Crop Insurance Rates and the Laws of Probability

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Working Paper 02-WP 298
April 2002

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Abstract

Increased crop insurance subsidies have increased the demand for insurance at coverage levels higher than the traditional level of 65 percent. Premium rates for higher levels of yield insurance under the Federal Actual Production History (APH) program equal the premium rate at the 65 percent coverage level multiplied by a rate relativity factor that varies by coverage level but not by crop or region. In this paper, we examine the consistency of these constant rate relativity factors with the laws of probability by determining the maximum 65 percent premium rate that is consistent with a well-defined yield distribution. We find that more than 50 percent of U.S. counties have premium rates for corn, soybeans, and wheat that are not consistent with the laws of probability for coverage levels up to 75 percent. For coverage levels up to 85 percent, almost 80 percent of corn counties, 82 percent of soybean counties, and 80 percent of wheat counties have rates that are not consistent. Adding the further restriction that at least 15 percent of probability falls between 85 percent and 100 percent of APH yields implies that 92 percent of corn counties, 90 percent of soybean counties, and 95 percent of wheat counties have APH rates that are not consistent with the laws of probability for coverage levels up to 85 percent. These results imply that crop insurance rates under the APH program in most U.S. production regions at high coverage levels exceed those that could be generated by a well-defined yield distribution.

Key words: crop insurance, premium rates, rate relativities.

CROP INSURANCE RATES AND THE LAWS OF PROBABILITY

Introduction

Crop insurance is by far the most popular risk management tool used by U.S. crop producers. Corn and wheat farmers insure more than 70 percent of all acres planted. The two most popular crop insurance products are Actual Production History (APH), which provides insurance against low yields, and Crop Revenue Coverage (CRC), which is a revenue insurance product. Both of these products offer coverage in 5 percent increments from 65 percent to 85 percent of expected yield or revenue. APH premium rates are calculated by multiplying the premium rate at the 75 percent coverage level by a rate relativity factor that is the same for all crops and for all regions. For example, to find an 85 percent APH premium rate, one simply multiplies the 75 percent rate by 1.60. The 65 percent rate equals the 75 percent rate multiplied by 0.65. CRC premium rates for a given coverage level are based on the corresponding APH rates. So calculating an 85 percent CRC premium requires knowledge of the 85 percent APH premium rate. Thus, changes in APH premium rates as coverage levels increase are directly reflected in changes in CRC premium rates.

The purpose of this paper is to examine whether these constant rate relativities are consistent with the laws of probability in the sense that the implied rates are consistent with a well-defined probability distribution of yields. This topic is relevant because changes in the federal subsidy program now encourage producers to increase their coverage level and have increased available coverage to levels for which there is no historical loss/cost data. Even if one accepts that the 65 percent rates are as accurate as they can be, there is no guarantee that the rates at higher coverage levels are accurate unless these rate relativities are also accurate.

The paper is organized as follows. First, we expand on the institutional detail and make the case that the changes in the subsidy program enacted in 2000 will encourage a gradual switch to higher coverage levels. Next, we provide a very basic set of rules that crop insurance rates must follow if they are to be consistent with the laws of probability. For example, we show that for symmetric and negatively skewed distributions, the maximum rate for coverage levels below 100 percent is 0.5. This maximum rate can be justified only if all of the mass of the yield distribution below the mean value is at zero. Hence, we argue that 0.5 puts a practical upper bound on premium rates, yet we find APH rates that exceed this maximum amount. We then gradually impose restrictions on the structure of the yield distribution and report the APH rates that violate these restrictions. Maps are used to show areas where APH rates and rate relativities might be consistent with the laws of probability. These maps show that the APH rates structure is particularly disadvantageous in wheat producing areas and in high-risk counties.

Finally, we add enough structure to calculate a set of rate relativities that are consistent with both the laws of probability and the literature on yield distributions. When we compare these rate relativities with those in the APH system, the source of the problem with the APH rate structure becomes clear. Changes in the 65 percent rate imply changes in the mass of the yield distribution to the left of this 65 percent level, which, in turn, has implications for the mass of the distribution to the right of the 65 percent level. Examination of these implications demonstrates that constant rate relatives are inconsistent with variations in the 65 percent rate across crops, producers, and counties. In other words, if there is evidence that rates should vary across producers, then there is evidence that rate relativities should vary across producers.

Institutional Background

The Agricultural Risk Protection Act (ARPA) of 2000 increased crop insurance premium subsidies significantly and changed them on coverage levels above 65 percent from a fixed per-acre dollar amount to a percentage of the premium. The percent subsidy depends on the coverage level as follows: 59 percent subsidy for coverage levels of 65 percent and 70 percent; 55 percent subsidy for coverage levels of 75 percent; 48 percent for 80 percent coverage; and 38 percent for 85 percent coverage. This policy change has two implications. First, the move to a subsidy expressed as a constant percentage for a given coverage level means that the per-acre subsidy increases with the per-acre premium, thus increasing the incentive for farmers to purchase more expensive products. The particular subsidy levels used for the different coverage levels cause the second effect. The decline in the percent subsidy associated with an increased coverage level is generally less than the increase in the insurance premium. Thus, per-acre subsidies also increase as coverage levels increase. This change encourages farmers to purchase higher coverage levels.

Crop insurance rates under USDA's APH program are empirically determined in that they depend upon the level of indemnities paid to farmers. They are set so that they would generate an adequate premium to cover average historical losses (Josephson, Lord, and Mitchell). Figure 1 shows that, until recently, the coverage level most in demand by farmers was the 65 percent level. The 1995 increase in acres insured at less than 65 percent was a result of a rule that made eligibility for commodity subsidies contingent on participation in the crop insurance program. The increase in popularity of coverage levels greater than 65 percent in 2000 and 2001 can be attributed to the increase in subsidies that were available on an emergency basis in 2000 and as part of ARPA in 2001. This increase in participation at coverage levels greater than 65 percent is consistent with the finding of Just, Calvin, and Quiggin that farmers' main motivation for purchasing crop insurance is to increase their net income by capturing the value of subsidies rather than to decrease risk.

Figure 1 suggests that the RMA has by far the most information about losses at the 65 percent coverage level. This means that the 65 percent level probably best reflects historical losses. The significant acreage covered at the 75 percent level suggests that there is some information about how losses, and hence rates, should increase as coverage levels increase. However, a substantial portion of the acreage insured at 75 percent comes from a few states. For example, in 1993, 40 percent of the acres insured at the 75 percent coverage level were in two states, Iowa and Illinois. This means that much of the knowledge about how losses increase as coverage increases above 65 percent resides in a relatively few states.

Insurance Rates and Probability Rules

Actuarially fair insurance rates are found by dividing expected indemnity by liability. For a yield insurance policy that covers against yield losses below some guaranteed level, Y_I, the actuarially fair rate is given by $r_I = (1/Y_I)\ \Pr(y < Y_I) \cdot$

$E[Y_I - y \mid y < Y_I]$, where the price paid per unit of yield loss is normalized to one, and $\Pr(y < Y_I)$ denotes the probability that yield is below the insurance level. Then

$$\Pr(y < Y_I) = \frac{r_I Y_I}{Y_I - E[y \mid y < Y_I]}. \tag{1}$$

Equation (1) shows that, given an insurance rate and a yield guarantee, there is a one-to-one relationship between conditional expected yields and the probability that yields are below the yield guarantee.

The laws of probability and the definitions of conditional expectation put bounds on the permissible values of probability and conditional expected yield. We know that if the insurance guarantee is less than the median yield, then $\Pr(y < Y_I) = 0.5$. For symmetric and negatively skewed distributions, we know also that the mean yield is no greater than the median. This implies that if the insurance yield is less than the median, it is also less than the expected value of yields and $\Pr(y < E[y]) = 0.5$. For positively skewed distributions, $\Pr(y < E[y]) > 0.5$, and it could be the case that $\Pr(y < Y_I) > 0.5$ if the insurance deductible is small enough.

