

Risk, Self-Protection, and Ex Ante Economic Value

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ABSTRACT

We develop three propositions about the ex ante value of reduced risk. If there is a continuous outcome distribution and if self-protection influences outcome probability and severity, then: (1) unobservable utility terms cannot be eliminated from the ex ante value expressions; (2) knowledge of the convexity or the nonconvexity of dose-response functions is insufficient to sign changes in these expressions; and (3) self-protection expenditures need not be a lower bound measure of these expressions. Therefore, many restrictions applied in recent empirical work on the economic value of risk changes are not immediately transferable to settings where endogenous risks prevail.

I. INTRODUCTION

Public agencies now feel considerable pressure to reduce risks to individuals' health and welfare through additional provision of security, personal safety, fire and flood prevention, auto safety, product safety, environmental protection, and emergency planning. Persons who might suffer harm from exposure to hazards can reduce their expected ex post costs by purchasing market insurance. However, Arrow [1] and Shavell [31] show that moral hazard compels private insurers to defray only a fraction of these costs. Moreover, adverse selection and nonindependence of risks cause contingent claims markets to be incomplete. Finally, many individuals are thought to be "...psychologically unable to cope with risk" (Oi [25]), causing them to misperceive it systematically. Collective attempts to overcome these limits to decentralized allocations and resolutions of risk can be more efficient if accurate estimates are available of individuals' choices and the ex ante economic values of risk reductions that these choices imply.

The empirical risk valuation literature typically assumes that: (i) risks are independent of individual actions; and (ii) individuals require progressively increasing compensation if they are to maintain constant expected utility when confronted by increasing risk. Jones-Lee et al. [16], for example, embodies both conditions. These conditions could be excessively restrictive in the sense that they excise common and plausibly significant features of the individual's decision problem. We investigate the structure of functions representing this individual's willingness to pay to reduce risk when these two restrictions are set aside.

Two bits of theoretical and empirical evidence suggest that the two restrictions lead to misleading results. First, Marshall [22] shows that

exogenous risk requires a complete set of Arrow-Debreu contingent claims contracts. Because the writing of contracts is costly, complete contracts rarely, if ever, exist: the individual must therefore choose between contractually defining states of nature or making an effort to alter states of nature. Spence and Zeckhauser [36] demonstrate that the ability to influence states of nature enhances both the ex ante and the ex post gains from adaptation. Ehrlich and Becker [10], Laffont [19], and Crocker [5] allow individual prior actions to influence ex post gains. Shogren and Crocker [33] show in a set of controlled experiments that these prior actions influence the individual's ex ante willingness to pay for collective risk reduction efforts.

Second, in a contingent valuation study of the risk valuations attached to hazardous waste exposures, Smith and Desvousges [34, 35] report increasing marginal valuations with decreasing exogenous risk. This finding is but the latest in a 15-year-long parade of analytical (Starett [40], Winrich [45]) and empirical (Crocker [6], Repetto [28]) papers which use prior information on physical dose-response relations, individual abilities to process information about these relations, or individual perceptions of the relations to produce an increasing marginal valuation result for more of a desirable commodity. However, when risk is endogenous, no one has yet asked whether convexity of the marginal value of risk follows when cognition is not an issue.

Berger et al. [2] appear to be among the first to consider endogenous risks in the context of human health. Our treatment differs from their seminal effort in two significant ways. First, though they state the general continuous distribution case of risks to human health, they examine ex ante value only in a world of two mutually exclusive and independent states of nature: survival or death. We extend the ex ante value concept to the

general continuous case, while presuming that individuals recognize that outcomes are stochastically related to actions, implying that predictions of behavior and the relative values that motivate it depend not only on preference orderings over outcomes, but also on preference orderings of lotteries over outcomes.¹

Second, Berger et al. [2] model only probability-influencing self-protection. They disregard the severity of the health outcome being risked, even though they concede that prior self-protection can influence both ex ante probability and ex post severity. Similarly, Lewis and Nickerson [21] work with self-insurance that influences ex post severity, but they do not allow the individual to affect ex ante probabilities. Ehrlich and Becker [10] point out that the distinction between self-protection that influences probability and self-protection that influences severity is somewhat artificial. The distinction is often made for theoretical convenience (see, for example, Hiebert [13]). In contrast, we model the effects of self-protection that influences both the probability and the severity of the undesired state, and we consider the effects on the ex ante value of reduced risk. This allows us to develop three propositions:

- 1) Given moral hazard, when self-protection influences the probability, the severity, or both of an undesirable state, unobservable utility terms cannot be eliminated from the individual's ex ante valuation expression. Consequently, empirical studies that attribute differences across groups in ex ante value estimates solely to unobserved differences in household health production technologies are misplaced.

