The Theory and Measurement of Producer Response under Quotas

Lilyan Fulginiti and Richard Perrin

Working Paper 92-WP 94
May 1992

Center for Agricultural and Rural Development
Iowa State University
Ames, Iowa 50011

Lilyan Fulginiti is assistant professor of economics, Iowa State University, and Richard Perrin is professor of economics, North Carolina State University.

Partial support was provided by the North Carolina Agricultural Experiment Station.

Abstract

Tobin and Houthakker's (1950-51) work on consumer behavior under quantity rationing has been extended by many authors, especially through the use of duality theory. This paper uses duality theory to extend the work on demand theory under rationing to the case of producer behavior under quotas. These results permit estimation of otherwise unobservable market supply and demand structures. The structure of the farm economy operating under a tobacco quota system is estimated, and the theory is utilized to infer that the supply elasticity of tobacco would be about 7.0 if the quotas were removed. Estimates such as this are not normally attainable without the theory outlined here, even though they are essential for the evaluation of policy changes.

The Theory and Measurement of Producer Response Under Quotas

1. Introduction

There has recently been a revival of interest in the implications of rationing, or more generally of quantity constraints, in a number of different branches of economic theory. Much of the earlier work on rationing was conducted during and immediately after World War II. The principal results establishing locally valid relationships between demand curve slopes under rationed and unrationed conditions were derived by Tobin and Houthakker (1950-51). Related works were surveyed by Tobin (1952), the results were later restated by Pollak (1969), and were extended by Howard (1977), Latham (1980), Neary and Roberts (1980), and Deaton (1981). In particular, the last two authors illustrate how duality theory can be used to generate empirically estimable demand functions under rationing in the same way that it can do so in the unrationed case.

In this paper we extend the work on demand theory under rationing to explore the implications of quantity constraints in the context of production theory. Because of the presence of short-run adjustment costs leading to short-run input fixity or because of regulatory or institutional constraints, quantity rationing often influences production decisions. Import licensing and quotas and the rationing of intermediate inputs are widespread in the developing world. In many developing countries, agricultural input, output, and credit markets are often targets of government intervention that results in dual markets. In Canada, the European Community, and the United States, production quotas have been implemented for dairy products, tobacco, peanuts, and poultry. Mandatory sales of agricultural output at below free market prices have been features of India, Indonesia, China, and many African nations. Quantity restrictions became widely used in international trade as substitute tariffs after the Tokyo round of the General Agreement on Tariffs and Trade (GATT) negotiations. All of these cases have a common attribute, kink points in the iso-cost sets of firms. These kink points arise from binding constraints on inputs or outputs or other types of restrictions that result in kink points in the interior (as opposed to the vertices) of iso-cost sets, the extreme case being a quantity constraint.

In empirical analysis, it is often important to be able to represent an unrationed supply/demand function in terms of a rationed one, and vice versa. Such functions are necessary if we wish to predict behavior under rationing where we have observations only on free supply; more important, they can be used in the converse situation of predicting unrationed behavior from observations on a market under rationing. Similarly, we may wish to estimate a system of firm supplies and derived demands for a cross section or time series of firms, some of which are rationed and some of which are not. Such functions can be estimated efficiently if a common technology with common parameters is assumed for all firms so that the same parameters appear in the two sets of functions. In this paper, section 2 characterizes the firm's behavior under rationing in terms of its unconstrained behavior when faced with virtual prices. Section 3 discusses the specification of flexible functional form models under rationing. Finally, an empirical example is presented in which the structure of the unrestricted supply curve of a quota-restricted commodity, tobacco, is retrieved from observations on the quota-restricted markets. This methodology provides the necessary information to simulate the effects of deregulation. Section 5 is a summary with conclusions.

2. Quota-constrained versus Unconstrained Behavior

In their classic treatment, Tobin and Houthakker manipulated the first-order conditions to obtain properties of the derivatives of the rationed demands. They obtained locally valid relationships between the derivatives of the rationed and unrationed functions; for example, the Le Chatelier result (Samuelson 1947, pp. 163-69) that at the price at which the ration would have been just bought, the compensated demand curve is no steeper with rationing than without it. Papers by Wales and Woodland (1983), Hausman (1985), and Lee and Pitt (1986) have proposed methods for estimating consumer demand systems in the presence of binding constraints. Wales and Woodland's approach is based upon the Kuhn-Tucker conditions associated with a direct utility function, while Lee and Pitt's is a dual approach beginning with an indirect utility function and showing how "virtual" price[1] relationships can take the place of Kuhn-Tucker conditions. We extend the analysis to that of production technologies where kink points may occur because of binding nonnegativity constraints on inputs or outputs or because of production quotas and rationing of inputs.

