

# **The Short-Run Behavior of Forward-Looking Firms**

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# THE SHORT-RUN BEHAVIOR OF FORWARD-LOOKING FIRMS

## Abstract

A theory of short-run competitive firm behavior allowing for nonmyopic risk aversion, randomness in input and output prices, as well as forward trading and storage of final good and material input is introduced. If the firm is a forward-looking risk-averse expected-utility maximizer, separation of production and storage from hedging decisions is obtained. Production and storage are shown to depend only upon forward and cash prices and to be independent of the agent's degree of risk aversion and the distribution of random prices. Comparative statics are derived regarding production, purchases, and sales. The hypotheses advanced are tested empirically with monthly data pertaining to the U.S. soybean-processing industry. The results support the model and suggest that in stationary equilibrium futures prices of the soybean complex have had little influence on crushings or production, but they have been important determinants of inventory levels. Both theoretical and empirical results indicate that short-run firm behavior is more complex than is generally assumed in the literature. They also suggest long-term firm behavior can be better understood by studying its short-term behavior rather than using the medium- to long-run models that are currently available.

## THE SHORT-RUN BEHAVIOR OF FORWARD-LOOKING FIRMS

The objective function used in the theory of the firm typically contains output price times output quantity and a quantity-dependent cost function. Implicit are the assumptions that the firm simultaneously sells output and buys inputs at known prices. In many firms, however, much of the managerial effort is targeted toward buying inputs when their prices are lowest, selling output when its price is highest, using input and output storage to take advantage of price movements, and employing forward and/or futures markets to hedge some of the risks associated with production and storage. These activities are particularly important in commodity-oriented firms, such as those involved in producing and processing food and natural resources.

In the medium and long terms, the observed short-term differences between production and output sales and between input purchases and usage are averaged out and seem trivial. The medium- and long-term behavior of the firm, however, may reflect the cumulative impact of short-term decisions. In this instance, a full understanding of medium- and long-term behavior will depend on how managers respond to short-term incentives.

Figures 1 and 2 illustrate these assertions with data from the U.S. soybean-processing industry. Figure 1 depicts the annual quantities of soybeans purchased and processed, and the annual sales (in soybean units) of soybean oil and meal for the years 1967/66 through 1986/85.<sup>1</sup> It can be seen that annual crushings, purchases, and sales are almost indistinguishable. The major departures from the common pattern are those of oil sales in the years 1977/76, 1980/79, and 1984/83. It is erroneous, however, to infer from Figure 1 that processors manage crushings, purchases, and sales as a single undifferentiated entity. Figure 2 displays the monthly quantities purchased, processed, and sold for two typical years (1980:8 through 1982:8). The sharp contrast between Figures 1 and 2 shows that behavioral rules derived from monthly data will be substantially different from those found with annual data. Behavioral rules estimated from

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<sup>1</sup>Soybean processors crush raw soybeans to produce oil and meal in fixed proportions.

monthly data should provide a better understanding of actual behavior than rules obtained from annual data, because real-world decisions are made at intervals much shorter than one year.

In this paper, we develop and test a theory of short-run competitive firm behavior under risk aversion in the presence of forward markets. One innovation in this model is that we allow the firm to be forward-looking or nonmyopic, that is, we assume that the firm's planning horizon exceeds the firm's decision horizon.<sup>2</sup> The study also shows how short-term parameters can be used to derive meaningful long-term response parameters.

Nearly all of the literature on the theory of the competitive firm under uncertainty is based on myopic or static models. The standard assumption is that the firm's decision and planning horizons are identical and equal to "one period" (Robert Merton 1982, p. 656), i.e., the firm's objective is to maximize the expected utility of wealth at the end of the current period, without concern about future periods. The main results from this theory are that the risk-averse firm will produce at a point such that production under price uncertainty is less than under certainty (Agnar Sandmo 1971) and that a marginal increase in price uncertainty reduces production if the firm's risk attitude is either decreasing or constant absolute risk aversion (DARA or CARA, respectively) (Yasunori Ishii 1977).

The framework set up by Agnar Sandmo led to a formal analysis of the behavior of the firm in the presence of forward and futures markets for the final good. Jean-Pierre Danthine (1978), Duncan Holthausen (1979), and Gershon Feder et al. (1980) further refined Agnar Sandmo's model by introducing a forward market for the final product. They proved that, in the presence of a forward market, the competitive firm facing price uncertainty (and nonstochastic production) will produce as if the price were certain and equal to the forward price. This result is usually referred to as the *separation* property. When there is basis risk, that is, the case of futures markets, Carl Batlin (1983) and Jacob Paroush and Avner Wolf (1989) showed that (i) the firm will produce less

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<sup>2</sup>Robert Merton (1982, p. 656) defines decision horizon as "the length of time between which the investor makes successive decisions, and it is the minimum time between which he would take any action," and planning horizon as "the maximum length of time for which the investor gives any weight in his utility function."

than in the absence of basis risk and that (ii) the separation property generally does not hold. Recently, however, Harvey Lapan et al. (1991) demonstrated that separation occurs even in the presence of basis risk if the utility function is CARA and the relationship between futures and cash prices satisfies some commonly assumed regularity conditions.

Edward Zabel (1971) pioneered the study of forward-looking behavior under risk aversion, allowing also for inventories of final good. Zabel proposed a CARA intertemporal utility function to characterize the preferences of the competitive firm. His line of research was not pursued until recently, when John Hey (1987) built a dynamic model of the competitive firm with a forward market for final good. In that theoretical paper, John Hey assumed a risk-averse firm with an additive intertemporal utility function. The firm was allowed to hold inventories of final good and also to trade in a forward market for final good. With this setting, Hey proved that the firm separated production from hedging and showed that it produced as if the cash price were known and equal to the forward price. In addition to postulating additive utility, a characteristic of Hey's paper is that the results depend crucially upon the production and hedging and sales decisions occurring sequentially. In his model, the firm chooses optimal production and hedging at one decision date and optimal sales at the next decision date. For the short-run analysis considered in this study, a more realistic scenario would involve simultaneous rather than sequential decision making.

A different approach to forward-looking behavior in the presence of futures markets was undertaken by Ronald Anderson and Jean-Pierre Danthine (1983). They allowed the firm to revise the futures position within the cash market holding period and found that there is separation between cash and futures decisions. However, they assumed a single production cycle, which seems overly restrictive to model the behavior of some types of firms (for example, commodity processors).

Our paper is organized as follows. In Section I, we lay out the theoretical model. We then test some of the theoretical results using data from the U.S. soybean-processing industry, and discuss our findings in Sections II and III. In Section IV, we summarize the major conclusions.

### I. The Theoretical Model

Consider a competitive firm characterized by a twice continuously differentiable von Neumann-Morgenstern utility function, utility being strictly increasing and concave in its argument terminal wealth ( $U(W_T)$ ,  $U' > 0$ ,  $U'' < 0$ ). Terminal wealth is defined as

$$(1.1) \quad W_T = r_{-1} r_0 r_1 \dots r_{T-1} W_{-1} + r_0 r_1 \dots r_{T-1} \pi_0 + r_1 \dots r_{T-1} \pi_1 + \dots + r_{T-1} \pi_{T-1} + \pi_T$$

where  $W_t$  is monetary wealth at the end of decision date  $t$ ,  $\pi_t$  is the cash flow at time  $t$ , and  $r_t$  equals one plus the one-period interest rate prevailing at  $t$ . Interest rates can change from period to period, but they are assumed to be nonstochastic. At each trading date  $t$ , the firm can borrow and lend unlimited amounts of money for one period at the prevailing interest rate. The time elapsed between successive decision dates represents the firm's decision horizon. The furthest in the future that the firm cares about is date  $T$ , hence the time comprised between dates  $t$  and  $T$  is the firm's planning horizon at date  $t$ .

It is further assumed that the firm's short-run production function is represented by the Leontief function

$$(1.2) \quad Q_t = \min[Q_t^s/\Phi, q(\cdot)], \quad \Phi > 0$$

where  $Q_t$  denotes production of final good at date  $t$ ,  $Q_t^s$  is material input use at date  $t$ ,  $\Phi$  is a fixed input-output coefficient, and  $q(\cdot)$  is a strictly increasing and concave production function for nonmaterial inputs such that  $q(0) = 0$ . According to (1.2), adding  $\Phi$  units of material input increases production by one unit over the range in which the set of variable nonmaterial inputs does not constrain production. Walter E. Diewert (1971) has shown that the cost function dual to the production function (1.2) is

$$(1.3) \quad C_t = \Phi s_t Q_t - c(Q_t; \cdot)$$

where  $C_t$  is variable cost at date  $t$ ,  $s_t$  is material input price at date  $t$ , and  $c(Q_t; \cdot)$  represents a strictly increasing and convex variable nonmaterial cost function.

For our purposes, the most important property of the Leontief function (1.2) is that there is no substitution between material and nonmaterial inputs. The absence of substitution between material and nonmaterial inputs allows us to introduce randomness in material input prices without having to face the extreme complications that would arise otherwise (Raveendra Batra and Aman Ullah 1974, Richard Hartman 1975, Marion Stewart 1978, Stylianos Perrakis 1980, Brian Wright 1984). This property also allows us to address the lack of input substitutability that exists in the short run.

Relaxation of the standard nonstorage constraint is one of the main contributions of our analysis. Hence, we make explicit allowance for storage of both output and material input. The presence of storage means that output sales and material input purchases will generally be different from the amount produced and the material input employed in the production process, respectively. We also allow for the presence of forward markets for both output and material input. This is the most general setting of our model; in situations where one or both forward markets are not available, the more general scenario can be adjusted by omitting the relevant variables from the objective function.

The assumption of forward markets as opposed to either futures markets or both forward and futures markets greatly simplifies the presentation. The major theoretical result used in the empirical section (i.e., the separation property) can also be derived from models that assume the existence of either only futures or both forward and futures markets, by applying the techniques used by Harvey Lapan et al. (1991), Jacob Paroush and Avner Wolf (1989), and Frances Antonovitz and Ray Nelson (1988). Harvey Lapan et al. (1991) showed that, when some regularity conditions are imposed on the utility function and on the relationship between futures

and cash prices, the separation property holds in the presence of futures instead of forward markets. Jacob Paroush and Avner Wolf (1989), and Antonovitz and Nelson (1988) demonstrated that the separation property holds in the presence of both forward and futures markets.

The studies mentioned in the previous paragraph omit the marking-to-market that takes place in futures as opposed to forward markets. Marking-to-market has been analyzed in the context of a single production cycle by Ronald Anderson and Jean-Pierre Danthine (1983). Under the same assumptions that we make about financing, these authors show that marking to market affects only the futures position and not the physical position. Extending our model to include marking-to-market as in Anderson and Danthine is straightforward, albeit tedious. This extension would involve modifying expressions (1.4) and (1.6) below, by including equations to reflect marking-to-market, and then obtaining first-order-conditions for the decision dates in which no physical decisions are taken. Our paper's key results, however, would remain unchanged because of the separation property.