2) With moral hazard and self-protection, knowledge of the convexity or nonconvexity of physical dose-response relations is insufficient to sign unambiguously the change in an individual's ex ante marginal valuation for a reduction in the level of the hazard, even when consumer cognition is perfect. Therefore, we do not support the traditional argument that those individuals exposed to greater risk with greater income must place a higher value on a given risk reduction.

3) With moral hazard, an increase in the level of the environmental hazard does not necessarily lead to an increase in the level of self-protection. Therefore, self-protection expenditures are not a consistent lower bound of the ex ante value a risk averse individual attaches to a reduction in risk.

These three statements imply that several propositions originally developed for cases of exogenous risk and which form the analytical basis for most recent empirical work on the value of health risk changes are not immediately transferable to settings where endogenous risks prevail.

2. SELF-PROTECTION AND RISK

Psychologists agree that individuals perceive that they have substantial control over uncertain events (Perlmutter and Monty [26]). Stallen and Tomas [39] conclude that "...the individual is not so much concerned with estimating uncertain parameters of a physical or material system as he is with estimating the uncertainty involved in his exposure to the threatening event and in opportunities to influence or control his exposure" (emphasis added). Starr [39], an engineer, makes much of the difference between voluntary (endogenous)

and involuntary (exogenous) exposures to risk. Indeed, rare is the noneconomic discussion of risk that does not consider "...measures that modify events or reduce the vulnerability to loss" (Kates [18, p. 7]). People move or reduce physical activities when air pollution becomes intolerable, they buy bottled water if they suspect that alternative supplies are polluted, they chelate children who have high blood lead concentrations, and they apply sunscreen to protect their skins from UV radiation. Finally, if one sets aside its risk valuation component, endogenous risk considerations are abundant in technical economic discourse.^{2/}

At the policy level, the success of collective safety mandates often depends upon individual choices. Auto seat belts, when worn, reduce both the probability and the severity of injury, but their mandatory installation cannot guarantee that passengers will choose to wear them. Workplace safety initiatives involving personal protective gear (e.g., hard hats) have the same problem. Highway speed limits are yet another example. In each case, individual decisions influence both the chance and the magnitude of harm.^{3/}

Individuals often substitute self-protection that is expected to reduce hazard probability or severity or both for collectively supplied safety programs. Burton et al. [3] enumerate numerous examples including the use of higher-strength building materials in response to prospective tornado, storm surge, and earthquake hazards, more thorough weeding and crop storage in response to the prospect of drought, sandbagging and evacuation in anticipation of floods, and improved nutrition and exercise regimens to cope with health threats. These and similar private coping strategies reduce the individual's chance of having a threat realized and its magnitude if it is realized.

Finally, recognition of the frequently endogenous nature of risk raises questions about the assessment-management bifurcation now common in scientific and policy discussions about environmental risks to human health and property. Broadly, risk assessment, because it defines what risk levels are, is considered to be the exclusive domain of the natural and the biomedical sciences, while risk management is left to the law, politics, philosophy, economics, and the sciences (National Academy of Sciences [24]). However, endogenous risk implies that observed risks are functions of natural science parameters and the self-protection decisions of individuals. Alternatively stated, the risks on the basis of which people make decisions will differ across individuals with the relative marginal productivities of their self-protection efforts, even though the properties of the natural phenomena that trigger these efforts may apply equally to everyone. It follows that attempts to assess observed risk levels solely in natural science terms may be highly misleading: costly self-protection is endogenous and may thus vary systematically in the observed risk data. Economic parameters enter and the manner in which they do so depends upon the relative values that people assign them. Some properties of these values for the case of endogenous risk are established in the next section.

