# Demand Uncertainty and Price Stabilization

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## 1. Introduction

Price stabilization is an important policy goal of government intervention in competitive markets. These policies are primarily directed at raising producer income and stabilizing market prices at levels acceptable to consumers and producers (Fox [1956], Turnovsky [1978], Newbery and Stiglitz [1979]). Many of the stabilization policy results have been developed from the study of agricultural commodity markets. In these markets, prices tend to be highly variable due to uncertain and inelastic supply and demand (Schultz [1945], Gardner [1981]).

Benefits and costs of price stabilization policies have been studied extensively.[1] Broadly speaking, these analyses have been for either the individual producer/consumer (e.g. Waugh [1944], Oi [1961], Pope, Chavas and Just [1983]) or market level responses (Massell [1969], Turnovsky [1976], Samuelson [1972]). Recently, the linkages between implications of stabilization policy impacts at micro and market levels have been explored systematically. However, with the exception of Kawai [1983], the results are limited to supply uncertainty (Newbery and Stiglitz [1979,1981], Wright [1979], Wright and Williams [1982]).[2]

This paper develops a framework for efficient price stabilization under demand uncertainty. Similar to Newbery and Stiglitz [1979] and Wright [1979], a rational expectations equilibrium model is employed. Stabilization policies may reflect whether the commodity is perishable or storable (Fox [1951a,1951b]). Following Kawai [1983], we assume that the commodity is nonstorable. Hence, the government or stabilization agency does not arbitrage in the commodity or hold inventories as buffer stocks. Instead, direct subsidies (taxes) are used to assure a minimum price for voluntary participants. To regulate excessive supply, a capacity restriction is imposed on the participating producers.

The plan of this paper is as follows. Section 2 describes a stylized stabilization program with voluntary participation. A rational expectations equilibrium model incorporating this stabilization program is then developed. Section 3 shows how stabilization policy parameters affect producer participation and factor intensity. In Section 4, risk premiums and factor use levels are aggregated and used to investigate the impacts of changes in stabilization policy parameters on market supply response. Section 5 provides an analysis of how certainty equivalent producer income are affected by program parameters. Section 6 introduces government cost and the efficiency of stabilization policies. A brief summary and some concluding remarks are contained in Section 7.

2. Government Intervention and Market Equilibrium

Consider a competitive industry producing a homogeneous commodity. Market demand is assumed stochastic and given by

$$p = p(Q,u), \ \partial p/\partial Q < 0$$

where p, Q and u denote market price, output and a random variable with a distribution function H(u), respectively. For a particular market output Q, the distribution H(u) completely determines a price distribution $F(p|Q)$. The expected inverse demand function is given by

$$\bar{p}(Q) = \int_0^\infty p dF(p|Q) = \int_{-\infty}^\infty p(Q,u) dH(u).$$

When the market price is uncertain, producers may sell their outputs at a high price but also face the risk of selling at a low price. Baron [1970] and Sandmo [1971] have shown that for a given price distribution, a risk averse firm will produce less than if the mean price were known with certainty.

Consider the stylized stabilization policy $(\alpha, p_m)$ where $\alpha$ is an acreage (capacity) parameter representing the portion of land cultivated by participating producers, $0 < \alpha < 1$, and $p_m$ is a guaranteed minimum price.[4] Since the commodity is not storable, the government uses a direct subsidy to assumre the minimum price for the participants. Assume that participation in a goverment stabilization program is voluntary, but that partial participation is not allowed. Nonparticipants sell at the random market price p with distribution $F(p|Q)$, whereas participants receive a random price v with distribution $G(v|Q)$.

To distinguish between $F(p|Q)$ and $G(v|Q)$, observe that the participant receives $v = p_m$, implying a per unit subsidy of $(p_m - p)$. Thus, the minimum price truncates the distribution $F(p|Q)$ at $p_m$, eliminating the downside price risk. For market prices above $p_m$, $v = p$. Thus, the probability density function of the price v for the participants, $g(v|Q) \equiv G'(v|Q)$, is obtained by truncating that of the market price $f(p|Q) \equiv F'(p|Q)$ at $p_m$. That is

$$g(v|Q) = \begin{cases} F(p_m|Q) & \text{for } v = p_m \\ f(p|Q) & \text{for } v = p > p_m \\ 0 & \text{for } v < p_m. \end{cases}$$

Participation

Voluntary participation has been a common feature of stabilization programs in U.S. agricultural commodity markets. Consider the participation decision of an individual firm. The production function is assumed to be identical for all producers. However, producers can be different in their attitudes toward risk. The production function for firm i is

$$q_i = q(K_i, L_i)$$

where $q_i$ is output, $K_i$ is a nonland input and $L_i$ is land. This production function $q(.)$, common to all producers, is assumed monotone increasing, quasiconcave and linearly homogeneous to permit the aggregation of firm output.

Let $\bar{v} = \int v dG(v|Q^e)$ and $\bar{p} = \int p dF(p|Q^e)$ be the conditional expected prices for participants and nonparticipants, respectively, where $Q^e$ is the estimated industry output. Let w and r denote the prices of the land and nonland inputs. Although the land price is ultimately endogenously determined in our market equilibrium framework, each producer is assumed to be a price taker in the factor markets. Note that in the production function, $L_i$ denotes the land actually utilized by producer i. Due to the acreage restriction, a participant using $L_i$ units of land must control $(L_i/\alpha)$ units of land.

For participant i, the expected utility of profit is

$$EU^i \equiv \int_{p_m}^{\infty} U^i[vq(K_i,L_i) - rK_i - (w/\alpha)L_i]dG(v|Q^e) = U^i[P^i]$$

where $U^i(.)$ is a monotone increasing and concave von Neumann-Morgenstern utility function, and $P^i \equiv (U^i)^{-1}\{EU^i\}$ is certainty equivalent (CE) profit. The CE profit can be explicitly written as

$$P^i(Q^e) = \bar{v}q(K_i,L_i) - rK_i - (w/\alpha)L_i - R_i \tag{1}$$

where $R_i$ is the Arrow-Pratt risk premium. Since $U^i(.)$ is concave, $R_i$ must be positive for all nondegenerate price distributions. Similarly, the expected utility of a nonparticipant j is

$$EU^j \equiv \int_0^{\infty} U^j[pq(K_j,L_j) - rK_j - wL_j]dF(p|Q^e) = U^j[N^j]$$

where $N^j \equiv (U^j)^{-1}\{EU^j\}$ is CE profit of the nonparticipant. More explicitly,

$$N^j(Q^e) = \bar{p}q(K_j,L_j) - rK_j - wL_j - R_j \tag{2}$$

where $R_j$ is the risk premium. The ith producer participates in the stabilization program if max $P^i \geq$ max $N^i$, and not participate if max $P^i <$ max $N^i$.

Rational Expectations Equilibrium

In their seminal papers Baron [1970] and Sandmo [1971] analyzed how uncertain prices affect the output decision of a risk averse competitive firm. In these studies, the producer is a stochastic price taker and makes production decisions before the market price is known. Although uncertain about the market price, the producer is presumed either to know the price distribution F(p) or to use a subjective price distribution $F^e(p)$. Thus, *ex ante* firm output depends on the distributon of the market prices, although it is independent of the market price ultimately revealed.

Critical to many of the results on the *ex ante* production model is the idea of rational expectations. For market equilibrium to be workable, the competitive firms must be capable of confirming their subjective price distributions *ex post*. Subjective price distributions and output levels are revised simultaneously. Newbery and Stiglitz [1981] suggest that, for the market to be in (short run) equilibrium, the firms must have rational expectations, i.e., the subjective and actual price distributions must be equal. If the actual price distribution deviates from the subjective price distribution, firms will adjust their outputs accordingly. Hence, the market will not be in equilibrium except if by perchance the output adjustments of firms happen to cancel one another. Once firms confirm their rational price expectations, further output adjustment is not stimulated and the market is in equilibrium.

However, it is not a simple task to verify that expectations are rational. At the end of the period, when the market price is realized, the firms observe only a single market clearing price, not the distribution $F(p)$. If the competitive firms were to confirm the distribution of market prices by this process, it would require repeated experiments. This confirmation procedure may not be tolerably timely in agricultural commodity markets with a production period of one year. Thus, indirect and more timely methods of confirming price expectations are necessary, if rational expectations are to be tractable in modeling ex ante production in competitive markets.

