

Bayesian Estimation of Technical Efficiency of a Single Input

Lyubov A. Kurkalova and Alicia L. Carrquiry

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Center for Agricultural and Rural Development

Iowa State University
Ames, Iowa 50011-1070
www.card.iastate.edu

Lyubov A. Kurkalova is an associate scientist, Center for Agricultural and Rural Development. Alicia L. Carriquiry is an associate professor, Department of Statistics, and associate provost, Iowa State University.

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For questions or comments about the contents of this paper, please contact, Lyubov A. Kurkalova, 483 Heady Hall, Ames, Iowa. Phone: 515-294-6307, Fax: 515-294-6336, Email: lyubov@iastate.edu.

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Abstract

We propose estimation of a stochastic production frontier model within a Bayesian framework to obtain the posterior distribution of single-input-oriented technical efficiency at the firm level. The proposed method is applicable to the estimation of environmental efficiency of agricultural production when the technology interaction with the environment is modeled via public inputs such as soil quality and environmental conditions. All computations are carried out using Markov chain Monte Carlo methods. We illustrate the approach by applying it to production data from Ukrainian collective farms.

Keywords: stochastic production frontier, Bayesian estimation, input efficiency, environmental efficiency

BAYESIAN ESTIMATION OF TECHNICAL EFFICIENCY OF A SINGLE INPUT

Introduction

Most existing stochastic production frontier models assume that technical efficiency is firm- or time-specific, but not input-specific. In many cases, however, researchers may have enough reasons to believe that a firm is especially inefficient in the use of a single input. For example, in the case of post-Soviet collective farming, one may believe that most of the measured inefficiency comes from the labor input: people work on their subsidiary household plots, but these hours get recorded as being spent in collective production. In well-established market economies, a sole input inefficiency is of interest when the input in question does not have a market price, as is the case of environmental quality or another public good. As an example, Reinard et al. (1999) (hereafter referred to as RLT) estimate environmental efficiency of Dutch dairy farms, where the environmental inefficiency is defined via the amount of nitrogen overused in the production process relative to a required technological minimum.

A well-known way to measure technical efficiency of an input use is via a mathematical programming approach (for a review of this methodology see, for example, Seiford and Thrall [1990]). Because the production frontier is constructed to envelope the observed data on inputs and outputs, this approach also is called data envelopment analysis. After the frontier is constructed, the inefficiency of a particular input is calculated as a multiplier by which the use of this input can be reduced while remaining in the feasible production set. The serious shortcoming of this technique lies in the inability to account for randomness in production, because all deviations from the frontier are assumed to be associated with inefficiency. Although this may be an acceptable assumption in some settings, it is hardly justifiable in the analysis of

agricultural production, which is intrinsically prone to random disturbances due to, for example, weather fluctuations, pests, and plant and animal diseases.

In the agricultural economics literature an alternative approach, the stochastic frontier approach, has been generally preferred; Battese (1992), Bravo-Ureta and Pinheiro (1993), and Coelli (1995) provide reviews of recent production frontier applications in agriculture. Stochastic frontier models have two error terms, one for inefficiency and another one associated with factors such as measurement error in output, combined effects of unobserved inputs in production, weather, etc. This intuitively appealing property of stochastic frontiers, however, comes at a price: assumptions must be made about the functional form of the production function and of the distributions of the two errors involved (for comprehensive reviews of stochastic frontier techniques, see Bauer [1990] and Greene [1993]).

A generic stochastic parametric production frontier model is a model of the form

$$y_i = F(x_{1i}, \dots, x_{Ni}; \bar{\beta}) + v_i - u_i, \quad (1)$$

where y_i is (a function of) the i -th firm output, and $F(\cdot)$ is a known function of the firm's inputs x_{1i}, \dots, x_{Ni} and a vector of parameters $\bar{\beta}$. The random component v_i is white noise representing, for example, errors in measurement of the firm's output and other events (such as weather variations and pest infestations) affecting the firm's output unobservable by econometricians. The nonnegative random variable u_i represents the firm's technical inefficiency measured in terms of forgone output, that is, by how much the firm's output falls short of the maximum possible output obtainable given the technology and the quantities of inputs available.

The model has been widely used to estimate an output-oriented technical efficiency defined as (a function of) $TE_{yi} \equiv y_i / y_i^F$, where y_i^F is the maximum possible output obtainable given the input quantities x_{1i}, \dots, x_{Ni} ; the maximum output obtainable is computed by replacing y_i with y_i^F and setting $u_i = 0$ in (1). By construction, $0 \leq TE_{yi} \leq 1$, and the measure of output efficiency is intuitively appealing as the higher values of TE_{yi} correspond to higher technical efficiency (lower values of u_i), with the case

$TE_{y_i} = 1$ representing full efficiency ($u_i = 0$ or $y_i^F = y_i$), and the case

$TE_{y_i} = 0$ corresponding to complete inefficiency ($u_i = \infty$ or $y_i^F \gg y_i$).

Instead of referring to the output-oriented scale, a firm's technical efficiency could also be measured on an input-oriented scale, i.e., by identifying how much less inputs could have been used to produce output y_i had the firm been technically efficient.

Assuming for the sake of presentation that the single input overused is x_1 , the single-input-oriented technical efficiency is defined as (a function of) $TE_{1i} \equiv x_{1i}^F / x_{1i}$, where x_{1i}^F is the minimum amount of input x_1 needed to produce y_i . As with the output counterpart TE_{y_i} , TE_{1i} changes between $TE_{1i} = 0$ (complete inefficiency, $x_{1i}^F \ll x_{1i}$) and $TE_{1i} = 1$ (full efficiency, $x_{1i}^F = x_{1i}$). For a two-input case, the output-oriented and the input- x_1 -oriented technical efficiency measures are illustrated in Figure 1.

Recently, RLT (1999) proposed a novel approach to the estimation of a single-input-oriented technical efficiency for the model (1). They estimated (1) using the method of maximum likelihood, and then they used the parameter and output-oriented efficiency estimates to compute single-input-oriented efficiency estimates for every firm in the sample. Specifically, RLT computed x_{1i}^F via subtracting (1) from the equation (1) in which x_{1i} was replaced with x_{1i}^F and u_i was set to zero.