The particular form of the firm's cash flow at any date  $t = 0, 1, \dots, T-1$  is

$$(1.4) \quad \pi_t = p_t P_t - s_t S_t - c(Q_t) - i(I_t - P_t) - i^s(I_t^s + S_t - Q_t^s) + (f_{t-1,t} - p_t) F_{t-1} + (f_{t-1,t}^s - s_t) F_{t-1}^s$$

$$\text{s.t.} \quad I_t = I_{t-1} - P_{t-1} + Q_{t-1} \geq P_t$$

$$I_t^s + S_t = I_{t-1}^s + S_{t-1} - Q_{t-1}^s + S_t \geq Q_t^s = \Phi Q_t \geq 0$$

where:  $p_t$  = cash price of final good at date  $t$

$P_t$  = sales of final good at date  $t$

$s_t$  = cash price of material input at date  $t$

$S_t$  = purchases of material input at date  $t$

$i(\cdot), i^s(\cdot)$  = strictly increasing and convex variable inventory cost functions of final good and material input, respectively



$I_t$  = beginning inventory of final good at date  $t$ ,  $I_t = I_{t-1} - P_{t-1} + Q_{t-1}$

$I_t^s$  = beginning inventory of material input at date  $t$ ,  $I_t^s = I_{t-1}^s + S_{t-1} - Q_{t-1}^s$

$f_{t-1,t}$  = forward price for delivery of final good at  $t$ , prevailing at  $t-1$

$F_{t-1}$  = net short position for delivery of final good at  $t$ , open at  $t-1$

$f_{t,t+1}^s$  = forward price for delivery of material input at  $t$ , prevailing at  $t-1$

$F_{t-1}^s$  = net short position for delivery of material input at  $t$ , open at  $t-1$

At any date  $t$  there are only two positions that can be traded in the forward market: one for delivery of the good at date  $t+1$ , and the other for immediate delivery (i.e., delivery at  $t$ ).<sup>3</sup> The cash flow due to the opening of the forward contract lags one period the actual decision to open the contract because forward trading does not involve cash flows until positions are closed. Forward prices prevailing at  $t$  for immediate delivery are identical to the corresponding current cash prices of output ( $p_t$ ) and material input ( $s_t$ ). Forward prices at  $t$  for delivery at the following date  $t+1$  ( $f_{t,t+1}$  and  $f_{t,t+1}^s$ ), however, will be generally different from the respective current cash prices  $p_t$  and  $s_t$ .

At any decision date  $t$ , the firm selects the levels of purchases and use of material input ( $S_t$  and  $Q_t^s$ ), production ( $Q_t = \Phi Q_t^s$ ), sales of final good ( $P_t$ ), and hedging ( $F_t$  and  $F_t^s$ ) that maximize expected utility, given available information. The optimal decision vector at the current date  $t = 0$  ( $d_0^* \equiv (P_0^*, Q_0^{s*}, S_0^*, F_0^*, F_0^{s*})$ ) is obtained by solving the following set of recursive equations

$$(1.5) \quad M_T(r_{T-1} W_{T-1}, I_T, I_T^s, F_{T-1}, F_{T-1}^s; \mathbf{p}_T) = \max_{d_T \in D_T} U(r_{T-1} W_{T-1} + \pi_T)$$

$$(1.6) \quad M_t(r_{t-1} \dots r_{T-1} W_{t-1}, I_t, I_t^s, F_{t-1}, F_{t-1}^s; \mathbf{p}_t)$$

$$= \max_{d_t \in D_t} E_t\{M_{t+1}[r_t \dots r_{T-1} (r_{t-1} W_{t-1} + \pi_t), I_{t+1}, I_{t+1}^s, F_t, F_t^s; \mathbf{p}_{t+1}]\}, t = 0, 1, \dots, T-1$$

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<sup>3</sup>We do not require that the good be actually delivered, but we still use the term *delivery* for clarity of exposition. Forward commitments may be canceled either by delivering the good or by undertaking an opposite transaction in the forward market.

where  $\mathbf{d}_t \equiv (P_t, Q_t^s, S_t, F_t, F_t^s)$  is the decision vector at date  $t$ ,  $D_t$  is the feasible decision set at date  $t$ ,  $E_t(\cdot)$  denotes the expectation operator based on information available at  $t$ ,  $\mathbf{p}_t \equiv (p_0, s_0, f_{0,1}, f_{0,1}^s, \dots, p_t, s_t, f_{t,t+1}, f_{t,t+1}^s)$  is a vector containing past and current prices, and terminal wealth and cash flows are given by (1.1) and (1.4), respectively. The solution to the problem summarized by expressions (1.5) and (1.6) can be obtained by recursively solving the Lagrangian functions

$$(1.7) \quad \mathcal{L}_T = U(r_{T-1} W_{T-1} + \pi_T) + \eta_T (I_T - P_T) + \eta_T^s (I_T^s + S_T - Q_T^s)$$

$$(1.8) \quad \mathcal{L}_t = E_t\{M_{t+1}[r_t \dots r_{T-1} (r_{t-1} W_{t-1} + \pi_t), I_{t+1}, I_{t+1}^s, F_t, F_t^s; \mathbf{p}_{t+1}]\}$$

$$+ \eta_t (I_t - P_t) + \eta_t^s (I_t^s + S_t - Q_t^s), t = 0, 1, \dots, T-1$$

where  $\eta_t$  and  $\eta_t^s$  are the Lagrangian multipliers corresponding to inventories of final good and material input, respectively.

The first order necessary conditions (FOCs) corresponding to the terminal date  $T$  are

$$(1.9) \quad \frac{\partial \mathcal{L}_T}{\partial P_T} = (p_T + i') M_T' - \eta_T = 0$$

$$(1.10) \quad \frac{\partial \mathcal{L}_T}{\partial Q_T^s} = -(s_T + c'/\Phi) M_T' \leq 0, Q_T^s \geq 0, Q_T^s \frac{\partial \mathcal{L}_T}{\partial Q_T^s} = 0$$

$$(1.11) \quad \frac{\partial \mathcal{L}_T}{\partial S_T} = -(s_T + i^s) M_T' + \eta_T^s = 0$$

$$(1.12) \quad \frac{\partial \mathcal{L}_T}{\partial \eta_T} = I_T - P_T \geq 0, \eta_T \geq 0, \eta_T \frac{\partial \mathcal{L}_T}{\partial \eta_T} = 0$$

$$(1.13) \quad \frac{\partial \mathcal{L}_T}{\partial \eta_T^s} = I_T^s + S_T - Q_T^s \geq 0, \eta_T^s \geq 0, \eta_T^s \frac{\partial \mathcal{L}_T}{\partial \eta_T^s} = 0$$

where  $M_T'$  represents  $U'$  evaluated at the optimum. The rationale for Kuhn-Tucker condition (1.10) is that the amount processed cannot be negative (i.e., production reversal is precluded). Neither (1.9) nor (1.11) are Kuhn-Tucker conditions because the firm is allowed to buy final good and sell material input. The first term of the derivative of the Lagrangian function with respect to sales is positive, hence the Lagrangian multiplier ( $\eta_T$ ) is also positive to satisfy (1.9). But if  $\eta_T > 0$ , then  $\partial \mathcal{L}_T / \partial \eta_T = I_T - P_T$  must equal zero to avoid violating the Kuhn-Tucker condition (1.12). By the same reasoning,  $\eta_T^s > 0$  and  $\partial \mathcal{L}_T / \partial \eta_T^s = I_T^s + S_T - Q_T^s = 0$ . Finally,  $\partial \mathcal{L}_T / \partial Q_T^s > 0$  because  $M_T' > 0$  and  $c' > 0$ , which requires  $Q_T^s = 0$  in order to satisfy (1.10). Therefore, the optimal decision vector at the terminal date  $T$  consists of liquidating inventories ( $P_T^* = I_T$  and  $S_T^* = -I_T^s$ ) and processing no material input ( $Q_T^{s*} = 0$ ). Consequently, the optimal cash flow reduces to  $\pi_T^* = p_T I_T$ , and the value function is

$$(1.14) \quad M_T(r_{T-1} W_{T-1}, I_T; p_T) = U(r_{T-1} W_{T-1} + p_T I_T)$$

For dates prior to the terminal date ( $t = 0, \dots, T-1$ ), the FOCs are (see Appendix A):

$$(1.15) \quad \frac{\partial \mathcal{L}_t}{\partial P_t} = r_{t+1} \dots r_{T-1} [r_t (p_t + i') M_t' - E_t(p_{t+1} M_{t+1}')] - \eta_t = 0$$

$$(1.16) \quad \frac{\partial \mathcal{L}_t}{\partial Q_t^s} = r_{t+1} \dots r_{T-1} [E_t(p_{t+1}/\Phi M_{t+1}') - r_t (s_t + c'/\Phi) M_t'] \leq 0, Q_t^s \geq 0, Q_t^s \frac{\partial \mathcal{L}_t}{\partial Q_t^s} = 0$$

$$(1.17) \quad \frac{\partial \mathcal{L}_t}{\partial S_t} = r_{t+1} \dots r_{T-1} [E_t(s_{t+1} M_{t+1}') - r_t (s_t + i^s) M_t'] + \eta_t^s = 0$$

$$(1.18) \quad \frac{\partial \mathcal{L}_t}{\partial F_t} = r_{t+1} \dots r_{T-1} [f_{t,t+1} M_t' - E_t(p_{t+1} M_{t+1}')] = 0$$

$$(1.19) \quad \frac{\partial \mathcal{L}_t}{\partial F_t^s} = r_{t+1} \dots r_{T-1} [f_{t,t+1}^s M_t' - E_t(s_{t+1} M_{t+1}')] = 0$$

$$(1.20) \quad \frac{\partial \mathcal{L}_t}{\partial \eta_t} = I_t - P_t \geq 0, \eta_t \geq 0, \eta_t \frac{\partial \mathcal{L}_t}{\partial \eta_t} = 0$$

$$(1.21) \quad \frac{\partial \mathcal{L}_t}{\partial \eta_t^s} = I_t^s + S_t - Q_t^s \geq 0, \eta_t^s \geq 0, \eta_t^s \frac{\partial \mathcal{L}_t}{\partial \eta_t^s} = 0$$

where  $M_t' = E_t(M_{t+1}')$  evaluated at the optimum corresponding to date  $t$  (note that  $M_t' > 0$ ). The solution to FOCs (1.15) through (1.21) is a unique absolute constrained maximum because the objective function is strictly concave, and the constraint set is convex.<sup>4</sup> These FOCs can be further manipulated to yield separation between "physical" decisions (i.e., purchases, production, and sales) and hedging. This assertion is readily shown by substituting (1.18) into (1.15) and (1.16), and (1.19) into (1.17), and rearranging, which yields the set of expressions (1.22) through (1.24) as an alternative to (1.15) through (1.17):

$$(1.22) \quad f_{t,t+1} - r_t [p_t - i'(I_t - P_t)] = \eta_t^s / (r_{t+1} \dots r_{T-1} M_t')$$

$$(1.23) \quad f_{t,t+1} - r_t [\Phi s_t - c'(Q_t^s/\Phi)] \leq 0, Q_t^s \geq 0, Q_t^s \{f_{t,t+1} - r_t [\Phi s_t - c'(Q_t^s/\Phi)]\} = 0$$

$$(1.24) \quad f_{t,t+1}^s - r_t [s_t - i^s(I_t^s + S_t - Q_t^s)] = \eta_t^s / (r_{t+1} \dots r_{T-1} M_t')$$