The price distribution $F(p|Q)$ is conditional on industry output $Q$. We assume that the competitive firms use their subjective estimates of mean industry output $Q^e$ to form subjective conditional price distributions $F(p)^e \equiv F(p|Q^e)$.[6] Since there are many price distributions with positive probabilities for any realized market price, the firms do not use the observed market price to evaluate their subjective price distributions. Instead, they compare the actual output and their subjective estimates of mean industry output. If different, these firms revise their industry output estimates and adjust the conditional price distributions accordingly. A sufficient condition for short run market equilibrium is that each firm's subjective estimate of market output degenerates at $Q$. Expected utility maximization leads the firms to revise their subjective estimates consistent with the equilibrium output $Q$.

Price Stabilization and Market Supply

Participation and production decisions are based on the price distributions, $G(v|Q^e)$ and $F(p|Q^e)$, conditional on estimates $Q^e$ of industry output. In addition, the output of a participant is directly affected by the policy parameters, $\alpha$ and $p_m$. Hence, for each participant $i \in \mathbb{P}$, optimal output can be written $q_i = q_i(Q^e;\alpha,p_m)$. Similarly, for each nonparticipant $j \in \mathbb{N}$, optimal output is $q_j = q_j(Q^e)$. Let $X = \sum_{i \in \mathbb{P}} q_i$ and $Y = \sum_{j \in \mathbb{N}} q_j$, respectively, denote the total output of the participants and the nonparticipants. The market supply function is

$$Q = X(Q^e;\alpha,p_m) + Y(Q^e) \equiv h(Q^e;\alpha,p_m). \tag{3}$$

When the market is not in equilibrium the firms may have different estimates of market output. Since firms can verify their subjective price distributions ex post, market equilibrium requires that all firms have identical and correct price distribution, $F(p|Q^e) = F(p|Q)$,[5] or more simply

$$Q^e = Q. \tag{4}$$

That is, for each firm $Q^e$ degenerates at $Q$. The rational expectations equilibrium output is thus the fixed point of the mapping $h(Q^e;\alpha,p_m)$.

The solution to (3) and (4) can be written as

$$Q^* = Q(\alpha,p_m). \tag{5}$$

Intuitively, expression (5) shows that the government can manipulate industry output by changing the program parameters. In the absence of government intervention, $\alpha = 1$ and $p_m = 0$. Thus, $Q(1,0)$ defines the free market equilibrium.

Figure 1 illustrates how supply response and market equilibrium are derived for a binary random demand function. The upper panel depics random market demands with two states of the world, $u_1$ and $u_2$, each with equal probability. The corresponding demand curves are denoted by $D_1$ and $D_2$, with expected (inverse) demand schedule $\bar{D}$. The lower panel is a graphic representation of equations (3) and (4).

To show how the supply curve $S[\bar{p}(Q^e);\alpha,p_m]$ is derived, assume that producers initially estimate aggregate output to be $Q_A$. The corresponding subjective price distribution for the participants is $F(p|Q_A)$ with expected price $\bar{p}(Q_A)$, shown in the upper panel of Figure 1. Similarly, the nonparticipants use price distribution $G(v|Q_A)$. Aggregate output $h(Q_A;\alpha,p_m)$ is $AQ_A$ in the lower panel or $OQ_A'$ in the upper panel. Thus, the point $A' = [\bar{p}(Q_A),h(Q_A;\alpha,p_m)]$ is on the supply curve, $S[\bar{p}(Q^e;\alpha,p_m)]$. The rational expectations equilibrium is attained at point E, the intersection of the expected market demand curve and the 45 degree line for $Q = Q^e$ in the lower panel. The equilibrium price distribution is $F^*(p) = F(p|Q^*)$. However, the actual market price will be either $p_1$ or $p_2$, depending on the state of the world.

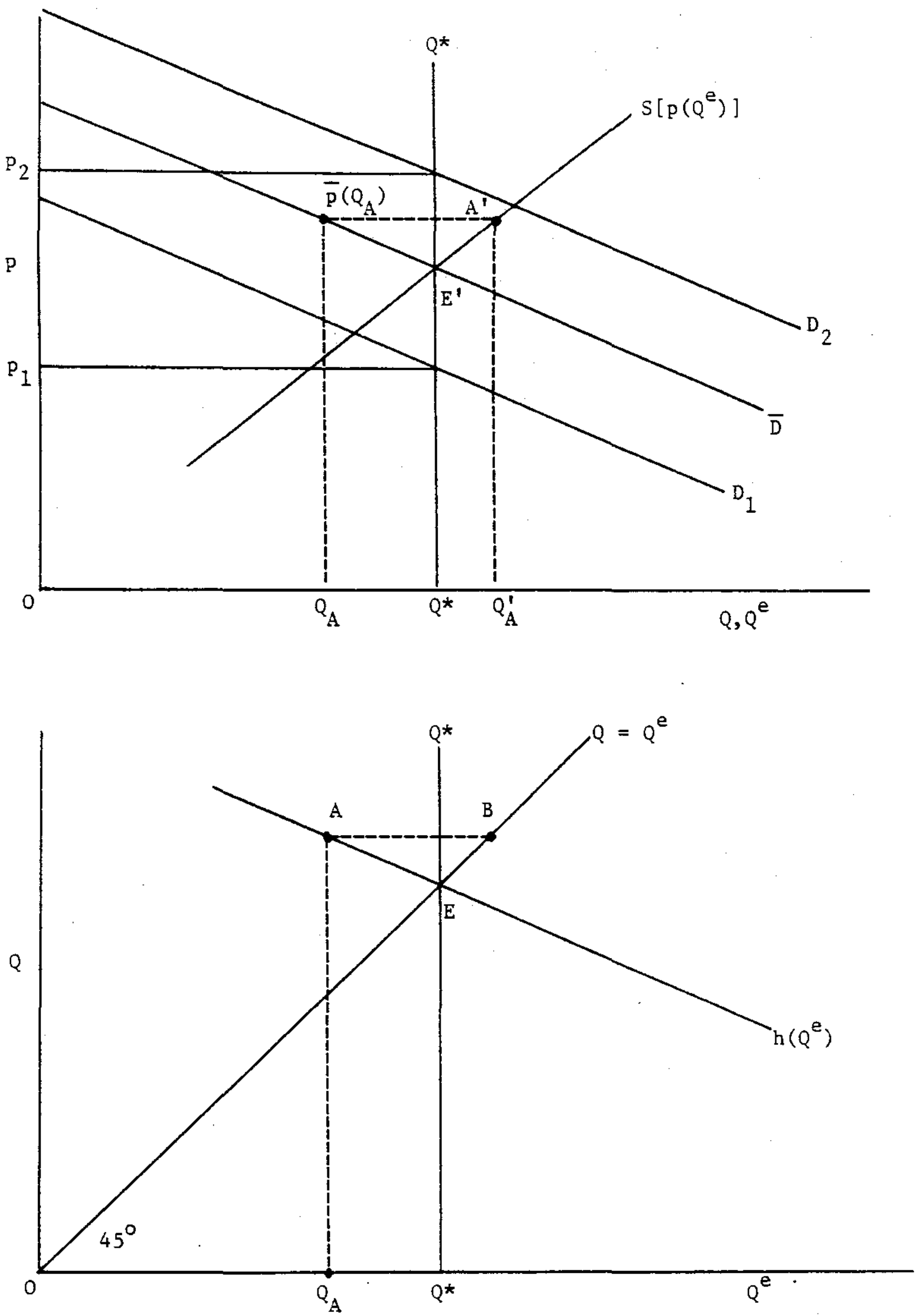


Figure 1. Rational Expectations and Market Equilibrium

## 3. Factor Intensities and Participation

It is well known that if the production function is linearly homogeneous, the choice of production technique depends only on the factor prices and not on the level of output. Moreover, if the market price is known, the output level of a competitive firm is indeterminate. If the market price is uncertain, the output level of a risk neutral firm is also indeterminate. However, indeterminacy of the output level disappears when the competitive firm is risk averse. Producers choose different output levels due to different attitudes toward risk. Baron [1970] demonstrated that the output of a competitive firm is a decreasing function of the firm's index of absolute risk aversion.

Policy parameters directly affect participation and production decisions. But aggregate supply response affects the price distributions, $G(v|Q)$ and $F(p|Q)$. Hence, secondary adjustments are necessary as producers revise their price expectations. The indirect adjustments may reinforce the direct responses or work against them. However, in the latter case the direct responses are likely to outweigh the indirect counterresponses.

Consider, for example, the effects of increases in $\alpha$ and $p_m$ on the equilibrium land use of the participants, $L_1^*$. Differentiating $L_1^*$ with respect to $\alpha$ and $p_m$ gives

$$dL_1^*/d\alpha = (\partial L_1/\partial\alpha)[1 + (\partial Q/\partial L_1)(dL_1/dQ^e)] \tag{7a}$$

$$dL_1^*/dp_m = (\partial L_1/\partial p_m)[1 + (\partial Q/\partial L_1)(dL_1/dQ^e)] \tag{7b}$$

where $\partial L_1/\partial\alpha$ and $\partial L_1/\partial p_m$ represent the direct responses, holding expectations $Q^e$ constant. The common second term in the brackets indicates that as producers adjust their estimates $Q^e$ of industry output, the price distribution $G(v|Q^e)$ and the land utilized $L_1$ are altered.

