Incorporation of Fixed-Flexible Exchange Rates in Econometric Trade Models: A Grafted Polynomial Approach

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Working Paper 86-WP 6
July 1986

Abstract

Endogenizing the exchange rate in trade models over the period of fixed and flexible exchange rate systems poses an econometric problem in the estimation because of the fixity of the exchange rate under the fixed exchange rate system. An econometric technique, the grafted polynomial approach, is used to solve this problem.

Introduction

A large increase in U.S. agricultural exports and volatile prices and incomes in the early 1970s caught the attention of many economists. Schuh (1974) suggested that the devaluation of the dollar by the Nixon administration in 1971 and the further decline in the value of U.S. dollar under the flexible exchange rate regime that was established in 1973 was the major cause for the unprecedented increase in U.S. farm exports. Following Schuh's article in 1974, there appeared a series of empirical studies to investigate the relationship between the exchange rate and agricultural commodity trade. Most of these studies treated the exchange rate as exogenous. Only a few studies (Chambers and Just 1982; Denbaly 1984; and Devadoss 1985) endogenized the exchange rate in their trade models.

Since the exchange rate is constant under a fixed exchange rate system and flexible under a floating exchange rate system, endogenizing the exchange rate poses an empirical problem in econometric trade models that include a data period covering both fixed and flexible regimes. The main objective of this study, which is based on Devadoss's dissertation work (1985), is to endogenously estimate the exchange rate covering both fixed and floating exchange rate periods using a grafted polynomial econometric technique and to incorporate the exchange rate equation into a large econometric trade model. The next section presents a monetary approach to exchange rate determination. Then the theory of grafted polynomial technique is described. Estimation procedures and empirical results are given, and the incorporation of the endogenous exchange rate into a large econometric trade model is explained. Finally, the conclusions and implications of the study are summarized.

Exchange Rate Determination

The approach taken in modeling the exchange rate is the monetary approach to exchange rate determination (see Frenkel 1976; and Johnson 1976). The monetary approach emphasizes the role of money in determining the balance of payments when the exchange rate is pegged, and in determining the exchange rate when it is flexible. Following the monetary approach, we assume the following three conditions to derive the exchange rate determination.

(1) Money market equilibrium in the U.S.:

$$M = P \cdot L(y, r), \qquad \frac{\partial L}{\partial y} > 0, \frac{\partial L}{\partial r} < 0$$

(2) Money market equilibrium in the rest of the world (ROW)

$$M^* = P^* \cdot L^* (y^*, r^*), \qquad \frac{\partial L^*}{\partial y^*} > 0, \frac{\partial L^*}{\partial r^*} < 0$$

(3) The purchasing power parity condition

$$P^* = E \cdot P,$$

where M is the money supply, P is the domestic price level, L(y, r) is demand for real money balances as a function of income (y) and interest rates (r). The variables in equations (2) and (3) follow similar definitions, except that they refer to ROW, and E is the exchange rate defined as SDR/US $.

Solving the above three equations for the exchange rate, we get

(4) $$E = \left(\frac{M^*}{M}\right) \cdot \left(\frac{L(y, r)}{L^*(y^*, r^*)}\right).$$

Equation (4) can be rewritten in functional form with expected signs for the explanatory variables as

(5) $$E = F[\overset{(-)}{M}, \overset{(+)}{M^*}, \overset{(+)}{y}, \overset{(-)}{y^*}, \overset{(-)}{r}, \overset{(+)}{r^*}].$$

The intuition behind these expected signs can be explained by analyzing the money markets. For example, an increase in the U.S. money supply brings about an excess supply of money, which leads to decline in the value of the dollar ($\frac{\partial E}{\partial M} < 0$). On the other hand, an increase in the ROW money supply puts an upward pressure on the value of the dollar ($\frac{\partial E}{\partial M^*} > 0$). Given that the income elasticity of money demand is positive for both the U.S. and ROW (since $\frac{\partial L}{\partial y} > 0$, and $\frac{\partial L^*}{\partial y^*} > 0$), an increase in the U.S. (ROW) income will increase (decrease) the value of the U.S. dollar, i.e., $\frac{\partial E}{\partial y} > 0$ ($\frac{\partial E}{\partial y^*} < 0$). Because the interest rate elasticity of money demand is negative (since $\frac{\partial L}{\partial r} < 0$, $\frac{\partial L^*}{\partial r^*} < 0$), an increase in the U.S. (ROW) interest rate will put downward (upward) pressure on the value of the U.S. dollar, i.e., $\frac{\partial E}{\partial r} < 0$ and ($\frac{\partial E}{\partial r^*} > 0$).