The RLT estimator of the input efficiency TE_{1i} is obtained for a translog frontier production function $F(\cdot)$ and is a cumbersome, nonlinear function of model parameters. A Cobb-Douglas specification simplifies the estimator, but the nonlinearity remains. The nonlinearity does not introduce difficulties when obtaining a point estimator of the input efficiency, because the maximum likelihood estimator (MLE) of TE_{1i} can be obtained in a straightforward manner. This is due to the invariance property of MLEs under certain transformations. But obtaining an estimate of the standard error of the MLE can be challenging, and only approximations are possible. To improve statistical inference for single-input-oriented technical efficiency, we propose a Bayesian approach for estimating the parameters of model (1).

We build on results of applications of Bayesian techniques in stochastic production frontier models estimation in the literature (van den Broeck et al. [1994], Koop et al. [1995], Osiewalski and Steel [1998]). But unlike these studies, which focus on the frontier- and output-oriented efficiency, our study employs Bayesian techniques to estimate a single-input-oriented technical efficiency.

The production model we postulate is a three-tier hierarchical model. We choose noninformative or diffuse prior distributions where possible. Markov chain Monte Carlo methods are used to obtain samples from the distributions of the parameters of interest. We illustrate the approach by applying it to production data from Ukrainian collective farms.

The rest of the paper is organized as follows. In the next section, we lay out our model together with prior distributions for all parameters. We then describe the Gibbs sampler used to obtain unconditional posterior distributions of the parameters and illustrate the method using production data from a sample of Ukrainian collective farms. We conclude with a discussion of results and directions for future research.

In this work, our aim is to emphasize the methodology rather than the application itself. Our goal is to describe an approach for modeling and estimation that is flexible and that provides insight beyond that arising from implementation of classical statistical techniques such as maximum likelihood. In particular, we argue later in this paper that one advantage of the Bayesian approach is that it permits estimation of firm-level parameters in a straightforward manner. It is not possible to do so from a classical statistical perspective.

Econometric Model

We postulate a three-level hierarchical model to describe the association of firm output and inputs.

In *level 1*, the farm's logarithm of output in tons, $\log(y_i)$, is modeled as a normally distributed random variable with mean equal to a linear combination of the logarithms of

production inputs $\log(x_{1i}), \dots, \log(x_{Ni})$ minus the amount of inefficiency u_i and with variance σ_v^2 :

$$\log(y_i) \mid \underbrace{u_i, \beta_0, \beta_1, \dots, \beta_N, \sigma_v^2}_{\text{parameters}}; \underbrace{x_{1i}, \dots, x_{Ni}}_{\text{data}} \sim N\left(\beta_0 + \beta_1 \log(x_{1i}) + \dots + \beta_N \log(x_{Ni}) - u_i, \sigma_v^2\right),$$

i.e.,

$$\log(y_i) = \beta_0 + \beta_1 \log(x_{1i}) + \dots + \beta_N \log(x_{Ni}) + v_i - u_i, \quad (2)$$

where $v_i \sim N(0, \sigma_v^2)$. Here the subscript i refers to the i -th farm, $i=1, \dots, M$. Conditional on the observable data and the parameters, the outputs $\log(y_i)$ are independent for all i .

That is, outputs are assumed to be conditionally exchangeable.

In level 2, the technical inefficiency u_i is modeled as an exponential random variable with an inverse scale parameter λ^{-1} :

$$u_i \mid \underbrace{\lambda^{-1}}_{\text{parameter}} \sim \text{Exponential}(\lambda^{-1}). \quad (3)$$

Conditional on the parameter, u_i are independent for all i .

In level 3, the priors for the parameters $\beta_0, \beta_1, \dots, \beta_N, \sigma_v^2$ and the hyper-parameter λ^{-1} are specified.

In the classical setting, model (2)-(3) has been introduced by Meeusen and van den Broeck (1977). Bayesian analysis of models such as (2)-(3) has been pioneered in van Broeck et al. (1994) and Koop et al. (1995). Osiewalski and Steel (1998) named model (2)-(3) a *common efficiency distribution model*, because all inefficiency terms constitute independent draws from the same distribution, as opposed to a varying efficiency distribution model, in which the distributions from which the u_i 's are drawn vary with firms.

The Bayesian approach to estimation combines the information about model parameters that is available from all sources. Information contained in the data is summarized in the likelihood function, just as is done in the classical approach to estimation. In the Bayesian approach, however, it also is possible to incorporate information from other sources. To do so, prior beliefs or prior knowledge (or lack

thereof) about model parameters are summarized into the prior distributions chosen for those parameters. Bayes' Theorem provides a mechanism to combine both sources of information into the posterior density of the quantities of interest, i.e., of $\beta_0, \beta_1, \dots, \beta_N, \sigma_v^2$, and λ^{-1} . In stochastic frontier models, we also are interested in the firm efficiencies, which are functions of u_i 's. The joint posterior distribution that results from combining the likelihood function and the prior distributions is an unwieldy multivariate function. Thus, derivation of the posterior marginal distributions of the parameters is not analytically tractable. We use a numerical approach, the Gibbs sampler, also called alternating conditional sampling, which permits obtaining a sample from the joint posterior distribution of all parameters by taking random draws from only full conditional distributions (see, for example, Gelman et al. [1995], for a detailed description of this technique). Following Koop et al. (1995), we include the u_i 's into the set of the random quantities for which we obtain the joint posterior distribution using the Gibbs sampler.

Prior Distributions

The following prior distributions were chosen for the parameters in model (2)-(3):

$$\begin{aligned}\beta_0 &\sim Unif(-\infty, \infty); \\ \beta_k &\sim Unif(0, \infty); k = 1, \dots, N; \\ \sigma_v^{-2} | p_1, p_2 &\sim Gamma(p_1, p_2); \\ \lambda^{-1} | r^* &\sim Exponential(-\ln(r^*)).\end{aligned}$$

A noninformative improper prior distribution is used for B_0 because the magnitude of this parameter varies with the units of measurement of the production inputs. From economic theory, β_k is the elasticity of output with respect to the k -th input ($k=1, \dots, N$). Thus, the uniform prior distributions are truncated below by zero. A Gamma prior distribution is a widely used choice for the inverse of the variance parameters in normal models (Gelman et al. [1995]). Fernandez et al. (1997) have shown that the parameter p_2 must be positive, because otherwise the posterior distribution in the inefficiency model does not exist. The chosen relatively noninformative prior distribution of

λ^{-1} implies the prior median of the efficiency distribution is τ^* (van den Broeck et al. [1994]).

The probability density functions of the prior distributions are provided in the Appendix.