Expression (1.23) allows us to solve for the optimal level of material input use ( $Q_t^{s*}$ ) independently from hedging, purchases, sales, and beginning inventories. A careful look at (1.22) and (1.20) reveals that optimal output sales ( $P_t^*$ ) are independent not only from the amounts hedged but also from use, purchases, and beginning stocks of material input. Output sales can take any value that does not exceed beginning stocks of final good. If sales equal beginning stocks, then  $P_t^* = I_t$ ; if sales are strictly less than beginning stocks then  $\eta_t = 0$ , and the precise level of sales is obtained from (1.22). Similarly, expressions (1.24) and (1.21) allow us to solve for the optimal level of material input purchases ( $S_t^*$ ) independently from sales and beginning stocks of final good.

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<sup>4</sup>We will assume for the remainder of the analysis that the solution to (1.6) exists. The conditions for existence are given in Dimitri Bertsekas (1976, p. 375).

In summary, the existence of forward markets for final good and material input leads to separation of purchases/processing/sales and speculative decisions for the forward-looking risk-averse firm. Moreover, optimal purchases, processing, and sales are independent of the agent's degree of risk aversion and the distributions of random cash prices. Sales of final good are obviously independent of the level of risk aversion and the random prices so long as sales equal beginning inventories (i.e.,  $P_t^* = I_t$ ). Alternatively, if sales of final good are less than beginning inventories (i.e.,  $P_t^* < I_t$ ), then the terms in which the risk attitude and the random prices appear collapse to zero, and again sales are independent of these variables. A similar analysis can be applied to show that purchases of material input are also independent of the decision maker's degree of risk aversion and the distribution of cash prices.

For interior solutions, the comparative statics corresponding to output sales, and purchases and use of material input can be obtained by setting the right-hand side terms in (1.22) through (1.24) equal to zero and totally differentiating the resulting expressions. This derivation is straightforward after recalling the properties imposed on the nonmaterial and storage cost functions ( $c(\cdot)$ ,  $i(\cdot)$ , and  $i^s(\cdot)$ , respectively). Comparative statics are summarized in Table 1.

The theoretical results reported in Table 1 indicate that the optimal use of material input should be negatively related to its current cash price and positively related to the forward price of final good. Beginning stocks of final good may or may not affect material input use, depending on the particular form of the nonmaterial variable cost function. Under some conditions, it can be shown that material input use adjusts negatively to increases in beginning stocks of final good.<sup>5</sup> The impact of the interest rate on material input use is negative if use is independent of output beginning stocks, and it is ambiguous otherwise. Purchases of material input respond in the same fashion as material input use but are also positively related to the current forward price of material input and negatively related to the beginning stock of material input. Sales of final good are independent of cash and forward prices of material input as well as beginning inventories of

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<sup>5</sup>This response to beginning inventories of final good is obtained by letting  $c = c(Q_t, I_t)$ ,  $c_1 > 0$ ,  $c_2 > 0$ ,  $c_{11} > 0$ ,  $c_{22} > 0$ ,  $c_{12} = 0$ .

material input. Output sales are positively related to current output cash price and beginning inventories of final good, and they are negatively associated to current output forward price. The interest rate has a positive effect on sales if input use does not depend on output beginning stocks, and an ambiguous effect otherwise.

The existence and direction of the causal relationships summarized in Table 1 are very different from those predicted by the standard myopic model. Processing, purchases, and sales are either identical or bear fixed relationships in the myopic model. It is interesting therefore to investigate whether the hypothesized relationships of Table 1 are supported by an appropriate data set. This investigation is the purpose of the remainder of the paper.

## II. Empirical Results and Discussion

The U.S. soybean-processing industry was chosen to test the theoretical propositions because there are highly liquid futures markets for both material input (soybeans) and final goods (soybean oil and meal) in the Chicago Board of Trade (CBOT). In addition, there are available high-quality data at a monthly frequency, which is the observation horizon employed in the empirical application.<sup>6</sup>

Before turning to the description of the methodology, data, and estimation procedures, it is worthwhile to summarize the empirical results from the econometric model in terms comparable to Table 1. This inversion of the standard presentation procedure allows a more direct linkage of theory and practice and is justified in part by the necessary complexity of the application of the model. Table 2 is entirely analogous to Table 1, but it contains the estimated partial elasticities corresponding to the U.S. soybean-processing industry.<sup>7</sup> A comparison of Tables 1 and 2 reveals that use and purchases of material input, as well as final good sales generally follow the

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<sup>6</sup>Robert Merton (1982, p. 656) defines observation horizon as "the length of time between successive observations of the data by the researcher."

<sup>7</sup>Soybean processors produce meal and oil in fixed proportions, and so there are two relevant output prices.

hypothesized pattern. The only exception is that beginning output stocks have a nonsignificant effect on material input purchases. Futures price of soybeans is significantly different from zero at the 5 percent level but not at the 1 percent level; this lack of significance at the 1 percent level, however, is due to multicollinearity (this point is discussed in more detail in the next section).

The most important feature of these results is that decisions regarding input use, input purchases, and output sales can be treated separately and in a predictable way when modeling the short-run behavior in these industries. In results presented later, it is shown that all but one of the relationships left blank in Table 2 are nonsignificant. In the absence of the preceding theoretical analysis, the lack of significance of these missing variables might seem counterintuitive. For example, one might (as the USDA does) use cash prices of oil and meal relative to the cash price of soybeans as a measure of processing profitability (USDA, Economic and Statistics Service, *Fats and Oils--Outlook and Situation*). A priori, any of the dependent variables could be used as a measure of the activity of the firm, and any one or set of the explanatory variables as the incentives to which the firm responds.

To emphasize the differences in relative magnitudes among input use, input purchases, and output sales, the short-term total elasticities of these variables with respect to prices and beginning inventories are summarized in Table 3. Table 3 differs from Table 2 in that it includes the indirect effect of prices and beginning inventories through the impact of input use (production) on input purchases (output sales).<sup>8</sup> The magnitudes of the total elasticities are directly comparable across the dependent variables and show, for example, that the soybean cash price causes a much greater change in soybean purchases than in soybean use. These elasticities indicate that the soybean cash price is the single most important factor affecting processors' behavior. Table 3 also indicates that in the short term processors adjust to changes in cash and futures prices mainly through their soybean purchases (a result consistent with the relative volatility of crushings, purchases, and sales depicted in Figure 2).

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<sup>8</sup>The rationale for having crushings (production) as an explanatory variable in the regression for purchases (sales) is discussed in the next section.

The monthly total elasticities of purchases and sales with respect to own cash prices are larger in absolute value than the analogous elasticities with respect to futures. For soybean purchases, this result happens because soybean cash prices affect the profitability of both storage and crushings, whereas soybean futures influence only the returns to soybean storage. The explanation for oil and meal sales is that futures have not only a direct impact on sales but also an opposite indirect effect through production. The indirect effect partially offsets the direct impact for oil (so that total response to own futures price is negative), but the former outweighs the latter for meal (leading to a positive total effect).

The largest (in absolute value) monthly total elasticity with respect to beginning inventories is that of soybean purchases with respect to soybean beginning stocks, which equals -0.33. The other elasticities with respect to beginning stocks are very small in absolute value, ranging from -0.05 to 0.014.

Long-term equilibrium elasticities are reported in Table 4.<sup>9</sup> According to these elasticities, the major long-term adjustment mechanism for processors are stocks rather than crushings (which in long-term equilibrium are identical to purchases and (weighted) sales). This difference in adjustment patterns is even larger when one examines long-term responses to futures prices. The long-term elasticity of crushings with respect to futures is small (0.16 for oil futures and 0.42 for meal futures), whereas the long-term elasticity of crushings with respect to cash prices ranges between 0.23 (oil and meal cash prices) and -1.03 (soybean cash prices). This finding may help explain why econometric models that use observation horizons longer than one month include cash prices but not futures in the set of variables explaining amounts processed. The main long-term impact of futures is on stocks, which are the endogenous variables with the worst fit in most econometric models.

The results presented above indicate that cash prices are important to explain crushings, not because firms ignore futures markets (as is implicitly or explicitly assumed in the standard

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<sup>9</sup>Note that we talk of long *term* and not of long *run*, because throughout the analysis crushing capacity is considered an exogenous variable.



literature) but because in the long term futures markets mainly influence inventory levels. Models that use cash prices when futures quotes are available may be correct in a reduced-form sense, but these models will inevitably do a poor job of explaining inventory levels. If one assumes that firms ignore futures prices in output decisions, then it is difficult to motivate the use of futures prices in inventory decisions.

### **III. Estimation and Derivation of the Empirical Results**

Ideally, the empirical estimation of the theoretical model should be conducted with data corresponding to a single firm, with an observation horizon coincidental with the planning horizon, and using that firm's forward prices. Existing data, however, precludes us from using such an ideal data set because series on physical decisions are available only in aggregate and because forward prices are not available.

Employing futures instead of forward prices in this application can be supported, both theoretically and empirically. The main theoretical results concerning the hypotheses to be tested hold (under some restrictions) in the presence of futures markets (see Section I). Futures are widely used by soybean processors for hedging purposes. Also, given the aggregate nature of data on physical quantities, futures prices are preferable to forward prices, as forward prices would be plant-specific. Furthermore, our sense is that for the U.S. soybean processing industry there is no substantial difference between futures and forward contracts in terms of their relative impact on the cash flow of the firms. First, only margins are required to operate in futures markets. Second, margins are usually held in the form of Treasury bills and/or other interest-bearing securities. Third, margins are relatively small for soybean processors because these firms are classified as hedgers. Finally, margin calls are made on net positions only.