Theory of Grafted Polynomial Technique

The data used in this study is annual data from 1950 to 1982, which includes both fixed and flexible exchange rate regimes. The explanatory variables in the exchange rate equation (5) had no effect on the exchange rate under the fixed exchange rate system. In view of this fact, the grafted polynomial approach developed by Fuller (1969) will be used to estimate the exchange rate equation under both fixed and flexible exchange rate systems.

To illustrate the use of grafted polynomials in the estimation of the mean function of the exchange rate time series, the series is divided into three segments: (1) fixed exchange rates (1950-1971), (2) adjustment or transition period (1971-1973), and (3) flexible exchange rates (1973-1982). These three segments are represented by the following three functional relationships:

(6) $E = A_0$ year < 1971

(7) $E = B_0 + B_1 X + B_2 X \cdot M + B_3 X \cdot M^* + B_4 X \cdot y + B_5 X \cdot y^* + B_6 X \cdot r + B_7 X \cdot r^*$

$1971 < \text{year} < 1973$

(8) $E = C_0 + C_1 M + C_2 M^* + C_3 y + C_4 y^* + C_5 r + C_6 r^*$ year > 1973

In equation (6), the exchange rate is constant corresponding to the fixed exchange rate regime. In equation (7), the transition period in 1972 and 1973 is illustrated. Even though the flexible exchange rate system was officially adopted in 1973, many countries started to revalue their currencies in terms of the U.S. dollar late in 1971 to break away from the fixed exchange rate system. Thereafter, according to the monetary approach to exchange rate determination, monetary factors started to have gradual influence on exchange rates. It is clear from Figure 1, the value of the exchange rate in this period changes significantly because of the revaluation and the gradual influence by monetary factors. The "grafted polynomial variable" X is included as a separate independent variable and also multiplied with all the independent variables to capture the sudden and significant change in the value of the exchange rate because of the revaluation and the influence of monetary factors. The grafted polynomial variable also connects these three equations, (6)-(8), into a single, continuous, and estimable equation. The definition of X is given below. Equation (8) fully captures the monetary approach to exchange rate determination that was described in Section II.

For the mean function to be estimable, it must be continuous. We shall show how it is possible to "graft" these three functions by incorporating X in the model so that we can obtain a single and continuous function for the whole period from 1950 to 1982. The restrictions which are required for the function to be continuous are that the value of the exchange rate in equation (6) and (7) will be equal in the year 1971; it will also be equal in equations (7) and (8) in the year 1973. For these continuity restrictions to hold and to capture the sudden change in the exchange rates in the transition period, X has to have the values year minus 1971 in the transition period from 1971 to 1973 as evident in the restrictions written below. These restrictions can be written as:

(9) $A_0 = B_0$ at year = 1971

(10) $B_0 + 2B_1 + 2B_2M_{73} + 2B_3M^*_{73} + 2B_4y_{73} + 2B_5y^*_{73} + 2B_6r_{73} + 2B_7r^*_{73} =$

$C_0 + C_1M_{73} + C_2M^*_{73} + C_3y_{73} + C_4y^*_{73} + C_5r_{73} + C_6r^*_{73}$ at year = 1973,

where the subscript 73 indicates the time period 1973.