Single-input-oriented Technical Efficiency

For the sake of presentation, we assume that the single input, in which the firms are technically inefficient, is x_1 . Following RLT (1999), we compute the x_1 -oriented inefficiency of the i -th firm as $TE_{1i} \equiv x_{1i}^F / x_{1i}$, where x_{1i}^F is obtained by replacing x_{1i} with x_{1i}^F and setting $u_i = 0$ in (2), i.e.,

$$\log(y_i) = \beta_0 + \beta_1 \log(x_{1i}^F) + \beta_2 \log(x_{2i}) + \dots + \beta_N \log(x_{Ni}) + v_i.$$

Subtracting the last equation from (2), we obtain, as in RLT, $TE_{1i} = \exp\left(-\frac{u_i}{\beta_1}\right)$, and

$TE_{1i} \in [0, 1]$ by construction.

Straightforward algebra ensures that the conditional probability density function of $z \equiv TE_{1i}$ is given by

$$p(z | \beta_1, \lambda^{-1}) = \beta_1 \lambda^{-1} z^{\beta_1 \lambda^{-1} - 1}, \quad z \in [0, 1],$$

$$E[z | \beta_1, \lambda^{-1}] = \frac{\beta_1 \lambda^{-1}}{1 + \beta_1 \lambda^{-1}}, \quad \text{and} \quad V[z | \beta_1, \lambda^{-1}] = \frac{\beta_1 \lambda^{-1}}{(1 + \beta_1 \lambda^{-1})^2 (2 + \beta_1 \lambda^{-1})}.$$

The case of the parameters $\beta_0, \beta_1, \dots, \beta_N, \sigma_v^2, \lambda^{-1}$ having degenerate distributions corresponds to a classical statistics model of Meeusen and van den Broeck (1977). Note that the mean of the x_1 -oriented-efficiency is a nonlinear function of model parameters. A point estimator of the mean x_1 -oriented-efficiency can be obtained in a straightforward manner from a classical viewpoint simply by plugging into the previous expressions the MLEs of the parameters. Obtaining a standard error for the estimator, however, is difficult from a classical perspective. This is because the estimators of interest are

nonlinear functions of model parameters. Consequently, drawing inference about input efficiency from a classical perspective is difficult, particularly in finite samples.

An alternative is to proceed with the estimation of model parameters from within a Bayesian framework. Although analytically deriving a Bayesian estimate is typically impossible (except in trivial applications), recent developments in numerical methods (see, e.g., Gelfand and Smith [1990]) have made it possible to implement Bayesian techniques in complex subject-matter problems. In particular, Markov chain Monte Carlo methods, of which the Gibbs sampler is but one variation, can be used to approximate marginal posterior distributions of model parameters and of smooth functions of the parameters. For example, it is possible to derive the posterior distribution of mean input-oriented technical efficiency. Given estimated posterior distributions, a wide array of inferences can be drawn about the parameter of interest (or about functions of it). We illustrate the approach later in this paper by applying the methods to production data obtained from a sample of collective farms in Ukraine.

Posterior Distributions and Gibbs Sampler

The conditional posterior distributions of the quantities of interest, $\bar{\beta}, \sigma_v^2, \lambda^{-1}$, and u_i 's are used to implement a Gibbs sampler. That is, the distributions of $\theta_i | \theta_{(-i)}; data$ are used, where $\theta \equiv (\bar{u}, \bar{\beta}, \sigma_v^2, \lambda^{-1})$, θ_i is a subvector of θ , $\theta_{(-i)}$ is θ without the element θ_i , and $data$ includes \bar{y} and \bar{x} . The conditional posterior distributions of $u_i, \bar{\beta}, \sigma_v^2, \lambda^{-1}$ are a truncated Normal, truncated multivariate Normal, Gamma, and Gamma distribution, respectively. They are reported in Koop et al. (1995) and Osiewalski and Steel (1998). The corresponding probability density functions are provided in the Appendix.

Briefly, the Gibbs sampler proceeds as follows. A value of each parameter in the model is drawn from the corresponding conditional distribution. The sequence of draws obtained in this manner form a Markov chain of each parameter, whose stationary distribution can be shown to be equal to the marginal posterior distribution of the parameter. Gelfand and Smith (1990) provide the proof for the result above and list the conditions that must be met for good performance of the method. In practice, we start

with “guesses” for the values of the parameters in the model and proceed sequentially, drawing a value from each of the full conditional distributions described earlier. After a suitably large number of Gibbs steps, the draws from the conditionals can be thought of as draws from the corresponding marginal posterior distributions. Although convergence of the chains to their stationary distributions is very hard to assess exactly, the behavior of the chains can be monitored and “convergence” can then be assumed. In the next section, we describe an approach to monitor the behavior of the Markov chains when discussing the application. If the full conditional distributions of all parameters are of standard form (e.g., of normal, gamma, or other similar form) then implementing the Gibbs sampler is easy, which is the case for our model. Further details on the Gibbs sampler, including convergence criteria and application to the estimation of technical efficiency models, are provided in Koop et al. (1995) and in Osiewalski and Steel (1998).

Implementation of the Gibbs sampler results in a (correlated) sample of draws of virtually any function of the model parameters. In each pass, draws from the distribution of the input- x_1 -oriented technical efficiency for the i -th farm are constructed as

$$TE_{1i} = \exp\left(-\frac{u_i}{\beta_1}\right).$$

Once the draws are obtained, the posterior distributions of the quantities of interest can be approximated and easily summarized via histograms and descriptive statistics such as means, variances, and percentiles. In contrast to the classical approach, the Bayesian allows straightforward estimation of probabilities of the form $\Pr[\theta \in \Delta]$, where θ is a random quantity of interest and Δ is a one-dimensional set. For example, in our application we can make statements such as “The probability that the technical efficiency of firm X is between 0.5 and 0.6 is Y percent.”

Application

To illustrate the method we have just described, we used the production data analyzed by Kurkalova and Jensen (1996) in an investigation of the technical efficiency of labor input in grain production on Ukrainian collective farms. The farm-level data

come from a random sample of 41 farms for the years 1989 to 1991. See Kurkalova and Jensen (1996) for details on the data.