3. THE MODEL

Consider an individual who is involuntarily exposed to a health risk under a particular liability regime. Assume the risk is created by exposure to an ambient concentration of given duration of an environmental hazard, r , taken from the real interval, R :

$$R = [\underline{r}, \bar{r}]. \quad (1)$$

Because of moral hazard, the individual cannot acquire enough market insurance to avoid the risk completely. If he were able to do so, the risk would be exogenous (Marshall [22]). The individual must decide from a real interval, S , how much self-protection, s , to undertake:

$$S = [\underline{s}, \bar{s}]. \quad (2)$$

Given exposure to the hazard, the individual is uncertain as to where in a continuum of health outcomes, h , he will be. Let $h(s, r)$ denote the outcome space, where outcomes are the individual's human health capital returns ordered from smallest to largest, given the individual's genetic and development history.

Let $f(h; s, r)$ denote the probability of a particular outcome occurring given that self-protection, s , is undertaken and that the exposure level to the environmental hazard is r . Assume the following about $f(\cdot)$:

Assumption 1: $f(h; s, r) > 0$ for every $s \in S$ and $r \in R$.

Let $F(h; s, r)$ denote the corresponding distribution function defined over the support $[a, b]$

$$F(h; s, r) = \int_a^b f(h; s, r) dh \quad (3)$$

where a and b are the minimum and maximum health outcomes.^{4/} We assume the following about $F(\cdot)$:

Assumption 2: $F(h; s, r)$ is twice continuously differentiable in $s \in S$ and $r \in R$ for every health outcome.

Assumption 3: $F_s(h; s, r) \leq 0$ for every $s \in S$ and $r \in R$ and every health outcome in the sense of first-order stochastic dominance, where a subscript denotes a partial derivative.^{5/}

Assumption 4: $F_r(h; s, r) \geq 0$ for every $s \in S$ and $r \in R$ and every health outcome in the sense of first-order stochastic dominance.

Assumption 5: No restrictions are placed on the convexity of the distribution function in the immediate neighborhood of an optimal level of self-protection, s^* , for all $s \in S$ and $r \in R$ and for every health outcome.

The individual is risk averse with a von Neumann-Morgenstern utility index over wealth W , $U(W)$. The following assumptions are made about $U(W)$:

Assumption 6: U is defined over the real interval $[\bar{W}, \infty)$ where \bar{W} is 0.

Assumption 7: $\lim_{W \rightarrow \bar{W}} U(W) = -\infty$.

Assumption 8: U is strictly increasing, concave, and thrice continuously differentiable.

For each health outcome the individual might realize, he selects a minimum cost combination of medical care and foregone work and consumption.

Let $C = C(h; s, r)$ (4)

be his ex ante expectation of realized costs which depend on the uncertain health outcome, self-protection, and the exposure level to the hazard. Assume the following about $C(\cdot)$:

Assumption 9: C is strictly decreasing, convex, and thrice continuously differentiable in $s \in S$ for every health outcome such that $C_s < 0$, $C_{ss} > 0$, and $C_{sh} \neq 0$ for all h .

Assumption 10: C is strictly increasing and thrice continuously differentiable in $r \in R$ for every health outcome such that $C_r > 0$, and $C_{rh} \neq 0$. No restrictions, however, are placed on C_{rr} for all h .

Given incomplete insurance purchases, intertemporally separable utility, and constant expected prices for medical care, the individual's choice problem is then

$$\text{Max}_{s \in S} \left[\int_a^b U(W - C(h; s, r) - s) dF(h; s, r) \right]. \quad (5)$$

Note that the price of self-protection has been normalized to unity.

Given the model, we are now able to develop the propositions stated in the introduction.

4. EX ANTE VALUE AND WILLINGNESS TO PAY

4.1 Endogenous Risk. A few very recent refinements to the willingness to pay approach to valuing environmental hazards have acknowledged the frequently endogenous form of the problem. For example, Rosen [30], Berger et al. [2], and Viscusi et al. [4] note that self-protection affects survival or injury probabilities, while Shibata and Winrich [32] and Gerking and Stanley [11] allow self-protection to influence the severity of ex post damages. In a nonstochastic world or in an uncertain world with only two feasible states, these studies demonstrate that marginal willingness to pay can be expressed solely in terms of the marginal rate of technical substitution between hazard concentrations and self-protection. This result cannot be generalized to a continuous world with endogenous risk.

Proposition 1: Given the model assumptions, when self-protection influences either the probability or the severity of health outcomes or both, the individual's marginal willingness to pay for reduced risk cannot be expressed solely in terms of the marginal rate of technical substitution between ambient hazard

concentrations and self-protection. In particular, unobservable utility terms cannot be eliminated from expressions for the ex ante value of reduced risk.