The behavioral hypotheses derived in Section I are applicable in the context of the firm's decision horizon. For soybean processors, the decision horizon may be roughly estimated as one week (Dah-Nein Tzang and Raymond Leuthold 1990). The observation horizon employed in the

empirical analysis, however, is one month because data on receipts, crushings, and shipments are not available covering periods shorter than one month. Data aggregated over periods longer than one month are purposely avoided because the dynamics of the firm's decisions becomes more difficult to analyze as the observation horizon lengthens. Averages of cash and futures prices tend to converge to each other as the observation horizon lengthens, and the same is true of purchases, crushings, and (weighted) sales. The underlying hypothesis is that this convergence hides much useful information on firm behavior (see Figures 1 and 2).

When the observation horizon is not the same as the decision horizon (i.e., the case of most empirical studies), there are problems that have to date been largely ignored in the literature. In this particular application, for example, production ( $Q_t$ ) and material input use ( $Q_t^s$ ) must be included as explanatory variables in the regressions for output sales ( $P_t$ ) and material input purchases ( $S_t$ ), respectively, because the observation horizon exceeds the decision horizon.<sup>10</sup> But because production and material input use are endogenous variables, ordinary least squares yield inconsistent estimates of the structural parameters in the regressions for  $S_t$  and  $P_t$  (William Greene 1990, p. 592). Consequently, the behavioral equations must be estimated by means of a simultaneous-equations model.

In this application, monthly rather than quarterly or annual data are employed because they provide a closer match between decision and observation horizons. But estimating high-frequency

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<sup>10</sup>For simplicity, assume that optimal production and sales levels at date  $t$  are  $Q_t^*$  and zero, respectively. Let all exogenous variables stay unchanged for the remaining  $\tau$  decision dates comprised in the observation horizon  $O$ . It follows from expression (1.23) that optimal production for all decision dates  $t+1$  through  $t+\tau$  will remain unchanged, so that production over the observation horizon will be  $Q_O = (\tau+1) Q_t^*$ . According to expression (1.22), optimal sales at decision dates  $t+1$  through  $t+\tau$  will be identical to the changes in beginning stocks, which are equal to optimal production at the previous decision date (i.e.,  $P_t^* = 0$ , and  $P_{t+1}^* = P_{t+2}^* = \dots = P_{t+\tau}^* = Q_t^*$ ). Hence, sales over the observation horizon are  $P_O = \tau Q_t^* = \tau/(\tau+1) Q_O$ , implying that observed sales asymptotically approach observed production as the observation horizon lengthens with respect to the decision horizon (i.e.,  $P_O \rightarrow Q_O$  as  $\tau \rightarrow \infty$ ). Because of this result, the sales regression must include production as an explanatory variable if  $\tau > 0$ , even though production is also an endogenous variable. Note also that the effect is from production on sales and not the other way around, so that sales ought not be included as an explanatory variable in the production regression.

The same reasoning can be applied to motivate the inclusion of processing as an explanatory variable in the regression for material input purchases.

(e.g., monthly) models introduces dynamic complexities because such models are more likely to present nonstationary variables, autocorrelated errors and/or dependent variables, heteroscedasticity, and autoregressive conditional heteroscedasticity. For this reason, tests to detect nonstationarity are performed before proceeding to the estimation of the model. The possibility of autocorrelated errors and/or dependent variables is explicitly dealt with by allowing autocorrelation structure in the errors and by allowing lags of the dependent variable to enter as additional explanatory variables. Heteroscedasticity is explicitly dealt with in a similar manner.

Prices always appear as margins in expressions (1.22) through (1.24):  $(f_{t,t+1} - r_t p_t)$ ,  $(f_{t,t+1}/\Phi - r_t s_t)$ , and  $(f_{t,t+1}^s - r_t s_t)$ . In the empirical estimation, these restrictions on prices are directly imposed to avoid multicollinearity, but using price ratios instead of price differences:  $f_{t,t+1}/(r_t p_t)$ ,  $(f_{t,t+1}/\Phi)/(r_t s_t)$ , and  $f_{t,t+1}^s/(r_t s_t)$ . Ratios are used for three main reasons. First, they are easy to interpret: the ratios are simply discounted end-of-period rates of return per unit of input. The ratios are positive and around unity, with large (small) values suggesting profits (losses). Second, the ratio specification eliminates the need for a price index to express the price series in real terms because cash prices are obvious deflators.<sup>11</sup> Third, the problem of not having delivery positions for all months in the futures market is easier to overcome, as discussed below.

The delivery months for soybean oil and meal in the CBOT are January, March, May, July, August, September, October, and December. Hence, in many months  $f_{t,t+k}$  ( $k > 1$ ) must be employed instead of  $f_{t,t+1}$  because  $f_{t,t+1}$  does not exist.<sup>12</sup> But using  $f_{t,t+k}$  implies that the price ratios for months with different  $k$  are not comparable. For example, the ratio  $(f_{t,t+k}/\Phi)/s_t$ , which involves a rate of return over  $k > 1$  months, cannot be compared with the ratio  $(f_{t,t+1}/\Phi)/s_t$ , which involves a one-month rate of return only. This lack of comparability among ratios for different months suggests converting them to the same base; and to facilitate the interpretation of the results, an annual base is chosen. Then, the corresponding annualized end-of-period rates of return are  $[(f_{t,t+k}/\Phi)/s_t]^{12/k}$ , where  $k$  is the number of months between the placement of the hedge and the

<sup>11</sup>For example, soybeans have accounted for more than 90 percent of the cost of producing oil and meal.

<sup>12</sup>Examples of nonexistent  $f_{t,t+1}$  are  $f_{\text{Jan, Feb}}$ ,  $f_{\text{Mar, Apr}}$ ,  $f_{\text{May, Jun}}$ , and  $f_{\text{Oct, Nov}}$ .

delivery month. This procedure is important because in practice the positions most used for hedging not always are the nearest ones. For example, in February most hedges are placed against the May position instead of the March position; therefore, the relevant futures price for our purposes is not  $f_{\text{Feb,Mar}}$  but  $f_{\text{Feb,May}}$ .

Soybean processing involves one material input and not one but two outputs in fixed proportions: oil and meal. Hence, the ratio  $[(f_{t,t+k}/\Phi)/s_t]^{12/k}$  must be modified to make it suitable to analyze the soybean complex. The ratio employed is  $[(f_{t,t+k}^o/\Phi^o + f_{t,t+k}^m/\Phi^m)/s_t]^{12/k}$ , where the superscripts o and m stand for oil and meal, respectively. This expression should be interpreted in the same way as for the single-output case, with the difference that its numerator consists of a composite index of two futures prices of final goods, each one weighted by its corresponding production share.

To account for seasonal factors, such as the difference in length among months and the plant closing for maintenance that takes place toward the end of the commercial year, monthly dummy variables are allowed in the regressions. Finally, the industry crushing capacity ( $CAP_t$ ) is also included as an explanatory variable in the regression for crushings. Crushing capacity is expected to be positively related to crushings because it limits the amount of soybeans that firms are able to process, and it also captures a time trend.

From the preceding discussion, the basic equations to be estimated for the soybean complex are

$$(3.1) \quad \ln(Q_t^s) = \beta_{0Q} + \beta_{1Q} \ln(\text{RETURN}_t^c) + \beta_{2Q} \ln(I_t^o) + \beta_{3Q} \ln(I_t^m) + \beta_{4Q} \ln(CAP_t) \\ + \sum_{i=1}^{n_Q} \lambda_{iQ} \ln(Q_{t-i}^s) + \sum_{i=2}^{12} \delta_{iQ} \text{MONTH}_{it} + u_{Qt} \quad \text{Crushings}$$

$$(3.2) \quad \ln(S_t) = \beta_{0S} + \beta_{1S} \ln(\text{RETURN}_t^s) + \beta_{2S} \ln(\text{RETURN}_t^c) + \beta_{3S} \ln(I_t^s) + \beta_{4S} \ln(I_t^o) \\ + \beta_{5S} \ln(I_t^m) + \beta_{6S} \ln(Q_t^s) + \sum_{i=1}^{n_S} \lambda_{iS} \ln(S_{t-i}) + \sum_{i=2}^{12} \delta_{iS} \text{MONTH}_{it} + u_{St} \quad \text{Purchases}$$

$$(3.3) \quad \ln(P_t^o) = \beta_{0o} + \beta_{1o} \ln(\text{RETURN}_t^o) + \beta_{2o} \ln(I_t^o) + \beta_{3o} \ln(Q_t^o)$$

$$+ \sum_{i=1}^{n_o} \lambda_{io} \ln(P_{t-i}^o) + \sum_{i=2}^{12} \delta_{io} \text{MONTH}_{it} + u_{ot} \quad \text{Oil Sales}$$

$$(3.4) \quad \ln(P_t^m) = \beta_{0m} + \beta_{1m} \ln(\text{RETURN}_t^m) + \beta_{2m} \ln(I_t^m) + \beta_{3m} \ln(Q_t^m)$$

$$+ \sum_{i=1}^{n_m} \lambda_{im} \ln(P_{t-i}^m) + \sum_{i=2}^{12} \delta_{im} \text{MONTH}_{it} + u_{mt} \quad \text{Meal Sales}$$

where  $\ln(\cdot)$  is the natural logarithm operator,  $u$  denotes error terms, and  $\beta$ ,  $\lambda$ , and  $\delta$  are constants. The variables  $\text{RETURN}_t^c$ ,  $\text{RETURN}_t^s$ ,  $\text{RETURN}_t^o$  and  $\text{RETURN}_t^m$  are rates of return per unit of input corresponding to crushings, soybeans, oil, and meal, respectively.<sup>13</sup> The coefficients associated with the RETURN variables are expected to be significantly greater than zero in the soybean crushings and purchases equations, and significantly less than zero in the oil and meal sales equations. The variable  $\text{MONTH}_{it}$  is a dummy variable that equals 1 in month  $i$ , and equals zero otherwise.<sup>14</sup> As stated earlier, the  $u$  errors are allowed to be autocorrelated and heteroscedastic:<sup>15</sup>

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<sup>13</sup>The expressions for the RETURN variables are

$$\text{RETURN}_t^c = 1/r_t [(f_{t,t+k}^o/\Phi^o + f_{t,t+k}^m/\Phi^m)/s_t]^{12/k}$$

$$\text{RETURN}_t^s = 1/r_t (f_{t,t+h}^s/s_t)^{12/h}$$

$$\text{RETURN}_t^o = 1/r_t (f_{t,t+k}^o/p_t^o)^{12/k}$$

$$\text{RETURN}_t^m = 1/r_t (f_{t,t+k}^m/p_t^m)^{12/k}$$

where  $k = 2$  if  $t = \text{January, March, May, June, October, and November}$ ;  $k = 3$  if  $t = \text{February, April, September, and December}$ ;  $k = 4$  if  $t = \text{August}$ ; and  $k = 5$  if  $t = \text{July}$ .  $h = 2$  if  $t = \text{January, March, May, June, September, and November}$ ;  $h = 3$  if  $t = \text{February, April, August, and October}$ ; and  $h = 4$  if  $t = \text{July}$ . Data on open interest reveal that, on average, these are the most used combinations of hedge-placement/delivery months.