Typical for Ukraine, the collective farms are very large by Western standards with more than 330 employees on average. Most of the collective farm members have two jobs: the official collective farm job, and an unofficial one on a family land plot. The production obtained from the family plot has always been a significant contributor to family income and, in the 1990s, has grown to provide more than one-half of a family's income in many post-Soviet countries (Van Atta [1998]). Collective farm members have little incentive to work as hard in the collective sector as they work on their private plots: the remuneration in the collective sector does not depend on the quality of the job, and shirking is widespread. Yet there is no distortion in the incentive to work on the family plot, because all the output from the plot belongs to the family and can be sold at farmers' markets.

The years 1989 to 1991 represent the beginning of economic reforms in the Soviet republics and the beginning of real economic hardship in rural Ukraine. Government subsidies to agriculture began to phase out, but the farms were not allowed any significant restructuring. Related to this, the state-provided farming infrastructure deteriorated sharply during this period. As a result, wages on the farms declined, and incentives to do a good job on the collective farms—already weak in Soviet times—all but vanished. As a consequence, it is reasonable to assume that a large proportion of the collective-farm-measured inefficiency comes from labor input because the hours of labor recorded as being spent in collective production may well have been spent on the family plots. In our analysis, and to illustrate the methods, we assume that all inefficiencies reside on the use of a single input, and we attribute the collective farm inefficiency to labor alone. Although this may be an assumption that is open for discussion, we proceed for illustration purposes. An extension of the model to the case in which only a part of the technical inefficiency is attributed to the input in question constitutes an interesting topic for future research.

The model we used in this application is given by

$$Y_{it} = \beta_0 + \beta_{90}d_{90} + \beta_{91}d_{91} + \sum_{j=1}^4 \beta_j x_{ijt} + V_{it} - U_{it}, \quad (4)$$

where the subscript i indicates the observation for the i -th farm in the survey ($i = 1, 2, \dots, 41$), and the subscript t indicates the observation for the t -th year ($t = 1, 2, 3$). Y represents the logarithm of the total grain production (in metric tons) on the given farm in the given year; β_i , ($i = 0, 90, 91, 1, \dots, 4$) represent the unknown parameters associated with the explanatory variables in the production function; d_{90} (d_{91}) is the dummy variable that has value 1 if $t = 2$ ($t = 3$) and value 0 otherwise; and x_i s ($i = 1, 2, \dots, 5$) represent the logarithms of the total amounts of land under grain production (in hectares), labor in grain production (in 1,000 hours), chemicals applied for grain production (in tons), and diesel fuel used in grain production (a proxy for machinery services) (in 1,000 liters), respectively. The error terms have the same meaning and distributions as before.

The summary statistics for the data used in estimation are given in Table 1. We estimated the parameters in model (4) using both a classical approach (ML) and a Bayesian approach (as described earlier). Maximum likelihood estimates of model parameters were obtained using the econometrics package LIMDEP (Greene 1991) and are presented in Table 2. The results imply mean output-oriented efficiency of $E[TE_{yi}] = 0.92$. The labor-oriented efficiency estimated has a mean of $E[TE_{li}] = 0.61$.

Table 1. Summary statistics for variables in the Production Frontier Model ^a

Variable	Units	Sample Mean	Sample St. Dev.	Minimum	Maximum
Production	Tons	4235	2477	1219	18574
Land	Hectares	1142	531	268	2850
Labor	1,000 hours	31	27	6	219
Chemicals	Tons	7.1	3.9	1.7	21.4
Fuel	1,000 liters	95	53	24	285

^a 41 farms, 3 years, 123 observations in total

For the Bayesian analysis, we generated five parallel chains, each containing 5,000 Gibbs sampler iterations, but we used only the last 1,000 draws to approximate the posterior distributions of interest. That is, we used the first 4,000 draws of each chain as “burn-in” draws to let the chains converge and assumed that the last 1,000 draws were draws from the stationary distributions of the chains. Because the chains were run independently, the last 1,000 draws from each can be combined into one sample, and all 5,000 can then be used to approximate the posterior distributions of the parameters. Values for the parameters of the priors were set as follows. For the prior distribution of the noise variance we set $p_1=1$, $p_2=0.01$, implying that a priori, the expected value of σ_v^{-2} was equal to 100. The parameter r^* was chosen to be 0.8, because this is the value reported in many studies of technical efficiency of (post-)Soviet agriculture (e.g., Sedik et al. (1999), Johnson et al. (1994), Sotnikov [1998]). For all other model parameters, we chose noninformative prior distributions. In particular, all of the regression coefficients were modeled, a priori, as uniform random variables bounded below by zero. Notice that this choice of prior distribution, even though improper, still results in an integrable posterior distribution.

The five independent Markov chains for all parameters were initialized by drawing the initial values from overdispersed distributions. The starting values for λ^{-1} , $\bar{\beta}$, and σ_v^{-2} were drawn from the corresponding prior distributions. The behavior of the chains was monitored by computing the statistic $\sqrt{\hat{R}}$. This statistic was proposed by Gelman and Rubin (1992). Intuitively, the statistic monitors convergence by comparing the within- and between-chain variances. If the chains have converged to their stationary distributions, the value of the statistic is approximately equal to 1. Values larger than 1 indicate that the “noise” in the draws can be reduced by an amount equivalent to the excess of 1 if the chains are allowed to proceed for additional steps. In our application, the values of the $\sqrt{\hat{R}}$ statistic were under 1.05 for all parameters after 5,000 iterations, indicating that additional Gibbs steps would not have resulted in increased precision of our estimates. From those values, we can reasonably assume that the chains have converged to their stationary distributions.

Bayesian estimation results are summarized in Table 2 and Figures 2 to 7. The point estimates of the frontier parameters and their standard deviations are very close to those that were obtained using the classical maximum-likelihood approach, suggesting that Bayesian estimation provides little advantage there. It is important to notice, however, that in general we cannot expect this to be the case, in particular for second order moments. The variances of the regression coefficients obtained from the classical approach to estimation are in this case very similar to the posterior variances of the regression coefficients. One reason for this is that the error with which σ_v^2 is estimated appears to be rather small (see, e.g., Figure 2) so that even when incorporating the uncertainty about σ_v^2 into the estimate of the uncertainty around the regression coefficients, the resulting variance is similar to that arising from the MLE.

The estimates of variance parameter σ_v^2 differ slightly between the classical and Bayesian approaches. The mean of technical efficiency, both output-oriented and input-oriented, is also estimated closely using the two methods.