We assume that the indirect effect, when negative, does not completely offset the direct effect. This is reminiscent to the argument insuring negatively sloped demand curves — despite lack of an a priori reason for it — that the negative income effect of an inferior good is presumed not to more than offset the substitution effect of a price change. Since the equilibrium response and the direct response have the same sign, we concentrate on the latter and investigate how changes in policy parameters affect participation decisions and factor intensities. Of course, in principle, the equilibrium responses can always be obtained from (7a) and (7b) or similar expressions by adjusting for changes in expectations. If expectations adjust slowly, producers are likely to exhibit the direct effects immediately after a policy change.

Participation

Suppose that producers cannot change their inputs and output, an assumption relaxed shortly. Consider a marginal participant i, indifferent between participation and nonparticipation, i.e., $P^i = N^i$. An increase in the minimum price results in a rightward first degree shift in the distribution $G(v|Q)$ and in the distribution of profit. Hadar and Russell [1969] showed that a first degree rightward shift in the distribution increases expected utility for all monotone increasing utility functions. Thus, an increase in $p_m$ increases CE profit $P^i$ of participant i. However, the increase in $p_m$ does not influence CE profit $N^i$, if the marginal producer is a nonparticipant. Thus, an increase in $p_m$ reduces nonparticipants' use of land $L_2$ and increases the participation rate, $L_1/\alpha L$. Alternatively, an increase in $\alpha$ reduces the effective land price $(w/\alpha)$ and increase participant CE profit $P^i$ without affecting nonparticipant CE profit $N^i$. Thus, an increase in $\alpha$ reduces $L_2$ and increases the participation rate, $L_1/\alpha L$.

Next, let producers adjust their inputs in response to changes in policy parameters. Then participant CE profits will be even higher than with the ouptut restriction. In contrast, the nonparticipants will not adjust their output levels and their CE profits will remain unaffected, if they stay outside the program. Thus, nonparticipants have an additional incentive to join the government program. Differentiating $L_1$ with respect to $\alpha$ and $p_m$ gives

$$\partial L_1/\partial\alpha = L_1/\alpha - \alpha(\partial L_2/\partial\alpha) > 0 \tag{8a}$$

$$\partial L_1/\partial p_m = -(\partial L_2/\partial p_m) > 0. \tag{8b}$$

Land Price and Participation

Policy parameters also induce changes in the market rental price of land, w. Equations (8a) and (8b) indicate how changes in program parameters shift the land demand schedule $L_1$, and not the equilibrium land use level after an adjustment in the land price. However, it can be demonstrated that the same inequalities hold, even when endogenous changes in the land price are incorporated. Figure 2 illustrates the impacts of an increase in the value of the acreage parameter on land use. The total eligible land L is measured by PN. The demand for

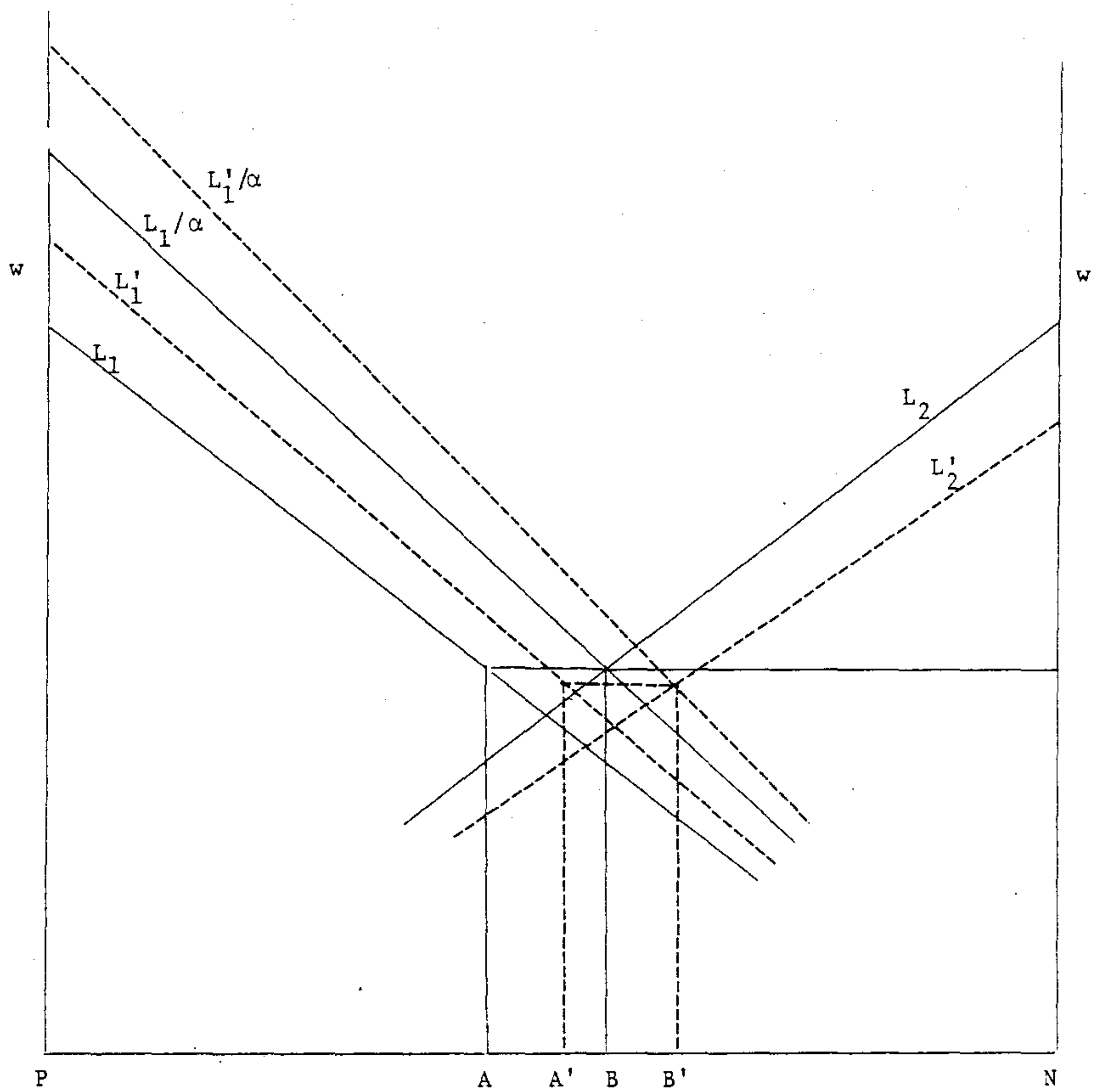


Figure 2. Land Allocation between Participants and Nonparticipants and Land Price

the land used by program participants, $L_1$, is obtained by summing $(\bar{v} - R_i')(\partial q/\partial L_i)$ horizontally. Due to the acreage restriction, the effective land price is $w/\lambda$, and not w, for the participants. Thus, the total land requirement for the program participants, $L_1/\alpha$, is obtained by dividing $L_1$ schedule by $\alpha$, thereby moving $L_1$ upward. The land demand $L_2$ of nonparticipants is obtained by summing $(\bar{p} - R_j')(\partial q/\partial L_j)$ horizontally. The intersection of $L_1/\alpha$ and $L_2$ determines the equilibrium land price, w. At this price, the equilibrium quantity of $L_1$ is PA and that of $L_2$ is BN. The horizontal gap, AB, represents the idle land of the participants complying with the acreage restriction.

An increase in the acreage parameter initially shifts the demand for the total land requirement for participants, $L_1/\alpha$, downward (not drawn). Since a higher value of $\alpha$ increases CE profits of participants without altering CE profits of nonparticipants, switching of producers to become program participants occurs. That is, producers initially outside the program now join the government program. The effect of shifting participation status on the demand for land used by participants, $L_1$, is captured by (8a). When the switching occurs, $(\partial L_1/\partial\alpha)$ is positive and hence both $L_1$ and $L_2$ shift to the right, $L_1'$ and $L_2'$, respectively, as shown in Figure 2. Since $L_1$ shifts to the right, the equilibrium land use $L_1$ at the new land price can decrease only if $L_2$ shifts to the left. But $L_2$ also shifts to the right. Thus, the equilibrium use level $L_1$ must rise with $\alpha$. That is, regardless of the adjustment in the land price, the equilibrium quantity of $L_1$ increases as $\alpha$ increases, $\partial L_1/\partial\alpha > 0$. Thus, the inequality (8a) also holds even if the land price is flexible.