Restriction (10) can be rewritten as

(11) $(B_0 + 2B_1 - C_0) + (2B_2 - C_1)M_{73} + (2B_3 - C_2)M^*_{73} + (2B_4 - C_3)y_{73}$

$+ (2B_5 - C_4)y^*_{73} + (2B_6 - C_5)r_{73} + (2B_7 - C_6)r^*_{73} = 0.$

From (11), C_is can be defined in terms of B_is

(12)
$$
\begin{aligned}
C_0 &= B_0 + 2B_1 \\
C_1 &= 2B_2 \\
C_2 &= 2B_3 \\
C_3 &= 2B_4 \\
C_4 &= 2B_5 \\
C_5 &= 2B_6 \\
C_6 &= 2B_7
\end{aligned}
$$

Substituting (9) and (12) into equations (6), (7), and (8), we get

(13) $E = A_0$ for $1950 \le \text{year} \le 1971$

(14) $E = A_0 + B_1X + B_2X\cdot M + B_3X\cdot M^* + B_4X\cdot y + B_5X\cdot y^* + B_6X\cdot r + B_7X\cdot r^*$

for $1971 \le \text{year} \le 1973$

(15) $E = A_0 + 2B_1 + 2B_2M + 2B_3M^* + 2B_4y + 2B_5y^* + 2B_6r + 2B_7r^*$

for $1973 \le \text{year} \le 1982$.

By definition that X is year minus 1971, it is equal to 0 in 1971, 1 in 1972, and 2 in 1973. Hence, the above three equations can be written in a matrix notation as:

$$
(16)\quad
\begin{bmatrix} E_{50} \\ \cdot \\ \cdot \\ \cdot \\ E_{70} \\ E_{71} \\ E_{72} \\ E_{73} \\ E_{74} \\ \cdot \\ \cdot \\ \cdot \\ E_{82} \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & M_{72} & M^*_{72} & y_{72} & y^*_{72} & r_{72} & r^*_{72} \\
1 & 2 & 2M_{73} & 2M^*_{73} & 2y_{73} & 2y^*_{73} & 2r_{73} & 2r^*_{73} \\
1 & 2 & 2M_{74} & 2M^*_{74} & 2y_{74} & 2y^*_{74} & 2r_{74} & 2r^*_{74} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
1 & 2 & 2M_{82} & 2M^*_{82} & 2y_{82} & 2y^*_{82} & 2r_{82} & 2r^*_{82}
\end{bmatrix}
\begin{bmatrix} A_0 \\ B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \\ B_7 \end{bmatrix}
$$

Thus, estimates of the parameters in (16) are obtained from the regression equation

$$
(17)\qquad E = A_0 + B_1X + B_2X\cdot M + B_3X\cdot M^* + B_4X\cdot y + B_5X\cdot y^* + B_6X\cdot r + B_7X\cdot r^*,
$$

where X for the whole period is defined as

$$
X = \begin{cases} 0 & \text{for } 1950 \leq \text{year} \leq 1970 \\ \text{year}-1971 & \text{for } 1971 \leq \text{year} \leq 1973 \\ 2 & \text{for } 1974 \leq \text{year} \leq 1982. \end{cases}
$$

Thus, equation (17) is a single, continuous, and estimable equation and also endogenizes the exchange rate determination covering the periods of both fixed and flexible exchange rate systems.

Estimation and Empirical Results

Equation (17), which incorporates grafted polynomial technique, is estimated by ordinary least squares (OLS). As mentioned previously, the time period of the estimation is from 1950 to 1982. The data for the variables used in the estimation were collected from various sources. The money supply, M1, was obtained from the Federal Reserve Board, Washington, D.C. The interest rate (AAA corporate bonds rate) and real gross national product in the United States were from the 1983 Economic Report of the President. The exchange rate, ROW money supply, and ROW real gross national product were obtained from various issues of International Financial Statistics. Since there is no single interest rate for the ROW, the ROW interest rate is the average of interest rates from West Germany, Canada, the United Kingdom, Italy, and France. The interest rates of these countries were obtained from International Financial Statistics. The estimated results of equation (17) are given below, with R^2, Durbin Watson (DW), t-statistics (in parentheses), and adjusted t-statistics [in brackets].