For U.S. yield insurance products, the maximum guarantee is 90 percent of the expected value of yields for the Group Risk Plan (GRP) and 85 percent of APH yields for the APH program. Given this built-in deductible and given that APH yields are generally less than expected yields (Just, Calvin, and Quiggin), 0.5 places a practical upper bound on $\Pr(y < Y_I)$.

In addition, from the definition of a conditional equation, we know that $E[y \mid y < Y_I] < Y_I$. From equation (1), if $r_I = 0.5$, then the upper limit (practical or absolute) on $\Pr(y < Y_I)$ of 0.5 implies that the only permissible value of $E[y \mid y < Y_I]$ is 0. Thus, for negatively skewed and symmetric yield distributions, yield insurance rates greater than 0.5 for insurance coverage less than expected yield cannot be supported by a well-defined yield distribution. For positively skewed distributions, if the insurance guarantee is less than the median yield, then the rates greater than 0.5 cannot be supported by a well-defined yield distribution.

This result may seem trivial, but the APH program often charges farmers premium rates that exceed this upper limit. For example, in Hettinger County, North Dakota, a safflower farmer with an APH yield of 470 lb/ac or less will be charged a crop insurance rate of greater than 0.5 for a yield guarantee equal to 75 percent of APH yields. Of course, the vast majority of APH rates do not exceed 0.5, so this result is a rather weak condition. However, it can be used to develop a stronger condition.

Suppose we have two crop insurance rates, $r_1$and r_2, and two corresponding yield guarantees, Y_1 and Y_2, with $Y_2 > Y_1$. Denoting $\Pr(y < Y_I)$ as $F(Y_I)$, and using equation (1), we can write

$$\begin{aligned} r_2Y_2 - r_1Y_1 &= F(Y_2)\ (Y_2 - \mathrm{E}[y \mid y < Y_2]) - F(Y_1)\ (Y_1 - \mathrm{E}[y \mid y < Y_1]) \\ &= Y_2\ F(Y_2) - Y_1\ F(Y_1) - F(Y_1)\ \mathrm{E}[y \mid y < Y_1] \qquad (2) \\ &\quad - F(Y_1 = y < Y_2)\ \mathrm{E}[y \mid Y_1 = y < Y_2] + F(Y_1)\ \mathrm{E}[y \mid y < Y_1] \\ &= Y_2\ F(Y_2) - Y_1\ F(Y_1) - F(Y_1 = y < Y_2)\ \mathrm{E}[y \mid Y_1 = y < Y_2] \end{aligned}$$

which can be rewritten as

$$r_2Y_2 - r_1Y_1 = Y_2F(Y_2) - Y_1F(Y_1) - (F(Y_2) - F(Y_1))\ \mathrm{E}[y \mid Y_1 = y < Y_2]. \qquad (3)$$

The left-hand side of equation (3) shows the increase in the premium as coverage from yield insurance increases. With actuarially fair rates, this increase is a function of two cumulative probabilities and the conditional expectation of yield, given that it falls between the two yield guarantees. Again, there are permissible limits on both. From equation (1) we know that $0.5 = F(Y_2) = F(Y_1)$ for symmetric and negatively skewed distributions, and $Y_2 = \mathrm{E}[y \mid Y_1 = y < Y_2] = Y_1$ for all distributions.

The usefulness of equation (3) is that, for any two insurance rates and corresponding yield guarantees, it defines the combinations of probabilities and conditional yields that are consistent with actuarially fair rates that are generated by yield losses that are generated from some probability distribution. If there is no combination of cumulative probabilities and condition expectations that solves (3), then there does not exist a yield distribution function that could support the given rates and yield guarantees. That is, from the perspective of generating premiums sufficient to cover yield losses, the rates would not be actuarially fair.

Analysis of Actual Production History Rates

APH rates depend on a farmer's APH yield. For a given APH yield, knowledge of one coverage level's APH base rate is sufficient to calculate all other coverage level rates because RMA uses constant rate relativity factors to calculate rates at different coverage levels. These factors do not vary across crops or regions. The ratio of 70 percent rates to 65 percent rates is 1.21. The ratio of 75 percent rates to 65 percent rates is 1.53. The ratio of 80 percent rates to 65 percent rates is 1.93. And the ratio of 85 percent rates to 65 percent rates is 2.44. Currently, for many crops and counties, available coverage levels are 65 percent, 70 percent, 75 percent, 80 percent, and 85 percent. In some locations, coverage levels are available only at 65 percent, 70 percent, and 75 percent. An example best illustrates the method that we use to examine the actuarial fairness of APH rates.

A barley farmer in Becker County, Minnesota, with an APH yield of 55 bu/ac would pay an APH rate of 0.103 for 65 percent coverage and 0.125 for 70 percent coverage. The corresponding yield guarantees are 35.75 and 38.5 bu/ac. Suppose the conditional expected yield in equation (3) is 37 bu/ac. Substituting these numerical values into equation (3) and expressing $F(Y_2)$ as a function of $F(Y_1)$ results in $F(Y_2) = 0.75 - 0.833\, F(Y_1)$. There are solutions to this equation that satisfy the restrictions that $0.5 = F(Y_2) = F(Y_1)$, for example, $F(Y_1) = 0.35$ and $F(Y_2) = 0.458$. So some underlying yield distribution exists that could support these rates, and we cannot conclude that the 65 percent and 70 percent APH rates violate the laws of probability.

Now suppose that we have a barley farmer in Hubbard County, Minnesota, with an APH yield of 40 bu/ac. This farmer faces an APH rate of 0.172 at 65 percent coverage and 0.210 at 70 percent coverage. Suppose the conditional expected yield in equation (3) is 27 bu/ac. Substituting these values into (3) results in $F(Y_2) = 1.408 - F(Y_1)$. Clearly, there is no solution to this equation that satisfies $0.5 = F(Y_2) = F(Y_1)$. Hence, there exists no yield distribution that supports these rates that is consistent with a conditional expected yield of 27 bu/ac. Suppose the conditional expected yield equals the lowest possible level of 26 bu/ac. Then equation (3) becomes $F(Y_2) = 0.704$, which is not admissible. No combination of conditional expected yield and cumulative probabilities can be found that solves equation (3) and that satisfies the two conditions

$$0.5 = F(Y_2) = F(Y_1) \text{ and } Y_2 = E[y \mid Y_1 = y < Y_2] = Y_1.$$

If we accept that the 65 percent rate in Hubbard County is actuarially fair, then we can conclude that the 70 percent rate is not fair. It is too high in that there is no conditional expected yield in Hubbard County that can satisfy equation (3). Is there a 70 percent rate that could satisfy equation (3)? Suppose the rate for 70 percent coverage is 0.18 and the conditional expected yield is 26.5 bu/ac. Then equation (3) becomes $F(Y_2) = 0.379 - 0.333\, F(Y_1)$ and solutions to this equation clearly exist. This counter-example illustrates that the problem with the 70 percent APH rate in this county is that it is simply greater than can be justified by the laws of probability. That is, the rate relativity factor is too high.

Equation (3) can be normalized by dividing through by unconditional expected yield. This results in

$$r_2C_2 - r_1C_1 = C_2\, F(C_2) - C_1\, F(C_1) - (F(C_2) - F(C_1))\;\; \mathrm{E}[y \mid C_1 = \; y < C_2], \qquad (4)$$

where C_1 and C_2 are coverage levels. As previously discussed, because the RMA has the most loss experience with the 65 percent rates, we treat it as being actuarially fair. We can then ask the question, Given RMA rate relativities, for what range of 65 percent pure premium rates does there exist the possibility that rates at higher coverage levels are actuarially fair? Here, we define pure premium rates by equation (1). Table 1 provides the answer to this question.