The inequality (8b) shows that an increase in $p_m$ also shifts both the $L_1$ and $L_2$ schedules to the right. The equilibrium use level of $L_1$ also increases with $p_m$, and the inequality (8b) also holds even if the land price is permitted to adjust for changed participation rate.

Factor Intensities and Participation

If a government policy has an acreage restriction, then participants will use different factor combinations to minimize their production costs. Let $\mathbb{P}$ and $\mathbb{N}$ be the group of all participants and nonparticipants, respectively. Then each participant $i \varepsilon \mathbb{P}$ maximizes CE profit and satisfies the first order condition

$$(\bar{v} - R_i')(\partial q/\partial K_i) - r = 0 \tag{9a}$$

$$(\bar{v} - R_i')(\partial q/\partial L_i) - w/\alpha = 0$$

where $R_i' \equiv dR_i/dq_i$ is the marginal risk premium. Similarly, each nonparticipant $j \in \mathbb{N}$ maximizes CE profit and satisfies the first order condition

$$(\bar{p} - R_j')(\partial q/\partial K_j) - r = 0 \tag{9b}$$

$$(\bar{p} - R_j')(\partial q/\partial L_j) - w = 0.$$

Baron [1970] and Sandmo [1971] have shown that a risk averse competitive firm produces less under uncertain price than if the mean price is known with certainty. Moreover, Hartman [1975] showed that the competitive firm minimizes production cost. That is, the expansion path is independent of risk aversion. Let MRTS = $(\partial q/\partial L)/(\partial q/\partial K)$ denote the marginal rate of technical substitution of K for L. From (9a), for each participant $i \varepsilon \mathbb{P}$

$$MRTS^i = w/\alpha r. \tag{10a}$$

Likewise, equation (9b) yields for each nonparticipant $j \varepsilon \mathbb{N}$,

$$MRTS^j = w/r. \tag{10b}$$

Since the production function is linearly homogeneous, the expansion paths are rays from the origin. Thus, $K_i/L_i = k_1$ for each nonparticipant $i \varepsilon \mathbb{P}$, and $K_j/L_j = k_2$ for each nonparticipant $j \varepsilon \mathbb{N}$. Clearly, for the linearly homogeneous production functions, factor intensities are independent of risk aversion. Attitudes toward risk only influence participation and the level of output. Regardless of the level of output, each participant chooses input levels along the expansion path $k_1$ while each nonparticipant chooses input levels along $k_2$, as shown in Figure 3. Moreover, since the isoquants are convex to the origin, factor intensities are increasing in MRTS. This implies that the common factor intensity $k_1$ of the participants decreases as  increases, ceteris paribus. Thus, every participant uses the nonland input more intensively than every nonparticipant, i.e., $k_1(\alpha) > k_2$, $\alpha < 1$.

Figure 3 illustrates the effect of the acreage restriction on factor intensities. For a given isoquant $q = q(K,L)$, a nonparticipant faces the factor price ratio, $w/r$, and accordingly chooses the expansion path $k_2$. The isocost curve $C_2$ is tangent to the isoquant at 2.

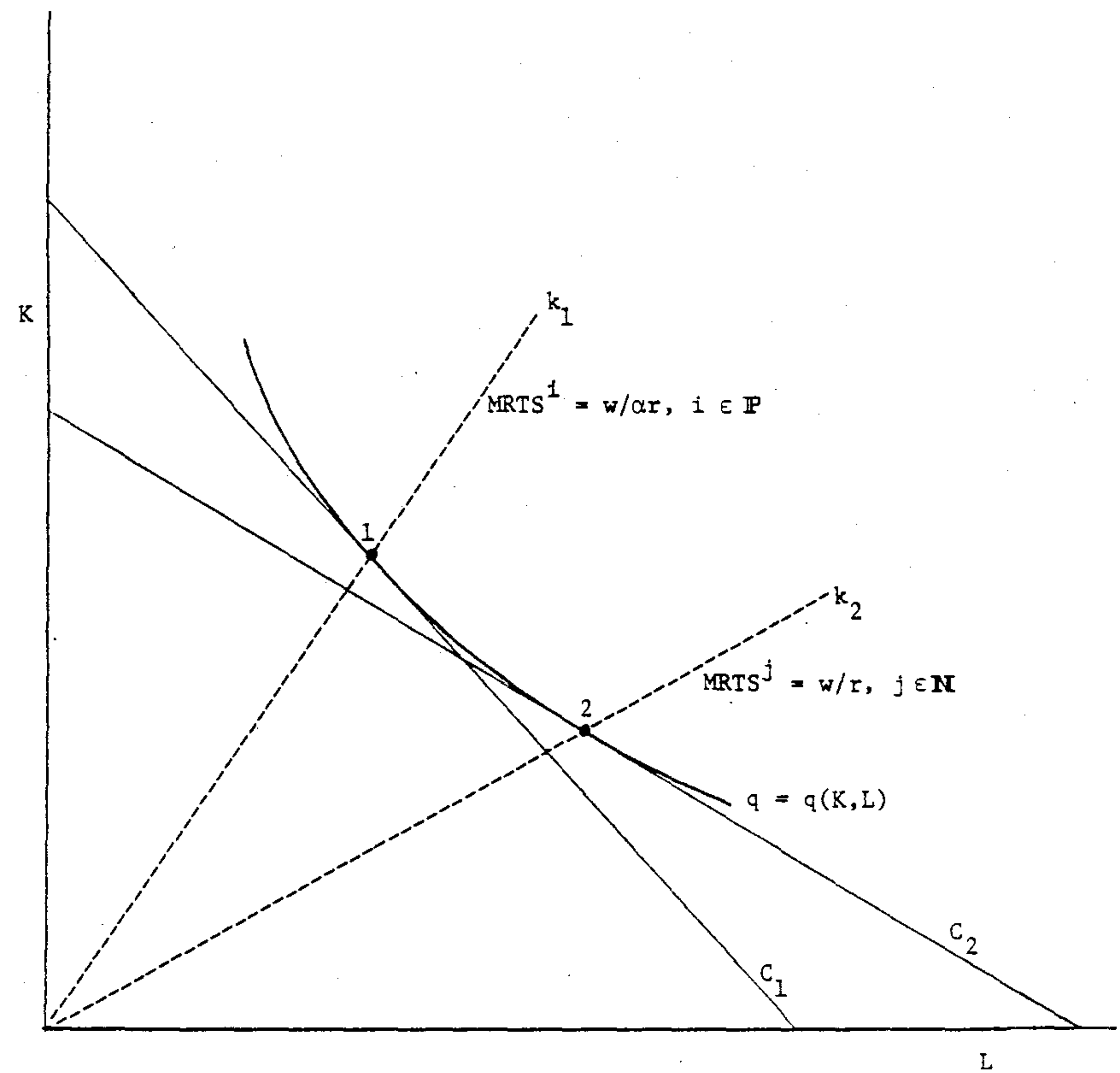


Figure . Factor Intensities

The acreage restriction raises the effective factor price ratio of the participant to $(w/\alpha r)$. Thus, the isocost curve $C_1$, tangent to the isoquant at 1, is steeper than $C_2$. Each participant chooses $(K_i, L_i)$ along the common expansion path $k_1$, depending on their attitude toward risk.

How does an increase in the minimum price affect the factor intensities? Recall that an increase in the minimum price changes the CE marginal revenue, $(\bar{v} - R_i')$, of each participant directly and also indirectly via a change in the industry output, which in turn changes the distributions, $G(v|Q^e)$ and $F(p|Q^e)$. However, this change in CE marginal revenue affects the level of output along the expansion path, but does not rotate the expansion path. Thus, an increase in the minimum price can change the factor intensities only by inducing a change in the land price.