<sup>14</sup>For example,  $\text{MONTH}_{2t}$  equals 1 for February observations, and equals 0 for non-February observations.

<sup>15</sup>An exponential function is employed to model heteroscedasticity because it precludes negative variances.

$$(3.5) \quad (1 - \rho_{1j} B - \rho_{2j} B^2 - \dots) (1 - \rho_{12j} B^{12}) u_{jt} = v_{jt} \exp(\alpha_j' z_{jt}), j = Q, S, o, m$$

where  $\rho$  denotes constants,  $B$  is the lag operator (i.e.,  $B u_{jt} \equiv u_{jt-1}$ ),  $\exp(\cdot)$  is the exponential operator,  $\alpha_j'$  is a vector of constants, and  $z_{jt}$  is a vector of variables explaining heteroscedasticity. The error vector  $v_t \equiv (v_{Qt}, v_{St}, v_{ot}, v_{mt})'$  is assumed to be independently multinormally distributed with zero mean vector and covariance matrix  $\Sigma$ .

The data cover September 1965 through December 1986. The period analyzed ends in 1986 because in the mid-1980s the processing sector suffered a profound concentration, raising doubts regarding its competitive performance (see Jean-Pierre Bertrand 1988, and Bruce Marion and Donghwan Kim 1991). All prices and quantities for the soybean complex are expressed in dollars per short ton and millions of short tons, respectively. Cash prices are quotations FOB Decatur published by the USDA, and data on crushings, receipts, and shipments are those reported by the U.S. Bureau of the Census.<sup>16</sup> Data sources for crushing capacity are USDA's *Fats and Oils--Outlook and Situation* for 1965 through 1980, and the American Soybean Association's *Soya Blue Book* for 1981 through 1986. These sources only report crushing capacity at the beginning of October, hence capacity for the remaining months is approximated by linear interpolation. Interest rate is the prime rate reported by the USDC's *Survey of Current Business*. Finally, futures prices are the monthly averages of daily closing futures prices for the selected delivery positions from the *Statistical Annual* of the CBOT.<sup>17</sup>

The results of the Phillips-Perron tests for unit roots are presented in Table 5. These tests lead to rejection of the null hypothesis of a unit root in all of the series of dependent variables and in six out of the eight series of explanatory variables at the 1 percent level of significance.<sup>18</sup> All

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<sup>16</sup>Available data correspond to receipts and shipments instead of actual purchases and sales, so that it is assumed that receipts and shipments are identical to purchases and sales, respectively.

<sup>17</sup>All data are available from the authors upon request.

<sup>18</sup>For completeness, the results of the Phillips-Perron tests are reported for all series employed. But the presence of nonstationarity is relevant only in the series of dependent variables, as the only plausible theoretical result of regressing a stationary variable on a nonstationary variable is that the coefficient associated with the latter equals zero (C. Granger 1986, p. 216).

four explanatory variables and the logarithms of oil and meal production are stationary with a deterministic time trend, whereas the logarithms of RETURNS and of soybean and meal beginning stocks are stationary without a trend. The Phillips-Perron tests cannot reject the null hypothesis of a unit root for the logarithms of oil beginning stocks and of crushing capacity. Because of the low power of the Phillips-Perron tests, however, nonrejection of the null hypothesis does not provide strong evidence that a series has a unit root. This assertion is supported by the results of the test recently developed by Denis Kwiatkowski et al. (1992), in which the null hypothesis is stationarity. The values of this test are 0.037 and 0.061 (0.136 and 0.215) for the logarithms of oil beginning stocks and crushing capacity with (without) a deterministic trend; the critical value at the 5 percent level of significance equals 0.146 (0.463) for the series with (without) a deterministic time trend.

The first step in estimating the model is to calculate the fixed input-output coefficients  $\Phi^o$  and  $\Phi^m$ . The monthly data on crushings and oil and meal production yield an average of 5.537 for  $\Phi^o$  and of 1.263 for  $\Phi^m$ . The coefficients of variation of the sample estimates of  $\Phi^o$  and  $\Phi^m$  are only 2.35 percent and 0.85 percent, respectively, lending strong support to the assumption that soybean processing is characterized by a Leontief production function.

Given the results of the nonstationarity tests, and using  $\Phi^o = 5.537$  and  $\Phi^m = 1.263$ , the behavioral regressions (3.1) through (3.4) (and the corresponding error structure (3.5)) are estimated as a simultaneous-equations model, subject to the accounting identities

$$(3.6) \quad I_t^s = I_{t-1}^s + S_{t-1} - Q_{t-1}^s \quad \text{Soybean Inventories}$$

$$(3.7) \quad I_t^o = I_{t-1}^o - P_{t-1}^o + Q_{t-1}^o \quad \text{Oil Inventories}$$

$$(3.8) \quad I_t^m = I_{t-1}^m - P_{t-1}^m + Q_{t-1}^m \quad \text{Meal Inventories}$$

and the fixed input-output restrictions

$$(3.9) \quad Q_t^o = Q_t^s / \Phi^o$$

Oil Production

$$(3.10) \quad Q_t^m = Q_t^s / \Phi^m$$

Meal Production

The estimation procedure employed is full information maximum likelihood.<sup>19</sup> The estimation results of the selected model are contained in Tables 6 through 8. The basic criteria employed for model selection are the properties of the  $v$  errors and the principle of parsimony.

Tests on the selected model's  $v$  errors are presented first (see Table 6) because the properties of these errors are key for model selection. Table 6 summarizes the results of the Lagrange multiplier test for normality, the Ljung-Box portmanteau test for autocorrelation, the Lagrange multiplier test for autoregressive conditional heteroscedasticity, and the White and Breusch-Pagan tests for heteroscedasticity. According to this collection of tests, there is insufficient evidence to reject the null hypotheses of normality, serial independence, and homoscedasticity in the selected model's  $v$  errors.

The estimated structural equations (i.e., the estimates of (3.1) through (3.4)) are reported in Table 7. The system's goodness-of-fit statistic is  $R_p^2 = 0.997$ , and the  $R^2$ s for the individual equations range from 0.946 to 0.991.<sup>20</sup> In the selected model, the coefficients associated with the monthly dummy variables in the oil and meal sales equations are constrained to equal zero. This restriction is imposed for parsimony and reduces the number of estimated parameters from 77 to 55. In the unrestricted model, none of the 22 omitted dummy variables is individually significant

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<sup>19</sup>Maximum likelihood is chosen over instrumental variables and feasible generalized least squares because instrumental variable estimators are biased in finite samples and have variances that are not easy to establish (John Johnston 1984, p. 363 and 365), and feasible generalized least squares preclude the usage of likelihood ratio tests and Lagrange multiplier tests (William Greene 1990, p. 392).

<sup>20</sup>The statistic  $R_p^2$  is the *pseudo*  $R^2$  introduced by Nevins Baxter and John Cragg (1970). This statistic is defined as  $R_p^2 = 1 - \exp[2(L_\omega - L_\Omega)/N]$ , where  $L_\omega$  is the maximum of the log likelihood function when only intercepts are used,  $L_\Omega$  is the maximum of the log likelihood function when all coefficients are included in the model, and  $N$  is the number of observations. The  $R^2$  for each individual equation is the squared correlation between predicted and actual dependent variables (G. S. Maddala 1988, p. 307).



at the 1 percent level and only one is significant at the 5 percent level. In addition, the likelihood ratio test indicates that the null hypothesis that all 22 monthly coefficients in the sales equations equal zero cannot be rejected. For similar reasons, the equations for crushings, purchases, and oil sales contain only a single lag in the dependent variable, and the meal sales equation contains no lags. Alternative specifications (e.g., with monthly dummies in all equations, without monthly dummies in any equation, and with different lags) yield no major changes in the signs or magnitudes of the structural parameters.

Production is a highly significant explanatory variable of sales, and crushings is a highly significant explanatory variable of purchases. The estimated elasticities of sales (purchases) with respect to production (crushings) are consistent with the discrepancy between decision and observation horizons (the meal sales elasticity, however, seems relatively large).<sup>21</sup>

Crushing capacity is a highly significant explanatory variable of the amount of soybeans processed. The short-run elasticity of crushings with respect to capacity is 0.245. Although this value is apparently small, it yields a long-term equilibrium elasticity of crushings with respect to capacity equal to 0.87.<sup>22</sup>

As hypothesized, beginning inventories of final goods have a significantly positive impact on their respective sales, and beginning stocks of soybeans have a significantly negative effect on material input purchases. In general, however, the corresponding total monthly elasticities are small in absolute value: 0.03 for meal sales, 0.14 for oil sales, and -0.33 for soybean purchases (see Table 3).<sup>23</sup> The empirical findings indicate that the elasticity of crushings with respect to beginning inventories of oil and meal is significantly negative (albeit very small in absolute value), ruling out separation between production and output storage in the U.S. soybean-processing industry.

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<sup>21</sup>With a decision horizon of one week and an observation horizon of one month,  $\tau \equiv 3$  and therefore  $\tau/(\tau+1) \equiv 0.75$  (see footnote 11).

<sup>22</sup>The methodology to obtain long-term elasticities from the structural parameters reported in Table 7 is explained in Appendix B.

<sup>23</sup>Note that the total monthly elasticities of oil and meal sales with respect to own beginning stocks include the indirect effect of beginning stocks on oil and meal production.

One of the most important empirical results regarding the theoretical model is that the RETURN variables have the expected effects and are significant. The likelihood ratio test for the null hypothesis that all five RETURN coefficients equal zero is 78.06, which strongly indicates rejection of the null hypothesis (the critical value at the 1 percent level of significance is  $\chi^2_{5;0.01} = 15.09$ ). The  $t$  statistics of the RETURN coefficients in the crushings and purchases equations indicate that, individually, the coefficients are significantly different from zero at the 5 percent level but not at the 1 percent level. This result, however, is misleading; the likelihood ratio test for the null hypothesis that these three coefficients are all zero equals 43.92, which is considerably larger than the 1 percent critical value of  $\chi^2_{3;0.01} = 11.34$ . In addition, the likelihood ratio test for the null hypothesis that the  $\text{RETURN}_t^c$  coefficient equals zero in the crushings (purchases) equation is 7.92 (10.44), which is significantly different from zero at the 1 percent level (the 1 percent critical value is  $\chi^2_{1;0.01} = 6.64$ ). In the case of the purchases equation, the problem is attributable to multicollinearity caused by high correlation between  $\text{RETURN}_t^s$  and  $\text{RETURN}_t^c$ . Evidence of this assertion is that deleting  $\text{RETURN}_t^s$  from the purchases equation yields a  $\text{RETURN}_t^c$  coefficient equal to 0.429 with a likelihood ratio test of 35.58; deleting instead  $\text{RETURN}_t^c$  from the same equation, the coefficient for  $\text{RETURN}_t^s$  becomes 0.411 with a likelihood ratio test of 31.58.