For each combination of coverage levels in Table 1, we conducted a grid search over all 65 percent rates to find the rates for which at least one feasible solution to equation (4) exists. For example, equation (4) at a 65 percent rate of 0.125, a 70 percent rate of 0.1515, and a conditional yield of 0.65001 results in a feasible solution of $F(Y_1) = 0.45$

TABLE 1. Range of 65 percent pure premium rate and APH rate relativities that are consistent with an upper limit of 0.5 on Pr($y < Y_I$) and the definitions of conditional probability

Rate Combination	Feasible Range
65% and 70%	0.010 to 0.126
65% and 75%	0.010 to 0.101
65% and 80%	0.010 to 0.083
65% and 85%	0.010 to 0.070
6%, 70%, and 75%	0.010 to 0.084
65%, 75%, and 85%	0.010 to 0.053
65%, 70%, 75%, 80%, and 85%	0.010 to 0.047

and $F(Y_2) = 0.497$. Of course, this solution means that there is almost no possibility of having yields between 0.85 and 1.0, which illustrates the weakness of the conditions we are imposing for actuarial fairness of rates.

The first four results in Table 1 show that as the coverage level increases, the range of feasible 65 percent rates is reduced. This reduced range implies that the rate relativity factors are becoming more restrictive as the coverage level increases. The 85 percent pure premium rates can only be actuarially fair if the 65 percent rate is less than 0.07.

The pairwise consideration of rates in the first four rows of Table 1 puts no restrictions on the underlying distribution function for intermediate coverage levels. If we require that there be the possibility of actuarial fairness for intermediate coverage levels as well, then the range of feasible 65 percent rates becomes even narrower. For example, in counties where the maximum coverage level is 75 percent, the range of feasible 65 percent rates for which there is the possibility of actuarial fairness for the 65 percent rate, the 70 percent rate, and the 75 percent rate is 0.01 to 0.084. If we require the possibility of actuarial fairness for 65 percent, 75 percent, and 85 percent rates, then the maximum 65 percent rate is 0.053. And, if we require the possibility of actuarial fairness for all coverage levels, then the maximum 65 percent rate is 0.047.

Effects of Further Restrictions

The maximum 65 percent rates reported in Table 1 were obtained by setting the conditional yield equal to the minimum possible. This is equivalent to assuming that the probability of yields between the considered coverage levels is zero. In addition, no restrictions were placed on yields between 0.85 and 1.00. At the upper end of the ranges reported in Table 1, there is no probability that yields would fall between 0.85 and 1.00. This is illustrated in Figure 2, which shows graphs of the cumulative distribution function (CDF) for yields given 65 percent insurance rates and conditional yields when yields falls below 65 percent (denoted by CY 0 – 65). The yield distributions at the 65 percent rate of 0.047, which is at the upper end of the potentially acceptable range of rates for coverage up to 85 percent, show little possibility of yields falling between 0.80 and 1.00, regardless of the conditional yield level. The yield distribution at a 65 percent rate of 0.025, which is in the middle of the Table 1 range, shows a more realistic distribution, with positive weight across all coverage levels. For most distribution functions, one would expect the bulk of probability to fall around the level of mean yields, although this is not necessarily the case for bimodal distributions.

Given that the implied CDFs at the upper end of the ranges reported in Table 1 may not conform to prior expectations about how yield distributions should look, we examine five further restrictions regarding the probability of yields between the highest allowed coverage level (0.75 or 0.85) and 1.00. These five scenarios are that the probability of yields between the highest allowed coverage level and 1.00 is at least 0.05, 0.10, 0.15, 0.20, and 0.25. These scenarios provide a range of cases where some weight is distributed just below the APH yield in the yield distributions. For example, a normal distribution with a mean of 1.00 and a standard deviation of 0.25 would have 22.57 percent of its weight between 0.85 and 1.00.

We also examine the effects of forcing convexity on the CDF of yields between 65 percent and 85 percent coverage. Convexity implies that the difference in cumulative probabilities between two coverage levels increases as coverage increases: $F(0.85) - F(0.80) > F(0.80) - F(0.75) > F(0.75) - F(0.70) > F(0.70) - F(0.65)$. In addition to this convexity restriction, we also assume that $0.5 - F(0.85) > F(0.85) - F(0.80)$. To impose these conditions, we need to relate observed crop insurance rates to these cumulative probabilities.

Rewriting equation (4) for $C_2 > C_1$ gives an expression for cumulative probability at one coverage level as a function of cumulative probability at a lower coverage level and the conditional expectation of yield given that yield is between the two coverage levels:

$$F(C_2) = \frac{r_2 C_2 - r_1 C_1 - F(C_1)\,(\mathrm{E}[y \mid C_1 \le y < C_2] - C_1)}{C_2 - \mathrm{E}[y \mid C_1 \le y < C_2]}. \quad (5)$$

Given a conditional yield for yields below the 65 percent coverage level, we can solve for $F(0.65)$ using

$$F(0.65) = \frac{r_{0.65}\ 0.65}{0.65 - E[y \mid y < 0.65]}. \quad (6)$$

Then, $F(0.70)$ as a function of the conditional yield between 65 percent coverage and 70 percent coverage can be obtained through direct substitution into equation (5).

Likewise, *F*(0.75), *F*(0.80), and *F*(0.85) can be obtained with subsequent substitutions. All of the scenarios require a value for the conditional expectation of yield for yields below the 65 percent coverage level and values for the conditional expectation of yields between coverage levels. Convexity in cumulative probability implies that the expected yield conditional on yield being between two coverage levels would typically be greater than the midpoint of the coverage levels.

The task is to determine the range of feasible 65 percent rates that are consistent with a given set of probability restrictions. This task can be accomplished by searching over all possible values for expected yield conditional on yield being below 65 percent for each given 65 percent rate. If any of the conditional yields are consistent with the restrictions, then we can maintain that there is an underlying yield distribution that could be consistent with APH rates.

Table 2 presents the range of feasible pure premium rates. For the first five scenarios, a grid search is performed across all possible conditional yields at 0.005 unit intervals. For the convexity scenario, we assume that expected yield conditional on yield being between coverage levels equals the midpoint between the two yields.

A comparison of the Table 1 results with the Table 2 results shows that adding reasonable requirements for an underlying yield distribution decreases the maximum rate substantially. For crops and counties where 75 percent is the maximum coverage level, rates could possibly be actuarially fair if the 65 percent rate is less than 0.055 given the

TABLE 2. Range of 65 percent pure premium rate and APH rate relativities for which actuarial fairness is possible

Restriction	Rate Range	
	65%, 70%, and 75 %	65%, 70%, 75%, 80%, and 85%
No further restrictions (see Table 1)	0.010 to 0.083	0.010 to 0.047
0.5 - *F*(Highest coverage level) > 0.05	0.010 to 0.075	0.010 to 0.042
0.5 - *F*(Highest coverage level) > 0.10	0.010 to 0.067	0.010 to 0.037
0.5 - *F*(Highest coverage level) > 0.15	0.010 to 0.058	0.010 to 0.033
0.5 - *F*(Highest coverage level) > 0.20	0.010 to 0.050	0.010 to 0.028
0.5 - *F*(Highest coverage level) > 0.25	0.010 to 0.042	0.010 to 0.023
Convex CDF plus 0.5 - *F*(0.85) > *F*(0.85) - *F*(0.80)	0.010 to 0.055	0.010 to 0.032

convexity restrictions. For crops and counties that have 85 percent coverage, the maximum 65 percent rate for which actuarial fairness is possible is only 0.032 when the convexity restrictions are put in place.

Which Crops and Counties Have Actuarially Unfair Rates?