An increase in the minimum price increases CE profits of participants without altering those of nonparticipants. Thus, switching of participation status may occur. For simplicity, we assume that when a marginal producer switches, the quantity of the land input demanded remains unaffected so that switching of the marginal participants/nonparticipants alone has no appreciable effect on the land price. Eeckhoudt and Hansen [1980] have shown that an increase in the minimum price increases the output of every risk averse firm. This implies that CE marginal revenue, $\bar{v} - R_i'$, and CE marginal revenue product of land, $(\bar{v} - R_i')(\partial q/\partial L_i)$, of each participant increases with the minimum price. Since every participant increases the demand for the program land, $L_1/\alpha$, while each nonparticipant does not alter his demand for $L_j$, the equilibrium land price must rise with the minimum price, i.e., $dw/dp_m > 0$. Moreover, factor intensities are increasing in the land price, and hence in the minimum price. That is,

$$\partial k_1/\partial p_m > 0, \qquad \partial k_2/\partial p_m > 0. \tag{11}$$

How does an increase in the acreage parameter affect the factor intensities? As the acreage parameter increases, the effective land price ($w/\alpha r$) for a partcicipant declines and the expansion path $k_1$ in figure 3 rotates clockwise, converging to $k_2$ as $\alpha$ approaches unity. Thus, if the land price were constant, each participant would use the nonland input more intensively as the acreage parameter $\alpha$ increased. The effects of the acreage parameter on the land price is generally ambiguous. Assuming that $dw/d\alpha = 0$, we have

$$\partial k_1/\partial\alpha < 0, \qquad \partial k_2/\partial\alpha = 0. \tag{12}$$

4. Supply Response

To investigate impacts of policy changes for industry output, it is necessary to aggregate input demands for the participants and nonparticipants separately. For this purpose, we first develop an important property of the rational expectations equilibrium. This property is also used to investigate the impact of policy changes on aggregate CEproducer income in section 5.

Since each producer is a price taker, the land price w is treated as constant; as if the supply of land input were perfectly elastic at w. The assumption is appropriate for analyzing firm behavior. However, the equilibrium rental value of land, w, is endogenously determined by the aggregate land demand and supply schedules. We assume that the total eligible land L for the government program is fixed in supply. The aggregate land constraint is thus

$$L - \frac{1}{\alpha} \sum_{i \varepsilon \mathbb{P}} L_i - \sum_{j \varepsilon \mathbb{N}} L_j = 0. \tag{13}$$

The CE producer income of all participants is

$$I_1 \equiv \sum_{i \varepsilon \mathbb{P}} [\bar{v} q(K_i, L_i) - rK_i - R_i].$$

Similarly, the CE producer income of all nonparticipants is

$$I_2 \equiv \sum_{j \quad \mathbb{N}} [\bar{p} q(K_j, L_j) - rK_j - R_j].$$

Consider the problem of maximizing the aggregate CE producer income, $I \equiv I_1 + I_2$, subject to the land constraint (13). The first order conditions are,

$$(\bar{v} - R_i')(\partial q/\partial K_i) - r = 0$$

$$(\bar{v} - R_i')(\partial q/\partial L_i) - \lambda/\alpha = 0, \text{ for all } i \in \mathbb{P}, \text{ and}$$

$$(\bar{p} - R_j')(\partial q/\partial K_j) - r = 0$$

$$(\bar{p} - R_j')(\partial q/\partial L_j) - \lambda = 0, \text{ for all } j \in \mathbb{N},$$

where $\lambda$ is the Lagrange multiplier or the shadow price of land. Note that these conditions are identical to (9a) and (9b), when $w = \lambda$, i.e., the market rental value of land is at its equilibrium value $\lambda$. Recall that identical expectations $Q^e = Q$ were used for the industry output to generate identical price distributions perceived by all firms.[4]

Proposition 1: Assume that producers have rational expectations but have different attitudes toward risk. Then individual producers independently maximizing their expected utilities behave as if they jointly maximize the aggregate CE producer income.

Recall that producers maximize expected utility if and only if they maximize CE producer income. Different attitudes toward risk among producers make it difficult to compare and aggregate expected utilities. However, CE producer income can be used to compare producer welfare under uncertainty. First, CE producer income is the "uncertainty counterpart" of producer surplus or quasirent under certainty. Since the producer receives a random output price, CE producer income is obtained by subtracting the risk bearing cost from the expected surplus. Second, not only interfirm comparisons but also aggregation of CE producer income have — whereas aggregation of expected utilities do not — economic meanings. If producers are risk neutral, they are jointly maximizing expected producer surplus. If they are risk averse, they behave as if they jointly maximize expected producer surplus less the risk bearing cost.

Unlike the producer surplus under certainty, however, CE producer income is not always observable since the implicit risk bearing cost $R_i$ is subjective, reflecting the subjectivity of expected utility. When the market is in long run equilibrium, the CE income and risk premium are observable, since the former is exhausted by compensating for the cost of the short run fixed input and risk bearing cost. Hence, the aggregate risk premium equals expected profit in long run equilibrium.

Aggregation of Risk Premiums and Outputs

Since we have demonstrated that the industry behaves as if producers collectively maximize CE income, and we are interested in analyzing aggregate supply response — rather than individual firm response — to stabilization policy, we will conjure up a super agent representing all producers and investigate its response to changes in program parameters. For this purpose, it is necessary to aggregate risk premiums and factors for the participants and the nonparticipants separately.

Note that since the factor intensity of a participant is completely determined by $(w/\alpha r)$, the CE income of a participant can be written as

$$I^i = \bar{v}L_i x(k_1) - rL_i k_1 - R_i[L_i x(k_1)]$$

where $x(k_1) = q_i/L_i$ is the average yield per acre of land used by the participant. Since there is a fixed relationship between the land input and output, the former is allocated efficiently among the participants if and only if the latter is. Let $c(q_i) = (rk_1/x)q_i$ be the total cost and $c_1 = rk_1/x(k_1)$ be the constant unit production cost of the participants. Then the CE income of participant i can be rewritten as

$$I^i = \bar{v}q_i - c_1 q_i - R_i(q_i). \tag{14}$$

Consider the problem of allocating outputs among the participants to maximize their aggregate CE income. The Lagrangian function is given by

$$\mathbb{L}_1 = \bar{v} \sum_{i\varepsilon \mathbb{P}} q_i - c_1 \sum_{i\varepsilon \mathbb{P}} q_i - \sum_{i\varepsilon \mathbb{P}} R_i(q_i) + \nu_1[X - \sum_{i\varepsilon \mathbb{P}} q_i]$$

where $\nu_1$ is the multiplier. The first order conditions are

$$\bar{v} - c_1 - R_i' - \nu_1 = 0, \text{ for all } i\varepsilon\mathbb{P}. \tag{15a}$$

This implies that marginal risk premiums, $R_i'(q_i)$, must be equalized among all participants, regardless of differences in their attitudes toward risk. Similarly,

$$\bar{p} - c_2 - R_j' - \nu_2 = 0, \text{ for all } j\varepsilon\mathbb{N}.$$

where $c_2 = rk_2/y(k_2)$ is the common average cost of nonparticipants and $\nu_2$ is the multiplier for the output allocation problem among the nonparticipants. The solutions to the first order conditions can be written as

$$q_i = q_i(X),\ i \in \mathbb{P}, \qquad q_j = q_j(Y),\ j \in \mathbb{N}.$$

Thus, the group risk premiums are given by

$$R_1(X) = \sum_{i \in \mathbb{P}} R_i[q_i(X)], \qquad R_2(Y) = \sum_{j \in \mathbb{N}} R_j[q_j(Y)].$$

Differentiating $R_1(X)$ with respect to X gives

$$R_1'(X) = R_i' \sum_{i \in \mathbb{P}} (dq_i/dX) = R_i', \text{ for } i \in \mathbb{P} \tag{16a}$$

Similarly, extending the argument for the nonparticipants,

$$R_2'(Y) = R_j', \text{ for } j \in \mathbb{N}. \tag{16b}$$

We have succeeded in expressing the aggregate risk premium of the participants in terms of their total output, rather than individual outputs. Moreover, the marginal aggregate risk premium is also equal to the common marginal risk premium of the participants. Similar results hold for the nonparticipants. Our next task is to express the aggregate output of the participants in terms of their total inputs. The total output of the participants is given by

$$X = \sum_{i \in \mathbb{P}} L_i x(k_1) = L_1 x(k_1) = q(K_1, L_1).$$

Similarly, the total output of the nonparticipant is given by

$$Y = L_2 y(k_2) = q(K_2, L_2).$$

Since the participants and the nonparticipants use different expansion paths, we distinguish the two group production functions,

$$X = X(K_1, L_1) = L_1 x(k_1), \quad Y = Y(K_2, L_2) = L_2 y(k_2).$$

Thus, the aggregate CE producer income can be written as

$$I = \bar{v}X(K_1, L_1) + p\bar{Y}(K_2, L_2) - r(K_1 + K_2) - R_1(X_1) - R_2(Y_2). \quad (17)$$

The aggregate CE income of all producers is now expressed in terms of total, not individual, input demands of the participants and nonparticipants. The advantage of (17) is that it can be used to analyze the impacts of policy changes on the aggregate supply response and CE producer income.