The variables labeled as *ad hoc* in Table 7 are those that could be included in an *ad hoc* model of the industry, but which are not predicted by the theoretical model. An unrestricted model including all these variables reveals that, individually, only one of the associated 16 coefficients is significantly different from zero at the 5 percent level. Furthermore, the likelihood ratio test for the null hypothesis that the 15 nonsignificant coefficients are all equal to zero is 15.74, yielding no evidence to reject the null hypothesis (the critical value at the 5 percent level equals  $\chi^2_{15;0.05} = 25.00$ ). The only significant *ad hoc* variable is  $\text{RETURN}_t^s$  in the meal sales equation; a possible rationale for this significance is that meal and soybeans compete for storage space. Interestingly, if one begins by searching the data for variables that are significant, one would with three exceptions (oil and meal stocks do not significantly influence soybean purchases, and  $\text{RETURN}_t^s$  significantly affects meal sales) arrive at the model structure predicted by the theory.

Table 8 shows the model's  $u$  errors structure (i.e., expression (3.5)). The selected structure is the result of preliminary analysis based on the properties of the  $v$  errors and parsimony. On this basis, most autocorrelation coefficients in the selected model are set to zero. Similarly, the heteroscedasticity structure is relatively simple. Again, it is reassuring that the magnitudes of the structural coefficients are largely unaffected when the autocorrelation coefficients are not constrained and when the heteroscedasticity coefficients are restricted to equal zero.

#### IV. Conclusions

The paper provides theoretical evidence that in the short run purchases and processing of material input and production and sales of final goods by typical manufacturing firms should, and do, respond to explanatory variables in different ways than is commonly accepted. These results are achieved by introducing the realistic assumption that firms make production, purchasing, and selling decisions in order to take advantage of cash and forward price differentials, while hedging the inherent risk in forward markets. Although the focus of the study is on short-run behavior, the results show that inferences made about long-term behavior derived by aggregating over short-term decisions are different in some respects from the inferences one would draw from medium- or long-run models.

For the particular industry analyzed, U.S. soybean processing, it is found that the long-term equilibrium elasticity of crushings with respect to price is close to unity. This estimate is distinctly larger than the standard elasticity estimates obtained in studies which use aggregate annual data. It is also found that U.S. soybean processors exhibit a very elastic long-term equilibrium response of stocks to prices, and in particular to futures prices. This finding is also in sharp contrast with the findings of models based on aggregate annual data. These results may help explain why the stock regressions in annual models generally exhibit poor fit: annual models largely ignore the role of futures (or forward) prices on processors' behavior.

The model presented is capable of identifying the individual effect of each cash and forward price on purchases, processing, and sales. The cash price of material input (soybeans) appears to be the single most important price affecting processors' decisions. Cash prices of output (i.e., oil and meal cash prices) do not affect material input use (i.e., soybean crushings) in the short term, but they do affect use in long-term equilibrium. Output futures prices, in contrast, affect crushings both in the short term and in long-term equilibrium.

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### Appendix A. Derivation of Expressions (1.15), (1.16), and (1.17)

The FOCs corresponding to dates  $t = 0, \dots, T-1$  are (A1) through (A3) plus (1.18) through (1.21).

$$(A1) \quad \frac{\partial \mathcal{L}_t}{\partial P_t} = E_t \left[ \frac{\partial M_{t+1}}{\partial I_{t+1}} \frac{\partial I_{t+1}}{\partial P_t} \right] + r_t \dots r_{T-1} (p_t + i') M_t' - \eta_t = 0$$

$$(A2) \quad \frac{\partial \mathcal{L}_t}{\partial Q_t^s} = E_t \left[ \frac{\partial M_{t+1}}{\partial I_{t+1}^s} \frac{\partial I_{t+1}}{\partial Q_t} \frac{\partial Q_t}{\partial Q_t^s} + \frac{\partial M_{t+1}}{\partial I_{t+1}^s} \frac{\partial I_{t+1}^s}{\partial Q_t^s} \right] - r_t \dots r_{T-1} (c'/\Phi - i^s) M_t' - \eta_t^s \leq 0,$$

$$Q_t^s \geq 0, Q_t^s \frac{\partial \mathcal{L}_t}{\partial Q_t^s} = 0$$

$$(A3) \quad \frac{\partial \mathcal{L}_t}{\partial S_t} = E_t \left[ \frac{\partial M_{t+1}}{\partial I_{t+1}^s} \frac{\partial I_{t+1}^s}{\partial S_t} \right] - r_t \dots r_{T-1} (s_t + i^s) M_t' + \eta_t^s = 0$$

But note that

$$(A4) \quad \frac{\partial I_{t+1}}{\partial P_t} = -1$$

$$(A5) \quad \frac{\partial I_{t+1}}{\partial I_t} = 1$$

$$(A6) \quad \frac{\partial I_{t+1}}{\partial Q_t} = 1$$

$$(A7) \quad \frac{\partial I_{t+1}^s}{\partial S_t} = 1$$

$$(A8) \quad \frac{\partial I_{t+1}^s}{\partial I_t^s} = 1$$

$$(A9) \quad \frac{\partial I_{t+1}^s}{\partial Q_t^s} = -1$$

$$(A10) \quad \frac{\partial Q_t}{\partial Q_t^s} = 1/\Phi$$

Also,

$$(A11) \quad \frac{\partial M_t}{\partial I_t} = E_t \left[ \frac{\partial M_{t+1}}{\partial I_{t+1}} \frac{\partial I_{t+1}}{\partial I_t} \right] - r_t \dots r_{T-1} i' M_t' + \eta_t$$

$$(A11') \quad = r_t \dots r_{T-1} p_t M_t'$$

$$(A12) \quad \frac{\partial M_t}{\partial I_t^s} = E_t \left[ \frac{\partial M_{t+1}}{\partial I_{t+1}^s} \frac{\partial I_{t+1}^s}{\partial I_t^s} \right] - r_t \dots r_{T-1} i^s M_t' + \eta_t^s$$

$$(A12') \quad = r_t \dots r_{T-1} s_t M_t'$$

where expressions (A11') and (A12') are obtained by using expressions (A1), (A3), (A4), (A5), (A7), and (A8). It follows from (A11') and (A12') that

$$(A13) \quad \frac{\partial M_{t+1}}{\partial I_{t+1}} = r_{t+1} \dots r_{T-1} p_{t+1} M_{t+1}'$$

$$(A14) \quad \frac{\partial M_{t+1}}{\partial I_{t+1}^s} = r_{t+1} \dots r_{T-1} s_{t+1} M_{t+1}'$$

By substituting (A4) through (A10) and (A13) through (A14) into FOCs (A1) through (A3), and by rearranging a little, we get expressions (1.15) through (1.17).

### Appendix B. Derivation of Long-Term Equilibrium Elasticities

In long-term equilibrium, beginning stocks must remain unchanged from date to date.

Hence,

$$I_t^S = I_{t-1}^S = I^S \quad \Rightarrow \quad (B1) \quad S = Q^S$$

$$I_t^O = I_{t-1}^O = I^O \quad \Rightarrow \quad (B2) \quad P^O = Q^O = Q^S/\Phi^O$$

$$I_t^M = I_{t-1}^M = I^M \quad \Rightarrow \quad (B3) \quad P^M = Q^M = Q^S/\Phi^M$$

where time subscripts are dropped when referring to long-term equilibrium values.

From the regression for oil sales we have

$$(B4) \quad \ln(P^O) = \alpha_1 - 0.088 \ln(\text{RETURN}^O) + 0.163 \ln(I^O) + 0.742 \ln(Q^O) - 0.132 \ln(P^O)$$

where  $\alpha_i$  ( $i = 1, 2, \dots, 6$ ) represents terms in the regression that are irrelevant for our purposes.

Substituting (B2) into (B4) and solving for  $\ln(I^O)$  in terms of  $\ln(Q^S)$  yields

$$(B5) \quad \ln(I^O) = \alpha_2 + 0.541 \ln(\text{RETURN}^O) + 2.388 \ln(Q^S)$$

By performing analogous operations for meal sales and soybean purchases we get

$$(B6) \quad \ln(I^M) = \alpha_3 + 0.343 \ln(\text{RETURN}^M) + 0.869 \ln(Q^S)$$

$$(B7) \quad \ln(I^S) = \alpha_4 + 0.860 \ln(\text{RETURN}^C) + 0.682 \ln(\text{RETURN}^S) + 0.288 \ln(Q^S)$$

$$+ 0.087 \ln(I^O) - 0.063 \ln(I^M)$$

Finally, by replacing (B5) and (B6) into the regression for crushings (B8)

$$(B8) \quad \ln(Q^s) = \alpha_5 + 0.061 \ln(\text{RETURN}^c) - 0.0246 \ln(I^o) - 0.039 \ln(I^m) + 0.812 \ln(Q^s)$$

and solving the resulting expression for  $\ln(Q^s)$  we obtain

$$(B9) \quad \ln(Q^s) = \alpha_6 + 0.216 \ln(\text{RETURN}^c) - 0.047 \ln(\text{RETURN}^o) - 0.048 \ln(\text{RETURN}^m)$$

Expression (B9) is the basic equation to calculate the long-term equilibrium elasticities for crushings. Substitution of (B9) in (B5) through (B7) allows us to obtain the long-term elasticities for inventories. The average values used in the calculations were  $12/k = 4.787$ ,  $12/h = 4.918$ ,  $(f_{t,t+k}^o/\Phi^o)/(f_{t,t+k}^o/\Phi^o + f_{t,t+k}^m/\Phi^m) = 0.372$ , and  $(f_{t,t+k}^m/\Phi^m)/(f_{t,t+k}^o/\Phi^o + f_{t,t+k}^m/\Phi^m) = 0.628$ .

Table 1. Theoretical Effect of Exogenous Variables on Material Input Use (Production), Material Input Purchases, and Output Sales over the Decision Horizon.

