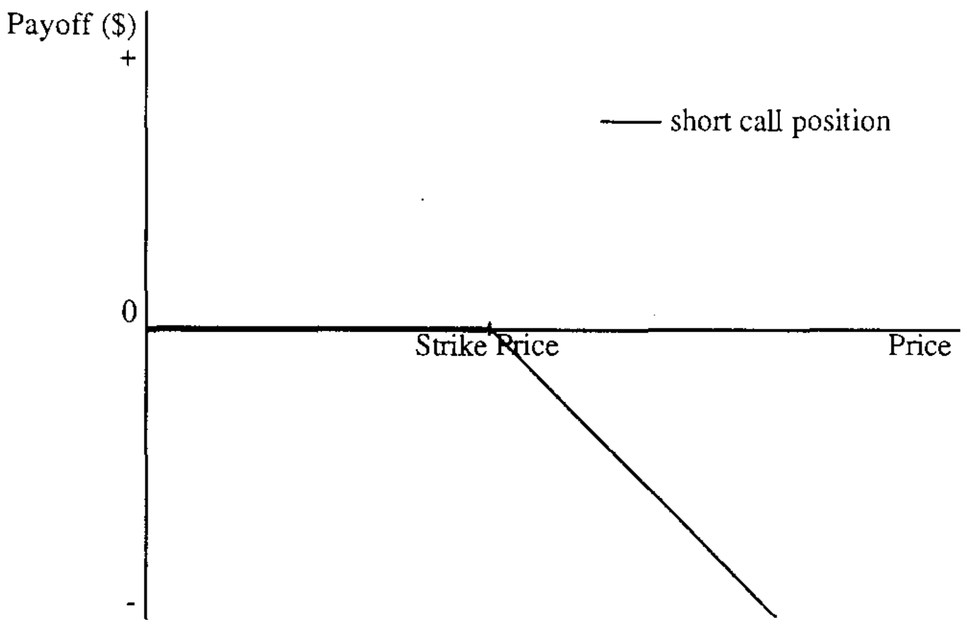
[3]These conditions on $q(\cdot)$ are standard assumptions in production models. These conditions are useful because in most scenarios they rule out the possibility of negative input demand.

[4]Note that $X_t < 0$ ($Z_t < 0$) means that the agent is buying forward (selling put) contracts at date t.

Figure 1. Payoff diagrams for short forward, short put, and short call positions.



a. Payoff diagram for short forward and short put positions.



b. Payoff diagram for short call position (equal to sum of short forward and short put positions).

where H_t denotes input price, and L_t is a binary variable such that $L_t = 1$ when the put option finishes in the money ($F_{t-1} > P_t$) and $L_t = 0$ when the put option finishes at or out of the money ($F_{t-1} \leq P_t$).

Because our purpose is to explore the consequences of relaxing the assumption of a single production cycle, and because a two-production cycle model captures the essentials, we limit the discussion to the two-cycle case. Denote the current decision date by t = 1, the next decision date by t = 2, and the terminal date (T) by t = 3. The optimal decisions regarding input, and forward and put positions at the current date t = 1 (i.e., I_1, X_1, and Z_1) must solve the following set of recursive equations:

$$(1.3)\quad M_3[W_2, q(I_2), X_2, Z_2; \mathbf{\underline{p}}_3] =$$

$$\max_{I_3 \geq 0} U[W_2 + P_3\, Q_3 + (F_2 - P_3)\, X_2 + (F_2 - P_3)\, L_3\, Z_2 - H_3\, I_3]$$

$$(1.4)\quad M_2[W_1, q(I_1), X_1, Z_1; \mathbf{\underline{p}}_2] = \max_{I_2 \geq 0, X_2, Z_2} E_2\{M_3[W_1 + c_2, q(I_2), X_2, Z_2; \mathbf{\underline{p}}_3]\}$$

$$(1.5)\quad M_1[W_0, q(I_0), X_0, Z_0; \mathbf{\underline{p}}_1] = \max_{I_1 \geq 0, X_1, Z_1} E_1\{M_2[W_0 + c_1, q(I_1), X_1, Z_1; \mathbf{\underline{p}}_2]\}$$

where: $\mathbf{p}_t = (P_t, H_t, F_t, R_t)$,

$\mathbf{\underline{p}}_t = (\mathbf{p}_0, \ldots, \mathbf{p}_t)$.

$E_t(\cdot)$ denotes the expectation operator based on information available at t, the matrix $\mathbf{\underline{p}}_t$ comprises the cash, forward, and put prices up to (and including) time t, and cash flows are given by (1.2). Equations (1.3) through (1.5) tell us that the decision maker at the current decision date chooses the levels of input, puts, and forward contracts that maximize expected utility over the entire planning horizon, assuming that the future levels of input, forward contracts, and puts will be chosen so as to maximize expected utility over the rest of the planning horizon.

The optimizing levels of forward contracts and puts at the current date are obtained by starting at the terminal date and working backwards. Simple inspection of (1.3) indicates that the value of I_3 that maximizes $U(\cdot)$ is zero ($I_3^* = 0$) because $U(\cdot)$ is a strictly decreasing function of I_3 and I_3 cannot be negative. Substituting $I_3^* = 0$ into the expression for terminal wealth yields

$$(1.6) \quad W_3 = W_1 + c_2 + P_3\, Q_3 + (F_2 - P_3)\, X_2 + (F_2 - P_3)\, L_3\, Z_2$$

At date 2, the optimum levels of input (I_2^*), puts (Z_2^*), and forward contracts (X_2^*) must satisfy the following necessary first order conditions (FOCs)

$$(1.7) \quad \frac{\partial E_2(M_3)}{\partial I_2} = E_2\{M_3'\,[P_3\, q'(I_2^*) - H_2]\} = 0$$

$$(1.8) \quad \frac{\partial E_2(M_3)}{\partial X_2} = E_2[M_3'\,(F_2 - P_3)] = 0$$

$$(1.9) \quad \frac{\partial E_2(M_3)}{\partial Z_2} = E_2\{M_3'\,[- R_2 + (F_2 - P_3)\, L_3]\} = 0$$

where M_3' denotes U' evaluated at I_3^*, I_2^*, Z_2^*, and X_2^*. Combining (1.7) and (1.8), we get $F_2\, q'(I_2^*) = H_2$, which means that the optimal level of input (and therefore of production) is determined separately from the optimal number of puts and forward contracts. The optimal input demand at date 2 is a function of the ratio of input and forward prices only, i.e., $I_2^* = q'^{-1}(H_2/F_2)$, where q'^{-1} is the inverse function of q'.[5]

It can be inferred that the decision problem at date 2 represents the standard myopic or static decision problem. Only one production cycle remains at date 2; therefore, the decision maker behaves as if he or she were to stop producing at the end of the current cycle. This myopic model has been extensively used in studying optimal production and hedging behavior; in particular, the case of production in the presence of futures and options has been recently analyzed by Lapan,

[5]The inverse function q'^{-1} exists because $q'' < 0$ by assumption.