Given the bounds on rates indicated in the previous section, we would like to compare these bounds to current crop insurance rates throughout the country. However, these bounds are based purely on yield distribution arguments and do not include adjustments for insurance loading and prevented planting, whereas the current crop insurance rates do contain such adjustments. To create comparable rates, we have adjusted our bounds to reflect the insurance loading and prevented planting adjustments by dividing by 0.88 to capture the insurance loading adjustment and adding 0.005 to the rates to capture the prevented planting adjustment. These adjustments follow the rate-setting procedure outlined in Josephson, Lord, and Mitchell.

For the “no further restriction” case, the adjusted insurance rates have maximum bounds of 0.099 for areas with up to 75 percent coverage and 0.058 for areas with up to 85 percent coverage. For the restriction of at least a 15 percent probability of yields falling between the highest coverage level and 1.0, the adjusted insurance rates have maximum bounds of 0.071 for the areas with up to 75 percent coverage and 0.043 for the areas with up to 85 percent coverage. We examined the crop year 2000 APH 65 percent coverage level insurance rates for corn, soybeans, and wheat for the typical producer in each county (as determined by the R05 yield span, the middle yield span per county for these crops). Figures 3-5 show maps of these rates across the country and whether they fall into the bounds previously outlined. The counties shaded in dark blue have 65 percent APH rates that fall within the bounds set by a yield distribution with the 15 percent restriction in place for coverage levels from 65 percent to 85 percent. The counties shaded in gold have 65 percent APH rates that fall within the bounds set by a yield distribution with the 15 percent restriction in place for coverage levels from 65 percent to 75 percent. The counties shaded in white have 65 percent APH rates that exceed either of these bounds. Counties that are not outlined do not have insurance coverage for that crop. RMA provides these rates through their Actuarial Data Master web site (http://www.rma.usda.gov/tools/utils/grepadm/). These 2000 rates are indicative of current rates that are calculated with a formula without reference to a yield span.

Table 3 shows that most of the counties where APH insurance is available do not have actuarially fair rates over all coverage levels. For corn, only 21.4 percent of the counties have potentially fair rates up to 85 percent coverage, and 48.5 percent have potentially fair rates up to 75 percent coverage. With the 15 percent probability restriction, the proportions fall to 8.2 percent and 28.7 percent. As shown in Figure 3, these counties primarily reside in the Corn Belt. The proportion of soybean and wheat counties with potentially fair rates is similar, as shown in Table 3.

For corn and soybeans (see Figures 3 and 4), because the counties that have APH rates that could be actuarially fair are located in the Corn Belt, the proportion of production that they represent is high. For corn, almost 90 percent of production comes from counties that could have actuarially fair rates up to 75 percent coverage, and almost 65 percent of production comes from counties with APH rates that could be actuarially fair up to 85

TABLE 3. Proportion of counties and production with possibly actuarially fair APH premiums

	Criterion[a]			
	Under 0.043	**Under 0.058**	**Under 0.071**	**Under 0.099**
Corn				
Number	207	540	725	1225
% of Counties	8.2	21.4	28.7	48.5
% of Production	29.1	64.8	77.5	89.7
Soybeans				
Number	190	351	473	808
% of Counties	9.7	17.8	24.0	41.0
% of Production	35.7	56.9	66.4	80.1
Wheat				
Number	121	291	500	1105
% of Counties	5.0	11.2	20.5	45.4
% of Production	11.2	23.0	41.9	70.2

[a] The level 0.043 corresponds to the upper limit on the APH rate for which actuarial fairness is possible for coverage levels up to 85% if at least 15% of probability is between 85% and 100% coverage levels. 0.058 is the upper limit on actuarially fair APH rates up to 85% coverage levels with no such probability restriction. 0.071 is the upper limit on actuarially fair APH rates up to 75% if at least 15% of probability is between 75% and 100% coverage levels. 0.099 is the upper limit on actuarially fair APH rates up to 75% coverage levels with no such probability restriction.

percent coverage. However, this latter number falls to 29 percent of production if the reasonable 15 percent restriction is placed on the yield CDF. For soybeans, 36 percent of production comes from counties with APH rates that could be fair up to 85 percent coverage even with the 15 percent restriction. This estimate increases to 80 percent of production for coverage to 75 percent and no other CDF restriction.

For wheat, the story is different. As shown in Figure 5, most of the major wheat growing areas in Kansas and North Dakota have 65 percent APH rates that fall outside the possible fair range for coverage levels up to 85 percent. Even without the 15 percent probability restriction, only 23 percent of production comes from the 12 percent of the counties with low enough 65 percent APH rates to be actuarially fair. Even at 75 percent coverage with the 15 percent restriction, only 20 percent of the counties and 42 percent of production have 65 percent APH rates that are low enough to be fair.

Based on the results of Table 3, one would expect that wheat farmers would have been less likely than corn and soybean farmers to buy 75 percent coverage before the additional ARPA subsidies were available for the simple reason that moving to 75 percent coverage meant that incremental costs exceeded incremental benefits. Examination of RMA data bears out this conjecture. In 1998, 5.7 percent of wheat crop insurance policies were at the 75 percent coverage level, as compared to 9.8 percent of corn policies and 10.7 percent of soybean policies.

Rate Relativities Derived from a Density Function

A comparison of APH rate relativities with those derived from a density function will illustrate why constant APH rate relativities are not consistent with the laws of probability. The starting point is to select a density function.

Characterizing the distribution of farm-level yields has been the focus of much effort by agricultural economists. Day demonstrated that crop yields are skewed, although Just and Weninger demonstrated how data used to measure skewed yields is subject to a number of possible problems. Day found that the beta distribution is an appropriate functional form for parametric estimation purposes. Applied studies that have found the beta distribution useful include Babcock and Blackmer, Borges and Thurman, Babcock and Hennessy, and Coble et al.

The beta density function that describes the distribution of yield *y*, can be written

$$g(y) = \frac{G(p+q)}{G(p)G(q)} \frac{(y - y_{\min})^{p-1}(y_{\max} - y)^{q-1}}{y_{\max}{}^{p+q-1}} \text{ where } y_{\min} \le y \le y_{\max}.$$

where p, q, $y_{\max}$, and $y_{\min}$ are the four parameters. Advantages of the beta distribution are that it can exhibit both negative and positive skewness; it has finite minimum and maximum values; and it can take on a wide variety of shapes, including J-shaped, and normal-like.

What we want to accomplish is specification of beta parameters that are consistent with a given pure premium rate at the 65 percent coverage level. We do this by first relating the shape parameters, p and q, to the mean and standard deviation of farm yields, and to the maximum and minimum yields. For a given $y_{\max}$ and $y_{\min}$, p and q can be obtained from mean yield μ (which was set to 1) and the standard deviation of yields, s, by the following two equations (Johnson and Kotz, p. 44):

$$p = \left(\frac{m - y_{\min}}{y_{\max} - y_{\min}}\right)^2 \left(1 - \frac{m - y_{\min}}{y_{\max} - y_{\min}}\right) \left(\frac{s^2}{(y_{\max} - y_{\min})^2}\right)^{-1} - \frac{m - y_{\min}}{y_{\max} - y_{\min}}$$

$$q = \frac{m - y_{\min}}{y_{\max} - y_{\min}} \left(1 - \frac{m - y_{\min}}{y_{\max} - y_{\min}}\right) \Big/ \left(\frac{s^2}{(y_{\max} - y_{\min})^2}\right) - 1 - p.$$

We normalized mean yield to 1 and searched for the standard deviation of yields that resulted in the 65 percent APH rate using Monte Carlo integration. Of course, the maximum and minimum yields must be defined to identify a one-to-one mapping of yield standard deviation to APH rate. This was accomplished with the following specifications: $y_{\min} = \max(1 - 4s, 0)$ and $y_{\max} = 1 + 2s$. Thus, the search for a standard deviation that generates the 65 percent APH rates is accomplished by imposing these two conditions on the minimum and maximum yields.

The next step is to generate the appropriate yield distribution for a range of possible APH rates. We chose a series of possible 65 percent APH rates ranging from 0.02 to 0.3. An APH rate of 0.02 represents an extremely low risk production situation such as might exist with irrigated corn. An APH rate of 0.3 represents the high-risk extreme.