Recall from Proposition 1 that when producers have rational price expectations individual expected utility maximization results in a market equilibrium in which the aggregate CE income is also maximized. The first order conditions for maximization of CE producer income are now expressed in terms of aggregate input demands and risk premiums:

$$(\bar{v} - R_1')(\partial X/\partial K_1) - r = 0 \quad (18)$$

$$(\bar{v} - R_1')(\partial X/\partial L_1) - \lambda/\alpha = 0$$

$$(\bar{p} - R_2')(\partial Y/\partial K\ ) - r = 0$$

$$(\bar{p} - R_2')(\partial Y/\partial L\ ) - \lambda = 0.$$

The solution to (18) gives the input demand functions,

$$K_I = K_I(\alpha, p_m), \; L_I = L_I(\alpha, p_m).$$

The aggregate outputs of the two groups and industry output can be written as

$$X(\alpha,p_m) = X[K_1(\alpha,p_m),L_1(\alpha,p_m)] = L_1(\alpha,p_m)x(k_1(\alpha,p_m)) \tag{19}$$

$$Y(\alpha,p_m) = Y[K_2(\alpha,p_m),L_2(\alpha,p_m)] = L_2(\alpha,p_m)y(k_2(\alpha,p_m))$$

$$Q(\alpha,p_m) = X(\alpha,p_m) + Y(\alpha,p_m).$$

Supply Response

Recall that a policy change initially affect participation and production decisions and hence secondary adjustments are necessary if producers are to have rational expectations. Consider the rational expectations equilibrium supply response to an increase in the minimum price. Differentiating $Q^e$ with respect to $p_m$ gives

$$\frac{dQ^e}{dp_m} = \frac{dQ}{dp_m}\bigg|_{Q^e} + \frac{dQ^e}{dp_m}\,\frac{dQ}{dQ^e}$$

where the first term refers the direct response, holding expectations constant, and the second term represents the indirect output adjustment due to a change in price expectation. Rearranging the terms, we have

$$\frac{dQ^e}{dp_m} = \frac{\dfrac{dQ}{dp_m}\bigg|_{Q^e}}{1 - (dQ/dQ^e)}$$

If a higher estimate of industry output or a lower price expectation reduces the output of every firm, then $dQ/dQ^e < 0$. Alternatively, if a higher expectation of industry output increases supply, then $dQ/dQ^e > 0$. However, when expectations do not adjust instantaneously in response to changes in program parameters, the market equilibrium will be stable only if $dQ/dQ^e > -1$. Thus, stability requires that $1 - (dQ/dQ^e) > 0$. If the market is stable, then the rational expectations equilibrium response and the direct response will always be in the same direction. Moreover, in the more likely situation, $dQ/dQ^e < 0$, the rational expectations equilibrium response will be a fraction of the direct response, since the adjustment in expectations partly offset the direct response. With this in mind, we now focus on the direct response to policy changes.

The direct responses to changes in program parameters can be obtained from (19). Differentiating X with respect to $p_m$ gives

$$\partial X/\partial p_m = (\partial L_1/\partial p_m)x + L_1 x'(\partial k_1/\partial p_m) > 0 \tag{20a}$$

since $(\partial L_1/\partial p_m) > 0$ by (8a) and $\partial k_1/\partial p_m > 0$ by (11). That is, an increase in the minimum price increases the output of participants. Differentiating Y with respect to $p_m$ gives

$$\partial Y/\partial p_m = (\partial L_2/\partial p_m)y + L_2 y'(\partial k_2/\partial p_m). \tag{20b}$$

Since $(\partial L_2/\partial p_m)$ is negative and $(\partial k_2/\partial p_m)$ is positive, the effect on the output of nonparticipants is ambiguous. Summing (20a) and (20b) gives

$$\partial Q/\partial p_m = \frac{1}{\alpha}(\partial L_1/\partial p_m)[\alpha x(k_1)-y(k_2)] + [L_1 x'(\partial k_1/\partial p_m) + L_2 y'(\partial k_2/\partial p_m)]. \tag{20c}$$

Note that $y(k_2) = x(k_1(\alpha))$ when $\alpha = 1$. If $x(k_1(\alpha))$ increases with the first bracketed terms are negative. Differentiating $\alpha x(k_1)$ with respect to $\alpha$ gives $\partial\alpha x/\partial\alpha = x(1 - \varepsilon_{x\alpha})$, where $\varepsilon_{x\alpha} \equiv -(\partial x/\partial\alpha)(\alpha/x) > 0$ is the elasticity of the average land yield of the participants with respect to the acreage parameter. Observe that $\varepsilon_{x\alpha} = \varepsilon_{xk}\,\varepsilon_{k\alpha}$, where $\varepsilon_{xk} \equiv (dx/dk)(k/x)$ and

$$\varepsilon_{k\alpha} = -(\partial k/\partial\alpha)(\alpha/k) = (\%\Delta k)/(\%\Delta \mathrm{MRTS}) = \sigma$$

is the familiar elasticity of substitution of K for L for the participants. If $x(k) = 0$ when $k = 0$, and $x(k)$ is monotone increasing and concave in k, then $\varepsilon_{xk}$ is less than unity. Thus, if the elasticity of substitution $\sigma$ is less than or equal to unity, then $\varepsilon_{x\alpha} < 1$ and the first bracketed terms in (20c) are negative. Since the second bracketed term is positive, the output effect of an increase in the minimum price is generally ambiguous. If the stabilization program has no acreage restriction ($\alpha = 1$), then $L_1 = L$ and the first bracketed term is zero while the second is positive. Thus, in the absence of the acreage restriction, an increase in the minimum price has a positive supply response.

Next, consider the effects of an increase in the acreage parameter on supply responses. Differentiating $X = L_1 x = (L_1/\alpha)(\alpha x)$ with respect to $\alpha$ gives

$$\partial X/\partial\alpha = [\partial(L_1/\alpha)/\partial\alpha]\alpha x + (L_1/\alpha)\,d\alpha x/d\alpha > 0 \qquad (21a)$$

since $\partial(L_1/\alpha)/\partial\alpha = -\partial L_2/\partial\alpha > 0$ and $\varepsilon_{x\alpha} < 1$ implies $\partial\alpha x/\partial\alpha > 0$. Differentiating Y with respect to $\alpha$ and holding $\lambda$ constant gives

$$\partial Y/\partial\alpha = (\partial L_2/\partial\alpha)y < 0. \qquad (21b)$$

Finally, adding (21a) and (21b) gives

$$\partial Q/\partial\alpha = (\partial L_2/\partial\alpha)(y - \alpha x) + (L_1/\alpha)[y - \alpha\varepsilon_{k\alpha}x] \tag{21c}$$

which indicates that the effect of an increase in $\alpha$ on the aggregate suppoy response is generally ambiguous. Evaluating (21c) at $\alpha = 1$,

$$\partial Q/\partial\alpha = L_1 x(1 - \varepsilon_{x\alpha})$$

since $x(k_1(1)) = y(k_2)$. If the elasticity of substitution $\sigma \leq 1$, then $\varepsilon_{x\alpha}$ is less than unity. Thus, as the acreage parameter decreases from unity, industry output declines.

<u>Proposition 2</u>: Assume that the elasticity of substitution $\sigma \leq 1$. Then

(i) an increase in the minimum price or the acreage parameter increases the total output of the participants, but the output effect on the nonparticipants is generally ambiguous, and

(ii) the acreage restriction initially reduces the industry output, and in the absence of the acreage restriction an increase in the minimum price increases the industry output.

## 5. Producer Income and Price Stabilization

A principal objective of price stabilization in commodity markets is to improve CE producer income by providing more stable prices. In this section we analyze the effects of changes in program parameters on CE producer income. Since the stabilization program influences industry output Q, expected prices can be written as

$$\bar{p}(\alpha,p_m) = \bar{p}(Q(\alpha,p_m)), \qquad \bar{v}(\alpha,p_m) = \bar{v}[Q(\alpha,p_m);p_m].$$

Note that when industry output is held constant, $\bar{v}$ increases with the minimum price but is unaffected by changes in $\alpha$. Changes in program parameters affect $\bar{v}$ also indirectly via a change in industry output. In contrast, the expected price of the nonparticipants are affected by program parameters only indirectly.