Moschini, and Hanson. Perhaps the most important (and striking) result obtained by these authors is that options are not used as hedging instruments if both forward and options prices are unbiased {i.e., $E_2(P_3) = F_2$ and $E_2[(F_2 - P_3) L_3] = R_2$}. Under unbiasedness, the optimal decisions are given by $I_2^* = q'^{-1}(H_2/F_2)$, $Z_2^* = 0$, and $X_2^* = Q_3^*$ [$= q(I_2^*)$], independently of the degree of risk aversion and the initial wealth.

One of the purposes of this study is to show that, in contrast with the myopic case, options are generally useful hedging instruments for forward-looking decision makers even if both forward and options prices are unbiased. The optimal amount of puts bought by a myopic decision maker is generally nonzero when prices are biased; hence, to demonstrate that our results are driven by forward-looking behavior rather than biased prices we will assume throughout that prices are unbiased {i.e., $E_t(P_{t+1}) = F_t$ and $E_t[(F_t - P_{t+1}) L_{t+1}] = R_t$}.[6] Then, substituting I_1^*, I_2^*, Z_2^*, and X_2^* under unbiased forward and options prices in the expression for terminal wealth (1.1) we get

$$(1.10)\quad W_3 = W_0 + c_1 + P_2\, Q_2 + (F_1 - P_2)\, X_1 + (F_1 - P_2)\, L_2\, Z_1 + \Pi$$

where: $\Pi \equiv F_2\, q(I_2^*) - H_2\, I_2^*$, $I_2^* = q'^{-1}(H_2/F_2)$

The term Π represents the profits arising from optimal behavior in the second production cycle; Π is nonnegative because it is zero if $I_2^* = 0$ and strictly positive if $I_2^* > 0$.

Finally, the necessary FOCs for optimality at the current date t = 1 are

$$(1.11)\quad \frac{\partial E_1(M_2)}{\partial I_1} = E_1\{M_2'\, [P_2\, q'(I_1^*) - H_1]\} = 0$$

$$(1.12)\quad \frac{\partial E_1(M_2)}{\partial X_1} = E_1[M_2'\, (F_1 - P_2)] = 0$$

[6] We assume that markets are unbiased throughout the paper because we are interested in hedging behavior. Any changes from the optimal positions caused by the introduction of biased markets would be speculative by definition. Therefore, we learn nothing about hedging behavior by introducing biased markets.

$$(1.13)\quad \frac{\partial E_1(M_2)}{\partial Z_1} = E_1\{M_2' [- R_1 + (F_1 - P_2) L_2]\} = 0$$

where M_2' denotes U' evaluated at I_3^*, I_2^*, Z_2^*, X_2^*, I_1^*, Z_1^*, and X_1^*. As it was the case for the myopic scenario (t = 2), the optimal input level for the forward-looking firm is determined separately from the optimal levels of puts and forward contracts. Optimal input demand at date 1 is given by $I_1^* = q'^{-1}(H_1/F_1)$.

To characterize the optimal forward and put positions, it is helpful to express FOCs (1.12) and (1.13) as (1.12') and (1.13'), respectively:[7]

$$(1.12')\qquad Cov_1(P_2, M_2') = 0$$

$$(1.13')\qquad Cov_1[(F_1 - P_2) L_2, M_2'] = 0$$

where $Cov_t(\cdot)$ denotes the covariance operator, given information at date t. Evaluate now $Cov_1(P_2, M_3')$ and $Cov_1[(F_1 - P_2) L_2, M_3']$ at the levels that would be optimal if the firm were myopic, that is, selling forward the entire output $\{X_1 = Q_2^* [= q(I_1^*)]\}$ and buying zero puts ($Z_1 = 0$). Plugging these values in (1.10) yields $W_3 = W_0 + c_1 + F_1 Q_2^* + \Pi$. This means that, in general, $X_1 = Q_2^* [= q(I_1^*)]$ and $Z_1 = 0$ are the optimal forward and put positions only if Π and P_2 are independently distributed. If Π and P_2 are not independent, setting $X_1 = Q_2^*$ and $Z_1 = 0$ generally will not render M_3' and P_2 independent, and consequently $Cov_1(P_2, M_3') \neq 0$ and $Cov_1[(F_1 - P_2) L_2, M_3'] \neq 0$. Hence, $X_1 = Q_2^*$ and $Z_1 = 0$ are not optimal because the necessary FOCs (1.12') and (1.13') do not hold for such values of X_1 and Z_1. The myopic case, which is the assumption upon which much of the literature is based, can be obtained by setting $\Pi = 0$ and can therefore be viewed as a special case of the model presented here. The direction and magnitude to which X_1^* differs from Q_2^* and Z_1^* differs from zero in the more realistic scenario where Π and P_2 are not independent is the focus of the following sections.

[7]Recall that E(x y) = Cov(x, y) + E(x) E(y) for any pair of random variables x and y.

The Role of Options as Hedging Instruments for Forward-Looking Producers

Inspection of $\Pi \equiv F_2\, q(I_2^*) - H_2\, I_2^*$ reveals that the only way for Π and P_2 to be independent is the very unrealistic instance in which both F_2 and H_2 are independently distributed from P_2. Thus, the key is whether at time one the agent believes that output price is related to input or forward prices at date two. Any price shock that is not viewed as transitory will change expected cash and forward prices at subsequent periods. Input prices at date t will respond to changes in output prices at date t-1 if input supply is other than perfectly elastic and if output prices at date t-1 alter optimal production at date t for at least a subset of producers. Producers who belong to this subset may, for example, bid up input prices because they view an output price increase as permanent or because higher output prices create more liquidity and allow them to expand production. For our purposes, it does not matter whether this input price response is caused by rational behavior, but only that the decision maker believes it may occur.