These four parameters were used to generate 5,000 draws from a beta density with a mean of 100 bushels. To calculate the fair premium, we found the average indemnity at each coverage level for each APH rate. These results are shown in Table 4.

Table 4 presents the actuarially fair pure premium rates for ten measures of yield uncertainty as expressed by a 65 percent pure premium rate shown in the first column. At low levels of yield uncertainty, the fair premiums are low at all coverage levels. However, the rate at which the premium rates increase is high. At higher levels of yield uncertainty, rates are high but the rate at which they increase in coverage levels is low. Figure 6 provides the intuition for this result. Presented are the cumulative distributions for two rates. The low rate of 0.03 shows the standard S-shaped curve that we expect when looking at a yield distribution. This curve is quite convex over the range of yields from 0.65 to 0.85 as shown. The probability of a yield being below 65 percent is about 13 percent. This probability rises to about 30 percent at a yield of 0.85. This means that the chances of receiving a crop insurance indemnity increase by a factor of 2.3.

Now observe the CDF associated with a 10 percent pure premium rate at the 65 percent coverage level. The probability of receiving an indemnity with 65 percent coverage is 32 percent. This probability does not grow rapidly as coverage increases because the CDF is concave over this range. The probability only grows by a factor of 1.3 as coverage increases to 85 percent. This demonstrates why crop insurance rates should increase by a lesser amount for crops and regions that have high initial rates compared to crops and regions in lower-risk areas.

TABLE 4. Actuarially fair pure premium rates

65% Rate	70% Rate	75% Rate	80% Rate	90% Rate
0.020	0.027	0.035	0.045	0.057
0.030	0.038	0.048	0.059	0.072
0.040	0.049	0.060	0.072	0.085
0.050	0.060	0.072	0.084	0.098
0.060	0.071	0.083	0.096	0.110
0.080	0.092	0.105	0.118	0.132
0.100	0.113	0.126	0.140	0.154
0.150	0.163	0.177	0.191	0.204
0.200	0.213	0.226	0.239	0.252
0.300	0.312	0.323	0.334	0.344

A Comparison with the APH Rate Relativities

Figure 7 compares the rate relativities calculated from Table 3 with the constant rate relativities used in the APH program. The APH rate relativities are close to those implied by a 0.03 pure premium rate, and this suggests that they are most accurate at this rate level. However, a constant set of rate relativities cannot be accurate at all rate levels. For example, with the 10 percent pure premium rate used in Figure 7, the actuarially fair 85 percent premium rate is 1.54 times as large as the 65 percent rate. However, the 85 percent APH rate is 2.13 times as large. That is, the APH rate is 38.3 percent greater than the actuarially fair rate calculated from a well-defined yield distribution. It is clear that the APH rate structure is not supportable by the laws of probability. The simple fact is that as 65 percent rates increase, more of the mass of the yield distribution must lie to the left of the 65 percent level, and this means that fewer outcomes are available for the area between the 65 percent level and the mean of the distribution. As mass is shifted to the left of the distribution, the probability of a yield outcome that would trigger an indemnity at 75 percent and not at 65 percent is reduced, and this in turn reduces the motivation for a higher 75 percent rate. Figure 7 shows that the potential for errors in premium rates increases as 65 percent APH rates increase and as coverage levels increase.

Implications and Conclusions

The use of fixed rate relativities to set APH rates cannot be supported if rates at all coverage levels are to be actuarially sound. As shown in Figure 7, farmers who plant crops in regions where the 65 percent pure premium rate is above 0.03 (which corresponds to an APH rate of about 0.04) face unsubsidized premiums that are too high at 80 percent and 85 percent coverage levels. The higher is the yield risk, the greater is the discrepancy between actuarially fair rates and actual APH rates for coverage levels greater than 65 percent. This discrepancy could explain why 65 percent coverage was the most popular coverage level in higher-risk areas. Farmers in high-risk areas were being charged too much at higher coverage levels. Farmers' reluctance to increase their coverage above 65 percent could also explain why Congress felt the need to move away from a fixed per-acre premium subsidy to the current subsidy structure that increases per-acre subsidies as coverage levels increase. That is, the higher subsidies are used to offset the excessive premium charge, making higher coverage levels more attractive. If APH rates were made actuarially fair by allowing rate relativities to vary by crop and region, then perhaps Congress could revert to fixed per-acre subsidies, which would induce farmers to select the crop insurance product that gives them the biggest risk management return per premium dollar rather than having that decision distorted by proportionate premium subsidies.

The increased subsidies under ARPA have brought increased public attention to the implications of an unsound APH rate structure. For example, Barnaby demonstrated that Revenue Assurance (RA) with the harvest price option could actually cost 20 percent less than simple yield insurance and 32 percent less than CRC for Kansas dryland corn production at 85 percent coverage. Farmers in these regions must settle for lower coverage, or they may find that increased subsidies available at higher coverage levels offset the excessive unsubsidized premium.