Consider a firm with netput vector $y=(y_1, y_2)'$, where y_1 is a vector of unconstrained netputs (with positive signs for outputs and negative for inputs) and y_2 is a vector of netputs that are traded in the market but are subject to quotas. In the short run (when a vector z of inputs is fixed), the variable profit function when y_2 is unconstrained by quotas is

$$\Pi^u(p_1, p_2; z) = \max_{y_1, y_2} \left(p_1' y_1 + p_2' y_2 : \ (y_1, y_2, z) \in \tau\right) , \tag{1}$$

where τ is the technology set, and p_1 and p_2 are netput prices. The properties we assume for this function are standard: nondecreasing in output prices and fixed inputs, nonincreasing in input prices, linear homogeneous and convex in prices, concave in fixed quantities, continuous and twice differentiable. When y_2 are constraining quota levels, the firm's constrained variable profit function is

$$\begin{aligned} \Pi^c(p_1, p_2; y_2, z) &= \max_{y_1} \left(p_1' y_1 + p_2' y_2 : \ (y_1, y_2, z) \in \tau\right) \\ &= \max_{y_1} \left(p_1' y_1 ; \ (y_1, y_2, z) \in \tau\right) + p_2' y_2 \\ &= \Pi^p(p_1; y_2, z) + p_2' y_2 , \end{aligned} \tag{2}$$

[1] "Virtual" prices (Rothbarth 1941) are the prices that would induce an unrationed household to behave in the same manner as when faced with a given vector of ration constraints.

where the function Π^p is a restricted profit function that we refer to as the "partial profit" function, independent of p_2. The partial profit function (2) shares the properties of the unconstrained variable profit function as described above.

To establish the relationship between the unconstrained profit function (1) and the quota-constrained profit function (2), we turn again to the concept of virtual price. We define virtual prices as the vector of prices p_v that would induce the firm to freely choose the netput vector y_2. Hence, p_v must be a function of p_1, y_2 and z, or

$$p_v = p_v \ (p_1; \ y_2, \ z) \quad . \tag{3}$$

We can now evaluate the unconstrained profit function (4) at $p_2 = p_v$ as

$$\begin{aligned} \Pi^u \ (p_1, \ p_v; \ z) &= \max_{y_1, \ y_2} \left(p_1' \ y_1 + p_v' \ y_2 : \ (y_1, \ y_2, \ z) \in \tau\right) \\ &= \Pi^p \ (p_1; \ y_2, \ z) + p_v' \ y_2 \quad , \end{aligned} \tag{4}$$

and from Hotelling's lemma, we may formally define p_v as the solution to

$$\Pi^u_{p_v} = y_2 \quad . \tag{5}$$

Now, at virtual prices for quota commodities, constrained and unconstrained profit must be equal,

$$\Pi^c \ (p_1, \ p_v; \ y_2, \ z) = \Pi^u \ (p_1, \ p_v; \ z) \ , \tag{6}$$

and from (2) and (4) we establish the relationship between constrained and unconstrained profit functions as

$$\Pi^c \ (p_1, \ p_2; \ y_2, \ z) = \Pi^u \ (p_1, \ p_v; \ z) + (p_2 - p_v)' \ y_2 \quad . \tag{7}$$

We can characterize the differences between the quota-constrained and unconstrained firm behavior by examining first and second derivatives of (7). Differentiating with respect to p_1 and using (5) we obtain

$$\Pi^c_{p_1} = \Pi^u_{p_1} + \left(\Pi^u_{p_v} - y_2\right)' \frac{\partial p_v}{\partial p_1} = \Pi^u_{p_1} \quad . \tag{8}$$

Applying Hotelling's lemma to (8), we conclude that

$$y_1^c \; (p_1, \; p_2; \; y_2, \; z) \; = y_1^u \; (p_1, \; p_v; \; z) \;\; ; \tag{9}$$

that is, the optimal vector of nonquota goods under a quota regime (y_1^c) is identical to the optimal unconstrained vector (y_1^u) if the latter is evaluated at virtual prices.

Differentiating (7) with respect to quota levels y_2, we obtain

$$\Pi^c_{y_2} = (p_2 - p_v)' + \left(\Pi^u_{p_v} - y_2\right)' \frac{\partial p_v}{\partial y_2} = (p_2 - p_v)' \quad . \tag{10}$$

Thus, the marginal effect of a change in the quota level is simply the difference between the market price and the virtual price for the quota input or output (see Figure 1). We refer to this value as quota rent, designated as $r = p_2 - p_v$.

Finally, differentiating (7) with respect to fixed inputs z,

$$\Pi^c_z = \Pi^u_z + \left(\Pi^u_{p_v} - y_2\right)' \frac{\partial p_v}{\partial z} = \Pi^u_z \quad . \tag{11}$$

Thus, the vector of shadow prices for the fixed inputs is the same under a quota regime as under a nonquota regime evaluated at $p_2 = p_v$.