Assume that F_2 and H_2 are each a differentiable function of P_2 and of other random variable independent of P_2, i.e.,

$$(2.1)\quad F_2 = F(P_2, f)$$

$$(2.2)\quad H_2 = H(P_2, h)$$

where f and h are random variables with finite variances. The random variables f and h are possibly related to each other but they are independent of P_2. Let $g(P_2)$ be the expectation of M_2' over f and h, i.e., $g(P_2) \equiv E_1(M_2'|P_2)$, where the notation $g(P_2)$ emphasizes that P_2 is the only random variable affecting the function $g(\cdot)$. Then, FOCs (1.12) and (1.13) can be rewritten as

$$(2.3)\quad E_1[g(P_2)\,(F_1 - P_2)] = 0$$

$$(2.4)\quad E_1\{g(P_2)\,[- R_1 + (F_1 - P_2)\, L_2]\} = 0$$

Given the assumptions of the model, $g(P_2)$ is continuous and differentiable everywhere except at $P_2 = F_1$, at which it is continuous but not differentiable if $Z_1^* \neq 0$. Under these conditions, it is possible to apply the Mean-Value Theorem to prove the following Proposition.

PROPOSITION. *Assume that forward output prices and input cash prices behave as in (2.1) and (2.2). Then, the forward-looking firm under unbiased forward and put prices will find it optimal to use puts as hedging instruments and to establish a forward position different from short selling total output.*

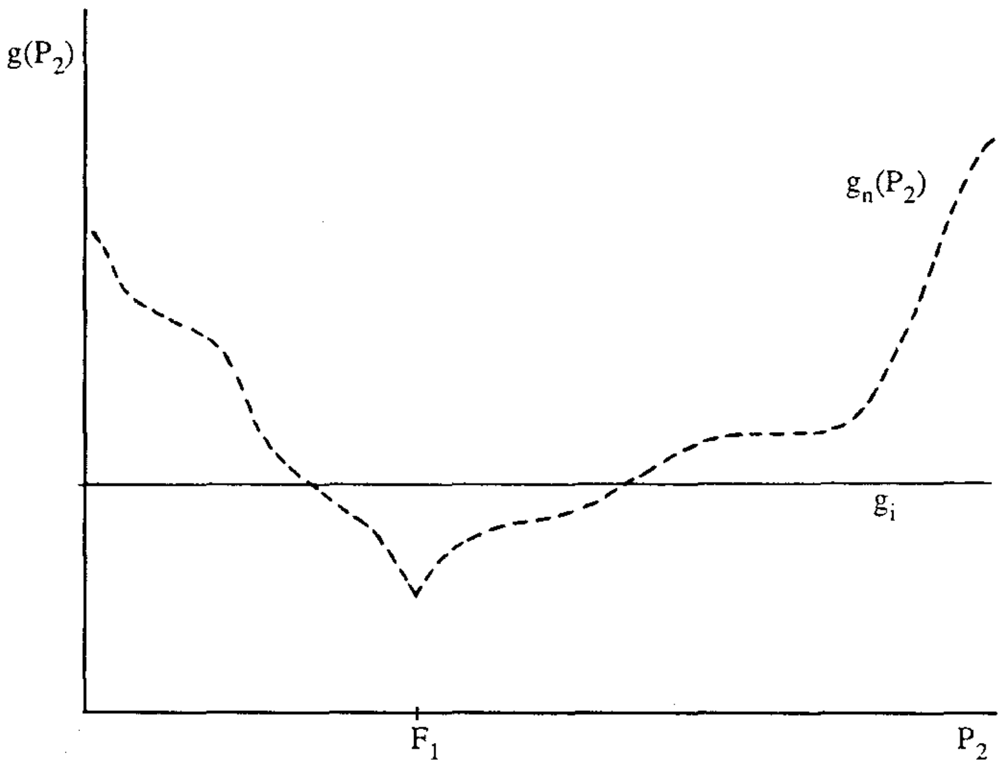
Proof. See Appendix.

This Proposition is important because it reveals that key results previously reported regarding optimal hedging behavior (i.e., optimality of a null position in options coupled with forward full hedging under unbiased prices) are due to the assumption of myopic behavior. Moreover, whether decision makers are forward-looking or myopic can be (at least conceptually) tested empirically because their behavioral differences are observable.

The reason for resorting to a nonstandard technique (i.e., the Mean-Value Theorem) to prove the above Proposition is as follows. If Π and P_2 were independently distributed, the optimal forward and put positions would be $X_1^* = Q_2^*$ and $Z_1^* = 0$, and the graph of $g(P_2)$ would look like the horizontal line in Figure 2. In contrast, when Π and P_2 are not independent the optimal forward and put positions are $X_1^* \neq Q_2^*$ and $Z_1^* \neq 0$ (see Proposition), and the graph of $g(P_2)$ looks something like the dashed line in Figure 2. If Π and P_2 are not independent, the function $g(P_2)$ is differentiable everywhere except where P_2 equals the strike price F_1. Moreover, the function $g(P_2)$ need not be concave or convex on either side of F_1 for the Proposition to hold.

As we already discussed, whether the optimal combination of forward and put contracts is ($X_1^* = Q_2^*$, $Z_1^* = 0$) or ($X_1^* \neq Q_2^*$, $Z_1^* \neq 0$) depends on whether Π and P_2 are independently

Figure 2. Graph of the function $g(P_2)$.



$g_i \equiv g(P_2)$ when Π and P_2 are independent.