The CE income of the participants is given by

$$I_1 = \bar{v}(\alpha,p_m)X[K_1(\alpha,p_m),L_1(\alpha,p_m)] - rK_1(\alpha,p_m) - R_1[X(\alpha,p_m);\alpha,p_m] \quad (22a)$$

This indirect CE income is now expressed in terms of stabilization program parameters. The expression for the risk premium, $R_1[X(\alpha,p_m);\alpha,p_m]$ indicates that the program parameters can affect risk premium directly and also indirectly via a change in output X. Regarding the <u>direct</u> effect of the program parameters on the risk premium, holding input/output constant, we note that (i) an increase in the minimum price results in a rightward first degree shift in the distribution of v while reducing its dispersion, and hence reduces the risk premium, and (ii) an increase in the acreage parameter increases profit and decreases risk premium, provided that the participants exhibit diminishing absolute risk aversion. Thus, the direct effects on the risk premium are negative for both parameters, i.e.,

$$\left.\frac{\partial R_1}{\partial p_m}\right|_{\overline{Q}} < 0, \qquad \left.\frac{\partial R_1}{\partial \alpha}\right|_{\overline{Q}} < 0.$$

The indirect CE income of the nonparticipants is given by

$$I_2 = \overline{p}(\alpha,p_m)Y[K_2(\alpha,p_m),L_2(\alpha,p_m)] - rK_2(\alpha,p_m) - R_2[Y(\alpha,p_m)]. \tag{22b}$$

Changes in program parameters affect the risk premium of the nonparticipants only indirectly through changes in output.

Minimum Price

Differentiating (22a) with respect to $p_m$ and using (18) gives

$$dI_1/dp_m = (\overline{v} - R_1')(\partial X/\partial L_1)(\partial L_1/\partial p_m) + X(d\overline{v}/dp_m) - (\partial R_1/\partial p_m)$$

$$= (\partial L_1/\partial p_m)\lambda/\alpha + X(d\overline{v}/dp_m) - (\partial R_1/\partial p_m). \tag{23a}$$

Recall that the direct effect on risk premium, $\partial R_1/\partial p_m$, is negative. An increase in the minimum price directly increases expected price $\overline{v}$. If the industry output Q increases, the direct impact is partly offset. If a government program is to give a consistent signal to the participants, expected price $\overline{v}$ of the participant must increase with the minimum price, i.e., $d\overline{v}/dp_m > 0$. Since an increase in the minimum price increases the land $L_1$ used by the participants (by (8b)) then the first term in (23a) is positive. Thus, an increase in the minimum price increases CE income of the participants.

Differentiating (22b) with respect to $p_m$ and using (18) gives

$$dI_2/dp_m = (\bar{p} - R_2')(\partial Y/\partial L_1)(\partial L_2/\partial p_m) + Y(d\bar{p}/dp_m)$$

$$= \lambda(\partial L_2/\partial p_m) + Y(d\bar{p}/dp_m). \tag{23b}$$

If an increase in the minimum price increases market output, then $d\bar{p}/dp_m$ is negative. Moreover, an increase in the minimum price reduces the land $L_2$ used by the nonparticipants (by (8b)). Thus, CE income of the nonparticipants decreases as the minimum price increases. Adding (23a) and (23b) gives

$$dI/dp_m = X(d\bar{v}/dp_m) + Y(d\bar{p}/dp_m) - (\partial R_1/\partial p_m). \tag{23c}$$

The first term is positive, regardless of supply response Q. The second term is positive only if $dQ/dp_m$ is positive. Thus, an increase in the minimum price has an ambiguous effect on the CE producer income.

<u>Proposition 3</u>: An increase in the minimum price increases CE income of the participants. If $dQ/dp_m$ is positive, an increase in the minimum price decreases CE income of the nonparticipants. If $dQ/dp_m$ is negative, an increase in the minimum price increases the aggregate CE producer income.

<u>Acreage Restriction</u>

Differentiating (22a) with respect to $\alpha$ gives

$$dI_1/d\alpha = (\bar{v} - R_1')(\partial X/\partial L_1)(\partial L_1/\partial\alpha) + X(d\bar{v}/d\alpha) - (\partial R_1/\partial\alpha) \tag{24a}$$

$$= (\partial L_1/\partial\alpha)(\lambda/\alpha) + X(d\bar{v}/d\alpha) - (\partial R_1/\partial\alpha).$$

The first term is positive by (8a) and $(\partial R_1/\partial\alpha)$ is negative. If the industry output Q decreases as $\alpha$ increases, then the second term is also positive. In this case, CE income of the participants increase with $\alpha$. However, if $dQ/d\alpha > 0$, then $dI/d\alpha$ is ambiguous.

Differentiating (23a) with respect to $\alpha$ gives

$$dI_2/d\alpha = (\bar{p} - R_2')(\partial Y/\partial L_2)(\partial L_2/\partial\alpha) + Y(d\bar{p}/d\alpha)$$

$$= \lambda(\partial L_2/\partial\alpha) + Y(d\bar{p}/d\alpha). \tag{24b}$$

The first term is negative by (8a). If $dQ/d\alpha$ is positive, the second term is also negative. In this case, an increase in the acreage parameter decreases CE income of the nonparticipants. Finally, adding (24a) and (24b) and differentiating the land constraint with respect to $\alpha$ gives

$$dI/d\alpha = [\underline{\lambda}L_1/\alpha^2 - (\partial R_1/\partial\alpha)] + X(d\bar{v}/d\alpha) + Y(d\bar{p}/d\alpha). \tag{24c}$$

The brackted term is positive. If the market output increases with ($dQ/d\alpha > 0$), then the second and the third terms are both negative. If an interior maximum of I exists for $\alpha < 1$, then the acreage restriction can increase CE producer income.

We now investigate the conditions under which the acreage restriction initially increases CE producer income. Evaluating (24c) at $\alpha = 1$, we have

$$dI/d\alpha = [\lambda L - \partial R_1/\partial\alpha] + Q(d\bar{v}/dQ)(dQ/d\alpha)$$

If the producers exhibit diminishing absolute risk aversion, then $\partial R_1/\partial\alpha < 0$. Moreover, $d\bar{v}/dQ < 0$ since an increase in industry output results in a leftward first degree shift in the price distribution.

Proposition 4: Assume that producers exhibit diminishing absolute risk aversion. If $dQ/d\alpha < 0$, then $dI/d\alpha > 0$, and hence the acreage restriction decreases the aggregate CE producer income.

If the industry output decreases with $\alpha$, the acreage restriction cannot improve the CE producer income. Thus, $dQ/d\alpha > 0$ is a necessary condition for an acreage restriction to improve aggregate CE producer income.

## 6. Government Cost and Efficiency

We have shown how the stabilization program parameters influence industry output and CE producer income. If price incentives are introduced to encourage voluntary participation, the program can be costly to the government. Since the government cost is financed by taxes imposed on consumers, it is important to control government cost within limits, while maintaining a high benefit-cost ratio (Fox [1951b]). In this section, we first investigate how program parameters affect expected government cost. Then we show how an efficient stabilization program can be designed to maximize CE producer income for a given expected government cost.

Expected per unit subsidy can be written as

$$\Theta = \int_0^{p_m} (p_m - p)\,dF(p|Q) = \overline{v} - \overline{p}.$$

The expected program cost to the government is

$$C(\alpha,p_m) = \Theta(Q;p_m)X(\alpha,p_m) = \Theta L_1(\alpha,p_m)x(k_1(\alpha,p_m)). \tag{25}$$

If the participation rate is positive, X is also positive. Thus, expected government cost is positive if the minimum price is effective, i.e., $F(p_m) > 0$.

Minimium Price

Differentiating (25) with respect to $p_m$ gives

$$\partial C/\partial p_m = (d\Theta/dp_m)X + \Theta(\partial X/\partial p_m). \tag{26}$$

Recall that $\partial X/\partial p_m$ is positive by Proposition 2. Differentiating $\Theta(Q;p_m)$ with respect to $p_m$ gives

$$d\Theta/dp_m = F(p_m) + (\partial\Theta/\partial Q)(\partial Q/\partial p_m).$$

Since an increase in market output Q results in a leftward first degree shift in the distribution $F(p|Q)$ for a given minimum price, the expected per unit government subsidy increases with industry output. If $\partial Q/\partial p_m$ is positive, then $d\Theta/dp_m$ is also positive. Even if the indirect effect were negative, due to a negative supply response, it is not likely to offset the direct effect, $F(p_m)$. In the anomolous case, an increase in the minimum price may give producers a wrong signal, since their expected per unit subsidy would be decreasing. Barring this anonomly, an increase in the minimum price increases the expected government cost, i.e., $\partial C/\partial p_m > 0$.

Acreage Restriction

Price Incentives not only encourage participation, but also increase expected government cost. Government cost can be moderated by a suitable acreage restriction. Differentiating (25) with respect to $\alpha$ gives

$$\partial C/\partial\alpha = (\partial\Theta/\partial Q)(\partial Q/\partial\alpha) + \Theta(\partial X/\partial\alpha). \tag{27}$$

Since $(\partial X/\partial\alpha) > 0$, an increase in $\alpha$ increases expected government cost, if the supply response is negative. Recall that $(\partial Q/\partial\alpha)$ is negative when evaluated at $\alpha = 1$ (Proposition 2).

Proposition 5: Assume that the elasticity of substitution $\sigma \leq 1$. Then

(i) the acreage restriction initially reduces expected government cost, and

(ii) an increase in the minimum price increases expected government cost.