The comparative statics of the nonquota and quota regimes can be further elaborated by deriving the Hessians of the former in terms of the latter and vice versa. To do this, we first differentiate (8) with respect to p_1 and y_2 to obtain

$$\Pi^c_{p_1 p_1} = \Pi^u_{p_1 p_1} + \left(\Pi^u_{p_1 p_v}\right)' \frac{\partial p_v}{\partial p_1} \; , \tag{12}$$

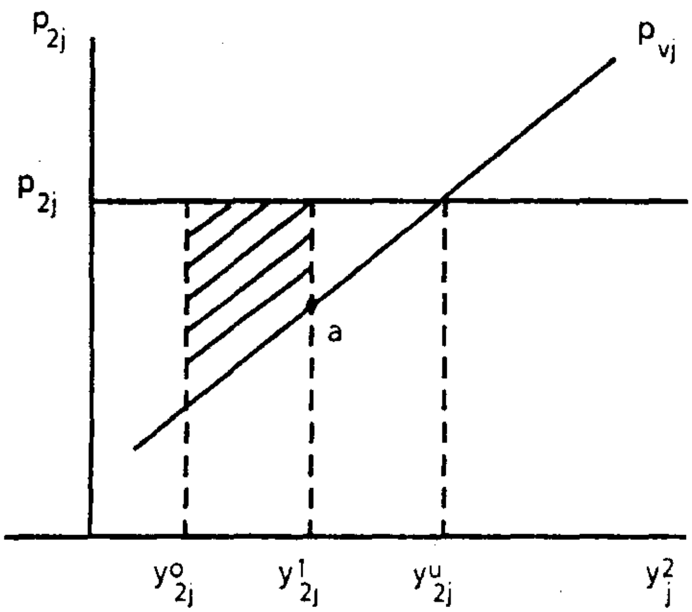

Figure 1. Virtual price (p_{vj}) and quota levels for output y_{2j}

and

$$\Pi^{c}_{p_1 y_2} = \left(\Pi^{u}_{p_1 p_v}\right)' \frac{\partial p_v}{\partial y_2} \quad . \tag{13}$$

Now differentiating (10) with respect to p_1 and y_2, we have

$$\Pi^{c}_{y_2 p_1} = - \frac{\partial p_v}{\partial p_1} \tag{14}$$

and

$$\Pi^{c}_{y_2 y_2} = - \frac{\partial p_v}{\partial y_2} \quad . \tag{15}$$

Finally, we differentiate (5) with respect to y_2, to obtain

$$\Pi^{u}_{p_v p_v} \frac{\partial p_v}{\partial y_2} = I \quad . \tag{16}$$

Equations (12) through (16) may be solved for the Hessians of the unconstrained equilibrium in terms of those of the constrained equilibrium as follows. First combine (15) and (16) to obtain

$$\Pi^{u}_{p_v p_v} = - \left(\Pi^{c}_{y_2 y_2}\right)^{-1} \quad . \tag{17}$$

Next, from (13) and (15)

$$\Pi^{u}_{p_1 p_v} = - \Pi^{c}_{p_1 y_2} \left(\Pi^{c}_{y_2 y_2}\right)^{-1} \quad . \tag{18}$$

Finally, from (12), (14), and (18),

$$\Pi^u_{p_1 p_1} = \Pi^c_{p_1 p_1} - \Pi^c_{p_1 y_2} \left(\Pi^c_{y_2 y_2}\right)^{-1} \Pi^c_{y_2 p_1} \quad . \tag{19}$$

In a similar fashion, the Hessian of the constrained profit function may be expressed in terms of the unconstrained Hessians as

$$\Pi^c_{y_2 y_2} = - \left(\Pi^u_{p_v p_v}\right)^{-1} \quad , \tag{20}$$

$$\Pi^c_{p_1 y_2} = \Pi^u_{p_1 p_v} \left(\Pi^u_{p_v p_v}\right)^{-1} \quad , \text{ and} \tag{21}$$

$$\Pi^c_{p_1 p_1} = \Pi^u_{p_1 p_1} - \Pi^u_{p_1 p_v} \left(\Pi^u_{p_v p_v}\right)^{-1} \Pi^u_{p_v p_1} \quad . \tag{22}$$

Given equation (2), the results in equations (17) to (22) are preserved if we replace Π^c by Π^p everywhere.

Equations (17) through (19) show how one may deduce the slopes of the supply and demand curves of a non-quota regime if slopes for a quota regime are known, while equations (20) through (22) provide the opposite transformation. Since these results are derived from Π^u evaluated at $p_2 = p_v$, the transformations are exact only at the quota-constrained equilibrium corresponding to quota level y_2. The results provide second-order approximations to the unconstrained profit function in the vicinity of the constrained equilibrium. This is equivalent to a first-order approximation of the supply and demand functions such as that shown in Figure 1. Here we can see that the estimates of a profit function for a firm constrained by a quota to output y^1_{2j} will provide estimates of the unconstrained equilibrium level y^u_{2j} via linear approximation through point *a*.