The Bayesian methodology, however, provides more informative results on individual firm inefficiency scores. Whereas from the ML approach all we are able to compute is an average (across-firms) efficiency, the Bayesian approach permits estimation of firm-level efficiency scores, both for output and for labor input orientation. Consider two of the firms in our study, labeled 5 and 6. These two firms appear to have very different efficiency in their use of inputs. As Figures 5 and 6 show, the technical efficiency scores for firms 5 and 6 differ substantially. In particular, the distributions of the labor-input-oriented technical efficiencies for these two firms hardly overlap. The firms differ both in the actual efficiency estimates (whether we choose the mean or the median of the posterior distributions as point estimate) and also in the precision with which efficiency can be assessed for the firm. For example, it appears that there is little information in the data to accurately estimate the technical efficiency for firm 6.

Table 2. Maximum-likelihood and Bayesian estimates for parameters of the Frontier Model

Variable	Parameter	Maximum Likelihood Estimate ^a	Bayesian Point Estimates ^b		
			25th Percentile	Median	75th Percentile
<u>Stochastic Frontier</u>					
Constant	β_0	4.49 (0.45*)	4.16	4.48 (0.48)	4.80
Year 1990 Dummy	β_{90}	-0.084 (0.034**)	-0.107	-0.085 (0.034)	-0.062
Year 1991 Dummy	β_{91}	-0.372 (0.036*)	-0.397	-0.374 (0.035)	-0.350
ln (labor)	β_1	0.141 (0.029*)	0.123	0.141 (0.028)	0.160
ln (land)	β_2	0.32 (0.12*)	0.24	0.32 (0.13)	0.41
ln (chemicals)	β_3	0.264 (0.041*)	0.227	0.262 (0.052)	0.297
ln (fuel)	β_4	0.181 (0.098***)	0.118	0.184 (0.099)	0.251
<u>Inefficiency model</u>					
The inverse of the mean of the output-oriented inefficiency	λ^{-1}	11.2 (4.9**)	8.9	13.3 (6.8)	15.7
Standard deviation of the white-noise error component	σ_v	0.126 (0.023*)	0.117	0.130 (0.020)	0.144
Mean output-oriented technical efficiency	$E[TE_{yi}]$	0.918 (0.033)	0.900	0.920 (0.028)	0.940
Mean labor-oriented technical efficiency	$E[TE_{li}]$	0.61 (0.18)	0.55	0.60 (0.10)	0.68
Log (Likelihood)		57.04			

^a Standard errors in parentheses; they are computed from analytic second derivatives.

Values below means of marginal posterior distributions are not standard errors of the means but rather posterior standard deviations.

^b Computed from 5,000 runs of the Gibbs sampler.

*, **, and *** indicate statistical significance at the 1, 5, and 10 percent level, respectively.

We can take this analysis a step further and obtain an estimate of the marginal posterior distribution of the percentage difference in efficiency across the two firms. Consider, for example, the indicator $TE_{1,5} / TE_{1,6}$. If the ratio approaches 1, then we would conclude that firms 5 and 6 have comparable efficiency in the use of labor. If the ratio is larger than 1, then we would conclude that firm 5 uses labor input more efficiently than firm 6. For illustration, we have obtained the marginal posterior distribution of the ratio of technical efficiency in the use of labor for the two farms. The histogram that represents that distribution is given in Figure 7. Notice that in our application, a large proportion of the mass of the distribution is to the right of 1. We can therefore say that the probability that firm 5 uses labor input more efficiently than firm 6 is approximately equal to 89.5 percent. Furthermore, the variance of this posterior distribution reflects the confidence with which we can conclude that firm 5 uses inputs more efficiently than firm 6.

Concluding Remarks

Our study is built upon two intuitively appealing techniques: the idea of recalculation of input efficiency from an estimated output efficiency of RLT (1999), and Bayesian estimation of an output-oriented frontier production function as presented, for example, in Osiewalski and Steel (1998).

Recently, there has been an increase in both academic and policy attention to the environmental consequences of agricultural production (see, for example, Weaver [1998]). The methodology we suggest for single-input-oriented technical efficiency estimation is applicable to the estimation of environmental efficiency of agricultural production when the technology interaction with the environment is modeled via public inputs such as soil quality and environmental conditions.

In the more traditional production economics research, or for environmental-type work, a next step in the analysis would include investigation of those firm-specific attributes that may affect efficiency in the use of an input. In our study, it is apparent that collective farms in Ukraine exhibit very different efficiencies in labor use. Which firm characteristics result in a more efficient use of inputs? Answering this question requires that the production model be extended and that an additional layer in the hierarchy be

added. In this level, the response variable would be the firm's efficiency, and the predictors in the model would then be firm characteristics that might be associated to input efficiency. Because firm efficiency is observed with error, a classical approach would suggest that methodology appropriate when variables are measured with error be employed. But most of the methods available for analysis assume that the error in the dependent variable is homogeneous across firms. As is clear from Figures 5 and 6, the uncertainty with which we measure efficiency for the firms is variable across firms, and thus a more general approach for modeling efficiency as a function of firm attributes needs to be developed. The Bayesian paradigm provides a flexible framework that permits the investigation of these types of questions in a natural and unambiguous fashion.

We have attempted to illustrate, using the production data from a small sample of collective farms in Ukraine, the wealth of results and inferences that can be drawn when adopting a Bayesian approach for analysis. Although we are not arguing that the classical approach to estimation is inappropriate in all cases, we stress the additional information that can be obtained from within the Bayesian framework. In addition, we demonstrate that the Bayesian approach is feasible, even in complex problems. In our case, we wanted to derive firm-level estimates of efficiency in the use of labor and to highlight the fact that the precision of the estimate is likely to be heterogeneous across firms. This fact would have been difficult to uncover from the usual ML analysis.

As a final note, we must comment that the use of the Cobb-Douglas specification is not crucial for the method suggested. Less parsimonious forms can be used for the frontier production function as long as it is possible to recalculate input inefficiency from estimated output inefficiency as RLT (1999) showed it for a Translog frontier production function.

Possible future extensions of the model include the abolishment of the common inefficiency effects model in favor of the model in which firm efficiency varies with firm-specific variables in the spirit of Battese and Coelli (1995) and Osiewalski and Steel (1998). Also, the exponential model has a drawback originating from the properties of Gamma distribution because the mean and the variance of the Gamma distribution are

positively related. In our setting, that means that the model structure imposes a positive relationship between the size and the dispersion of inefficiency. Whether this assumption is plausible or not is something to be determined in future work.