$g_n(P_2) \equiv g(P_2)$ when Π and P_2 are not independent.

distributed or not. If forward and input cash prices behave as in (2.1) and (2.2), the first derivative of Π with respect to P_2 is[8]

$$(2.5) \quad \frac{\partial \Pi}{\partial P_2} = Q_3^* \frac{\partial F_2}{\partial P_2} - I_2^* \frac{\partial H_2}{\partial P_2}$$

Clearly, the sign of $\partial\Pi/\partial P_2$ is ambiguous. Assuming that $\partial^2 F_2/\partial P_2^2 = \partial^2 H_2/\partial P_2^2 = 0$ (i.e., that forward and input prices are linearly related to contemporaneous output prices), the second derivative of Π with respect to P_2 is[9]

$$(2.6) \quad \frac{\partial^2 \Pi}{\partial P_2^2} = [q'(I_2^*) \frac{\partial F_2}{\partial P_2} - \frac{\partial H_2}{\partial P_2}] \frac{\partial I_2^*}{\partial P_2}$$

$$(2.6') \qquad = - \frac{1}{F_2\, q''(I_2^*)} \; [q'(I_2^*) \frac{\partial F_2}{\partial P_2} - \frac{\partial H_2}{\partial P_2}]^2 \geq 0$$

The second derivative $\partial^2\Pi/\partial P_2^2$ is nonnegative because $q'' < 0$ by assumption. This assumption means that there are decreasing returns to scale, which is the most realistic scenario because firms under perfect competition will only produce at a point at which $q'' < 0$. The linear relationship between output prices and payoff in the myopic model exists because production levels and costs are predetermined. When we introduce a second relevant production cycle, the convexity of the profit function re-emerges.

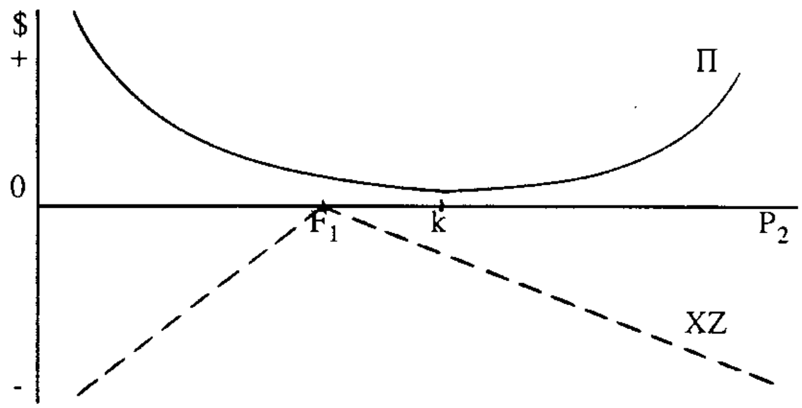
Figure 3 depicts the range of possible impacts of next period output price (P_2) on the profits from the next production cycle (Π), assuming that $\partial^2\Pi/\partial P_2^2$ is strictly positive. Figure 3 also shows schematically how options and forward contracts can be used to hedge the additional risk attributable to the next production cycle. It can be seen that Π is nonnegative and that the relationship between Π and P_2 is nonlinear. Options allow producers to create hedged positions

---

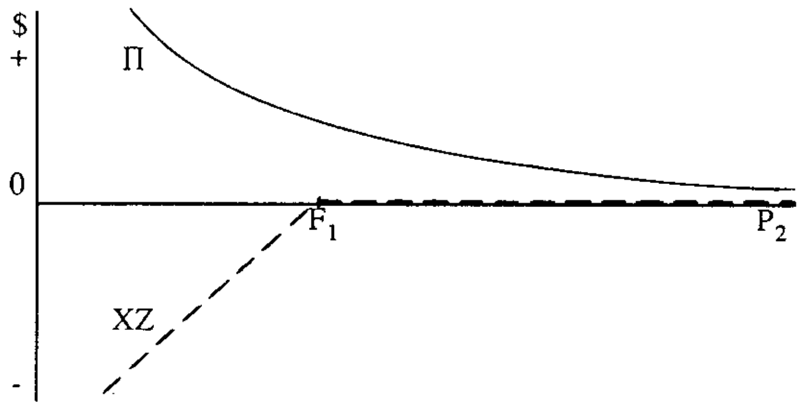
[8] Note that $F_2\, q'(I_2^*) = H_2$ from the FOCs corresponding to date $t = 2$ [see discussion following FOCs (1.7) through (1.9)].

[9] The expression for $\partial I_2^*/\partial P_2$ can be obtained by differentiating $F_2\, q'(I_2^*) = H_2$ with respect to P_2 and solving for $\partial I_2^*/\partial P_2$, which yields $\partial I_2^*/\partial P_2 = -\, [q'(I_2^*)\, \partial F_2/\partial P_2 - \partial H_2/\partial P_2]/[F_2\, q''(I_2^*)]$.

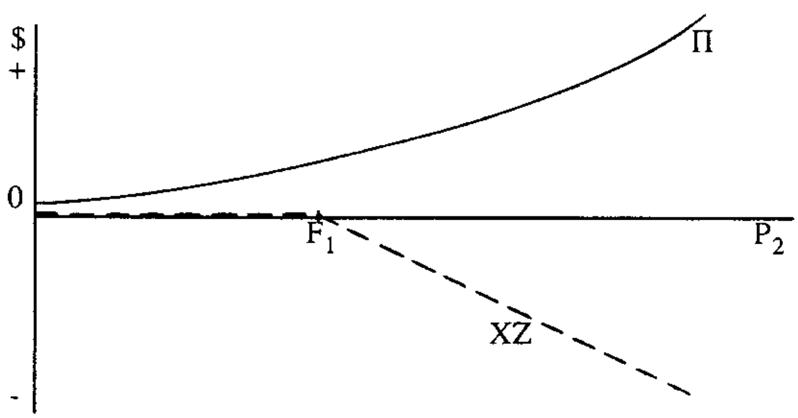
Figure 3. Profits from the second production cycle (Π) and payoffs from combined forward-put positions (XZ) as functions of output cash prices at date t = 2.



a. Situation in which $\partial\Pi/\partial P_2 < 0$ for $P_2 < k$ (arbitrary) and $\partial\Pi/\partial P_2 > 0$ for $P_2 > k$.



b. Situation in which $\partial\Pi/\partial P_2 < 0$ everywhere.



c. Situation in which $\partial\Pi/\partial P_2 > 0$ everywhere.

with kinked payoffs. This kink allows producers to better hedge the additional risk than the linear forward position. It is clear from Figure 3 that to hedge the additional risk attributable to the next production cycle, the agent will sell puts if $\partial\Pi/\partial P_2 < 0$, and sell puts and forward contracts (which is equivalent to selling calls) if $\partial\Pi/\partial P_2 > 0$. It is also clear that if options are not available, the forward-looking firm will sell forward more (less) than its entire production if $\partial\Pi/\partial P_2 < (>) 0$.