Some additional interpretation of these results is useful. The last term of (19) is negative semidefinite, and the last in (22) is positive semidefinite (Lau 1976). Thus, under quota constraints, the quantity responses to price changes are smaller than those in the unrationed case; i.e., the LeChatelier effect. For the case of a single rationed output commodity such as the one considered later in this paper, equation (19) shows that the own-price supply elasticity of a variable output under a nonquota regime is equal to its own-price elasticity under a quota regime plus a nonnegative term. The nonnegative term is the product of three subterms: the response of variable outputs to the quota level, the response of the quota commodity to its virtual price, and the response of virtual prices to the price of variable commodities. The second term is nonpositive due to concavity of the profit function, and the first and third have the same sign.

From (21), if there is but one rationed commodity, the effect of a quota on output (input) y_{2j}, i.e., a decrease in y_{2j}, on a nonquota output is to increase the supply (demand) of the latter if they are gross substitutes and to decrease it if they are gross complements. Since the order of differentiation is irrelevant, (21) also indicates that the effect on nonquota outputs of relaxing the constraint is equal to the effect of a decrease in the price of the nonquota output on the virtual price of the quota commodity. Therefore, an increase in the price of the nonquota commodity causes the virtual price of quota commodities to rise if they are gross substitutes and to fall if they are gross complements.

Two extensions of the results (17) through (22) are in order at this point. The first has to do with the relationship between the Hessian of the partial profit function and that of the unrestricted profit function. Note that from (2), $\Pi^p_{p1p1} = \Pi^c_{p1p1}$, $\Pi^p_{p1p1} = \Pi^c_{p1y2}$, and $\Pi^p_{y2y2} = \Pi^c_{y2y2}$; thus, the transformations between the Hessians of the partial profit and unconstrained profit are the same as those between constrained profit and unconstrained profit as shown in (17) through (22).

The second extension is to show transformations between the elasticities associated with the unconstrained, constrained, and partial profit functions. The notation for elasticities is as follows. Let y represent the vector of netputs as before, or any subset of y that is of interest, and let p represent the corresponding vector of prices. Let q represent any arbitrary subvector of arguments with respect to which elasticities are to be calculated. Elasticities of optimal netput values, y, with respect to q can be expressed as

$$E_{yq} = D_y^{-1} \, \Pi_{pq} \, D_q \quad , \tag{23}$$

where E_{yq} is the matrix of elasticities of netputs y with respect to q, and D_y, D_q are diagonal matrices with the diagonal consisting of y and q, respectively.

From (10) and (23) it is evident that the elasticity of quota rent with respect to quota levels can be expressed as

$$E^c_{ry_2} = D_r^{-1} \, \Pi^c_{y_2 \, y_2} \, D_{y_2} \quad . \tag{24}$$

Also,

$$E^u_{y_2 \, p_v} = D_{y_2}^{-1} \, \Pi^u_{p_v \, p_v} \, D_{p_v} \quad . \tag{25}$$

Solving these for the derivatives of the profit function, substituting into (17) and simplifying, we obtain

$$E^u_{y_2 \, p_v} = - \left(E^c_{ry_2}\right)^{-1} D_r^{-1} \, D_{p_v} = - \left(E^p_{p_v \, y_2}\right)^{-1} . \tag{17a}$$

Similarly, we obtain

$$E^u_{y_1 \, p_v} = - \, E^c_{y_1 \, y_2} \left(E^c_{r \, y_2}\right)^{-1} D_r^{-1} \, D_{p_v} = - \, E^p_{y_1 \, y_2} \left(E^p_{p_v \, y_2}\right)^{-1} \tag{18a}$$

and

$$E^u_{y_1 \, p_1} = E^c_{y_1 \, p_1} - E^c_{y_1 \, y_2} \left(E^c_{r \, y_2}\right)^{-1} E^c_{rp_1} = E^p_{y_1 \, p_1} - E^p_{y_1 \, y_2} \left(E^p_{p_v \, y_2}\right)^{-1} E^p_{p_v \, p_1} . \tag{19a}$$

The constrained profit function (2) represents variable producer profits under rationing and it is particularly useful in welfare analysis of rationing. It provides a basis for an empirical measurement of the willingness of the decision maker to pay for a particular change in some parameter, say, from α^o to α^1. The cost or willingness to pay for such a change can be measured as

$$W = \int_{\alpha_0}^{\alpha_1} \Pi_\alpha^c \, d\alpha \quad . \tag{26}$$

If $\alpha = p_{1i}$, then using Hotelling's lemma, the amount by which the firm must be compensated for a price change is given by

$$W_i = \int_{p_{1i}^o}^{p_{1i}^1} \Pi_{p_{1i}}^c \, dp_{1i} \quad . \tag{27}$$

This provides a measure of the change in producer surplus due to a price change. The presence of rationing poses no new difficulties for the calculation of valid measures of producer surplus. Using the restricted profit function in (2), and with $\alpha = y_i^o$, some useful additional welfare results can be obtained. Using (10), we have

$$\begin{aligned} W_j &= \int_{y_{2j}^o}^{y_{2j}^1} \Pi_{y_{2j}}^c \, dy_{2j} = \int_{y_{2j}^o}^{y_{2j}^1} (p_{2j} - p_{vj}) \, dy_{2j} \\ &= p_{2j} \left(y_{2j}^1 - y_{2j}^o\right) - \int_{y_{2j}^o}^{y_{2j}^1} p_{vj} \,(p_1 ; \, y_2 , \, z) \, dy_{2j} \quad . \end{aligned} \tag{28}$$

This expression provides an exact measure of the firm's willingness to pay for a change in the quota level of output i. The shaded area of Figure 1 illustrates this change in variable profits due to additional units of y_{2j} produced.