. Proof: To show that for a continuous distribution the individual's compensating variation statement of willingness to pay for reduced risk includes the unobservable utility terms, we examine self-protection that influences either the probability distribution or the severity (costs) of the health outcomes or both.

First, maximize the expected utility index (5) by selecting an optimal level of self-protection $s^* \in S$ yielding the following first-order condition for an interior solution

$$EU_w = -E[U_w C_s] + \int_a^b U_w C_h F_s dh. \quad (6)$$

The left-hand side of (6) represents the marginal cost of increased self-protection in terms of the utility of foregone wealth. The right-hand side reflects two types of marginal self-protection benefits: the first term is the direct utility effect of enhanced wealth resulting from reduced expected ex post costs; the second term is the indirect utility effect of a stochastically dominating change in the distribution of health outcomes.

The indirect effect was derived by integrating by parts the effect of self-protection on the distribution

$$\begin{aligned} \int_a^b U(\cdot) dF_s(\cdot) &= U F_s \Big|_a^b + \int_a^b U_w C_h F_s dh \\ &= \int_a^b U_w C_h F_s dh, \end{aligned}$$

since $F_s(a; \cdot) = F_s(b; \cdot) = 0$. Assume that improved health outcomes will decrease the ex post costs, $C_h < 0$.

Solve for the compensating variation statement of the willingness to pay for reduced risk by totally differentiating the expected utility index (5), and then applying the first-order condition (6). When self-protection influences both the probability and severity of health outcomes such that $F_s < 0$ and $C_s < 0$, the willingness to pay expression is

$$\frac{dW}{dr} = - \left[\frac{\int \frac{UC}{w} \frac{F}{h} \frac{dh}{r} - \int \frac{UC}{w} \frac{dF}{r}}{\int \frac{UC}{w} \frac{F}{h} \frac{dh}{s} - \int \frac{UC}{w} \frac{dF}{s}} \right] > 0, \quad (7)$$

where all integrals are evaluated over the support $[a, b]$. Obviously, the unobservable utility indexes cannot be removed from the individual's willingness to pay expression (7).

Even the assumption of a simple two-state world fails to remove the utility terms from (7). For example, let $\pi(s, r)$ and $(1 - \pi(s, r))$ respectively represent the subjective probabilities of healthy and of sick states. Let $U_0(W - s)$ and $U_1(W - s - C(s, r))$ be the expected utility of being healthy or sick, where $U_0 > U_1$. The individual thus chooses $s \in S$ to maximize

$$EU = \pi(s, r)U_0(W - s) + (1 - \pi(s, r))U_1(W - s - C(s, r)). \quad (8)$$

Following the same steps as before, the willingness to pay expression is

$$\frac{dW}{dr} = - \left[\frac{\pi_r [U_0 - U_1] - (1 - \pi) U_1' C_r}{\pi_s [U_0 - U_1] - (1 - \pi) U_0' C_s} \right] > 0, \quad (9)$$

where $\pi_r < 0$, $\pi_s > 0$, $U_1' = \partial U_1 / \partial W$, and $U_0' = \partial U_0 / \partial W$. Again, utility terms cannot be removed irrespective of state independence or dependence.

Next allow, as do Gerking and Stanley [11], self-protection to influence the severity, $C_s < 0$, but not the probability, $F_s = 0$, of health outcomes. Further assume that $F_r = 0$, which, with $F_s = 0$, implies that neither

collective nor individual actions will influence the probability of a particular health outcome; i.e., hazard concentrations resemble sunspots or the phases of the moon. With these assumptions, expression (7) reduces to

$$\frac{dW}{dr} = - \frac{E[U_w C_r]}{E[U_w C_s]} = - \left[\frac{EU_w EC_r - \text{cov}(U_w, C_r)}{EU_w EC_s - \text{cov}(U_w, C_s)} \right] > 0. \quad (10)$$

For the unobservable utility terms to be absent from (10), the two covariance expressions must be zero; however, our model assumptions do not allow them to be zero. Therefore the two utility terms cannot be removed.