In order to find the relative size of the optimal forward and put positions, it is necessary to specify the utility function (U), the production function (q), and the relationships between P_2 and F_2 and H_2 [i.e., $F_2 = F(P_2, f)$ and $H_2 = H(P_2, h)$]. The next section presents simulated examples that show how the optimal forward and put positions respond to some of the parameters characterizing U, q, $F_2 = F(P_2, f)$, and $H_2 = H(P_2, h)$.

Numerical Simulations

In this section, we present and discuss the results of numerical simulations regarding the theoretical Proposition derived previously. Although these simulations correspond to particular scenarios, they are helpful in assessing the orders of magnitude of the forward and options positions involved.

The simulations are performed assuming that the decision maker is constant absolute risk averse (CARA), i.e.,

(3.1) $U = - \exp(- \lambda W_3)$

where λ is the coefficient of absolute risk aversion, and that forward and input prices are linearly related to output cash prices,

(3.2) $F_2 = \alpha_F + \beta_F P_2 + f$

(3.3) $H_2 = \alpha_H + \beta_H P_2 + h$

where P_2 n.i.d. (μ, σ_P^2), f n.i.d. $(0, \sigma_f^2)$, and h n.i.d. $(0, \sigma_h^2)$. To be as unrestrictive as possible while maintaining tractability, a second-order Taylor expansion of Π $[\Pi \equiv F_2\, q(I_2^*) - H_2\, I_2^*$, $I_2^* = q'^{-1}(H_2/F_2)]$ around the means of the random variables P_2, f, and h is used instead of assuming a particular production function (q), i.e.,

$$(3.4) \quad \Pi \cong \hat{\Pi} + (P_2 - \mu)\, \hat{\Pi}_P + f\, \hat{\Pi}_f + h\, \hat{\Pi}_h + (P_2 - \mu)^2\, \hat{\Pi}_{PP}/2 + f^2\, \hat{\Pi}_{ff}/2 + h^2\, \hat{\Pi}_{hh}/2$$

$$+ (P_2 - \mu)\, f\, \hat{\Pi}_{Pf} + (P_2 - \mu)\, h\, \hat{\Pi}_{Ph} + f\, h\, \hat{\Pi}_{fh}$$

where the symbol ^ represents variables measured at the means ($P_2 = \mu$, f = 0, h = 0), and the subscripts along with Π denote the derivatives of Π with respect to the respective random variables.

The optimal forward and put positions can be calculated after (i) substituting expression (3.4) into (1.10) and the resulting expression into the utility function (3.1), (ii) calculating the expected utility over the random variables P_2, f, and h, and (iii) finding the combination of decision variables that maximizes the expected utility. At stage (ii), the combined assumption of CARA utility and normally distributed random variables coupled with the second-order Taylor expansion (3.4) proves very helpful because it allows a substantial simplification of the calculations. Under these conditions, two out of the three integrals comprised in the expression for expected utility have closed-form solutions, thus greatly simplifying the numerical integration.[10]

The simulation exercise can be performed for any decision setting for which the theoretical model provides a reasonable approximation. Because of data availability, the particular example chosen is the production of fat cattle (Q_{t+1}) from feeder cattle (I_t). The decision maker is assumed to be a representative producer, for whom $\hat{Q}_3$ = 120,000 pounds of fat beef and $\hat{I}_2$ = 60,000 pounds of feeder cattle, where $\hat{Q}_3$ and $\hat{I}_2$ equal production and input demand evaluated at the means

[10]Details about the simulations are available from the authors upon request.

of the random variables. The output elasticity of factor demand (η_{QI}) is assumed to equal 0.9, and the transformation of feeder cattle to fat cattle is hypothesized to last 8 months. Based on price data over the period 1974-1986,[11] it is postulated that $F_1 = \mu = \hat{F}_2 = 70$, $\sigma_P{}^2 = \sigma_F{}^2 = 70$, and $\sigma_H{}^2 = 250$, where prices are expressed in dollars of December 1986 per 100 pounds, and $\sigma_F{}^2$ and $\sigma_H{}^2$ denote the variances of F_2 and H_2, given information at date t = 1.

Sensitivity analysis was performed for the degree of absolute risk aversion (λ), the elasticity of input demand with respect to forward price (η_{IF}), the derivative of input prices with respect to output cash prices (β_H),[12] and the relationship between forward and output cash prices (β_F).[13] When $\beta_F = 0$, the underlying hypothesis is that output cash price shocks are transitory, i.e., price shocks at a particular date have no effect on prices at later dates. In contrast, $\beta_F = 1$ represents the case in which price shocks at a particular date are permanent because they affect prices forever. Note that different values of β_H and β_F imply different values of $\sigma_h{}^2$ and $\sigma_f{}^2$ because $\sigma_h{}^2 = \sigma_H{}^2 - \beta_H{}^2\,\sigma_P{}^2$ and $\sigma_f{}^2 = \sigma_F{}^2 - \beta_F{}^2\,\sigma_P{}^2$.

The results of the simulations are presented in Table 1. The optimal levels of the decision variables are expressed in thousands of pounds. Optimal production ($Q_2{}^*$) equals 120,000 pounds in all instances. Therefore, for a myopic decision maker it would be optimal to sell forward 120,000 pounds and to have a zero put position. In contrast, a forward-looking agent will find it optimal to sell the number of forward contracts reported in the column labeled ($X_1{}^* | Z_1 = Z_1{}^*$) and to buy the amount of puts indicated in the column labeled ($Z_1{}^*$). The amount of puts bought is negative in all instances, which means that the optimal strategy in this example is to sell puts.

[11]Cash prices used were monthly averages of daily prices corresponding to medium frame one steer calves at Kansas City and 900-1,100 pounds choice slaughter steers at Omaha, reported by the U.S. Department of Agriculture. Prices were expressed in real terms by using the Producer Price Index.

[12]Empirical estimates of β_H for different months ranged from 1.05 to 1.40 without correcting for autocorrelation, and from 0.93 to 1.30 when the correction was imposed.

[13]Using futures as proxies for forward prices, we obtained sample estimates of β_F ranging between 0.82 and 1.07 depending on the month being considered. Futures prices were monthly averages of daily futures prices for live cattle at the Chicago Mercantile Exchange, reported in the *Statistical Yearbook* of the Chicago Mercantile Exchange. The October contract was used in January and February, the December contract in March and April, the February contract in May and June, the April contract in July and August, the June contract in September and October, and the August contract in November and December.