From (28), the compensation required for a change in quantity constraints can be measured from price and quantity data and knowledge of the virtual price functions p_{vj} defined above. Such information is particularly useful in the economic evaluation of changes in quota policies.

3. A Translog Specification

The foregoing theory suggests that an unconstrained supply and demand system can be derived from a partial profit function estimated under a quota regime (or vice versa). We specify a translog structure for the partial profit function,

$$\ln \Pi^{p} = \alpha_{o} + \alpha' X + \frac{1}{2} X' BX \quad , \tag{29}$$

where $X' = (\ln p_1, \ln y_2, \ln z)'$ and α_0, α' and B are parameters to be estimated (a scalar, a vector, and a matrix, respectively). A convenient partitioning consists of $\alpha' = (\alpha_p, \alpha_y, \alpha_z)'$, and

$$B = \begin{bmatrix} B_{pp} & B_{py} & B_{pz} \\ B_{yp} & B_{yy} & B_{yz} \\ B_{zp} & B_{zy} & B_{zz} \end{bmatrix} \quad .$$

Using Hotelling's lemma, the share equations for the n nonquota-constrained variable inputs and outputs are

$$s_1 = \alpha_p + B_{pp} \ln p_1 + B_{py} \ln y_2 + B_{pz} \ln z \tag{30}$$

where s_1 is an n x 1 vector of optimal shares $s_i = p_{1i}\, y_{1i} / \Pi^p$. Note that B_{yy} and B_{yz}, which are needed to evaluate (17) through (22), cannot be estimated from this set of share equations. The partial profit function itself must be estimated, either alone or jointly with the share equations.

Given the assumptions stated earlier, the profit function must satisfy the properties of symmetry, monotonicity, linear homogeneity, and convexity in prices, and concavity in fixed quantities. Appropriate restrictions on the parameters are imposed in the estimation procedure so that the translog profit function satisfies symmetry and linear homogeneity in prices. Monotonicity, convexity and concavity are not general properties of the translog. They cannot be conveniently imposed with linear restrictions on parameters of equations (29) and (30). Instead, the consistency of the estimated share equations with these properties must be evaluated after estimation. To satisfy the monotonicity condition, the estimated shares must be positive. For convexity in prices, the Hessian implied by the estimated B_{pp} submatrix must be positive semidefinite, and for concavity in fixed quantities, the Hessians implied by B_{yy} and B_{zz} must be negative semidefinite.

Once the parameters of (29) are estimated, the virtual shares (defined as $p_{vj}\, y_{2j} / \Pi^p$) for the quota commodities may be estimated as

$$- s_v = \alpha_y + B_{yp} \ln p_1 + B_{yy} \ln y_2 + B_{yz} \ln z \quad . \tag{31}$$

The full response elasticity matrix consists of responses of netputs, virtual prices, and shadow prices (for fixed inputs) with respect to netput prices, quota levels, and fixed input levels. This elasticity matrix can be evaluated for a given set of values of the exogenous variables by using the estimated coefficients and the predicted shares as

$$E^p = (B - D_s + s\, s') \, D_s^{-1} \quad , \tag{32}$$

where E^p is the matrix of elasticities of netputs, virtual prices and shadow prices of inputs with respect to prices, quota levels and fixed inputs, and s is a vector of predicted shares for the given values of exogeneous variables.

4. An Application: Estimating Tobacco Supply Elasticity

The production of U.S. tobacco has been subject to federal output restrictions since the 1930s, first in the form of acreage controls, and later in the form of production quotas (since 1965 for flue-cured tobacco, and since 1971 for burley, the other major tobacco type)[2]. In this section we utilize

---

[2] Quotas are allocated to firms that could sell or rent them to firms within their county but in most years not to firms across county lines. This implies different marginal costs across counties. The rationing problem should then be modeled allowing for as many rations as counties. In this paper we abstract from this to simplify the model. In a recent study, Rucker, Thurman, and Sumner (1990) conclude that the welfare effects associated with removal of the cross-country restrictions is small. This suggests that the misspecification implied by our simplification may not be serious.

the theory developed to estimate the supply elasticity of this crop, a crucial parameter in evaluating potential changes in tobacco policy.