$$
\begin{aligned}
E = \underset{\substack{(362.83)}}{100.04} &+ \underset{\substack{(10.75)\\ [4.30]}}{189.18\, X} - \underset{\substack{(-3.50)\\ [-1.40]}}{0.00055(X \cdot M)} + \underset{\substack{(6.54)\\ [2.62]}}{0.46(X \cdot M^*)} + \underset{\substack{(5.89)\\ [2.36]}}{0.0096(X \cdot y)} \\
&- \underset{\substack{(-8.23)\\ [-3.29]}}{1.79(X \cdot y^*)} - \underset{\substack{(-3.03)\\ [-1.21]}}{2.22(X \cdot r)} + \underset{\substack{(5.78)\\ [2.31]}}{2.15(X \cdot r^*)}
\end{aligned}
$$

$$R^2 = 0.99 \qquad DW = 1.90$$

The actual t-statistics are adjusted for the degrees of freedom, since the explanatory variables and the grafted polynomial variable take zero values and hence fix the exchange rates to the actual values prior to 1971.[1] The estimated results of the exchange rate equation with the grafted polynomial technique meet the theoretical description given in Section II. All the coefficients are consistent with the a priori expectations. The actual t-statistics are highly significant; however, the adjusted t-statistics for the U.S. money supply and interest rate are insignificant at the 5 percent level. This is because of the smaller degrees of freedom (4) used in the calculation of adjusted t-statistics. Therefore, the sign and magnitude of these estimates are more important than the significant level per se. The Durbin Watson statistics of 1.90 indicates no first order autocorrelation. The actual and predicted values of the exchange rate are plotted in Figure 1. Because of the grafted polynomial technique, the predicted values of the exchange rate exactly coincide with the actual values of the exchange rate for the fixed exchange rate regime. The explanatory variables derived from the monetary approach to exchange rate determination help to explain the movements of exchange rate under a flexible exchange rate regime. Various specifications of equation (17) were specified with minor changes in the grafted polynomial variable. The grafted polynomial variable in the estimated equation above provided better results, however.

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[1]Adjusted t-statistics = actual t-statistics * $\sqrt{\frac{n_1-k_1}{n-k}}$, where n - k is the original degrees of freedom and $n_1 - k_1$ is the corrected degrees of freedom. In this case, $n_1 - k_1$ is equal to 4, i.e., (12-8).