Explanatory Variable	Dependent Variable		
	Mat. Input Use	Mat. Input Purchases	Output Sales
Cash prices: input	–	–	0
output	0	0	+
Forward prices: input	0	+	0
output	+	+	–
Interest rate	–/?	–/?	+/?
Beginning stocks: input	0	–	0
output	0/–	0/–	+

Table 2. Empirical Estimates of the Average Partial Elasticities of Monthly Material Input Use (Production), Material Input Purchases, and Output Sales of U.S. Soybean Processors with Respect to Selected Explanatory Variables, 1965:9-1986:12.

Explanatory Variable	Dependent Variable			
	Mat. Input	Mat. Input	Output Sales	
	Use	Purchases	Oil	Meal
Cash prices: soybeans	-0.29**	-2.50**		
oil			0.42**	
meal				0.11**
Futures prices: soybeans		1.12*		
oil	0.11**	0.51**	-0.42**	
meal	0.18**	0.86**		-0.11**
Interest rate	-0.005**	-0.04**	0.008**	0.002**
Beginning stocks: soybeans		-0.334**		
oil	-0.025**	0.029	0.163**	
meal	-0.039**	-0.021		0.069**

Note: The partial elasticities contained in this table were obtained from the coefficients reported in Table 7, by taking an average of  $12/k = 4.787$ ,  $12/h = 4.918$ ,  $r_t = 1.095$ ,  $(f_{t,t+k}^o/\Phi^o)/(f_{t,t+k}^o/\Phi^o + f_{t,t+k}^m/\Phi^m) = 0.372$ , and  $(f_{t,t+k}^m/\Phi^m)/(f_{t,t+k}^o/\Phi^o + f_{t,t+k}^m/\Phi^m) = 0.628$ . For example, the partial elasticity of soybean purchases with respect to oil futures price was calculated as  $(0.372 \times 4.787 \times 0.29) = 0.51$ .

\* (\*\*) Significantly different from zero at the 0.05 (0.01) level on the basis of the likelihood ratio test.

Table 3. Monthly Total Elasticities of Crashings, Purchases, and Sales with Respect to Prices and Beginning Stocks.

Explanatory Variable	Dependent Variable			
	Mat. Input	Mat. Input	Output Sales	
	Use	Purchases	Oil	Meal
Cash prices: soybeans	-0.29	-2.71	-0.22	-0.27
oil			0.42	
meal				0.11
Futures prices: soybeans		1.12		
oil	0.11	0.59	-0.34	0.10
meal	0.18	1.00	0.14	0.06
Beginning stocks: soybeans		-0.33		
oil	-0.02	0.01	0.14	-0.02
meal	-0.04	-0.05	-0.03	0.03

Note: The total elasticities contained in this table were obtained from the coefficients reported in Table 7, by taking an average of  $12/k = 4.787$ ,  $12/h = 4.918$ ,  $(f_{t,t+k}^o/\Phi^o)/(f_{t,t+k}^o/\Phi^o + f_{t,t+k}^m/\Phi^m) = 0.372$ , and  $(f_{t,t+k}^m/\Phi^m)/(f_{t,t+k}^o/\Phi^o + f_{t,t+k}^m/\Phi^m) = 0.628$ . For example, the total elasticity of oil sales with respect to soybean cash price was calculated as  $[(-1) \times 0.061 \times 4.787 \times 0.742] = -0.22$ .

Table 4. Long-Term Equilibrium Elasticities of Crashings and Stocks with Respect to Cash and Futures Prices.

Explanatory Variable	Dependent Variable			
	Crashings (Sales, and Purchases)	Soybean Stocks	Oil Stocks	Meal Stocks
Cash prices: soybeans	-1.03	-7.92	-2.47	-0.90
oil	0.23	-0.13	-2.05	0.20
meal	0.23	0.20	0.54	-1.45
Futures prices: soybeans		3.35		
oil	0.16	1.83	2.97	0.14
meal	0.42	2.67	1.01	2.01

Note: The derivation of the elasticities reported in this table is explained in Appendix B.



Table 5. Phillips-Perron Tests for Unit Roots.<sup>a</sup>

Alternative Hypothesis:	i. $y_t = \hat{\mu} + \hat{\alpha} y_{t-1} + \hat{u}_t$ ii. $y_t = \bar{\mu} + \bar{\beta} t + \bar{\alpha} y_{t-1} + \bar{u}_t$				
Test Statistic	$Z(\hat{\alpha})$	$Z(t_{\hat{\alpha}})$	$Z(t_{\hat{\beta}})$	$Z(\hat{\alpha})$	$Z(t_{\hat{\alpha}})$
Null Hypothesis	$\bar{\alpha} = 1$	$\bar{\alpha} = 1$	$\bar{\beta} = 0$	$\hat{\alpha} = 1$	$\hat{\alpha} = 1$
ln(soybean crushings)	-70.4	-6.38	5.36		
ln(soybean purchases)	-84.2	-8.72	4.46		
ln(sales): oil	-166.8	-9.76	8.60		
meal	-79.9	-6.77	5.76		
ln(RETURN): crushings	-91.7	-7.38	0.17	-91.9	-7.39
soybeans	-91.1	-7.40	-2.04	-90.5	-7.30
oil	-42.9	-4.91	0.45	-42.4	-4.91
meal	-61.8	-5.98	1.78	-53.1	-5.66
ln(beg. stocks): soybeans	-52.6	-6.07	-0.11	-52.9	-6.11
oil	-20.1	-3.22	2.11	-10.2	-2.35
meal	-61.7	-5.71	3.27	-30.6	-4.12
ln(production): oil	-74.8	-6.55	5.62		
meal	-70.1	-6.34	5.32		
ln(crushing capacity)	1.4	0.86	-2.36	-2.3	-3.65
Critical Value: 5%	-21.3	-3.43	2.79	-14.0	-2.88
1%	-28.4	-3.99	3.49	-20.3	-3.46

Note: The test statistics are calculated using the Bartlett window  $w_{st} = 1 - s/(l+1)$  suggested by Peter Phillips and Pierre Perron (1988). Following the analysis of G. William Schwert (1989), the value of the lag truncation parameter  $l$  is set to  $l = \text{integer}[12 (\text{number of observations}/100)^{1/4}] = 15$ .

<sup>a</sup>See Peter Phillips and Pierre Perron (1988).

Table 6. Tests for Normality, Autocorrelation, Autoregressive Conditional Heteroscedasticity (ARCH), and Heteroscedasticity of  $v$  Errors.

Test for	$v$ Error of Equation for	Evaluation Statistic	$p$ Value
Normality	Crushings	LMN = 1.72	0.42
	Purchases	LMN = 2.82	0.24
	Oil Sales	LMN = 1.65	0.44
	Meal Sales	LMN = 3.54	0.17
Autocorrelation	Crushings	$Q'(1) = 1.04$	0.31
		$Q'(3) = 1.89$	0.60
		$Q'(12) = 15.3$	0.22
	Purchases	$Q'(1) = 0.23$	0.63
		$Q'(3) = 1.32$	0.72
		$Q'(12) = 8.48$	0.75
	Oil Sales	$Q'(1) = 0.56$	0.45
		$Q'(3) = 3.43$	0.33
		$Q'(12) = 11.6$	0.48
	Meal Sales	$Q'(1) = 0.07$	0.79
		$Q'(3) = 1.10$	0.78
		$Q'(12) = 13.5$	0.33

Note: LMN is the Lagrange multiplier test for normality (Carlos Jarque and Anil Bera 1980, William Greene 1990, p. 329).  $Q'(i)$  is the Ljung-Box portmanteau test or modified-Q statistic for  $i$ -order autocorrelation (G. M. Ljung and G. E. P. Box 1978). LMA is the Lagrange multiplier test for first order autoregressive conditional heteroscedasticity (Robert Engle 1982).  $W$  is the White test for heteroscedasticity (Halbert White 1980).  $LMH_j$  is the Breusch-Pagan Lagrange multiplier test for heteroscedasticity (T. S. Breusch and A. R. Pagan 1979), obtained by regressing  $v_i^2/(\text{average } v_i^2)$  on the  $j$ th variable of the set comprising the 14 nondummy explanatory variables of the system plus the estimated dependent variable. To save space, only the  $LMH_j$  with the greatest  $p$  value for each  $v$  error (i.e.,  $\max(LMH_j)$ ) is reported.

Table 6. Continued.

Test for	v Error of Equation for	Evaluation Statistic	<i>p</i> Value
ARCH	Crushings	LMA = 0.56	0.45
	Purchases	LMA = 0.55	0.46
	Oil Sales	LMA = 1.95	0.33
	Meal Sales	LMA = 0.01	0.91
Heteroscedasticity	Crushings	W = 117.87	0.17
		max(LMH <sub>j</sub> ) = 1.48	0.22
	Purchases	W = 128.55	0.80
		max(LMH <sub>j</sub> ) = 1.21	0.27
	Oil Sales	W = 13.77	0.84
		max(LMH <sub>j</sub> ) = 2.02	0.16
	Meal Sales	W = 8.68	0.85
		max(LMH <sub>j</sub> ) = 0.83	0.36

Table 7. Estimated System of Structural Equations for U.S. Soybean Processors, 1965:9-1986:12.<sup>a</sup>

Explanatory Variable	Dependent Variable			
	ln(Soybean Crushings)	ln(Soybean Purchases)	ln(Oil Sales)	ln(Meal Sales)
Intercept	-0.216** (-4.11)	0.24 (1.69)	-0.093 (-1.58)	0.127** (7.82)
ln(RETURN): crushings	0.061* (2.33)	0.29* (2.58)	<i>ad hoc</i>	<i>ad hoc</i>
soybeans	<i>ad hoc</i>	0.23* (2.11)	<i>ad hoc</i>	<i>ad hoc</i>
oil	<i>ad hoc</i>	<i>ad hoc</i>	-0.088** (-4.11)	<i>ad hoc</i>
meal	<i>ad hoc</i>	<i>ad hoc</i>	<i>ad hoc</i>	-0.0238** (-2.78)
ln(beg. stocks): soybeans	<i>ad hoc</i>	-0.334** (-5.86)	<i>ad hoc</i>	<i>ad hoc</i>
oil	-0.0246** (-2.86)	0.029 (1.28)	0.163** (7.45)	<i>ad hoc</i>
meal	-0.039** (-3.49)	-0.021 (-0.63)	<i>ad hoc</i>	0.0694** (7.99)
ln(crushings)		0.73** (6.63)		
ln(production): oil			0.742** (18.48)	
meal				0.940** (68.79)
ln(crushing capacity)	0.245** (5.04)			
ln(lagged dependent variable)	0.812** (19.74)	0.370** (5.18)	-0.132** (-2.64)	

Note: The term *ad hoc* signifies explanatory variables that could be included in an *ad hoc* model, but which are not predicted by the theory presented earlier. These variables were excluded from the system reported here.