Table 1. Optimal decisions at date t = 1

Response of Cash to Output Prices (β_H)	Response of Forward to Output Prices (β_F)	Coefficient of Absolute Risk Aversion (λ)	Elast. of Input Demand with Respect to Forward Prices (η_{IF})	Optimal Decisions at Date t = 1 (unit: 1,000 pounds)			
				Production (Q_2^*)	Forward Contracts Sold in the		Puts Bought (Z_1^*)
					Absence of Puts ($X_1^* \mid Z_1 = 0$)	Presence of Puts ($X_1^* \mid Z_1 = Z_1^*$)	
0.5	0	0.00003	1	120	92.0	93.0	-2.0
0.5	0	0.00003	3	120	95.2	97.9	-5.5
0.5	0	0.00012	1	120	96.6	97.4	-1.7
0.5	0	0.00012	3	120	103.9	105.8	-3.7
0.5	1	0.00003	1	120	207.5	214.6	-14.3
0.5	1	0.00003	3	120	202.9	223.0	-40.2
0.5	1	0.00012	1	120	200.8	207.4	-13.0
0.5	1	0.00012	3	120	187.8	203.4	-31.4
1.0	0	0.00003	1	120	63.5	67.6	-8.3
1.0	0	0.00003	3	120	69.5	80.6	-22.3
1.0	0	0.00012	1	120	72.0	75.6	-7.1
1.0	0	0.00012	3	120	86.2	93.9	-15.4
1.0	1	0.00003	1	120	178.8	181.5	-5.5
1.0	1	0.00003	3	120	176.6	184.4	-15.6
1.0	1	0.00012	1	120	175.5	178.1	-5.1
1.0	1	0.00012	3	120	168.7	175.1	-12.8
1.5	0	0.00003	1	120	34.2	43.6	-18.8
1.5	0	0.00003	3	120	41.7	67.5	-51.7
1.5	0	0.00012	1	120	45.0	53.3	-16.6
1.5	0	0.00012	3	120	64.4	83.2	-37.7
1.5	1	0.00003	1	120	149.8	150.2	-0.8
1.5	1	0.00003	3	120	149.3	150.4	-2.3
1.5	1	0.00012	1	120	149.1	149.5	-0.8
1.5	1	0.00012	3	120	147.5	148.6	-2.0

Because one can create any desired forward/options payoff with any two contracts of the set of forward, put, and call contracts, we restricted the choice to forward and put contracts. But this restriction does not prevent the decision maker from creating a written call position. To do so, the producer sells forward more than 120,000 pounds and sells puts. The forward contracts in excess of 120,000 pounds coupled with the written puts create a written call position that hedges against Π when $\partial\Pi/\partial P_2$ is positive everywhere (see Figure 3c). This situation occurs when $\beta_F = 1$, i.e., when the producer perceives price shocks as permanent.

From Table 1, it can also be seen that

(i) $Q_2^* - X_1^*(Z_1 = Z_1^*) > 0 > Z_1^*$ when $\beta_F = 0$

(ii) $0 > Z_1^* > Q_2^* - X_1^*(Z_1 = Z_1^*)$ when $\beta_F = 1$

where $[Q_2^* - X_1^*(Z_1 = Z_1^*)]$ denotes the difference between the optimal myopic hedge ($X_1 = Q_2^*$) and the optimal forward-looking hedge in the presence of puts $[X_1^*(Z_1 = Z_1^*)]$. Inequalities in (i) hold because, for given values of the random variables f and h, $\partial\Pi/\partial P_2$ is negative everywhere when $\beta_F = 0$ [see expression (2.5)]. Hence, the position that offsets $\partial\Pi/\partial P_2$ must be positively sloped with respect to P_2, which is achieved by buying forward contracts $[Q_2^* - X_1^*(Z_1 = Z_1^*) > 0]$. The explanation of inequalities in (ii) obeys the same logic: $\partial\Pi/\partial P_2$ is positive everywhere when $\beta_F = 1$ and $\beta_H < 2$; therefore, the decision maker sells additional forward contracts $[Q_2^* - X_1^*(Z_1 = Z_1^*) < 0]$.

In the simulations reported, the amount of puts bought is positively related to the coefficient of absolute risk aversion (λ) and negatively related to the elasticity of input demand with respect to forward prices (η_{IF}). Although in unreported simulations both relationships also held for all other values of the exogenous variables that were tried, we were unable to prove that they always hold. The relationship between Z_1^* and η_{IF} should be negative in most situations because increasing η_{IF} increases the convexity of Π as a function of P_2:[14]

$$(3.5)\quad \hat{\Pi}_{PP} = (\beta_F\, \eta_{QI}\, \hat{Q}_3 - \beta_H\, \hat{I}_2)^2\, \eta_{IF}/(\eta_{QI}\, \hat{F}_2\, \hat{Q}_3)$$

[14]The derivation of (3.5) from (2.6') is available from the authors upon request.

The effect of η_{IF} on the convexity of Π is attributable to the fact that Π depends essentially on input usage at date 2, and the absolute changes in input usage due to P_2 are larger the larger is the elasticity η_{IF}.

Expression (3.5) helps explain the negative (positive) relationship between β_H and the amount of puts bought when $\beta_F = 0$ ($\beta_F = 1$). When $\beta_F = 0$, the convexity of Π is positively related to the absolute magnitude of β_H ($|\beta_H|$). Intuitively, $\beta_H \neq 0$ means that the agent perceives future input cash prices (H_2) to be associated with future output cash prices (P_2). But future profits (Π) are a convex function of H_2 because of the response of future input use (I_2) to H_2. Hence, the larger $|\beta_H|$, the larger the convexity of Π as a function of P_2. For similar reasons, the convexity of Π is positively related to the absolute magnitude of β_F, given $\beta_H = 0$.