Finally, assume, as does Rosen [30], that self-protection affects probability, $F_s < 0$, but not severity, $C_s = 0$. In Rosen's [30] terms, one cannot be more severely dead. For similar reasons, $C_r = 0$. Under these conditions, expression (7) reduces to

$$\frac{dW}{dr} = - \frac{\int U_w C_r F dh}{\int U_w C_s F dh} \quad (11)$$

and again the willingness to pay expression cannot be rid of the unobservable utility terms, which concludes the proof.

We could examine additional cases. For example, self-protection might influence only the probability of a health outcome, but hazard concentrations could affect probability and severity, or vice versa. The results would not change: utility terms would loom up in the willingness to pay expressions, implying empirical efforts that use observed behavior data, and that policy efforts to aggregate across individuals and to account simultaneously for the reality of probability and severity unavoidably involve interpersonal utility comparisons.^{6/}

Given the assumptions of the model and our above development of it, the sufficient conditions under which Proposition 1 would not hold can be stated as a corollary.^{7/}

Corollary 1: Utility terms will not appear in ex ante willingness to pay expressions for endogenous risk changes if and only if at least one of the following conditions is true:

- a) A two-state world exists where ex ante self-protection affects only ex ante probability;
- b) A two-state world exists where ex ante self-protection affects only ex post severity, and the marginal utilities between states are equal;
- c) States are discrete, ex post severity is independent of ex ante self-protection, and a unique self-protection activity exists that exerts no cross-partial effects across states.

Corollary 1a clearly fits some stark life and death situations. In addition, self-protection can reduce the probability of diseases like cancer without changing its severity. Substantial imagination is required to think of real situations corresponding to 1b. Corollary 1c might apply where there are multiple forms of a disease like cancer. It requires that actions taken to avoid skin cancer, for example, do not change the probability of lung cancer.

4.2 Nonconvex Dose-Response Relations. Proposition 1 poses hurdles to procedures which use observed behavior data or which would establish a social risk-benefit test by summing unweighted compensating or equivalent variations across individuals.^{8/} Yet another problem for these procedures is the ambiguous effect that a change in hazard concentrations has on the sign of

compensating variation.

An individual's marginal compensating variation can be shown to be ambiguous in sign even if the strongest possible case for negative effects of increased hazard exposure is imposed. To illustrate, define strong convexity as follows. Definition 1: Strong convexity of risk is defined as: convex ex post cost, $C_{\pi} > 0$; convexity of the distribution function, $F_{\pi} > 0$; and declining marginal productivity of self-protection, $C_{sr} > 0$, $C_{hr} > 0$, $C_{sh} > 0$ and $F_{sr} > 0$.

Strong convexity describes the conditions most favorable for the traditional argument that increased risk requires progressively increasing compensation to maintain a constant level of expected utility. Increased exposure increases the probability and the expected ex post costs of undesirable health outcomes to the hazard at an increasing rate; moreover, the marginal productivity of self-protection is decreasing across the board.

The opposite case is strong nonconvexity. Strong nonconvexity defines the weakest case for negative effects of increased exposure to the hazard. Definition 2: Strong nonconvexity of risk is defined as: nonconvex ex post cost, $C_{\pi} < 0$; concavity of the distribution function, $F_{\pi} < 0$; and increasing marginal productivity of self-protection, $C_{sr} < 0$, $C_{hr} < 0$, $C_{sh} < 0$ and $F_{sr} < 0$.^{9/}

The following proposition states the result:

Proposition 2: Even in the absence of cognitive illusions or failure to consider all scarcity dimensions of the risk-taking problem, a maintained hypothesis of strong convexity of risk is insufficient to guarantee that increased exposure to a hazard requires progressively increasing compensation to maintain a constant level

of expected utility. Similarly, strong nonconvexity is insufficient to guarantee progressively decreasing compensation.

The proposition is supported by Dehez and Drèze [8, p. 98], who show that the sign of the marginal willingness to pay for safety given an increase in the probability of death is generally ambiguous. Drèze [9, p. 172] concludes that any assertions about this sign given a change in safety "...must be carefully justified in terms of underlying assumptions."