<sup>a</sup> *t* statistics are shown in parentheses.

\* (\*\*) Significantly different from zero at the 0.05 (0.01) level on the basis of the *t*-test.

Table 7. Continued.

Explanatory Variable	Dependent Variable			
	ln(Soybean Crushings)	ln(Soybean Purchases)	ln(Oil Sales)	ln(Meal Sales)
Dummy: February	-0.087** (-5.23)	-0.041 (-0.64)		
March	0.058** (3.12)	0.007 (0.09)		
April	-0.049** (-2.93)	-0.156 (-1.96)		
May	0.036 (1.70)	-0.134 (-1.86)		
June	-0.061** (-3.57)	-0.157* (-2.14)		
July	0.002 (0.09)	-0.209** (-2.77)		
August	-0.017 (-0.73)	-0.334** (-4.24)		
September	-0.091** (-5.22)	-0.268** (-3.18)		
October	0.164** (6.63)	0.35** (2.91)		
November	0.018 (1.16)	0.042 (0.48)		
December	0.032* (2.00)	-0.154* (-2.26)		
STATISTICS:				
R <sup>2</sup>	0.972	0.946	0.974	0.991
Mean value of dep. variable	0.745	0.668	-0.967	0.512
Std. error of regression	0.0422	0.127	0.0400	0.0224
SYSTEM STATISTICS:				
R <sub>D</sub> <sup>2</sup> = 0.997	Log Likelihood Function = -1266.16		Observations = 239	

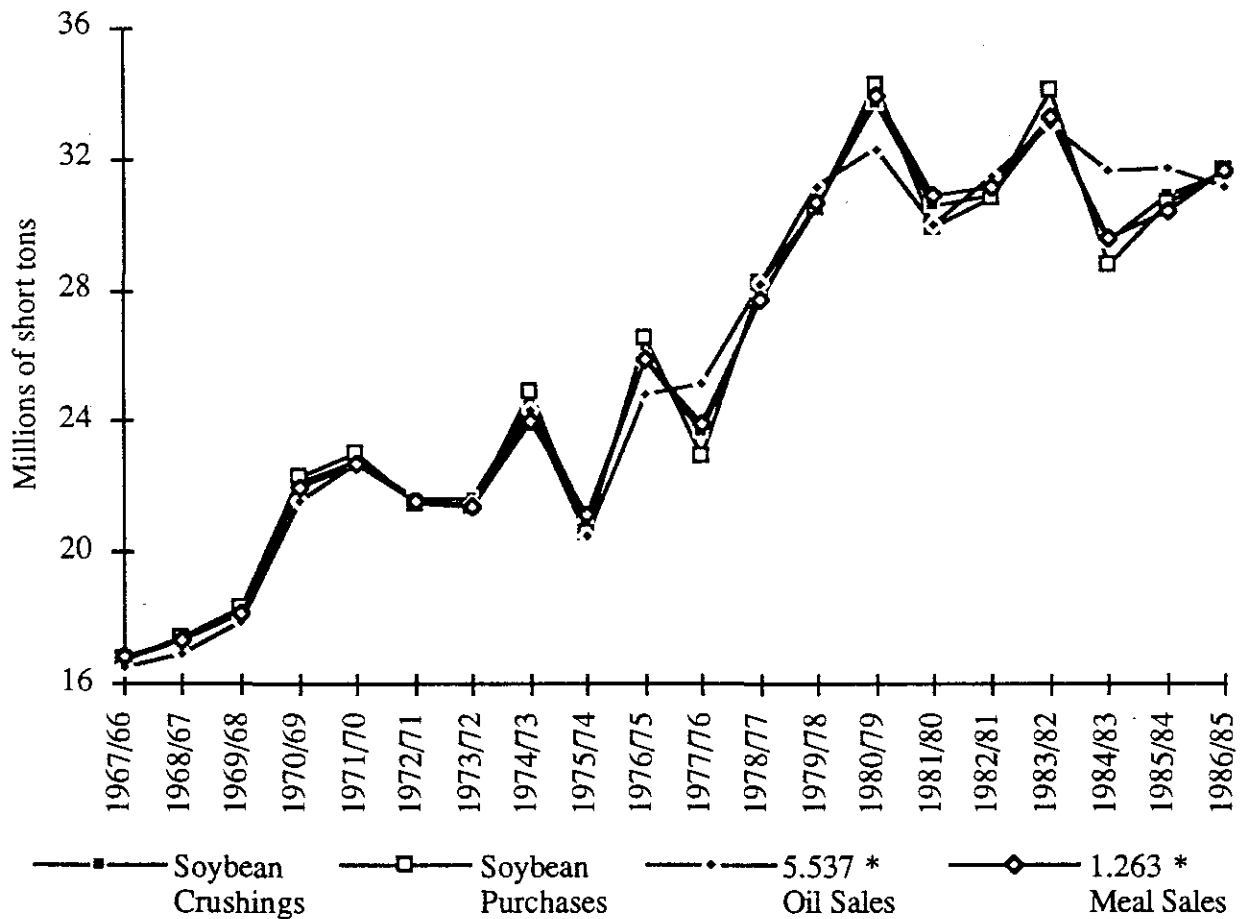
Table 8. Structure of u Errors of the Estimated System of Equations for U.S. Soybean Processors, 1965:9-1986:12.<sup>a</sup>

	Structure of u error corresponding to equation for			
	ln(Soybean Crushings)	ln(Soybean Purchases)	ln(Oil Sales)	ln(Meal Sales)
Autocorrelation Coefficients:				
$\rho_1$			0.57** (4.71)	
$\rho_2$			0.254** (2.66)	0.199** (2.76)
$\rho_3$				0.474** (6.58)
$\rho_{12}$		0.196** (2.68)		0.139 (1.84)
Variables Explaining Heteroscedasticity:				
ln[lagged(soybean crushings)]	-1.94** (-5.32)			
ln(crushing capacity)	1.94** (5.32)			
ln[lagged(soybean purchases)]		-0.27* (-2.11)		
ln(oil beginning stocks)			0.425** (4.81)	
ln(meal beginning stocks)				0.38* (2.23)

<sup>a</sup> *t* statistics are shown in parentheses.

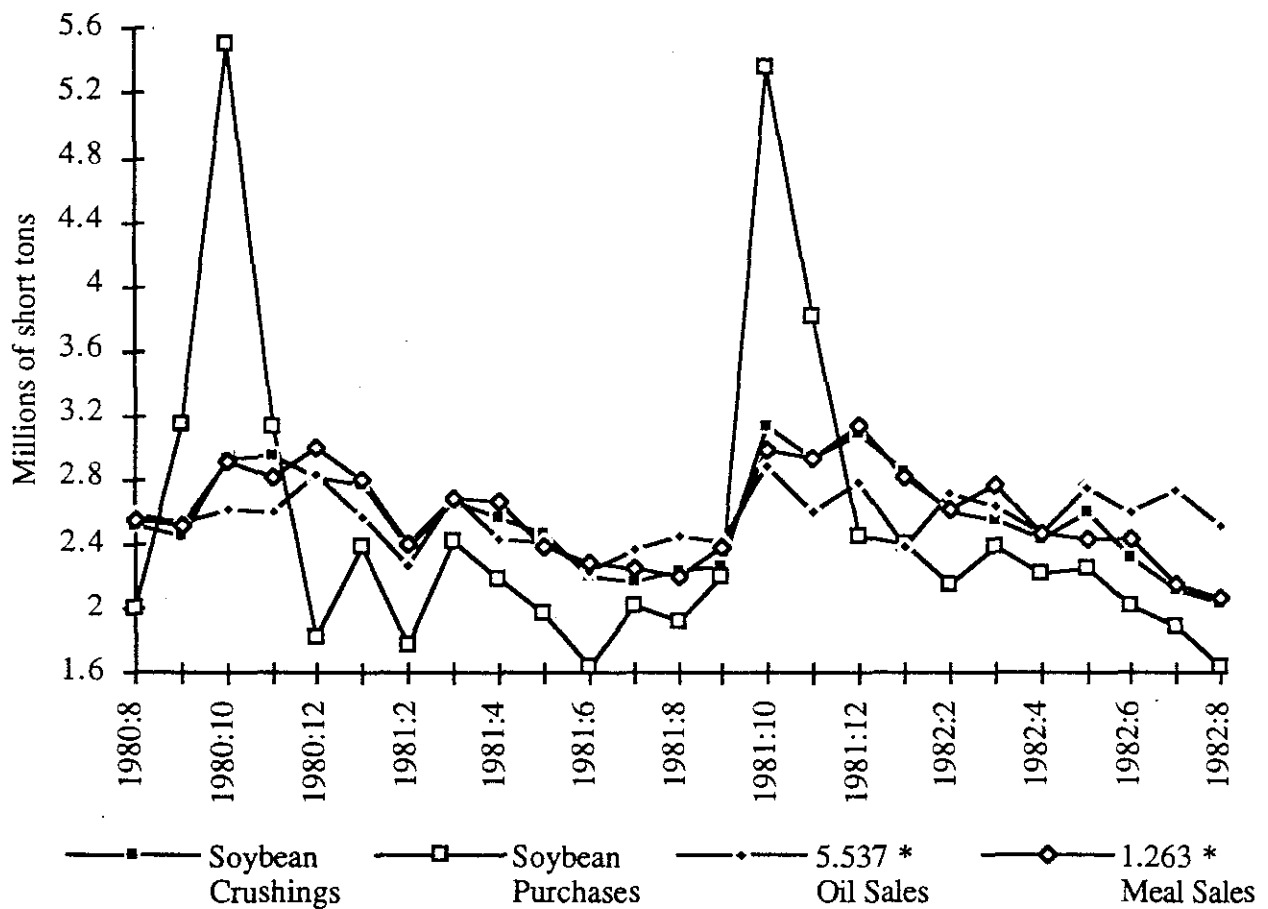
\* (\*\*) Significantly different from zero at the 0.05 (0.01) level on the basis of the *t*-test.

Figure 1. Annual Crushings and Purchases of Soybeans, and Sales of Soybean Oil and Meal,  
U.S. Soybean Processors, 1967/66-1986/85.



Note: The constants 5.537 and 1.263 are fixed input-output coefficients; 5.537 (1.263) short tons of soybeans yield one short ton of oil (meal).

Figure 2. Monthly Crushings and Purchases of Soybeans, and Sales of Soybean Oil and Meal,  
U.S. Soybean Processors, 1980:8-1982:8.



Note: The constants 5.537 and 1.263 are fixed input-output coefficients; 5.537 (1.263) short tons of soybeans yield one short ton of oil (meal).