Efficiency

The notion of efficient stabilization program has been developed by Theil [1965] and Fox, Sengupta and Thorbecke [1966]. Since the government cost is borne by consumers, we focus on CE producer income and expected government cost. Define an efficient stabilization program as one that maximizes CE producer income at a given expected government cost. With a voluntary program, the government decision problem is to choose $\alpha$ and $p_m$ to

$$\begin{aligned}&\text{maximize} && I(\alpha, p_m)\\ &\text{subject to} && C^o - C(\alpha, p_m) = 0\end{aligned}$$

where $C^o$ is a constant government cost. The first order conditions are given by

$$(\partial I/\partial\alpha) - \beta(\partial C/\partial\alpha) = 0 \tag{28}$$

$$(\partial I/\partial p_m) - \beta(\partial C/\partial p_m) = 0$$

$$C^o - C(\alpha, p_m) = 0.$$

With empirical information on the technical and behavioral relationship that underly CE producer income and government cost, condition (28) can be solved for the optimal or efficient program parameters.

Even without the empirical information for an explicit solution, the above optimization problem can provide useful insights. The solution is illustrated in Figure 4. First, consider the locus Aa along which participation is zero. To the right (left) of this "zero" participation curve, program output X is positive (zero). Next, consider the locus Ba along which the expected per unit subsidy $\Theta$ is zero. Differentiating $\Theta(Q;p_m)$, subject to $Q = Q^o$ along the zero participation curve, yields

$$\left.\frac{\partial\Theta}{\partial p_m}\right|_{Q^o} = F(p_m) > 0. \tag{29}$$

This implies that $\Theta$ increases with $p_m$ along the zero participation curve Aa, although expected government cost is still zero, due to zero participation. Thus, Aa lies above Ba. The area to the right of Aa defines combinations of $(\alpha, p_m)$ that yield a higher CE producer income than with no program. However, expected government cost is also positive in this region.

The efficiency locus or the contract curve, aed, in Figure 4, shows how $\alpha$ and $p_m$ should change to maximize CE producer income with increasing expected government cost. These points are determined by tangency between the isocost and isoincome curves. For example, e in Figure 4 is the tangency point between the isocost curve C* and the isoincome curve I*. Note that the multiplier $\beta = dI/dC^o$, which

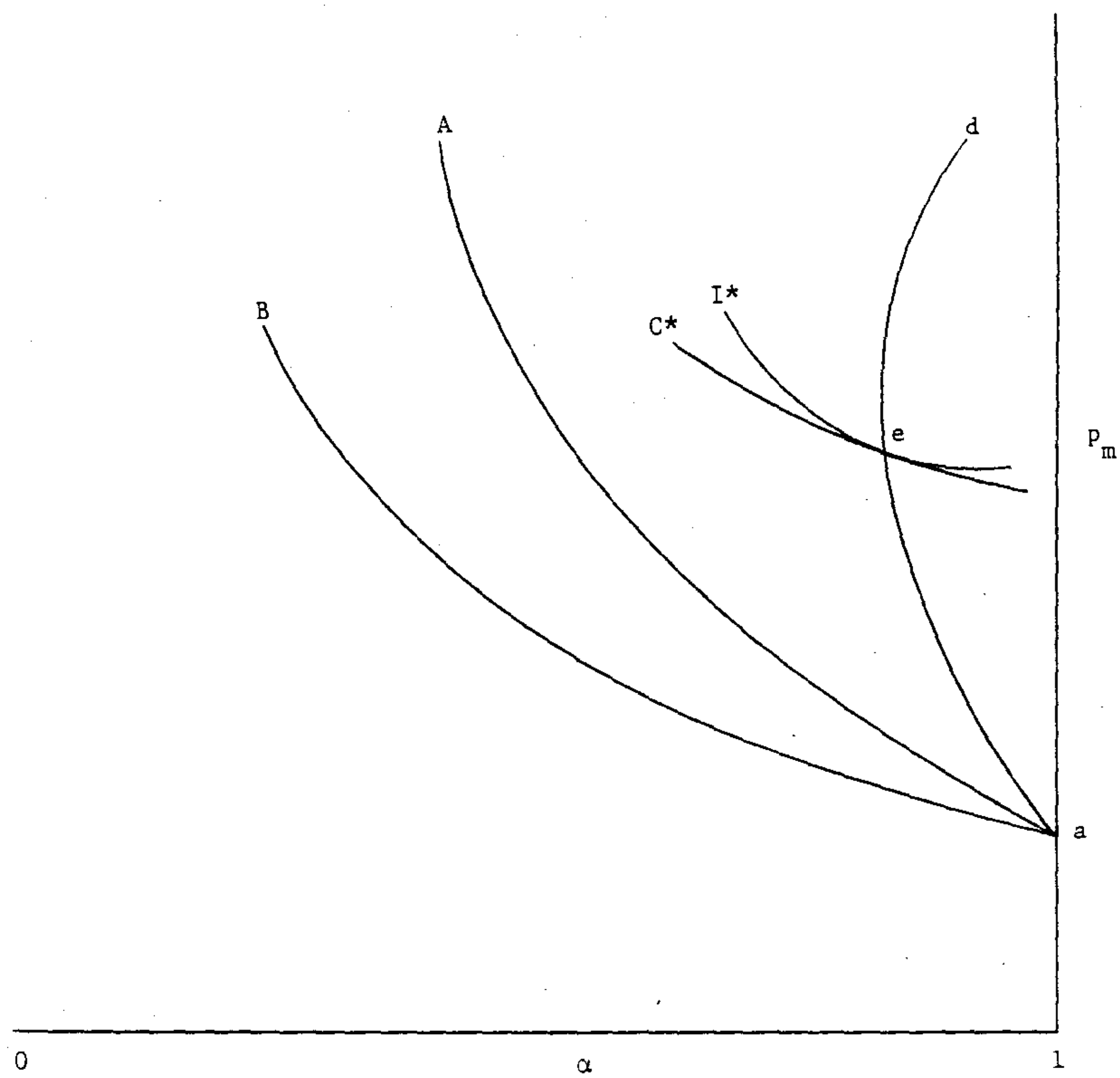


Figure 4. An efficient program with a maximum price ($p_M$ = b).

measures the marginal efficiency of government spending, varies along the contract curve. If producers were risk neutral, then the marginal efficiency of government spending would be less than unity, since the program distorts resource allocation. If producers are risk averse, however, government program reduce the risk bearing cost and hence will be higher than if producers are risk neutral.

If the government has a fixed (expected) budget, then the benefit-cost ratios, I/C, can be compared to choose the commodity markets for price stabilization. Moreover, if the government were to stabilize the prices of several commodities, the associated expenditure will be efficiently allocated if the multipliers are equalized. If not, reallocating government resources from a commodity market with a lower multiplier into another with a higher multiplier can improve producer welfare.

## 6. Concluding Remarks

This paper investigated the effect of a stylized stabilization program for supply response and producer income. The risk averse producers are assumed to face random market demand. The commodity is nonstorable and the government uses a direct price subsidy to assure the minimum price for program participants. However, to reduce industry output and improve producer income, an acreage restriction is imposed for participating producers.

We dmonstrated that if producers have rational expectations they behave as if they jointly maximize the aggregate CE producer income. This property of the rational expectations equilibrium permits aggregation of input demands, outputs and the risk premium for both participants and nonparticipants. An increase in the minimum price or the acreage parameter encourages participation, and increases the output of the participants. But, the output effect on the nonparticipants is generally ambiguous, due to endogenous changes in the land price.

A simultaneous increase in the minimum price and a decrease in the acreage parameter can increase CE producer income. The trade-off is an increase in expected government program cost. An efficient stabilization program can be designed to maximize CE producer income for a given expected government cost. Moreover, the multiplier for the constrained optimization problem can be used to compare efficiency of government programs. If the government has a fixed budget for stabilization, this efficiency analysis can be useful in selecting commodity markets for government intervention.

FOOTNOTES

1. See Turnovsky [1978], Newbery and Stiglitz [1981] and Just and Hallam [1982] for extensive references.

2. Kawai [1983] introduces a futures market into the model. If the speculators in the futures are risk neutral, expected (discounted) spot price will be equal to the present futures price and no producers will participate in government programs with acreage restrictions. However, if the speculators are risk averse, the futures market does not provide actuarially fair price insurance.

3. Since the production function is linearly homogenenous, the input demands are also decreasing in the absolute risk aversion index.

4. If producers have different expectations, the aggregate CE income is not meaningful. Although producers maximize their own CE incomes, based on their subjective expectations, all other firms with different expectations appear to be behaving suboptimally.

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