Proposition 2 contradicts the argument of Weinstein et al. [44] and others that individuals at greater risk must have a greater demand for safety. Consequently, contrary to Rosen [30], individuals at greater risk with greater wealth cannot necessarily be weighted more heavily when risk reductions are valued. Similarly, the assertions by Kahneman and Tversky [17] and Smith and Desvousges [35] that increasing marginal willingness to pay for reduced risk constitutes a lapse from rational economic behavior are not supported.¹⁰

Proof: To demonstrate that an increase in hazard concentration has an ambiguous effect on an individual's compensating variation, differentiate the compensating variation in expression (7) with respect to the hazard exposure:

$$\begin{aligned} \frac{d(dW/dr)}{dr} = & -\frac{1}{\Omega} \left[E[U_{ww} C_r^2 - U_{wrr} C_r] - 2 \left\{ [U_{ww} C_r C_h - U_{whr} C_r] F_r dh + [U_{wh} C_r F_{rr} dh] \right\} \right. \\ & + \frac{1}{\Omega^2} \left[E[U_{ww} C_s C_r - U_{w sr} C_r] + [U_{whr} C_s - U_{ww} C_h C_r] F_s dh \right. \\ & \left. \left. + [U_{ww} C_s C_r - U_{w sr} C_r] F_r dh + [U_{wh} C_s F_{sr} dh] \right] \right], \end{aligned} \quad (12)$$

where

$$\Omega = \int U_{wh} C_s F_s dh - \int U_{ws} dF > 0,$$

$$\Delta = \int U_{wh} C_r F_r dh - \int U_{wr} dF < 0,$$

and all integrals are evaluated over the support [a, b].

The terms on the right-hand side of (12) can be defined in terms of

direct and indirect utility effects given an increase in exposure to a hazard. $\Omega > 0$ and $\Delta < 0$ represent the combined first-order direct and indirect utility effects of s and r . The first and fourth terms in (12) represent second-order direct utility effects on expected costs with an increase in exposure. Given strong convexity, the sign of the first term is negative. The sign of the fourth term is ambiguous in the sense that alternative parameterizations are conceivable in which either $U_{ww}C_sC_r$ or U_wC_{sr} dominates in absolute magnitude. The second, fifth, and sixth terms are second-order direct and indirect utility effects weighted by the marginal effect on the distribution of either s or r . Given strong convexity, the signs of all three terms are ambiguous in the above sense. Without prior information on the magnitude of the marginal effects on the expected cost function, there is no reason to expect one term to dominate. The third and seventh terms represent the second-order indirect and cross-indirect utility effects of increased exposure. By the definition of strong convexity, the sign on both terms is negative. Without knowing the relative magnitude of all the direct and indirect utility effects, however, strong convexity is insufficient to sign (12) unambiguously. Likewise, the assumption of strong nonconvexity is also insufficient to sign (12). Whether one imposes strong convexity or strong nonconvexity the sign of (12) is ambiguous. Although numerous sufficient conditions for increasing or decreasing marginal willingness to pay can be determined, there is, in the absence of prior information or simple ad hoc assumptions, no reason to expect that one or two terms will dominate expression (12). This concludes the proof. Intuitively, the results occur because a changed exposure that induces self-protection may have productivity effects on probability that differ from those on severity. The only clear-cut sufficient condition for signing (12)

is the absence of all severity effects. This is stated as Corollary 2.

Corollary 2: Assuming no severity effects ($C_s = C_r = 0$), then the assumption of strong convexity is sufficient to guarantee increasing marginal ex ante valuations with increasing exposures.

Again, diseases like cancer or events like death seem the only apt examples that clearly fit the corollary.

4.3 Self-Protection Expenditures as a Lower Bound. Consideration of self-protection has not been limited to problems of ex ante valuation under uncertainty. A substantial literature has emerged, e.g., Courant and Porter [4] and Harrington and Portney [12], which demonstrates that under perfect certainty the marginal benefit of a reduction in a health threat is equal to the savings in self-protection expenditures necessary to maintain the initial health state. This result cannot be extended to the uncertainty case when self-protection influences both ex ante probability and ex post severity.

Proposition 3: Neither strong convexity nor strong nonconvexity of risk is sufficient to sign the effect of a risk change upon self-protection expenditures. Therefore these expenditures cannot be used to determine the welfare effect of a risk change.

Proposition 3 contradicts Berger et al.'s [2] argument that if increased exposure increases the marginal productivity of self-protection, $F_{s,r} < 0$, then self-protection will increase with exposure. Consequently, Berger et al.'s [2, p. 975] sufficient conditions for "plausible" results do not hold when self-protection influences both probability and severity.