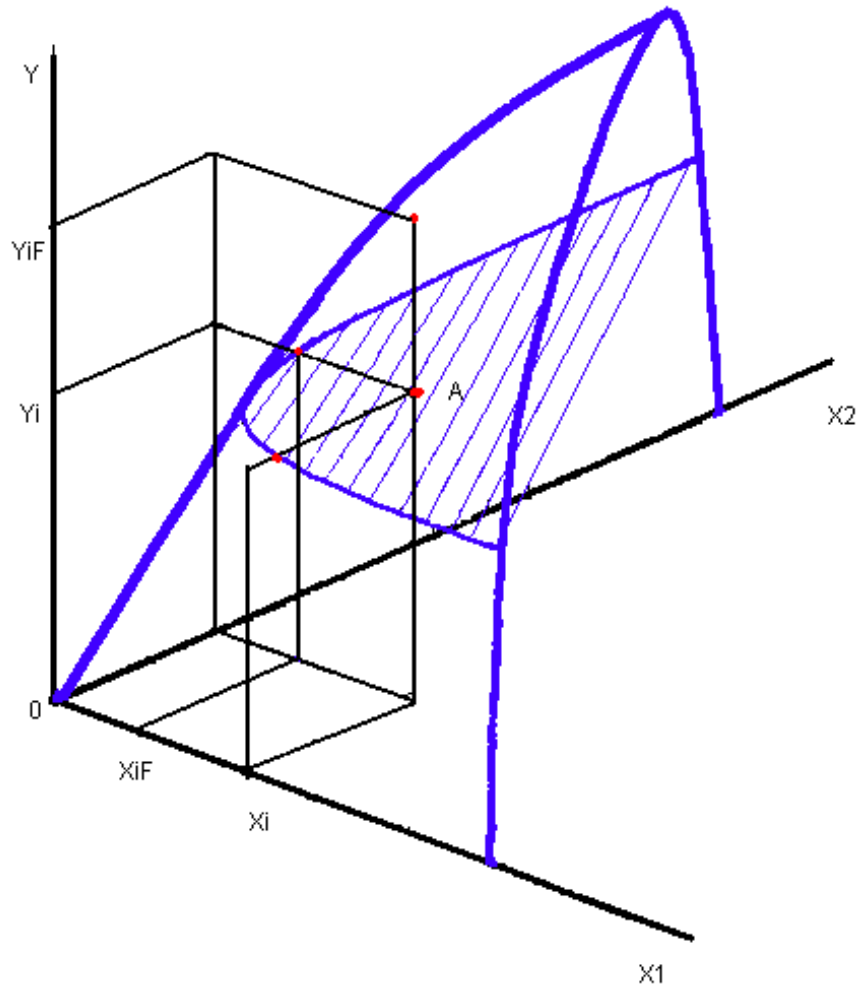


FIGURE 1. Output-oriented and single-input-oriented technical efficiency.

For the point A in the production space (y, x_1, x_2) , the output-oriented technical efficiency is $TE_{y_i} \equiv y_i / y_i^F$, and the input- x_1 -oriented technical efficiency is $TE_{x_1} \equiv x_{1i}^F / x_{1i}$.

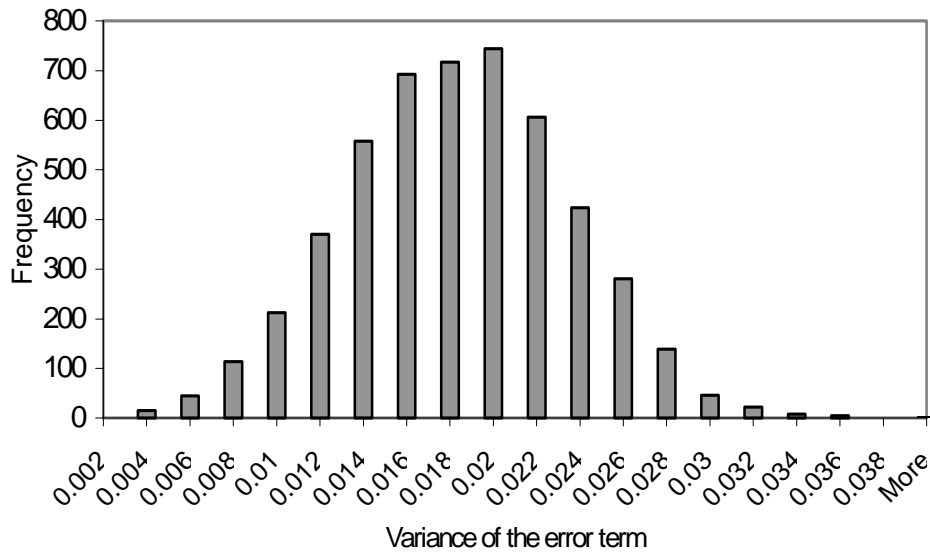


FIGURE 2. Marginal posterior distribution of σ_v^2 , the variance of the error term.

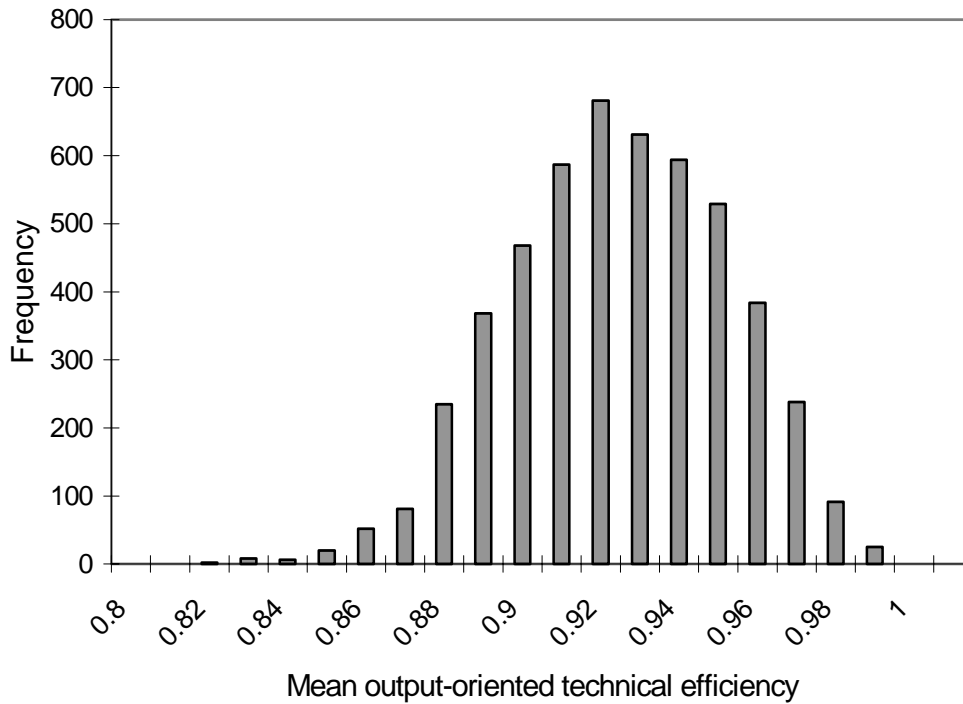


FIGURE 3. Marginal posterior distribution of the mean output-oriented technical efficiency. The mean is computed over all farms.