(a) The Data

We have chosen to estimate the tobacco supply elasticity for North Carolina, which is the largest tobacco-producing state, accounting for about one-third of total U.S. production. The primary reason for estimation at the state level is that tobacco constitutes a substantial share of agricultural production value in that state (between 20 percent and 50 percent over the 1950-1984 data period), thus providing a richer empirical base than would be the case for U.S. agriculture as a whole, in which tobacco's share of revenues is less than 4 percent during this period. We estimate a structure with two outputs (tobacco and all other crop and livestock products), one variable input (production inputs including hired labor), and three fixed inputs (land, capital and the stock of research knowledge). Table 1 describes these variables.

Among the data required for estimation of the profit function are expected prices, which are not directly observable. Our proxy for expected prices is a set of predictions from ARIMA (p, d, q) models estimated from the time series of realized prices. Using Akaike's (1974) information criterion and the Q-statistic (Ljung and Box 1978), the accepted models were an AR (1) for output price and an AR (2) for variable input price.

(b) Econometric Estimation

We estimate equations (29) and (30), with slight modifications for estimation purposes. First, random disturbance terms (ϵ_i) were added to the profit and share equations. These disturbances represent the effects of random weather conditions and approximation error; they are assumed to be homoscedastic and uncorrelated within equations. Contemporaneous cross-equation correlation of the disturbance terms is permitted.

If, besides satisfying the above assumptions, the vector of disturbances is multinormally distributed, maximum likelihood estimation can be performed. Under the stated stochastic assumptions, the maximum likelihood estimators are consistent, asymptotically normal, and asymptotically efficient. In addition, they provide estimates invariant to the choice of equation deleted. The ITSUR option of the SYSNLIN procedure in SAS was used for estimation.

Using the expected prices fitted with the AR models and the data described in the previous section, the equations (29) and (30) are estimated by the maximum likelihood method. Cross-equation symmetry and identity restrictions are imposed along with linear homogeneity in prices. Aggregation consistency requires homogeneity of degree one in fixed commodities, so these restrictions are also imposed. The system has two equations, with the dependent variables being the logarithm of profits and the variable output share. The stacked model has 64 observations and 16 estimated parameters.

Collinearity diagnostics developed by Belsley, Kuh, and Welsch (1980) indicate an absence of strong multicollinearity. Because time-series data are used, the presence of autocorrelation in the residuals is possible. Simple Durbin-Watson statistics for each of the equations in the system fall in the inconclusive range. A test for the joint hypothesis that the autocorrelation parameter in each equation is equal to zero, proposed by Judge et al. (1980), does not reject the null hypothesis (for this problem $q \sim X_2^2$ is calculated to be 4.09 and the 0.05 critical value is 5.99). Both of these procedures test for the existence of serial correlation occurring within a single equation but do not consider the more general case in which errors may also be serially correlated between equations.

Guilkey's (1974) Wald test statistic for a system of simultaneous equations that do not contain lagged endogenous variables as regressors is calculated as 6.51. For four degrees of freedom, the chi-square critical value at the 5 percent level is 9.48. Therefore, this statistic does not lead to rejection of the hypothesis that the matrix of first-order vector autoregressive coefficients is zero.

Estimation proceeded under the assumption of serially independent errors. R^2 obtained from OLS residuals are 0.78 for the profit equation and 0.71 for the output share equation. Table 2 presents the parameter estimates of the restricted model. The table contains a total of 28 parameters, six of which are significant at the 1 percent level, five at the 5 percent level, and six at the 10 percent level.

In addition to the imposed properties of symmetry and homogeneity, monotonicity and convexity in prices are additional properties of a profit function that cannot be satisfied globally with the translog function. However, they may hold at the specific data points used in estimating the function. For the estimates in Table 2, monotonicity is satisfied at the point of expansion, but is violated for two out of six predicted shares at the mean of the data, and for 39 of the 192 predicted shares at the individual data points. Convexity is violated if own-price elasticities have the wrong sign. There are no such violations at the average of the data points, but there are at 44 of the 192 data points.

(c) Estimates of Supply and Demand Elasticities

We use equation (32), with predicted shares evaluated at the mean values of variables, to calculate the estimated elasticities of optimal production decisions in response to changes in prices and fixed quantities. The results, shown in Table 3, indicate a nontobacco output supply elasticity of .24 and a derived variable input demand elasticity of -.41, estimates that are lower than we expected but consistent with other estimates of aggregate agricultural supply and demand elasticities. The key elasticity of interest in this study is the price elasticity of the latent tobacco supply curve, which is the inverse of the third element on the diagonal of Table 3. This estimated price elasticity is about 7.0. This is a large elasticity, larger than the recent estimates of 4.0 to 5.6 by Goodwin and Sumner (1990), who used a different approach with cross-sectional county-level data for a recent ten-year period. These large elasticity estimates are quite plausible because tobacco utilizes only 7 percent of harvested cropland and perhaps higher proportions of other inputs, virtually all of which can be reallocated between tobacco and other products.