Proof: To demonstrate that strong convexity is insufficient to determine the effect increased hazard exposure has on self-protection, take

the first-order condition in equation (6) and apply the implicit function theorem. The effect of increased exposure on self-protection is

$$\frac{ds}{dr} = - \left[E[U_{ww} C_r (1 + C_s) - U_{wr} C_s] + \int [U_{wsh} C - U_{wh} C (1 + C_s)] F_r dh \right. \\ \left. + \int [U_{whr} C - U_{wr} C C_h] F_s dh + \int U_{wh} F_{sr} dh \right] / D \quad (13)$$

where

$$D = E[U_{ww} C_s (1 + C_s) - U_{ws} C_s] + 2 \int [U_{wsh} C - U_{wh} C C_s] F_s dh \\ - \int U_{wh} C F_s dh + \int U_{wh} F_{ss} dh < 0 \quad (14)$$

and all integrals are evaluated over the support $[a, b]$. D is the second-order sufficient condition of the maximization problem (5), and is assumed to hold whenever (6) holds.

Given $D < 0$, the sign of (13) depends on the sign of its right-hand-side numerator. The first term in the numerator of (13) is the direct utility effect of increased exposure on expected costs. Given strong convexity of risk and $(1 + C_s) > 0$ from the first-order condition, the sign of the first term is negative. The second term reflects the indirect utility effect of increased exposure on the distribution. Given strong convexity, its sign is ambiguous in the earlier defined parameterization sense. The third term is a direct utility effect weighted by the marginal effect of self-protection on the distribution ($F_s < 0$), and its sign is also ambiguous. The signs for the second and third effect are ambiguous since there is no a priori reason to believe that any one set of terms dominates the others. The fourth term in the numerator is the cross-indirect utility effect of increased exposure. Given strong convexity, its sign is negative. Therefore, without prior information on the relative magnitudes of the four direct and indirect utility

effects, strong convexity is insufficient to sign (13) unambiguously. Given the conditions most favorable to the traditional argument that increased risk will increase self-protection, we still require prior information on the impact that increased exposure has on the marginal productivity of self-protection to support the argument.

Following the logic above, an assumption of strong nonconvexity of risk leads to a similar conclusion of an ambiguous effect of increased exposure on self-protection. Consequently, since self-protection may decrease as exposure to a hazard increases, self-protection expenditures cannot be considered a consistent lower bound on the ex ante value a risk averse individual attaches to a reduction in risk. This concludes the proof. The only clear-cut cases in which these expenditures would be a lower bound can be stated as a corollary.

Corollary 3: Sufficient conditions for self-protection expenditures being a lower bound on the ex ante value of risk reductions include:

- a) $C_{sr} < 0$, which is true under strong nonconvexity, and $F_s = F_r = 0$.
- b) $F_{sr} > 0$, which is true under strong convexity, and $C_s = C_r = 0$.

Examples of Corollary 3a are not obvious; cancer and death provide the best examples for Corollary 3b.

5. CONCLUSIONS AND IMPLICATIONS

Individuals and policymakers use self-protection activities to influence both their ex ante risks and their expected ex post consequences. Given, as we have argued, that both forms of self-protection jointly occur in practice with great frequency, the implications of this for efforts to value risks to human health and property are unequivocally negative. Only the corollaries

provide a rather pinched basis for optimism about the efficiency of traditional risk valuation efforts. With a parsimonious model in which only wealth provides direct utility, we show that unobservable utility terms cannot be eliminated from marginal willingness to pay expressions, implying that empirical efforts which identify marginal rates of substitution with willingness to pay are misdirected. We also show that even under the most favorable restrictions increased risk need not imply progressively increasing levels of compensation in order to restore initial utility levels. Consequently the traditional argument that those who are exposed to greater risk and have greater wealth must value a given risk reduction more highly does not follow. Finally, we demonstrate that increased risk need not imply increased self-protection expenditures; thus, changes in these expenditures may not bound the value of a risk change.