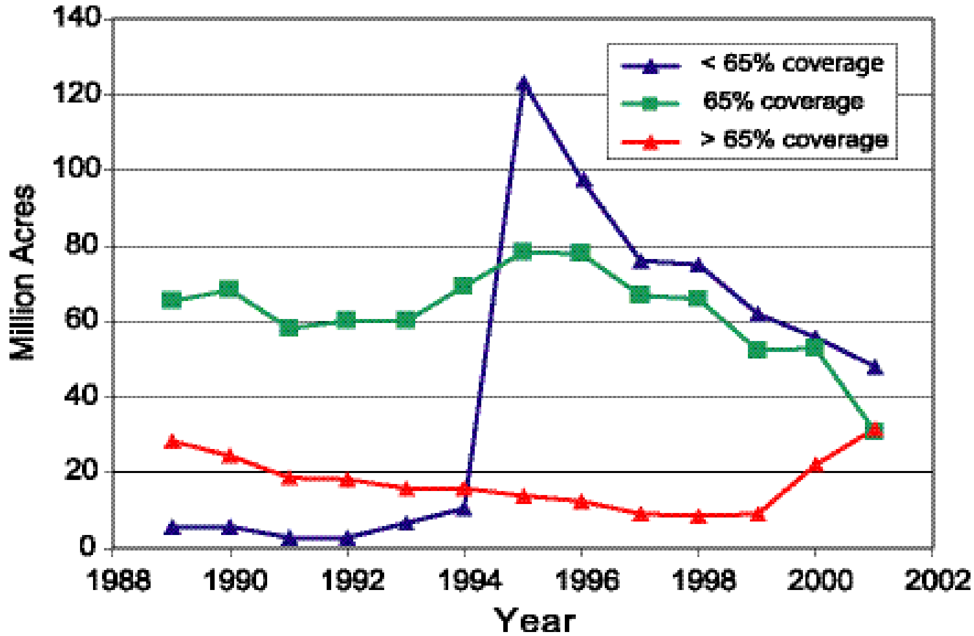


FIGURE 1. Net acres insured under the APH program

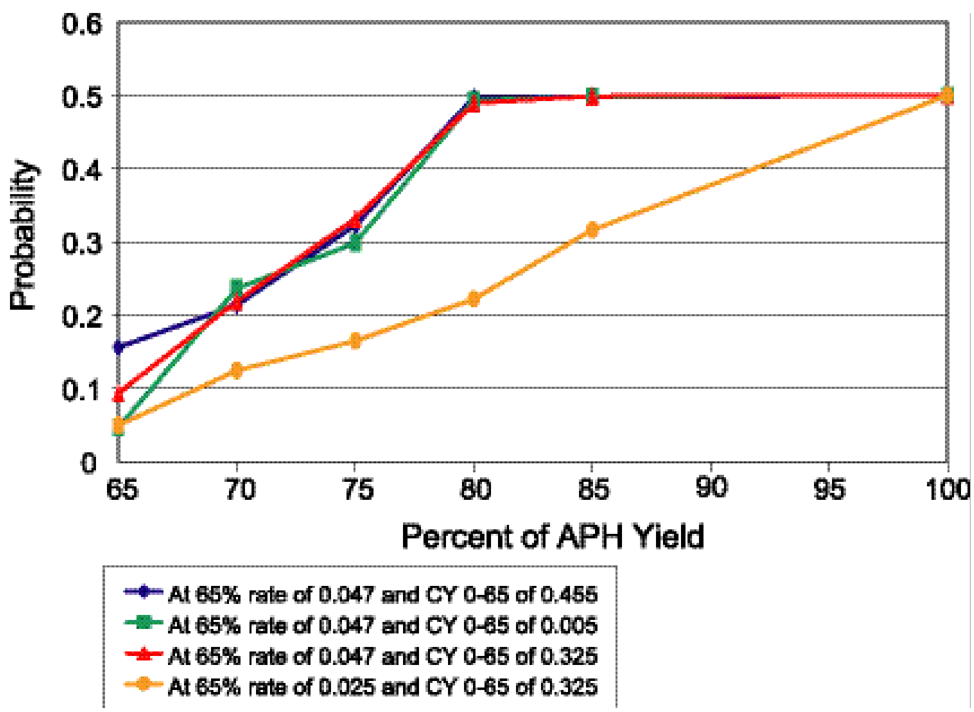
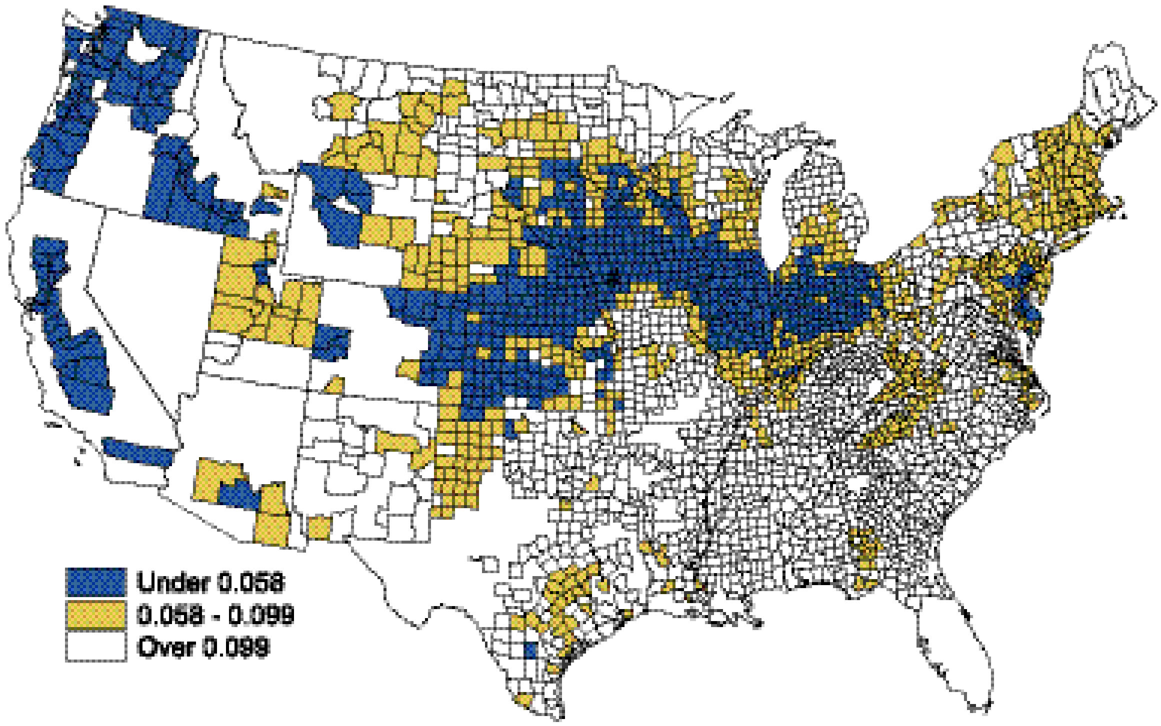
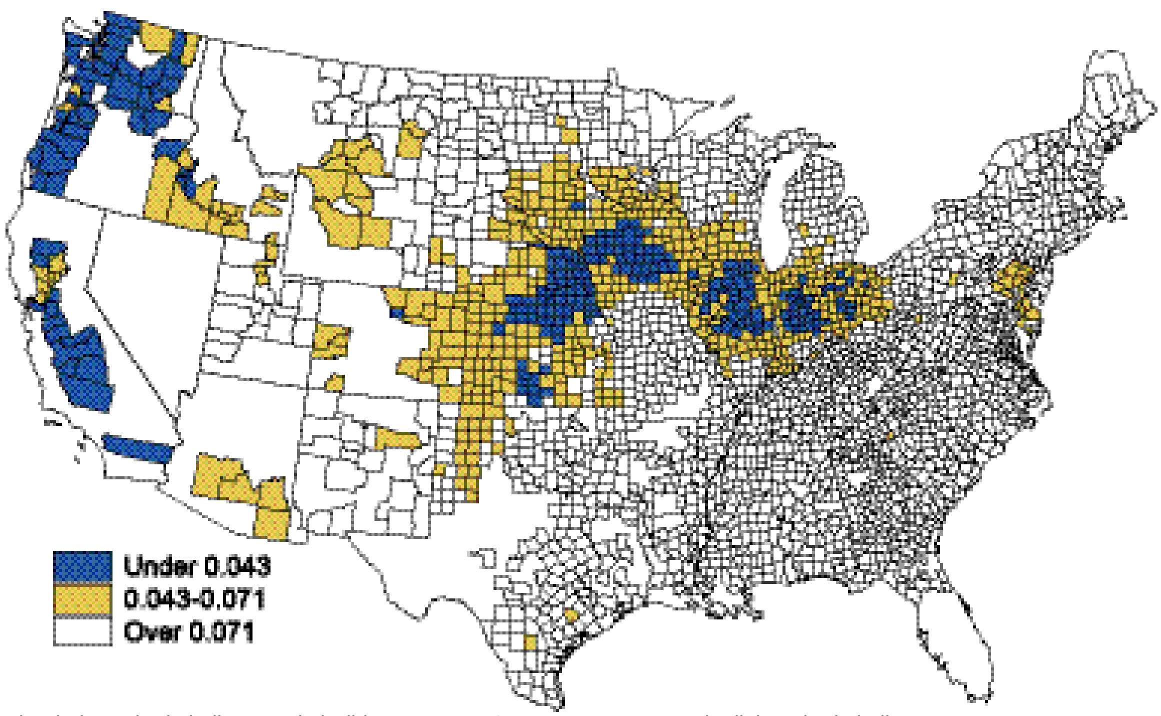


FIGURE 2. Graphs of cumulative distribution functions at various insurance rates and conditional yields