When β_H and β_F have the same signs, the effect that P_2 exerts on the curvature of Π through H_2 tends to offset the effect of P_2 on the curvature of Π through F_2. If β_H is positive, the producer will tend to reduce I_2 (and therefore Q_3) as P_2 increases. But if β_F is also positive, the agent will tend to increase Q_3 (and therefore I_2) with increases in P_2. These two responses of I_2 to P_2 work in opposite directions, tending to cancel with each other. The maximum offsetting effect occurs when $\beta_H = \beta_F \, \eta_{QI} \, \hat{Q}_3/\hat{I}_2$, at which $\hat{\Pi}_{PP} = 0$ [see (3.5)]. At this value of β_H, Π is approximately a linear function of P_2, thus rendering puts unnecessary. This observation makes sense, there must be a set of circumstances under which the advantages of forward price increases are exactly offset by increases in input costs. In Table 1, the closest scenario to $\hat{\Pi}_{PP} = 0$ is when $\beta_H = 1.5$ and $\beta_F = 1$, at which the optimal amounts of puts bought are near zero (Z_1^* ranges between -2.3 and -0.8).

Unreported simulations reveal that the optimal amount of puts purchased is negatively related to σ_P^2, and positively related to σ_F^2 and σ_H^2. The negative effect in the quantity of puts bought caused by an increase in σ_P^2 outweighs the positive effect due to the same percentage increase in σ_F^2 and σ_H^2. Within reasonable bounds, however, the optimal put position does not seem very sensitive to changes in the magnitudes of the variances.

In Table 1, the column labeled $(X_1^* | Z_1 = 0)$ shows the optimal forward positions in the absence of puts, and the column labeled $(X_1^* | Z_1 = Z_1^*)$ reports the optimal forward positions in the presence of puts. These two columns reveal that the amount of forward contracts sold when puts are available is greater than the amount sold when puts are not available. The explanation of this result is that forward contracts are used for two purposes: (i) offsetting the convexity of Π, and (ii) offsetting the average slope of Π. To offset the convexity of Π, it is necessary to sell both forward and put contracts; but because forward contracts alone are not useful for this purpose, the absence of options leads to a net loss in the number of forward contracts sold. In contrast, to offset the average slope of Π, forward contracts alone are used; therefore, the amount of forward contracts employed for this purpose is unaffected by the availability of options. The use of forward contracts to offset the average slope of Π explains why the amount of forward contracts sold is positively associated with β_F (given a particular value of β_H), and negatively related to β_H (given a particular value of β_F). From expression (2.5), it can be seen that $\partial\Pi/\partial P_2$ (and therefore the average slope of Π) increases with β_F, for a given β_H.

The increase in the amount of forward contracts sold attributable to the availability of puts suggests a synergetic effect between the two types of contracts from a hedger's perspective. In other words, the scenario analyzed depicts a situation in which puts complement rather than compete with forward contracts. The opposite is true if the options available are calls rather than puts. That is, in the absence of puts, the decision maker will sell less forward contracts if calls can be traded than if calls cannot be traded. Hence, the example is such that calls compete with forward contracts. The reason for this result is that the payoff of any position involving the sale of x forward contracts and y puts can be replicated by a combination of selling $(x - y)$ forward contracts and selling y calls. For example, if calls were available but puts were not, the simulation in the first row of Table 1 would involve selling calls for 2,000 pounds and forward contracts for 91,000 (= 93,000 - 2,000) pounds. It can be seen that the amount of forward contracts sold in the presence of calls and the absence of puts (91,000 pounds) is less than that sold in the absence of options (92,000 pounds).

In this example, the producer always sells puts because the simulations assumed decreasing returns to scale. Had it been assumed increasing returns to scale, puts would always be purchased [see (2.6')]. Decreasing returns to scale is a more realistic assumption, yet the optimality of writing puts seems counterintuitive. This result occurs because the producer uses the forward market to reduce price risk on the long physical position. Were we to make the forward market redundant (instead of the call market), the producer would achieve the exposure depicted in the sixth column of Table 1 by purchasing puts and writing calls to create a synthetic forward position.

Concluding Remarks

In a standard one-period (or myopic) model under nonstochastic production and unbiased forward and options prices, it is optimal to sell the entire production in the forward market for output and to hold a null position in options. Relaxing the restrictive assumption of a single production cycle (i.e., allowing for forward-looking behavior), however, will generally change the optimal forward position and create a hedging role for options. The reason for the behavioral difference between myopic and forward-looking agents is that the latter realize that this period's (random) output prices will be associated in a nonlinear manner with prices in subsequent periods.

Appendix

The function $g(P_2)$ is continuous and differentiable everywhere except at $P_2 = F_1$, at which it is continuous but not differentiable if $Z_1^* \neq 0$. Then, according to the Mean-Value Theorem, there exists at least one number a in the interval (c, F_1) at which

$$\text{(A1)} \quad g'(a) = \frac{g(F_1) - g(c)}{F_1 - c}$$

Denote the set of such numbers by A, i.e., $A \equiv \{a\colon a \in (c, F_1) \text{ and } g'(a) = [g(F_1) - g(c)]/(F_1 - c)\}$. Similarly, there exists at least one number in the interval (F_1, d) at which

$$\text{(A2)} \quad g'(b) = \frac{g(d) - g(F_1)}{d - F_1}$$

Let $B \equiv \{b\colon b \in (F_1, d) \text{ and } g'(b) = [g(d) - g(F_1)]/(d - F_1)\}$, and define the following strictly increasing function of P_2:

$$\text{(A3)} \quad p \equiv \begin{cases} \inf(A) \text{ if } P_2 = c < F_1 \\ \sup(B) \text{ if } P_2 = d > F_1 \end{cases}$$

Because p is a one-to-one mapping of P_2, the first derivative of $g(P_2)$ with respect to P_2 evaluated at $P_2 = p$ equals

$$\text{(A4)} \quad g'(p) = E_1[m_2''\,(Q_2^* - X_1^* - L_2\,Z_1^* + \pi')|p]$$

where $m_2'' \equiv M_2''(P_2 = p)$ and $\pi' \equiv \partial\Pi/\partial P_2$ evaluated at $P_2 = p$. But from (A1) through (A3) we also have

$$\text{(A5)} \quad g(P_2) = g(F_1) - g'(p)\,(F_1 - P_2)$$

By substituting (A4) into (A5) and the resulting expression into FOC (2.3) we get

$$\text{(A6)}\quad E_1[g(P_2)\,(F_1 - P_2)] = E_1[g(F_1)\,(F_1 - P_2)] - E_1[m_2''\,(Q_2^* - X_1^* - L_2\,Z_1^* + \pi')\,(F_1 - P_2)^2]$$