Some succor for risk valuation efforts could be obtained by stepping outside professional boundaries to draw upon prior information from psychology, biomedicine, and other disciplines. Insight might therefore be gained into the signs and the relative magnitudes of many terms in expressions (12) and (13). It is odd that the field of economics, which explicitly recognizes the policy relevance of incomplete markets, has historically been reluctant to use information from other disciplines in order to simulate the valuation results of a complete market. We recognize that there is a growing trend to incorporate restrictions about structure, functional forms, and parameter values from other disciplines into the behavioral postulates of economic models.¹¹ The results of this paper suggest that the incorporation process should be accelerated. With nonexperimental data, the Bayesian diagnostic techniques of Leamer [20] could be used to establish systematically

the restrictions to which estimates of (12) and (13) are especially sensitive. Controlled experiments could be used for the same purpose. We report elsewhere (Shogren and Crocker [33]) results of controlled experiments showing empirically that self-protection increases the willingness to pay for risk reduction, where, by definition, the reductions are collectively and self-supplied. Although this result conforms neatly to the Le Chatelier principle as well as to Spencer and Zeckhauser [37], similar empirical rather than purely theoretical analyses are likely to be required if the complexities offered by the three propositions in this paper are to be overcome.

Incorporation and more empirical analysis will not overcome, however, the aggregation problems posed by the presence of utility terms in individuals' willingness to pay expressions. Approaches to aggregate risk-benefit analysis do exist other than the mechanical summation of consumer surpluses calculated from the singular value judgement that social welfare and aggregate total income are synonymous. Given that individual consumer surpluses can be estimated, one possibility is to draw upon the extensive equivalence scale literature, e.g., Deaton and Muellbauer [7], in order to weight each individual or household. Tradeoffs can then be evaluated using an explicit social welfare function which recognizes that personal health is in part self-produced and inalienable. Alternatively, utilities might be calculated directly.

List of Symbols

0 is zero.

= is equal to.

∫ is an integral sign.

≥ is greater than or equal to.

≤ is less than or equal to.

ε is script Epsilon.

∞ is infinity.

≠ is not equal to.

> is greater than.

< is less than.

Ω is Omega.

Δ is print Delta.

π is script Pi.

FOOTNOTES

1. The ability of our individuals to take intervening actions implies that we are working within the "temporal risk" context of Spence and Zeckhauser [37]. Because we directly incorporate these intervening actions into our model, we do not violate the independence axiom of expected utility theory.
2. For example, the moral hazard literature deals with the effect of insurance on an individual's incentives to self-protect, the bidding literature recognizes that the probability of winning depends upon the bid submitted, and the resource depletion literature accounts for the effect that the amount extracted has upon knowledge of additional reserves.
3. The folk truth that "you can lead a horse to water but you can't make it drink" seems appropriate.
4. The $[a, b]$ interval could also be influenced in subsequent periods by self-protection. We disregard this issue.
5. The distribution $G(h)$ first-order stochastically dominates the distribution $F(h)$ when $G(h) \leq F(h)$ for all $h \in [a, b]$, which is equivalent to obtaining $G(h)$ from $F(h)$ by shifting the probability mass to the right.
6. Assumptions of a risk-neutral individual with an identity map of ex post costs would eliminate the unobservable utility terms. These assumptions seem excessively restrictive. Alternatively, one might eliminate the utility terms by using the pointwise optimization technique that Mirrlees [23] and Holmstrom [14] employ. However, pointwise optimization evaluates self-protecting choices individually at each and

- every health state rather than in terms of lotteries over health states. It thus adopts an ex post rather than an ex ante perspective.
7. Proofs of this and of subsequent corollaries are available from the authors upon request.
 8. See Polemarchakis, et al. [27] for recent thinking on aggregation under exogenous risk.
 9. Rogerson [29] assumes that the distribution function must generally satisfy the convexity of the distribution function condition (CDFC). Therefore, the assumption of a concave distribution in r and s is perhaps restrictive. As shown by Jewitt [15], however, the CDFC assumption is not universally required in that it satisfies very few of the standard distributions set forth in statistics textbooks.
 10. Close inspection of the Smith and Desvousges [35, pp. 110-111] questionnaire reveals that respondent opportunities to influence the chance of death and the time to death were not fully controlled. Given the enhanced adjustment opportunities that self-protection provides, the exogenous risk valuations that Smith and Desvousges [35] presume they are reporting would be underestimates of risk reduction values and overestimates of risk increase values. Effects on changes in marginal willingness-to-pay depend upon the manner in which the marginal productivity of self-protection varies with risk.
 11. See Warneryd [42], Weinstein and Quinn [43], and Smith and Johnson [36], for example.

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