The remaining diagonal elements in Table 3 indicate that the derived demand elasticities for land and capital are -.25 and -1.66, respectively (with other prices constant and tobacco quota fixed), and that there are increasing marginal returns to the research variable. Other key results from Table 3 related to the existence of a quota commodity are the negative unit elasticities of output and variable input use with respect to changes in the tobacco quota (the first is plausible, the second is surprising but plausible). The elasticity of tobacco supply price with respect to the price of other output is 2.47 and with respect to the price of variable inputs is -1.47 (an unexpected and implausible sign). This partial review of the econometric results indicates that the diagonal elements of the elasticities in Table 3 have appropriate signs and expected magnitudes, while the off-diagonal elements contain some estimates that are difficult to rationalize, though theoretically possible.

Since this approach to estimating the latent tobacco supply elasticity rests on measuring the economic effects of reallocating resources between tobacco and other jointly produced outputs, it is useful to test this jointness property. For the restricted profit function, nonjointness between aggregate output and tobacco requires that the second-order cross coefficient between these two variables (-.135 in our case) be equal to the negative of the product of the corresponding first-order coefficients (4.75 and -11.94 in our case). A likelihood ratio test, conditional on the maintained hypothesis of symmetry, homogeneity in prices and in fixed commodities, rejects this null hypothesis at the 5 percent level.

Equation (19a) provides a measure of how the supply elasticity of nontobacco products would change if the tobacco quota system were eliminated. We obtain the surprising result that eliminating quotas would increase the nontobacco supply elasticity from .24 to 17.67. To see why this effect is so large, recall that the last matrix expression of (19a) augments the elasticity matrix for a quota regime to obtain the corresponding portion of the elasticity matrix for an unconstrained regime. For the case of a single rationed commodity and a single aggregate of other commodities, the augmentation of output supply elasticity consists of the negative of the following product: elasticity of tobacco virtual price with respect to other output price (2.47) times the elasticity of tobacco output with respect to tobacco virtual price (6.97) times the elasticity of other output with respect to tobacco output (-1.01), which equals 17.43. The comparable LeChatelier effect on input demand is to increase elasticity from -.41 to -1.97, also a very large effect. These large elasticities and LeChatelier effects could be valid at the average of our data set but seem unlikely to hold over the range between the constrained and unconstrained equilibrium points, so we are more cautious in making inferences from those results than from the estimated supply elasticity of tobacco itself.

5. Summary and Conclusions

We have discussed the theory of producer response under quotas and have shown how duality theory and the concept of virtual prices may be used to simplify and extend this theory. Among the implications of our results are the fact that behavior under rationing may be predicted from a knowledge of behavior in an unrationed regime and vice versa. This information is important in evaluating policies that either impose quotas on a previously unconstrained sector or eliminate quotas in a sector in which they have long obscured unconstrained market responses. We examine an issue of the latter type, in which we estimate the market supply elasticity of tobacco from a time series of data during a quota regime that totally obscured producer response to tobacco price. The estimated supply elasticity is about 7.0, higher than estimated by others. This difference has implications for measuring the welfare effects of changes in the tobacco quota program. We conclude that the approach we develop may be useful in empirical evaluation of other quota and rationing policies where data permit estimation of restricted profit functions.

Table 1. Variables describing the agricultural sector

Π^P,	partial profit: the value of crops and livestock produced, not including tobacco, minus the value of variable inputs described below.
y_{11},	variable output: the value of production of all crop and livestock products other than tobacco, deflated to 1950 dollars using the GDP deflator. Realized price is a Tornquist-Theil index of deflated prices received by North Carolina farmers. Expected price is from an ARIMA estimator described in the text.
y_{12},	variable input: total farm production expenses, less depreciation, property taxes and net rent to nonoperator landlords, deflated to 1950 dollars using the GDP deflator. Realized price is a Tornquist-Theil index of U.S.-wide price indexes weighted by North Carolina expenditure shares, deflated by the GNP deflator. Expected price is from an ARIMA estimator described in the text.
y_2,	tobacco: millions of pounds produced
z_1,	land: millions of acres of harvested cropland
z_2,	capital: the value of machinery and motor vehicles on North Carolina farms deflated to 1950 dollars. For the period 1950-1970, this value was available only for the United States as a whole. For this period, the N.C. share of this U.S. value was estimated to be the same as the share of N.C tractors on farms to U.S. tractors on farms, as available from the agricultural censuses and interpolated linearly between census years.
z_3,	stock of research knowledge: a distributed lag of deflated state and federal funds expended by the N.C. Agricultural Research Service. The lag distribution consisted of a 13-year inverted-V.