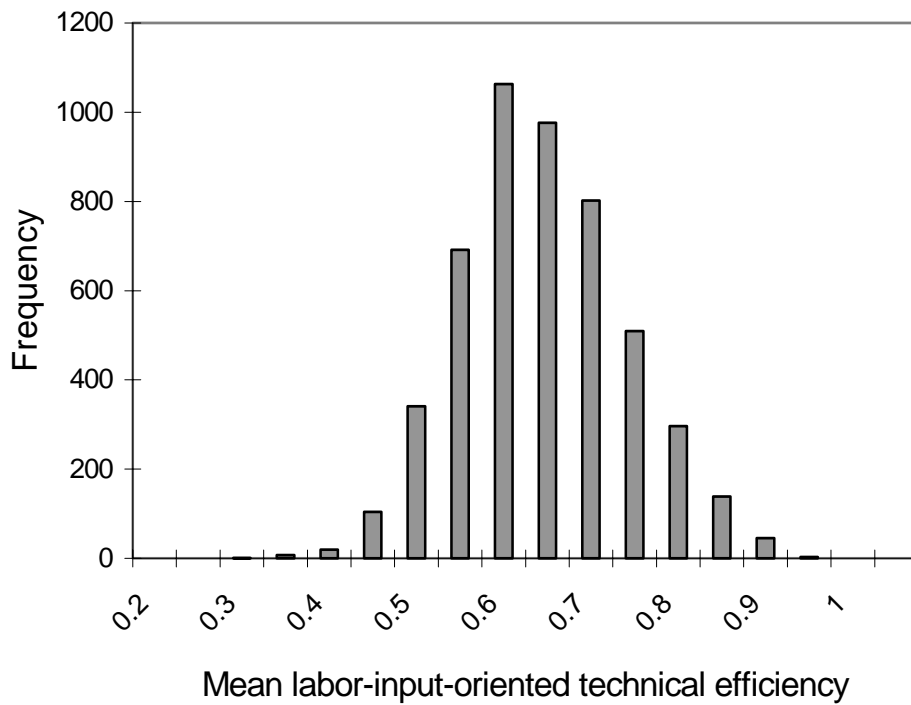


FIGURE 4. Marginal posterior distribution of the mean labor-input-oriented technical efficiency. The mean is computed over all farms.

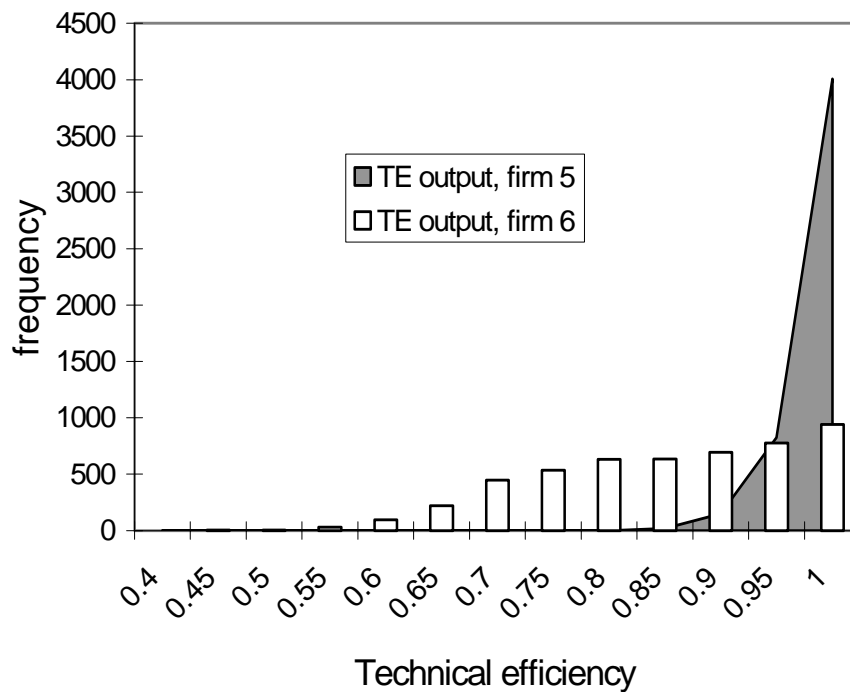


FIGURE 5. Marginal posterior distributions of $TE_{y,5}$ and $TE_{y,6}$, the output-oriented technical efficiency for firms #5 and #6.

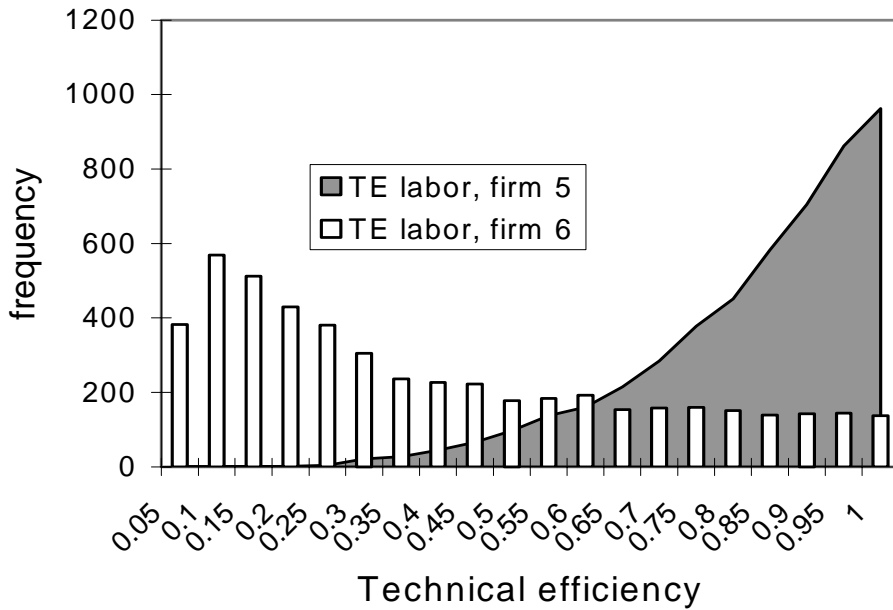


FIGURE 6. Marginal posterior distributions of $TE_{1,5}$ and $TE_{1,6}$, the labor-input-oriented technical efficiency for firms #5 and #6.