$$\text{(A6')}\quad = - Q_2^*\,£_{PP} + X_1^*\,£_{PP} + Z_1^*\,£_{PPL} - £_{PP\pi} = 0$$

where: $£_{PP} = E_1[m_2''\,(F_1 - P_2)^2]$

$£_{PPL} = E_1[m_2''\,(F_1 - P_2)^2\,L_2]$

$£_{PP\pi} = E_1[m_2''\,(F_1 - P_2)^2\,\pi']$

Expression (A6') follows from (A6) because F_1, Q_2^*, X_1^*, and Z_1^* are nonstochastic at date t = 1, and $F_1 = E_1(P_2)$ if forward prices are unbiased. By proceeding in a similar manner with FOC (2.4) we obtain

$$\text{(A7)}\quad E_1\{g(P_2)\,[- R_1 + (F_1 - P_2)\,L_2]\} = - R_1\,E_1[g(P_2)] + E_1[g(P_2)\,(F_1 - P_2)\,L_2]$$

$$\text{(A7')}\quad = - R_1\,E_1[g(P_2)] + E_1[g(F_1)\,(F_1 - P_2)\,L_2]$$

$$- E_1[m_2''\,(Q_2^* - X_1^* - L_2\,Z_1^* + \pi')\,(F_1 - P_2)^2\,L_2]$$

$$\text{(A7'')}\quad = - R_1\,£_\Delta - Q_2^*\,£_{PPL} + X_1^*\,£_{PPL} + Z_1^*\,£_{PPL} - £_{PP\pi L} = 0$$

where: $£_\Delta = E_1[g(P_2)] - g(F_1)$

$£_{PP\pi L} = E_1[m_2''\,\pi'\,(F_1 - P_2)^2\,L_2]$

In deriving (A7'') from (A7') we used the facts that $E_1[(F_1 - P_2)\,L_2] = R_1$ (because put prices are assumed to be unbiased) and that $L_2^2 = L_2$ (from the definition of L_2). Finally, FOCs (A6') and

(A7") can be combined to yield the following expressions for the optimal forward and put positions:

$$\text{(A8)}\quad X_1^* = Q_2^* + \frac{£_{PP\pi}}{£_{PP} - £_{PPL}} - \frac{R_1\, £_\Delta + £_{PP\pi L}}{£_{PP} - £_{PPL}}$$

$$\text{(A9)}\quad Z_1^* = - \frac{£_{PP\pi}}{£_{PP} - £_{PPL}} + \frac{R_1\, £_\Delta + £_{PP\pi L}}{£_{PP} - £_{PPL}}\ \frac{£_{PP}}{£_{PPL}}$$

It must be true that $£_{PP} < £_{PPL} < 0$ because $m_2'' < 0$ (from the assumption that $U'' < 0$) and $(F_1 - P_2)^2 > 0$ for $P_2 \neq F_1$. For the same reasons, $£_{PP\pi} < £_{PP\pi L} < 0$ if $\pi' > 0$ everywhere, and $£_{PP\pi} > £_{PP\pi L} > 0$ if $\pi' < 0$ everywhere. If π' is not either positive or negative everywhere, then the signs and relative magnitudes of $£_{PP\pi}$ and $£_{PP\pi L}$ are ambiguous. Finally, the sign of $£_\Delta$ is ambiguous in general, but it is strictly positive in the most realistic case of decreasing absolute risk aversion. Hence, barring pathological cases, it will be true that $X_1^* \neq Q_2^*$ and $Z_1^* \neq 0$. Q.E.D.

References

Anderson, Ronald W., and Jean-Pierre Danthine. "The Time Pattern of Hedging and the Volatility of Futures Prices." *Review of Economic Studies* 50 (1983): 249-266.

Anderson, Ronald W., and Jean-Pierre Danthine. "Hedging and Joint Production: Theory and Illustrations." *Journal of Finance* 35 (1980): 487-498.

Batlin, Carl A. "Production under Price Uncertainty and Imperfect Time Hedging Opportunities in Futures Markets." *Southern Economic Journal* 49 (1983): 682-692.

Benninga, S., R. Eldor, and I. Zilcha. "The Optimal Hedge Ratio in Unbiased Futures Markets." *Journal of Futures Markets* 4 (1984): 155-159.

Chicago Mercantile Exchange. *Statistical Yearbook.* Chicago: annually (1974-1986).

Feder, G., R. E. Just, and A. Schmitz. "Futures Markets and the Theory of the Firm under Price Uncertainty." *Quarterly Journal of Economics* 94 (1980): 317-328.

Hey, John D. "The Dynamic Competitive Firm under Spot Price Uncertainty." *Manchester School of Economics and Social Studies* 55 (1987): 1-12.

Holthausen, Duncan M. "Hedging and the Competitive Firm under Price Uncertainty." *American Economic Review* 69 (1979): 989-995.

Karp, Larry S. "Methods for Selecting the Optimal Dynamic Hedge When Production is Stochastic." *American Journal of Agricultural Economics* 69(1987):647-657.

Kerkvliet, Joe, and Michael H. Moffett. "The Hedging of an Uncertain Future Foreign Currency Cash Flow." *Journal of Financial and Quantitative Analysis* 26 (1991): 565-578.

Lapan, H., G. Moschini, and S. Hanson. "Production, Hedging, and Speculative Decisions with Options and Futures Markets." *American Journal of Agricultural Economics* 73 (1991): 66-74.

Losq, Etienne. "Hedging with Price and Output Uncertainty." *Economics Letters* 10 (1982): 65-70.

Paroush, Jacob, and Avner Wolf. "Production and Hedging Decisions in the Presence of Basis Risk." *Journal of Futures Markets* 9 (1989): 547-563.

Sakong, Y., D. Hayes, and A. Hallam. "Hedging Production Risk with Options." *American Journal of Agricultural Economics* 75(1993):408-415.

Sandmo, Agnar. "On the Theory of the Competitive Firm under Price Uncertainty." *American Economic Review* 61 (1971): 65-73.

Smith, Clifford W., and René M. Stulz. "The Determinants of Firms' Hedging Policies." *Journal of Financial and Quantitative Analysis* 20 (1985): 391-405.

U.S. Department of Agriculture. "Livestock and Meat Statistics, 1984-88." Washington, D.C.: U.S. Department of Agriculture, ERS Statistical Bulletin No. 784 (September 1989).