Table 2. Maximum likelihood estimates of the translog profit function coefficients

Dependent Variable	First-Order Coefficient	Price of: output	Price of: inputs	Quantity of: tobacco	Quantity of: land	Quantity of: capital	Quantity of: research
	Explanatory Variables	Second-Order Coefficients					
output	4.75411	-2.53356	2.53356	-0.13513	0.71337	0.84852	-0.98913
	(1.95623)	(0.29508)	(.29508)	(.40117)	(0.34628)	(0.38929)	(0.19681)
inputs	-3.75411		-2.5336	0.13515	0.71337	-0.84852	2.95917
	(1.95623)		(.29508)	(0.40117)	(0.34628)	(0.38929)	(0.68700)
tobacco	-11.93763			-2.00720	6.81942	-4.81220	2.68471
	(16.51615)			(3.96009)	(3.04188)	(2.96793)	(1.45328)
land	3.92327				-11.86200	5.04214	-4.47800
	(12.49898)				(3.64586)	(2.76176)	(1.67087)
capital	9.01436					-0.22991	1.79325
	(18.09359)					(3.88699)	(1.98842)
research	1.26446						-0.92058
	(8.97853)						(1.20012)

NOTE: Standard errors in parentheses.

Table 3. Estimated elasticities, evaluated at the mean of the variables

Variable	Elasticities with respect to					
	Output Price	Input Price	Tobacco Quota	Land Quantity	Capital Quantity	Research
Output	.24	-.24	-1.01	1.97	.04	1.76
Input	.41	-.41	-1.06	1.74	.32	-.05
Supply Price Tobacco	2.47	-1.47	.14	-4.86	4.71	-.63
Shadow Price Land	2.01	-1.01	2.04	-3.94	1.90	.21
Shadow Price Capital	-.32	1.32	14.07	-13.47	-.60	-3.42
Shadow Price Research	1.87	.03	.27	.22	.50	-.76

Bibliography

Akaike, H. 1974. A new look at the statistical model identification, IEEE Transaction on Automatic Control, AC-19,716-723.

Belsley, D.A., D. Kuh, and R.E. Welsch, 1980. Regression Diagnostics. New York: John Wiley & Sons.

Deaton, A. 1981. Theoretical and empirical approaches to consumer demand under rationing. In Essays in the Theory and Uses of Consumer Behavior, A. Deaton, Ed. Cambridge U. Press.

Diamond, P. and M. Yaari. 1972. Implications of the theory of rationing for consumer choice under uncertainty. Am. Econ. Rev. 62:333-43.

Fulginiti, L.E. and R.K. Perrin. 1987. Measuring tobacco supply elasticity. Paper presented at the AAEA meetings, Lansing, Michigan.

Goodwin, B.K. and D.A. Sumner. 1990. Estimation of market supply parameters under mandatory production quotas. Unpublished manuscript, Dept. of Ag. and Resource Economics, N.C. State University, Raleigh.

Guilkey, D.K. 1974. Alternative tests for a first-order vector autoregressive error specification. J. of Econometrics 2:95-104.

Houseman, J.A. 1985. "The econometrics of nonlinear budget sets. Econometrica 53:1255-1282.

Howard, D. 1977. Rationing, quantity constraints and consumption theory. Econometrica 45:399-412.

Judge, G., W. Griffiths, R.C. Hill, and T.C. Lee. 1980. The Theory and Practice of Econometrics, New York: John Wiley & Sons.

Lau, L. 1976. A characterization of the normalized restricted profit function. Journal of Economic Theory, 12:131-63.

Latham, R. 1980. Quantity constrained demand functions. Econometrica 48:307-314.

Lee, L. and M. Pitt. 1986. Microeconometric demand systems with binding nonnegativity constraints: the dual approach. Econometrica 54:1237-1242.

Ljung, G.M. and G.E.P. Box. 1978. On a measure of lack of fit in time series models. Biometrika 65:297-303.

Moschini, G. 1988. A model of production with supply management for the Canadian agricultural sector. Am. J. of Ag. Econ. 70(2):318-329.

Neary, J. and K. Roberts. 1980. The Theory of household behavior under rationing. European Economic Review 13:25-42.

Perrin, R.K. and D. Hoover. 1985. Tobacco policy and alternative enterprises. Paper presented at the Southern Agricultural Economics Association meeting, Biloxi, Mississippi.

Pollak, R. 1969. Conditional demand functions and consumption theory. Quarterly J. of Econ. 83(1):60-78.

Rothbarth, E. 1941. The measurement of change in real income under conditions of rationing. Rev. of Ec. Stud. 8:100-107.

Rucker, R., W. Thurman and D. Sumner. 1990. Production rights with limited transferability: A Case study of the U.S. tobacco and peanut programs. Unpublished manuscript, July 1990.

Samuelson, P. 1947. Foundations of Economic Analysis. Cambridge, MA: Harvard University Press.

Tobin, J. 1952. A survey of the theory of rationing. Econometrica 20:512-53.

Tobin, J. and H. Houthakker. 1950-51. The effects of rationing on price elasticities. Rev. of Ec. Stud. 18:140-53.

Wales, T.J. and A.D. Woodland. 1983. Estimation of consumer demand systems with Binding Non-negativity constraints. J. of Econometrics 21:263-285.