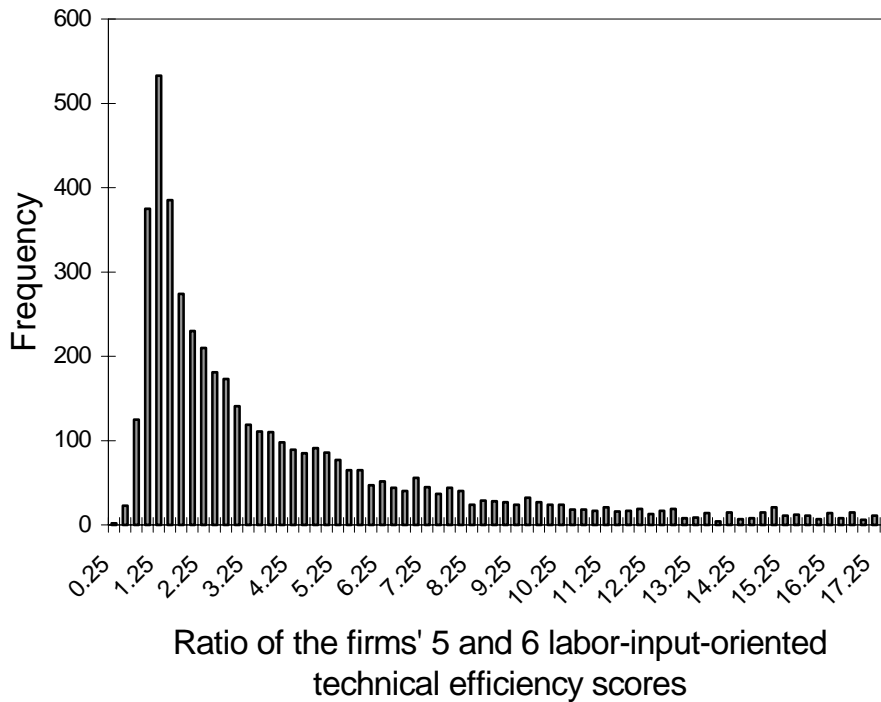


FIGURE 7. Marginal posterior distribution of $TE_{1,5} / TE_{1,6}$, the ratio of the labor-input-oriented technical efficiency of firm 5 to that of firm 6.

Appendix

Throughout the Appendix, $p(x)$ denotes the probability density function of a random variable X . The notation \bar{x} is used for a vector/matrix x of the appropriate dimension.

Probability density functions of *prior distributions* are given by

$$p(\beta_0) = 1, \quad \beta_0 \in (-\infty, \infty);$$

$$p(\beta_k) = 1, \quad \beta_k \geq 0, \quad k = 1, \dots, N;$$

$$p(\sigma_v^{-2} | p_1, p_2) = \frac{p_2^{p_1}}{\Gamma(p_1)} (\sigma_v^{-2})^{p_1-1} \exp(-p_2 \sigma_v^{-2}), \quad \sigma_v^{-2} \geq 0;$$

$$p(\lambda^{-1} | r^*) = (-\log(r^*)) \exp(\log(r^*) \lambda^{-1}), \quad \lambda^{-1} \geq 0.$$

The *full conditional posterior* of \bar{u} is a truncated Normal distribution:

$$p(u_i | \bar{\beta}, \sigma_v^2, \lambda^{-1}; y_i, x_{1i}, \dots, x_{Ni}) = \frac{1}{\sqrt{2\pi} \sqrt{\sigma_v^2} \Phi\left(\frac{m_i}{\sqrt{\sigma_v^2}}\right)} \exp\left\{-\frac{1}{2\sigma_v^2} (u_i - m_i)^2\right\}, \quad u_i \geq 0,$$

where

$$m_i \equiv \beta_0 + \beta_1 \log(x_{1i}) + \dots + \beta_N \log(x_{Ni}) - \log(y_i) - \lambda^{-1} \sigma_v^2.$$

The *full conditional posterior* of $\bar{\beta}$ is an $(N+1)$ -variate Normal distribution truncated to the subspace $\mathbf{K}(\bar{\beta})$:

$$\begin{aligned} p(\bar{\beta} | \bar{u}, \sigma_v^{-2}; \bar{x}, \bar{y}) &\propto \\ &\propto \exp\left\{-\frac{1}{2} \sigma_v^{-2} (\bar{\beta} - \bar{\mu})' (\overline{\log(x)}' \overline{\log(x)}) (\bar{\beta} - \bar{\mu})\right\}, \quad \bar{\beta} \in \mathbf{K}(\bar{\beta}) \end{aligned}$$

where

$$\bar{\mu} = \left(\overline{\log(x)}' \overline{\log(x)}\right)^{-1} \overline{\log(x)}' (\overline{\log(y)} + \bar{u}), \text{ and}$$

$$\mathbf{K}(\bar{\beta}) = (-\infty, \infty) \times [0, \infty) \times \dots \times [0, \infty).$$

The full conditional posterior of σ_v^{-2} is

$$p(\sigma_v^{-2} | \bar{u}, \bar{\beta}; data) \propto (\sigma_v^{-2})^{\frac{M}{2} + p_1 - 1} \times \exp \left\{ -\sigma_v^{-2} \left\{ \frac{1}{2} \sum_{i=1}^M (\log(y_i) - \beta_0 - \beta_1 \log(x_{1i}) - \dots - \beta_N \log(x_{Ni}) + u_i)^2 + p_2 \right\} \right\}, \quad \sigma_v^{-2} \geq 0.$$

Finally, the full conditional distribution of λ^{-1} is also a Gamma distribution:

$$p(\lambda^{-1} | \bar{u}) \propto \lambda^{-M} \exp \left(-\lambda^{-1} \left(\sum_{i=1}^M u_i - \log(r^*) \right) \right).$$

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