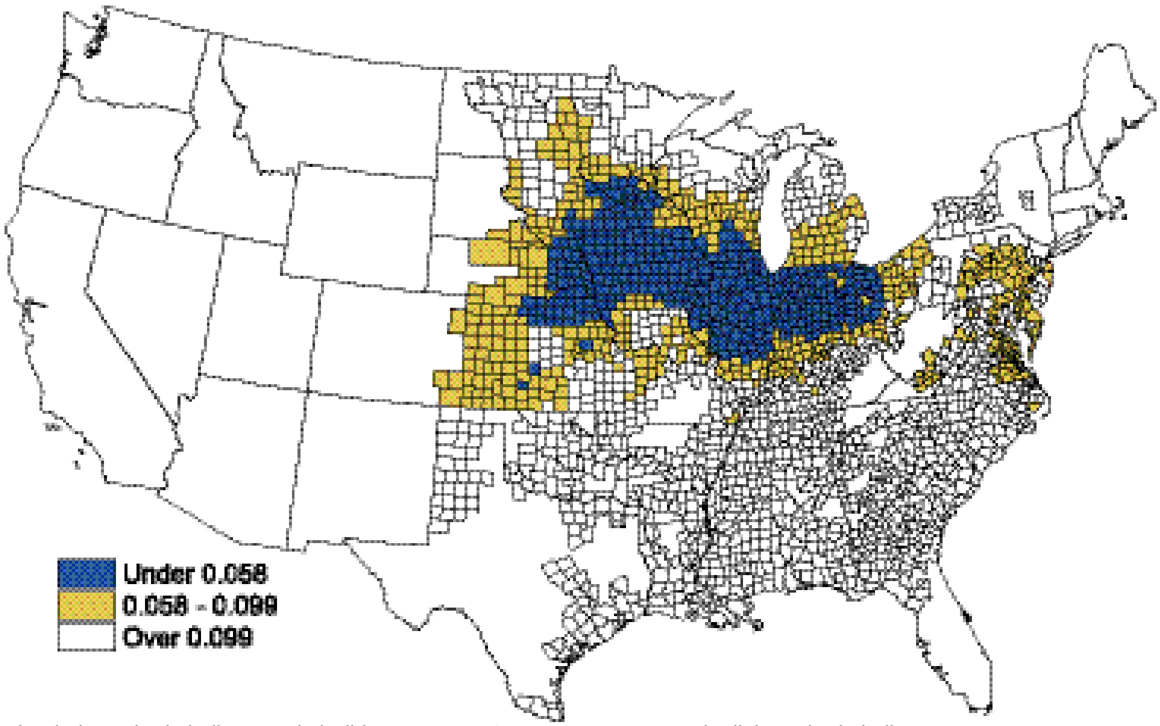


The darkest shade indicates admissible rates up to 85 percent coverage; the lighter shade indicates admissible rates up to 75 percent coverage with no restrictions on the distribution function.

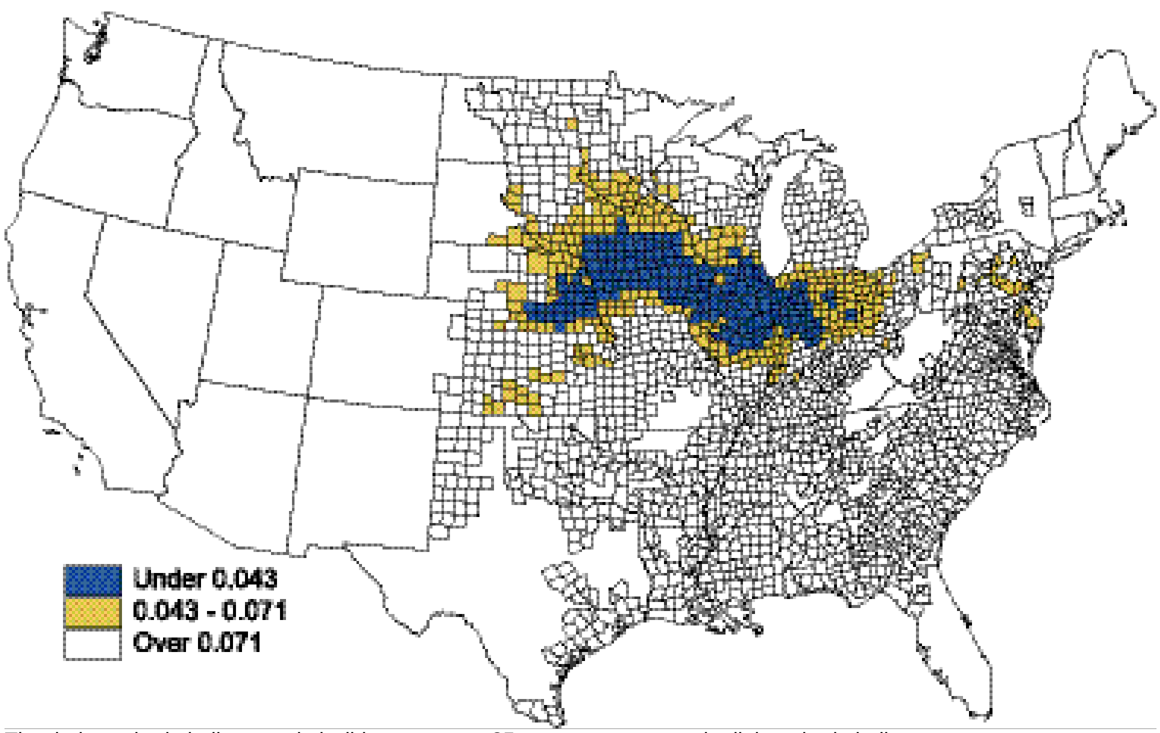


The darkest shade indicates admissible rates up to 85 percent coverage; the ligher shade indicates admissible rates up to 75 percent coverage with the restriction that 0.5 - *F* (highest coverage level) > 0.15.

FIGURE 3. Sixty-five percent APH rates for corn

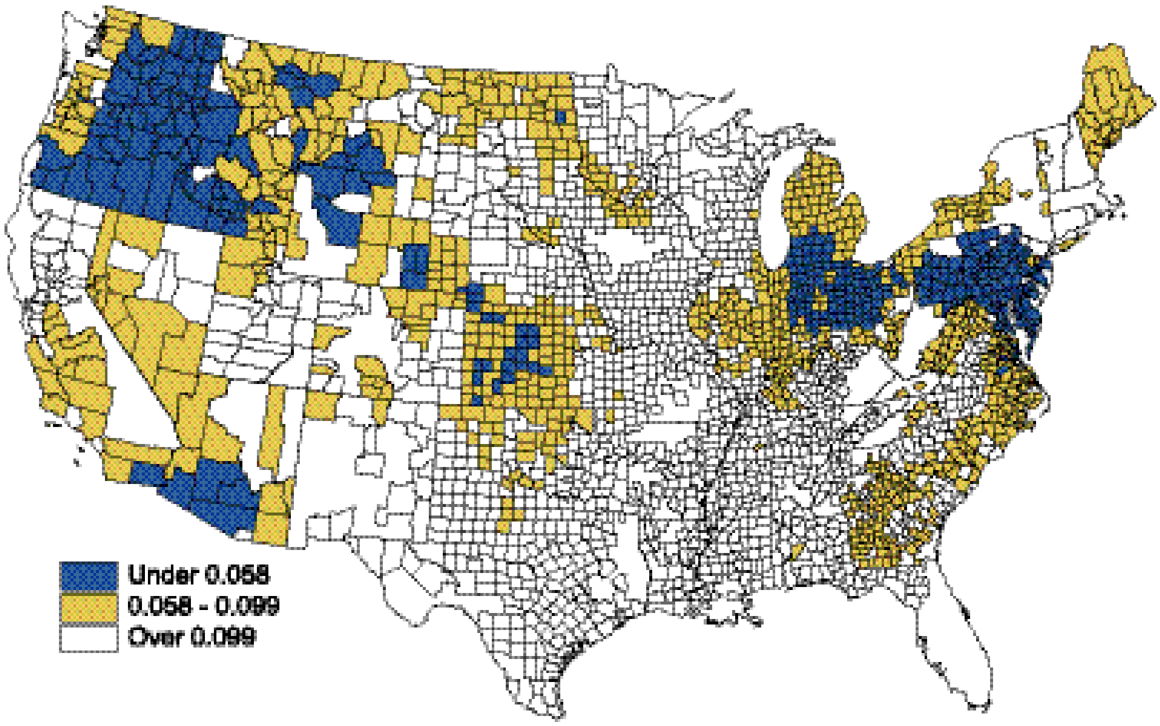


The darkest shade indicates admissible rates up to 85 percent coverage; the lighter shade indicates admissible rates up to 75 percent coverage with no restrictions on the distribution function.