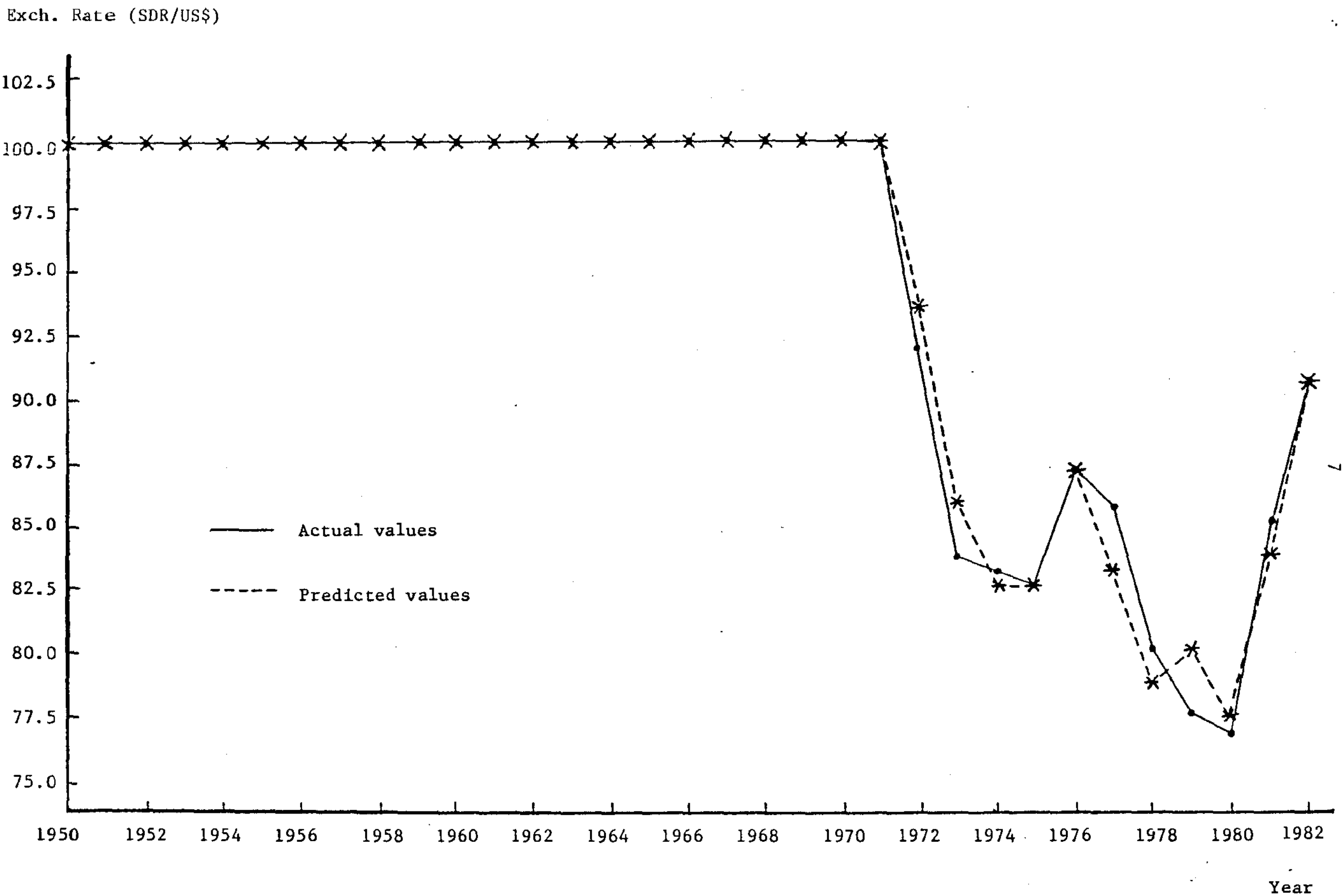

Figure 1. Actual and predicted values of the exchange rate

Incorporation of Endogenous Exchange Rates in Econometric Trade Model

This section describes the results of endogenizing the exchange rate in a trade model based on the grafted polynomial approach. These results emanate from the large econometric model developed by Devadoss (1985), in which the exchange rate was endogenized to examine the effect of U.S. money supply on farm commodity exports and other farm aggregate variables through the exchange rate. The model was simultaneous and nonlinear; in view of these facts, nonlinear three-stage least squares (N3SLS) was used to estimate the parameters. The study period was the same as the one used in the above OLS estimation. The estimated exchange rate equation from this model is given below.

$$E = \underset{(363.47)}{99.96} + \underset{\substack{(19.41)\\ [7.92]}}{196.97}\, X - \underset{\substack{(-6.42)\\ [-2.62]}}{0.00055}(X \cdot M) + \underset{\substack{(11.88)\\ [4.85]}}{0.47}(X \cdot M^*) + \underset{\substack{(11.62)\\ [4.74]}}{0.01}(X \cdot y) - \underset{\substack{(-16.25)\\ [-6.63]}}{1.93}(X \cdot y^*)$$

$$- \underset{\substack{(-7.54)\\ [-3.07]}}{2.57}(X \cdot r) + \underset{\substack{(14.14)\\ [5.77]}}{2.52}(X \cdot r^*)$$

$R^2 = 0.99$ $\quad\quad$ $DW = 1.95$

The above results not only confirm that all the coefficients are consistent with theoretical expectation but also indicate that the adjusted t-statistics, unlike in the OLS estimation, are significant at the 5 percent level. Thus, we show that the grafted polynomial technique can be easily applied to endogenize the exchange rate in large scale econometric models.

Implications and Conclusions

In this study, we have shown that the grafted polynomial technique can be applied very effectively to model the exchange rate that had structural change going from a fixed exchange rate regime to a flexible exchange rate regime. Thus, this technique provides a very useful solution for the researchers trying to model the exchange rate in a trade model analysis. The implication of this study is that the economic variables with structural change over a long period can be efficiently modeled by using the grafted polynomial technique. For example, Fuller (1976) applied this technique to estimate the wheat yields from 1908 to 1971.

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