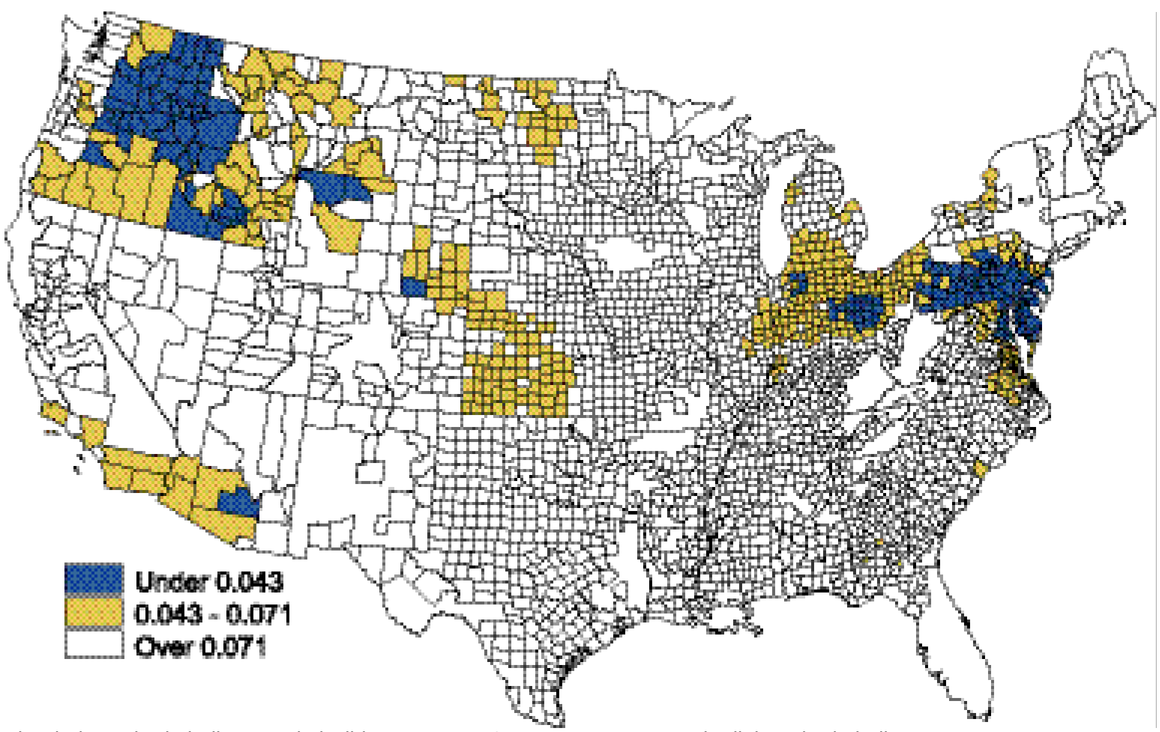


The darkest shade indicates admissible rates up to 85 percent coverage; the ligher shade indicates admissible rates up to 75 percent coverage with the restriction that 0.5 - *F* (highest coverage level) > 0.15.

FIGURE 4. Sixty-five percent APH rates for soybeans



The darkest shade indicates admissible rates up to 85 percent coverage; the lighter shade indicates admissible rates up to 75 percent coverage with no restrictions on the distribution function.



The darkest shade indicates admissible rates up to 85 percent coverage; the ligher shade indicates admissible rates up to 75 percent coverage with the restriction that 0.5 - *F* (highest coverage level) > 0.15.

FIGURE 5. Sixty-five percent APH rates for wheat

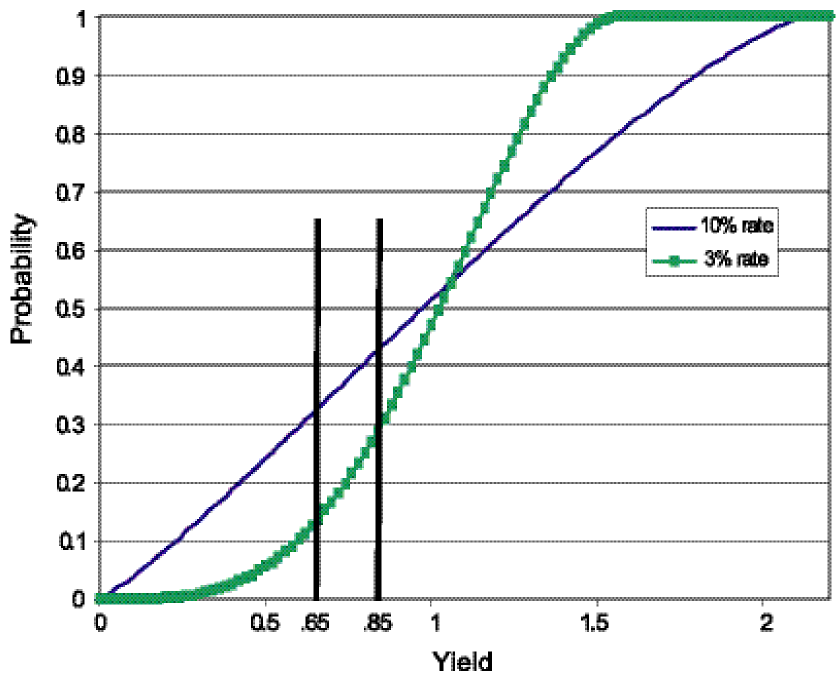


FIGURE 6. Beta cumulative distribution functions for two pure premium rates and an expected yield equal to 100

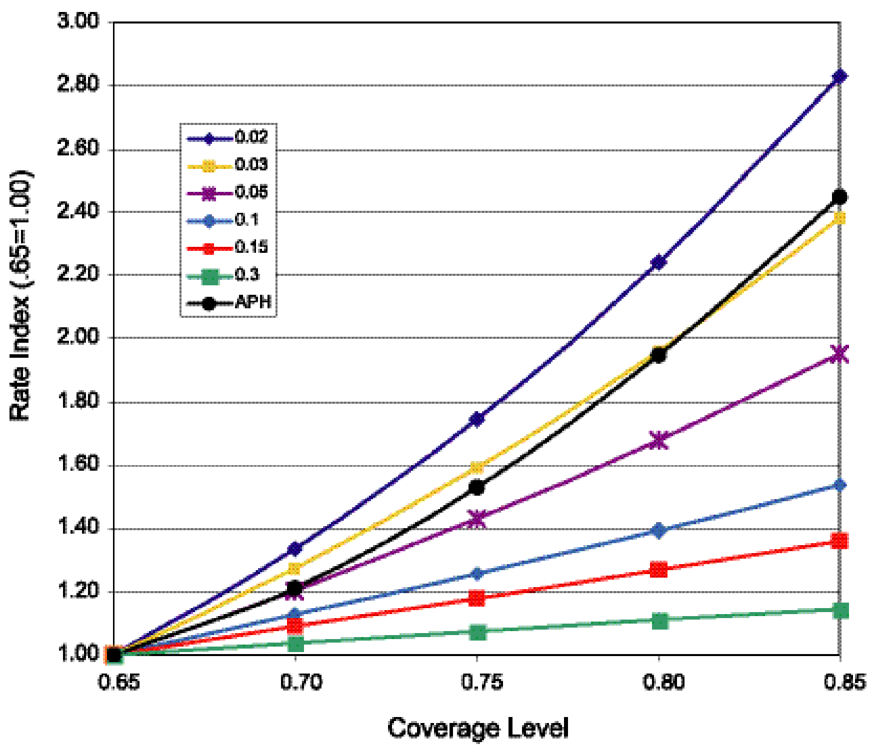


FIGURE 7. Comparison of APH rate relativities to those derived from a beta distribution at various 65 percent pure premium rates

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